Holographic description of boundary and interface CFTs

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Outline

- Introduction
- Six dimensional supergravity
- Half BPS solutions
- Holographic Interface CFTs
- Generalized solutions: Holographic Boundary CFTs
- Discussion

Introduction

based on the following two papers:

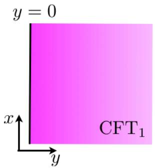
1107.1722 M. Chiodaroli, E. D'Hoker, Y. Guo and M.G.

1111.6912 M. Chiodaroli, E. D'Hoker and M.G.

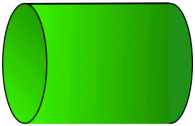
and work in progress together with M. Chiodaroli and E. D'Hoker.

2 dim Boundary CFT

- A boundary CFT is a CFT living in a space with boundary (i.e. the half plane)
- boundary conditions preserve a single copy of the *Virasoro* $\otimes$ *Virasoro* conformal symmetry of the CFT in the bulk.
- Modular transformation maps half plane to semi-infinite cylinder. Boundary conditions are imposed by boundary state.
- Applications: Open string vacua, D-branes, condensed matter and stat-mech etc.



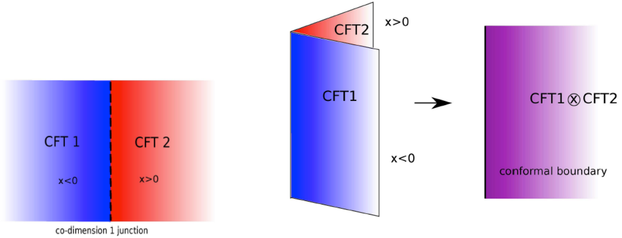
$T_{xy} = 0 \leftrightarrow T(z) = \bar{T}(\bar{z})$



$(L_n - \bar{L}_{-n}) \mid B\rangle = 0$

2 dim Interface CFT

- A conformal interface is a junction betwen two CFTs preserving part of the conformal symmetry.
- Gluing conditions need to be scale invariant. In 2d continuity of T_{xy} across the interface is sufficient.



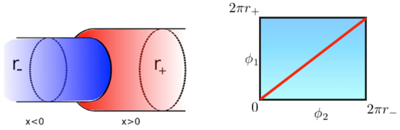
- The folding trick relates interface CFT to boundary CFT: conformal boundary conditions in tensor product $CFT_1 \otimes CFT_2$. Affleck and Ludwig; Bachas, de Boer, Dijkgraaf and Ooguri

Toy model of Interface CFT

- A simple toy model is given by a compact boson $\phi \sim \phi + 2\pi$ with a jump in the coupling/tension

$$S = 2r_-^2 \int_{x<0} dx\, dy\ \partial_a \phi\, \partial^a \phi + 2r_+^2 \int_{x>0} dx\, dy\ \partial_a \phi\, \partial^a \phi$$

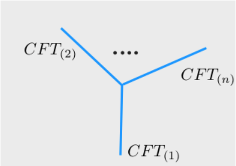
- After rescaling the boson has a jump in the radius $\phi \sim \phi + 2\pi r_\pm$
- After folding two bosons ϕ_1, ϕ_2 different radii conformal boundary conditions correspond to a diagonal D1 brane in a 2 torus spanned by ϕ_1, ϕ_2 with radius $r_\pm$.

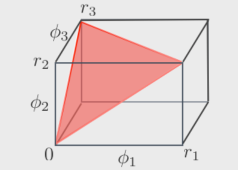


- Calculate g-factor $g = \langle 0 \mid B \rangle$ or boundary entropy $S_{bd} = \ln g$ of the interface

Junctions of CFTs (or strings)

- Interfaces between 2 CFTs can be generalized to junctions of 3 or more CFTs, for example a star graph.
- conformal boundary conditions: $\sum_i T^{(i)}_{xy} = 0$.
- Example: n bosons ϕ_i with radius r_i on half spaces, joined at interface.
- conformal boundary conditions:p dim D brane in n-torus g-factors calculated in BCFT.
- Reminiscent of (p, q)-string junctions.
- Boundary entropy can be calculated as before.





