

# Holographic Entanglement Entropy for Surface Operators and Defects

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# Overview

- Holographic description of defects (Wilson lines, Wilson surfaces, surface defects) with large backreaction: Bubbling supergravity solutions
- Calculate the Ryu-Takanagi entanglement entropy for these bubbling solutions for spherical entangling surfaces
- Lewkowycz-Maldacena formula for entanglement entropy in the presence of a defect:

$$\Delta S = \log \langle W \rangle - \int_{S^1 \times H^{d-1}} d^d x \sqrt{g} \Delta \langle T_{\tau\tau} \rangle_W$$

- Calculate  $\log \langle W \rangle$  and  $\langle T_{\tau\tau} \rangle_W$  on the holographic side and compare LM and RT
- The defects we will focus on here are disorder type surface defects in  $N = 4$  SYM.

# Review of defect operators in field theory

Non local operators are ubiquitous in field theories. They are distinguished by several features:

- The world volume dimension:  $d = 1$  for line operators,  $d = 2$  for surface operators.
- Wilson loop: can be written as an extended operator in terms of local fields

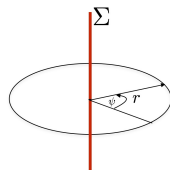
$$W(\mathcal{C}) = \text{tr}_R \mathcal{P} \exp \left( \int_{\mathcal{C}} d\tau (iA_\mu \dot{x}^\mu + \phi_i |\dot{x}| n^i) \right)$$

- t' Hooft loop: Singular behavior of fundamental fields  $A_\mu, \phi_i$  near defect  $\mathcal{C}$ .
- Amount of (superconformal) symmetry preserved. For us half-BPS defects.
- Representation  $R$
- breaking of gauge symmetry

# Review of defect operators in field theory

- Surface defects in  $N = 4$  SYM were constructed by Gukov and Witten (hep-th/0612073): 2 dim surface  $\Sigma = R^2$  in  $R^4$ .

geometry (one dimension of  $\Sigma$  suppressed)



- Preserve  $PSU(1, 1|2) \times PSU(1, 1|2)$  inside  $PSU(2, 2|4)$  superconformal symmetry.
- characterized by breaking of  $U(N)$  to Levi group  $L = \prod_{i=1}^M U(N_i)$  with  $\sum_i N_i = N$ .
- Holonomy of  $A_\theta$  around defect  $\Sigma$

$$A_\psi = \text{diag} \{ \alpha_1 \mathbf{1}_{N_1}, \alpha_2 \mathbf{1}_{N_2}, \dots, \alpha_M \mathbf{1}_{N_M} \}$$

- $\theta$  angles for gauge field on  $\Sigma$

$$\exp \left( i \sum_{i=1}^M \eta_i \int_{\Sigma} \text{tr}(F_i) \right)$$

# Review of defect operators in field theory

- Singular behavior of complex scalar  $\Phi = \phi_5 + i\phi_6$ .

$$\Phi = \frac{e^{-i\psi}}{\sqrt{2}r} \text{diag} \{(\beta_1 + i\gamma_1) \mathbf{1}_{N_1}, (\beta_2 + i\gamma_2) \mathbf{1}_{N_2}, \dots, (\beta_M + i\gamma_M) \mathbf{1}_{N_M}\}$$

- Data:  $4M$  continuous parameters  $\alpha_i, \beta_i, \gamma_i, \eta_i$  and discrete set  $N_i$  with  $i = 1, 2, \dots, M$ .
- Alternative description: Conformal map from  $R^4$  to (euclidean)  $AdS_3 \times S^1$ . Defect is located at boundary of  $AdS_3$ . Holonomy around  $S^1$ .
- Drukker et al (arXiv:0805.4199) calculated field theory observables. Expectation value of defect and one point function of stress tensor

$$\langle \mathcal{O}_\Sigma \rangle = e^{-S_{YM}}|_{\text{surface}} = 1$$

$$\langle T_{\mu\nu} \rangle_\Sigma dx^\mu dx^\nu = h_\Sigma \left( ds_{AdS_3}^2 - 3 d\psi^2 \right)$$

where

$$h_\Sigma = -\frac{2}{3g_{YM}^2} \sum_{i=1}^M N_i (\beta_i^2 + \gamma_i^2)$$

# Probe brane description of defects in $AdS_5 \times S^5$

- Defect realized as probe D3 branes in  $AdS_5 \times S^5$  (Constable et al hep-th/0211222)
- Use  $AdS_3 \times S^1 \times S^3$  fibration

$$ds^2 = \cosh^2 u \frac{dz^2 + d\tau^2 + dx^2}{z^2} + \sinh^2 u d\psi^2 + du^2 + \cos^2 \theta ds_{S^3}^2 + \sin^2 \theta d\phi^2 + d\theta^2$$

- Worldvolume of probe  $AdS_3 \times S^1$ ,  $z, \tau, x, \psi$  with

$$\sinh u_0 = \kappa, \quad \psi + \phi = \phi_0$$

- Map of field theory and and probe brane parameters:  $M$  stacks of  $N_i$  coincident D3

$$\beta_i + i\gamma_i = \frac{\sqrt{\lambda}}{2\pi} \sinh u_{0,i} e^{i\phi_{0,i}}, \quad \alpha = \frac{1}{2\pi} \int A, \quad \eta = \frac{1}{2\pi} \int \tilde{A}$$

# Bubbling geometry for surface defect operators

- Solution dual to surface defect in  $N = 4$  SYM is given by a double Wick rotation of the Lin-Lunin-Maldacena solution
- $AdS_3 \times S_3 \times S_1$  fibration over 3 dim space  $\Sigma_3$  with  $x_1, x_2, y$  with  $y > 0$ .

$$ds^2 = y \sqrt{\frac{2f+1}{2f-1}} ds_{AdS_3}^2 + y \sqrt{\frac{2f-1}{2f+1}} ds_{S^3}^2 + \frac{2y}{\sqrt{4f^2-1}} (d\chi + V)^2 + \frac{\sqrt{4f^2-1}}{2y} ds_{\Sigma_3}^2$$

- One form  $V$  is determined by

$$dV = \frac{1}{y} \star_3 df$$

- Only other nontrivial field is the self-dual five form

$$F_5 = -\frac{1}{4} \left( d \left[ y^2 \frac{2f+1}{2f-1} (d\chi + V) \right] - y^3 \star_3 d \left[ \frac{f+1/2}{y^2} \right] \right) \wedge \omega_{AdS_3} \\ - \frac{1}{4} \left( d \left[ y^2 \frac{2f-1}{2f+1} (d\chi + V) \right] - y^3 \star_3 d \left[ \frac{f-1/2}{y^2} \right] \right) \wedge \omega_{S_3}$$

# Bubbling geometry for surface defect operators

- Function  $f$  characterizes solution and satisfies PDE with source term and boundary condition  $f = +\frac{1}{2}$  at  $y = 0$ .

$$\partial_1^2 f + \partial_2^2 f + y \partial_y \left( \frac{\partial_y f}{y} \right) = \sum_{i=1}^M 2\pi y_i \delta(y - y_i) \delta^2(\vec{x} - \vec{x}_i)$$

- Most general solution for  $f$  which is asymptotically  $AdS_5 \times S^5$

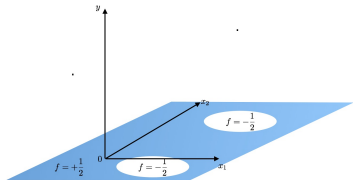
$$f = \frac{1}{2} + \sum_{i=1}^M f_i$$

with

$$f_i = -\frac{1}{2} + \frac{(\vec{x} - \vec{x}_i)^2 + y^2 + y_i^2}{2\sqrt{[(\vec{x} - \vec{x}_i)^2 + y^2 + y_i^2]^2 - 4y^2 y_i^2}}$$

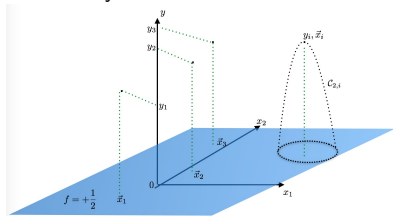
# Bubbling geometry for surface defect operators

- LLM solution  $S^3 \times \tilde{S}^3 \times S^1$  over  $\Sigma_3$ . Coloring of 2 plane at  $y = 0$



$f = +1/2$  3-sphere  $S^3$  shrinks.  
 $f = -1/2$  3-sphere  $\tilde{S}^3$  shrinks.

- Double analytic continuation has sources in the interior of  $\Sigma_3$



$f = +1/2$  on the boundary  
 since  $AdS_3$  cannot shrink.