Janus solution

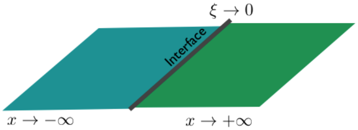
- The Janus solution is a simple example of a holographic realization of a interface CFT. Consider d+1 dim AdS gravity and massless scalar (dilaton)
- AdS_d slicing of AdS_{d+1} preserves $SO(2, d-1)$ of the $SO(2,d)$ conformal symmetry.

$$ds^2 = dx^2 + f(x) ds^2_{AdS_d}, \quad \phi = \phi(x)$$

- AdS vacuum: $f(x) = \cosh^2 x, \phi(x) = \phi_0$. Janus solution: As $x \to \pm\infty$ $\phi \to \phi_\pm$
- asymptotic form of metric as $x \to \pm\infty$ (AdS_d in Poincare coord, $\xi \in [0, \infty]$).

$$ds^2 = dx^2 + \frac{e^{2|x|}}{\xi^2}\left(d\xi^2 - dt^2 - d\vec{y}^2_{d-2}\right) + o(1)$$

- Three boundary components: $x \to \pm\infty$ (two d-dim half spaces), $\xi \to 0$ (a d-1 dimensional interface).
- Dilaton is dual to coupling constant of CFT. Interface CFT with jumping coupling.



Janus solution

- For AdS_3 It is possible to relate the boundary entropy to entanglement entropy Cardy

$$S_A = \frac{c}{6} \log \frac{L}{\epsilon} + \log g_B$$

- Holographic calculation of entanglement entropy Ruy and Takayanagi
- When embedded into supergravity Janus solution breaks all super symmetries (Since $\delta\lambda \neq 0$ for all ϵ).

Goals

- Can the Janus solution be generalized to preserve supersymmetries ?
- Can one find holographic duals of junctions of CFTs ?
- Can one find holographic duals of boundary CFTs ?

Six dimensional Type 4B supergravity

We consider the so called $(0,4)$ or type 4B supergravity in 6 dimensions. (Townsend; Romans). It has the following properties:

- It has 16 chiral supersymmetries
- It can be obtained by compactifying type IIB on $K3$.
- The scalar fields parameterize the coset $SO(5,21)/SO(5)\times SO(21)$.
- The field content is given by

Field	$g_{\mu\nu}$	$B^I_{\mu\nu}$	$B^R_{\mu\nu}$	$V^i{}_R$	ψ^α_μ	$\chi^{r\alpha}$
SO(5)	**1**	**5**	**1**	**5**	**4**	**4**
SO(21)	**1**	**1**	**21**	**21**	**1**	**21**

- The supersymmetry transformations are given by

$$\delta\psi^\alpha_\mu = D_\mu\epsilon^\alpha - \frac{1}{4}H^I_{\mu\nu\rho}\gamma^{\nu\rho}\,(\Gamma^I)^\alpha{}_\beta\,\epsilon^\beta$$

$$\delta\chi^{R\alpha} = \frac{1}{\sqrt{2}}\gamma^\mu P^{IR}_\mu(\Gamma^I)^\alpha{}_\beta\,\epsilon^\beta + \frac{1}{12}\gamma^{\mu\nu\rho}H^R_{\mu\nu\rho}\,\epsilon^\alpha$$

Probe branes in $AdS_3 \times S^3$

- Vacua: 6 dim Minkowski and $AdS_3 \times S_3$ supported by self-dual 3 form $H^i = *H^i$.
- $AdS_3 \times S_3$ is the near horizon limit of self-dual string solution in 6 dimensions.
- From 10 dim point of view: D5/D1 bound state wrapped on K3.
- AdS vacuum preserves 16 super symmetries. Half BPS probe branes preserve 8. κ-symmetry can be used to find all of these:

probe brane	AdS_3	S_3	$K3$
D1	AdS_2	$\cdot$	$\cdot$
D3	AdS_2	S_2	$\cdot$
D5	AdS_2	$\cdot$	$K3$
D7	AdS_2	S_2	$K3$
D3	AdS_2	$\cdot$	C_2