# Bubbling geometry for surface defect operators

- $AdS_5 \times S^5$  vacuum corresponds to one source  $\vec{x}_1 = 0, y_1 = 1$  with

$$\chi = \frac{1}{2}(\psi - \phi), \quad y = \sqrt{\rho^2 + 1} \cos \theta, \quad x_1 = \rho \sin \theta \cos(\psi + \phi), \quad x_2 = \rho \sin \theta \sin(\psi + \phi)$$

- Function  $f$  and one form  $V$  become

$$f = \frac{1}{2} \frac{\rho^2 + \cos^2 \theta + 1}{\rho^2 + \sin^2 \theta} \quad \text{and} \quad V = \frac{1}{2} \frac{\rho^2 - \sin^2 \theta}{\rho^2 + \sin^2 \theta} d(\psi + \phi)$$

- $AdS_5 \times S^5$ , boundary at  $\rho \rightarrow \infty$ .

$$ds^2 = L^2 \left[ (\rho^2 + 1) ds_{AdS_3}^2 + \frac{d\rho^2}{\rho^2 + 1} + \rho^2 d\psi^2 + d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta ds_{S^3}^2 \right]$$

# Bubbling geometry for surface defect operators

- Near  $AdS$  boundary, the metric can be brought to Fefferman Graham form

$$ds^2 = \frac{1}{u^2} \left( du^2 + \alpha_1 ds_{AdS_3}^2 + \alpha_2 d\tilde{\psi}^2 \right) + \alpha_3 ds_{S^3}^2 + \alpha_4 d\tilde{\theta}^2 + \alpha_5 d\tilde{\phi}^2 \\ + \alpha_6 d\tilde{\theta} d\tilde{\psi} + \alpha_7 d\tilde{\psi} d\tilde{\phi} + \alpha_8 d\tilde{\theta} d\tilde{\phi}$$

- One form  $V$  only defined up to  $V \rightarrow V + d\omega$ , there is a gauge choice for FG coordinates.
- $u = \epsilon$  defines FG cutoff

$$\rho_c(\epsilon, \psi, \theta, \phi) = \frac{1}{\epsilon} + \frac{F_\rho^{(2)} - 1}{4} \epsilon + \frac{F_\rho^{(3)}}{6} \epsilon^2 + O(\epsilon^3)$$

- All functions are expanded in inverse powers of  $\rho$  and depend on moments

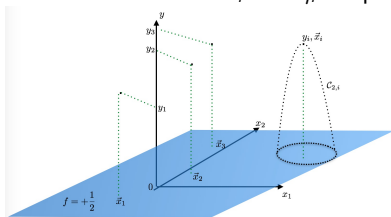
$$m_{abc} \equiv \sum_{i=1}^M y_i^a x_{i1}^b x_{i2}^c$$

# Map of bubbling supergravity solution and field theory

- In Gomis and Matsuura (arxiv:0704.1657) the parameters of the supergravity solution were identified with the parameters of the gauge theory surface defect as follows:

$$\frac{1}{2\pi l_s^2} (x_{i1} + ix_{i2}) = \beta_i + i\gamma_i, \quad \frac{y_i^2}{L^4} = \frac{N_i}{N}$$

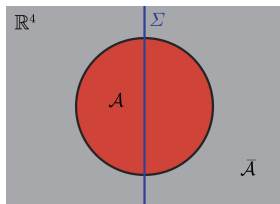
- Identification of  $\alpha_i$  and  $\eta_i$ . Topological flux of 2 form AST potentials



$$\alpha_i = \int_{C_i} B_{NSNS}$$

$$\eta_i = \int_{C_i} B_{RR}$$

# Holographic Entanglement entropy



- In  $R^4$  we consider a defect which is  $\Sigma = R^2$  and a spherical entangling region  $\mathcal{A}$ .
- Unlike for Wilson line, the boundary of the entangling surface and the defect intersect
- 2 coordinate systems:  $R^4$  in spherical coordinates and  $AdS_3 \times S^1$  (related up to a Weyl rescaling by a coordinate change).

$$\text{spherical} \quad ds_{\mathbb{R}^4}^2 = dt^2 + dx^2 + x^2 \left( d\vartheta^2 + \sin^2 \vartheta d\psi^2 \right)$$

$$AdS_3 \times S^1 \quad ds_{AdS_3 \times S^1}^2 = \frac{dt^2 + dl^2 + dz^2}{z^2} + d\psi^2$$

- Geometry of  $\Sigma$  and  $\partial\mathcal{A}$  for the two coordinate systems

	$\Sigma$	$\partial\mathcal{A}$
$AdS_3 \times S^1$	fills $t$ , fills $l, z = 0$	$t = 0, l^2 + z^2 = R^2$
spherical	fills $t$ , fills $x, \vartheta = 0, \pi$	$t = 0, x = R$