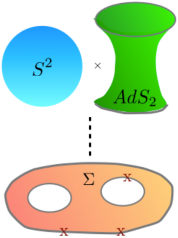
Ansatz

Half BPS solutions dual to interface or junction CFTs

- Janus ansatz: AdS_2 slicing of AdS_3.
- Global super algebra of vacuum $PSU(1,1|2) \times PSU(1,1|2)$ reduced to $PSU(1,1|2)$. Isometries reduced from $SO(2,2) \times SO(4)$ to $SO(2,1) \times SO(3)$.
- Ansatz: $AdS_2 \times S_2$ fibration over 2dim Riemann surface Σ

$$ds^2 = f_1^2 ds^2_{AdS_2} + f_2^2 ds^2_{S^2} + \rho^2 (dx^2 + dy^2)$$

- Σ has boundary where $vol(S_2) \to 0$. Scale factor of AdS_2 diverges at isolated points at boundary $\partial\Sigma$.



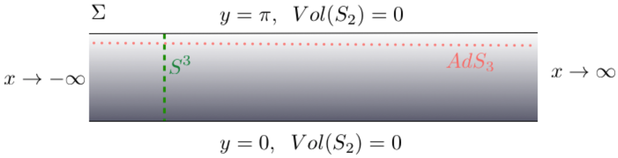
Ansatz

- For $AdS_3 \times S^3$ vacuum Σ is the strip, $\Sigma = \{x + iy, x \in R, y \in [0, \pi]\}$
- metric is given by

$$ds^2 = f_1^2 ds^2_{AdS_2} + f_2^2 ds^2_{S^2} + \rho^2 (dx^2 + dy^2)$$

with

$$f_1^2 = \cosh^2 x, \quad f_2^2 = \sin^2(y), \quad \rho^2 = 1$$



Ansatz

Ansatz for other bosonic fields which depend on coordinates of Σ and respect the $AdS_2 \times S_2 \times \Sigma$ fibration.

- Anti-symmetric tensor 2 form potentials

$$B^A = \Psi^A \omega_{AdS_2} + \Phi^A \omega_{S^2}, \quad A = 1, 2, \cdots, 26$$

- The scalar fields take values in the coset space $SO(5,m)/(SO(5) \times SO(m))$ via a frame field V

$$\eta = \begin{pmatrix} I_5 & \\ & -I_{21} \end{pmatrix}, \quad V = V^{(i,r)}{}_A = \begin{pmatrix} V^i_I & V^i_R \\ V^r_I & V^r_R \end{pmatrix}$$

- $A = (I, R)$ labels the fundamental representation of $SO(5,m)$. (i,r) label the fundamental representation of $SO(5) \times SO(m)$, with $I, i = 1, \cdots, 5$ and $R, r = 6, \cdots, m+5$.
- The coset representative satisfy

$$\eta = V^t \eta V$$

Solving the BPS equations

General strategy for solving the BPS equation (Details can be found in arXiv:1107.1722).
Find nontrivial solution of

$$\delta\chi^{R\alpha} = 0, \qquad \delta\psi_\mu^\alpha = 0$$

For some infinitesimal susy transformation parameter ϵ^α.

- Bosonic fields which respect $AdS_2 \times S_2 \times \Sigma$ fibration
- Expand susy parameter ϵ in terms of a basis of Killing spinors on AdS_2 and S_2.
- Reduced BPS equations: Algebraic projector equations and diff. equations on Σ.
- Projection conditions can be solved by choosing a $SO(5) \times SO(21)$ gauge. Sets some scalars and AST to zero.
- Differential equation equation can be solved in terms of harmonic function H.
- Solve Bianchi identities in terms of meromorphic function λ^A.
- Check that equations of motion are satisfied.