# Holographic Entanglement entropy

- Ryu-Takayanagi prescription. Find the minimal surface in bulk which ends on  $\partial A$  on the boundary

$$S_{\mathcal{A}} = \frac{A_{\min}}{4G_N^{(10)}}$$

- $AdS_3$  in Poincare coordinates

$$ds_{AdS_3}^2 = \frac{-dt^2 + dl^2 + dz^2}{z^2}$$

The ansatz minimal surface is at fixed  $t$ , extending over  $S^3$  and  $\Sigma_3$  and  $U(1)$  fiber coordinate and has profile  $z = z(l, \chi, y, x_1, x_2)$ . The area functional from the induced metric is

$$A(\mathcal{M}) = 2\pi^2 \int dl d\chi dy dx_1 dx_2 \frac{(f - \frac{1}{2})y}{z} \left\{ 1 + \left( \frac{\partial z}{\partial l} \right)^2 + \frac{y^2}{(f - \frac{1}{2})z^2} \left[ \left( \frac{\partial z}{\partial y} \right)^2 + \left( \frac{\partial z}{\partial x_1} - v_1 \frac{\partial z}{\partial \chi} \right)^2 + \left( \frac{\partial z}{\partial x_2} - v_2 \frac{\partial z}{\partial \chi} \right)^2 + \frac{(f + \frac{1}{2})(f - \frac{1}{2})}{y^2} \left( \frac{\partial z}{\partial \chi} \right)^2 \right] \right\}^{\frac{1}{2}}$$

# Holographic Entanglement entropy

- The function is minimized by dropping all dependence on  $\Sigma_3$  and the  $U(1)$  fiber coordinate  $\chi$

$$z^2 + l^2 = R^2$$

- The minimal area becomes

$$A_{\min} = 2\pi^3 \int dl \frac{R}{R^2 - l^2} l, \quad l = \int_X dy dx_1 dx_2 \left(f - \frac{1}{2}\right) y$$

- large  $\rho$  cutoff  $\rho = 1/\epsilon$  asymptotic  $AdS_5$  boundary FG cutoff for integral  $l$
- Small  $z$  cutoff  $z = \eta$  regularizes infinite volume of  $AdS_3$ .

$$\int_{-\sqrt{R^2-\eta^2}}^{\sqrt{R^2-\eta^2}} dl \frac{R}{R^2 - l^2} = 2 \log\left(\frac{2R}{\eta}\right) - \frac{\eta^2}{2R^2} + O(\eta^4)$$

# Holographic Entanglement entropy

- To evaluate integral  $I$ : First consider  $AdS_5 \times S^5$  vacuum (no defect)

$$\begin{aligned}
 I &= \int_{\Sigma_3} dy dx_1 dx_2 \left(f - \frac{1}{2}\right) y = \int_0^{2\pi} d\alpha \int_0^{\pi/2} d\theta \cos^3 \theta \sin \theta \int_0^{\rho_c^{(0)}} d\rho \rho \\
 &= L^8 \left( \frac{\pi}{4\varepsilon^2} - \frac{\pi}{8} + \frac{\pi\varepsilon^2}{64} \right)
 \end{aligned}$$

- Result for entanglement entropy for spherical entangling surface in  $AdS_5 \times S^5$  with  $AdS_3 \times S^1$  boundary

$$S_{\mathcal{A}}^{(0)} = N^2 \left[ \frac{1}{\varepsilon^2} - \frac{1}{2} + O(\varepsilon^2) \right] \log \left( \frac{2R}{\eta} \right)$$

# Holographic Entanglement entropy

- For general bubbling solution integral  $I$

$$I = \int_{\Sigma_3} dy dx_1 dx_2 \left( f - \frac{1}{2} \right) y$$

- Recall form of  $f$

$$f = \frac{1}{2} + \sum_{i=1}^M f_i$$

with

$$f_i = -\frac{1}{2} + \frac{(\vec{x} - \vec{x}_i)^2 + y^2 + y_i^2}{2\sqrt{[(\vec{x} - \vec{x}_i)^2 + y^2 + y_i^2]^2 - 4y^2y_i^2}}$$

- Integral invariant under shifts  $\vec{x} \rightarrow \vec{x} + \vec{x}_i$  and rescaling of  $y \rightarrow yy_i$ . Is the  $i$ -th integral the same as for vacuum ?