General local solution

The general local solution of the BPS equations is given by harmonic and meromorphic functions

- A real positive harmonic function H on Σ
- meromorphic functions $\lambda^A, \quad A = 1, 2, \cdots, 26$
- Projection conditions set $\lambda^3 = \lambda^4 = \lambda^5 = 0$
- The meromorphic functions satisfy constraints on Σ

$$\lambda \cdot \lambda = (\lambda^1)^2 + \cdots (\lambda^5)^2 - (\lambda^6)^2 - \cdots (\lambda^{26})^2 = 2, \qquad \bar{\lambda} \cdot \lambda \geqslant 2$$

- All bosonic fields can be expressed in terms of H, λ^A. Metric factors:

$$\begin{aligned} f_1^4 &= H^2 \frac{\bar{\lambda} \cdot \lambda + 2}{\bar{\lambda} \cdot \lambda - 2} \\ f_2^4 &= H^2 \frac{\bar{\lambda} \cdot \lambda - 2}{\bar{\lambda} \cdot \lambda + 2} \\ \rho^4 &= \frac{|\partial_w H|^4}{16 H^2} (\bar{\lambda} \cdot \lambda + 2)(\bar{\lambda} \cdot \lambda - 2) \end{aligned}$$

General local solution

- Scalar coset in gauge fixed form: $i, I = 1, 2, r, R = 6, 7, \cdots 26$.

$$V = \begin{pmatrix} V^i{}_I & 0 & V^i{}_R \\ 0 & I_3 & 0 \\ V^r{}_I & 0 & V^r{}_R \end{pmatrix}$$

- The combinations $V^{\pm}_{I,R} = V^1_{I,R} \pm i V^2_{I,R}$ can be expressed as

$$\begin{aligned} V^+{}_A &= X(\bar\lambda_A - |X|^2 \lambda_A)/(1 - |X|^4) \\ V^-{}_A &= \bar X(\lambda_A - |X|^2 \bar\lambda_A)/(1 - |X|^4) \end{aligned}$$

where $|X|^2 + 1/|X|^2 = \bar\lambda \cdot \lambda$.

- AST potential $B^3 = B^4 = B^5 = 0$ and

$$\Phi^A = -\sqrt{2}\,\frac{H\,Re(\lambda^A)}{\bar\lambda \cdot \lambda + 2} + \tilde\Phi^A \qquad \tilde\Phi^A = \frac{1}{2\sqrt{2}} \int^w \partial_w H\,\lambda^A + \text{c.c.}$$

$$\Psi^A = -\sqrt{2}\,\frac{H\,Im(\lambda^A)}{\bar\lambda \cdot \lambda - 2} + \tilde\Psi^A \qquad \tilde\Psi^A = \frac{i}{2\sqrt{2}} \int^w \partial_w H\,\lambda^A + \text{c.c.}$$

Regularity conditions

The local solution solves BPS equations, Bianchi identities and equations of motion. However the metric and other fields are regular only if the following **regularity conditions** are satisfied:

- In the interior of Σ we have $H > 0$ and $\bar{\lambda} \cdot \lambda > 2$;
- On the boundary $\partial \Sigma$ of Σ we have $H = 0$ and $Im(\lambda^A) = 0$, except at isolated points;
- The one-forms $\lambda^A \partial_w H$ are holomorphic and nowhere vanishing in the interior of Σ, forcing the poles of λ^A to coincide with the zeros of $\partial_w H$;
- The functions λ^A are holomorphic near $\partial \Sigma$, thereby allowing for poles in $\lambda^A \partial_w H$ on $\partial \Sigma$ only at those points where $\partial_w H$ has a pole.

Regularity conditions

The constraint and regularity conditions involving $\lambda \cdot \lambda$ are conveniently solved by a light cone parameterization singling out the $A = 2$ and $A = 6$ direction. L^A are a new set of meromorphic functions

$$\begin{aligned} \lambda^A &= \frac{\sqrt{2}\, L^A}{L^6} \\ \lambda^2 &= \frac{1}{L^6}\left(\frac{1}{2} - L^1 L^1 + L^R L^S \delta_{RS}\right) \\ \lambda^6 &= \frac{1}{L^6}\left(-\frac{1}{2} - L^1 L^1 + L^R L^S \delta_{RS}\right) \end{aligned}$$

where $A = 1, 7, 8, \cdots, 26$, and $R, S = 6, 7, 8, \cdots, 26$. The regularity condition $Im(\lambda^A) = 0$ on $\partial\Sigma$ implies

$$Im(L^A) = 0, \quad \text{for } w \in \partial\Sigma$$

Regular Solution on the half plane

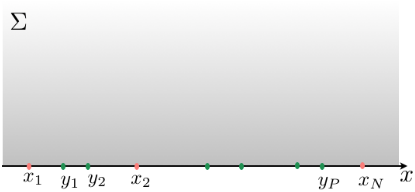
The simplest choice of Σ is the upper half plane where $\partial\Sigma$ is the real line

- Conditions on harmonic function H: N simple poles on the real line

$$H(w,\bar{w}) = \sum_{n=1}^{N} \left(\frac{i\, c_n}{w - x_n} - \frac{i\, c_n}{\bar{w} - x_n} \right)$$

- L^A satisfying boundary conditions can be given by a sum of P auxiliary poles on the real line

$$L^A(w) = l^A_\infty + \sum_{p=1}^{P} \frac{l^A_p}{w - y_p}$$



Regular Solution on the half plane

- For regular solutions, avoiding curvature singularities requires L^6 and $\partial_w H$ to have common zeros. Therefore, we take L^6 to have the form,

$$L^6(w) = i l_\infty^6 \frac{\prod_n^N (w - x_n)^2}{\prod_p^P (w - y_p)} \partial_w H(w)$$

- To avoid an auxiliary pole or an extra zero at infinity, we need the number of auxiliary poles to be related to the number of physical poles

$$P_{disk} = 2N - 2$$

- Regularity of the λ^A at the auxiliary poles implies

$$l_p^1 = \sqrt{\sum_R l_p^R l_p^R}$$

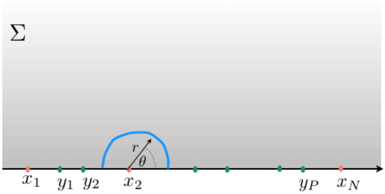
Regular Solution on the half plane

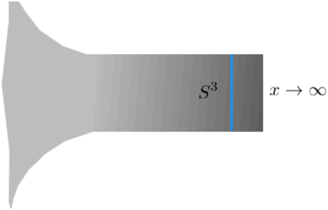
- Near a pole of H, the metric takes an asymptotic $AdS_3 \times S^3$ form: $w = x_n + re^{i\theta}$ with $r \to 0$ asymptotics.

$$ds^2 \sim \sqrt{2\mu_n \cdot \mu_n} \left(\frac{dr^2}{r^2} + \frac{2c_n^2}{\mu_n \cdot \mu_n} \frac{1}{r^2} ds^2_{AdS_2} + d\theta^2 + \sin^2\theta \, ds^2_{S^2} \right) + o(r^2)$$

- The μ^A are directly related to the conserved charges on the asymptotic S^3

$$Q_n^A \equiv \oint_{S^3} dB^A = i\sqrt{2}\pi \int_{x_n} \lambda^A \partial_w H + \text{c.c.} = 2\sqrt{2}\pi^2 \mu_n^A$$





Holographic interface solutions

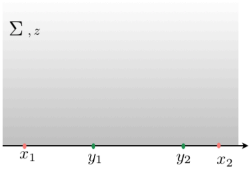
- N=2 is minimal because of charge conservation, also P=2

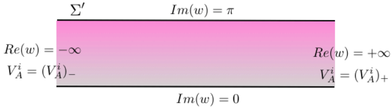
$$H(w,\bar{w}) = \left(\frac{i\,c_1}{w-x_1} + \frac{i\,c_2}{w-x_2}\right) + c.c, \quad L^A(w) = l^A_\infty + \frac{l^A_1}{w-y_1} + \frac{l^A_2}{w-y_2}$$

- Half plane can be mapped to the strip visa $z = e^w$.
- Conserved charge Q^A is charge carried by self dual string in 6 dim. Central charge of dual CFT $c \sim Q_n \cdot Q_n$
- as $Re(w) \to \pm\infty$ scalars approach $V^I_A \to (V^I_A)_\pm$

  (a) Subset of scalars are "fixed" whose limit $Re(w) \to \pm\infty$ is determined by the charge vector Q^A (attractor mechanism), i.e $(V^I_A)_+ = (V^I_A)_-$

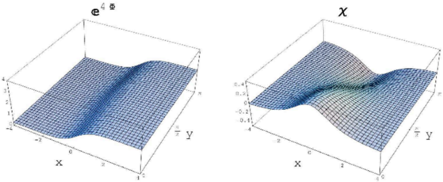
  (b) The other scalars are "free" and can jump as $Re(w) \to \pm\infty$, i.e $(V^I_A)_+ = (V^I_A)_-$





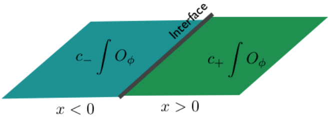
Holographic interface solutions

- Consider the truncation to only two scalars (”dilaton” and ”axion”)



- ”dilaton” ϕ is dual to dimension $(h, \bar{h}) = (1, 1)$ operator O_ϕ deformation of the dual CFT by source which jump across the interface

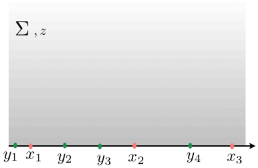
$$S_{CFT} \to S_{CFT} + c_+ \int_{x<0} dx\, dy\, O_\phi + c_- \int_{x>0} dx\, dy\, O_\phi$$

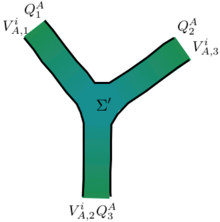


- Exact match of holographic and CFT boundary entropy. Chiodaroli, Hung, M.G. arXiv:1005.4433.

Holographic junction solutions

- A solution where H has N poles has N asymptotic AdS regions, corresponding to N half spaces in the dual CFT
- Each asymptotic AdS space is characterized by the charge Q^A and the value of the free (i.e. non attracted) scalars

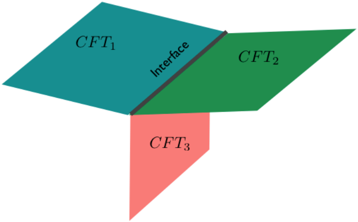




- Exact match between parameters of solution (x_i, y_j, c_i, l_k^A etc.) and charges Q_k^A and values of free scalars V_k^{iR} for each asymptotic AdS region
- Each asymptotic AdS region dual to a CFT defined by the near horizon limit of a self dual string with charge Q^A, deformed by operator source dual to unattracted scalars.

Holographic junction solutions

- Holographic dual: Junction of N CFTs (defined by charge Q) living on half-spaces glued together at a one dimensional interface. (See figure for the case N=3).
- Near Horizon limit of a half-BPS junction of N self dual strings in six dimensions.



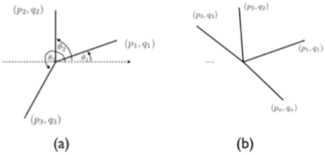


Figure 2: (a) Three string junction (b) multi-string junction

Generalized solutions

- Since the charge Q^A is conserved, it seems impossible to have a solution with a single pole in H
- Such a solution would only have one asymptotic AdS region, i.e would be dual to a CFT living on a single half space: Boundary CFT

Question

Can one find holographic duals of boundary CFTs in our class of solutions ?

AdS-cap

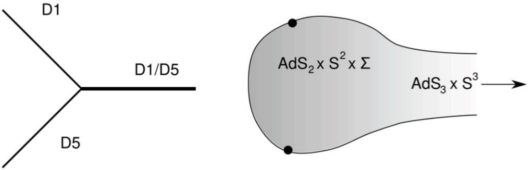
- The charge vector Q_i^A determines the central charge of the dual CFT

$$c_i = \frac{3\,Q_i \cdot Q_i}{16\pi^2 G_N}$$

- charge conservation implies

$$\sum_i Q_i^A = 0 \tag{1}$$

- we can consider charges for $Q \cdot Q = 0$. These leads to a singular solution (since $c = 0$ this implies $R_{AdS} = 0$) and call it "AdS-cap".
- Albeit singular some calculations like the holographic boundary entropy still make sense, giving a realization of a BCFT.
- $Q \cdot Q = 0$ means that we do not have a self dual string but a pure D1 or D5 brane.



(a) (b)

Higher genus solutions

- It's natural to consider a Riemann surface with more than one boundary. The simplest case is the annulus or cylinder.

$$\Sigma = \left\{ w \in \mathbf{C},\ 0 \leqslant Re(w) < 1,\ 0 \leqslant Im(w) \leqslant \frac{t}{2} \right\}, \quad w \sim w + 1 \tag{2}$$

- The harmonic and meromorphic functions can be constructed using

$$\zeta_0(w) \equiv \frac{\partial_w \theta_1(w|it)}{\theta_1(w|it)} + \frac{2\pi w}{t}$$

- Important ζ_0 has non vanishing monodromy around cylinder

$$\zeta_0(w+1) = \zeta_0(w) + \frac{2\pi}{t}$$

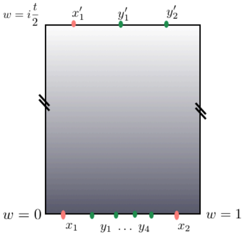
Higher genus solutions

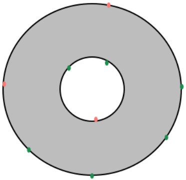
- H and L^A are given by

$$
\begin{aligned}
H &= -2\,Im \sum_{n=1}^{N} c_n\, \zeta_0(w - x_n) - 2\,Im \sum_{n'=1}^{N'} c'_{n'}\, \zeta_0(w' + x'_{n'}) \\
L^A &= L^A_\infty + \sum_{p=1}^{P} l^A_p\, \zeta_0(w - y_p) + \sum_{p'=1}^{P'} l^A_{p'}\, \zeta_0(w' + y'_{p'})
\end{aligned}
$$

- Here $w' = \frac{it}{2} - w$ and hence x_p, y_p are the location of poles on one boundary and x'_p, y'_p on the other boundary.
- Boundary and regularity conditions can be solved using the properties of ζ_0. Regular is

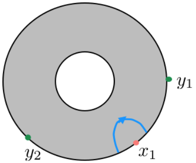
$$P = 2N, \quad P' = 2N'$$

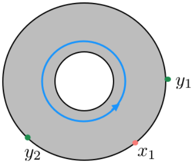




Higher genus solutions

- Holographic BCFT with only one asymptotic region $N = 1, P = 2$ and $N' = 0, P' = 0$.





- What happens to conservation of charge $Q_n^A \equiv \oint_{C\times S^2} dB^A$?. Nontrivial cycle on annulus.
- Monodromy of ζ_0 leads to nontrivial monodromy of scalars as $w \to w+1$. Compare monodromy of axion around a D7 brane.
- Conjecture: Such solutions correspond to the near horizon limit of a self dual string ending on a D7 brane wrapped on K3.

Discussion

- Calculation of observables like the entanglement entropy is (relatively) easy
- Calculation of correlation function (bulk/boundary in CFT)
- Generalization of solution to Σ with an arbitrary number of boundaries and handles is straightforward.
- Is it possible to have solutions with no asymptotic AdS regions ? (Cf Bachas talk last week).
- Do these solution correspond to string networks ?