# Holographic Entanglement entropy

- No ! Since integral has to be regularized by FG cutoff and the cutoff is not invariant under the transformation of variables.
- For the  $i$ -th integral, perform the transformation which makes  $I$  look like the vacuum integral and transform the FG cutoff function  $\rho_c$ . The integrals can then be easily performed.
- Result for  $I$

$$I = \sum_{i=1}^M I_i = \frac{\pi L^8}{4\epsilon^2} + \frac{\pi L^8}{24} [1 - 4(m_{220} + m_{202} + m_{400})] + O(\epsilon)$$

- Calculate difference in entanglement entropy in the presence of a defect by subtracting vacuum result. Eliminates  $\epsilon$  divergences.

$$\Delta S_A = \frac{2N^2}{3} (1 - m_{220} - m_{202} - m_{400}) \log \left( \frac{2R}{\eta} \right)$$

# A 2d CFT interpretation of $\Delta S_A$

- $AdS_3$  factor in bubbling geometry

$$ds^2 = L^2 \left( e^{2W} ds_{AdS_3}^2 + ds_Z^2 \right)$$

- Brown-Henneaux central charge for  $AdS_3$  (with vacuum subtraction)

$$\begin{aligned} \Delta c_{\text{eff}} &= \frac{3L}{2G_N^{(3)}} = \frac{1}{16\pi G_N^{(10)}} \Delta \left( \int_Z d^7x \sqrt{g_Z} e^W \right) \\ &= \frac{3}{2G_N^{(10)}} \text{Vol}(S^3) \text{Vol}(S^1) \Delta I \end{aligned}$$

- This has same form as entanglement entropy  $\Delta S_A$  ( $\log \eta$ )

$$\Delta S_A = \frac{\Delta c_{\text{eff}}}{3} \log \left( \frac{2R}{\eta} \right)$$

- In terms of field theory quantities

$$\Delta c_{\text{eff}} = 2 \left( N^2 - \sum_{i=1}^M N_i^2 \right) - \frac{8\pi^2}{g_{\text{YM}}^2} \sum_{i=1}^M N_i \left( \beta_i^2 + \gamma_i^2 \right)$$

# Entanglement entropy a la Lewkowycz-Maldacena

- Casini-Huerta-Myers (arxiv:1102.0440) Entanglement entropy of spherical  $\mathcal{A}$  in  $R^d$  can be mapped to thermal entropy in  $S^1 \times H^{d-1}$ .
- Lewkowycz and Maldacena (arxiv:1312.5682) applied this to calculate the contribution to the entanglement entropy due to the presence of a Wilson line operator

$$\Delta S = \log \langle W \rangle - \int_{S^1 \times H^{d-1}} d^d x \sqrt{g} \Delta \langle T_{\tau\tau} \rangle_W$$

Quantities can be calculated for half-BPS Wilson loop in terms of Matrix model via localization

- Gentle and Gutperle (arxiv:1407.5629): Entanglement entropy  $\Delta S$ ,  $\langle W \rangle$  and  $\Delta \langle T_{\tau\tau} \rangle_W$  from holography using bubbling solutions of type IIB dual to half BPS Wilson loops in very large representation  $R$ . Main result: LM formula holds for holographic quantities (for large  $\lambda, N$ ).

## LM formula for holographic surface defects

- Third coordinate system  $S^1 \times H^3$  related by Weyl transformation

$$AdS_3 \times S^1 \quad ds_{AdS_3 \times S^1}^2 = \frac{dt^2 + dl^2 + dz^2}{z^2} + d\psi^2$$

$$\text{spherical} \quad ds_{\mathbb{R}^4}^2 = dt^2 + dx^2 + x^2 \left( d\vartheta^2 + \sin^2 \vartheta d\psi^2 \right)$$

$$\text{hyperboloid} \quad ds_{S^1 \times H^3}^2 = d\tau^2 + R^2 \left( d\rho^2 + \sinh^2 \rho \left( d\vartheta^2 + \sin^2 \vartheta d\psi^2 \right) \right)$$

- geometry of defect  $\Sigma$  and entangling surface  $\partial\mathcal{A}$

	$\Sigma$	$\partial\mathcal{A}$
$AdS_3 \times S^1$	fills $t$ , fills $l, z = 0$	$t = 0, l^2 + z^2 = R^2$
spherical	fills $t$ , fills $x, \vartheta = 0, \pi$	$t = 0, x = R$
hyperboloid	fills $\tau$ , fills $\rho, \vartheta = 0, \pi$	$\rho \rightarrow \infty$

## LM formula for holographic surface defects

- Calculate  $\Delta\langle T_{\tau\tau}\rangle_{\Sigma}$  on  $AdS_3 \times S^1$  (Drukker et al, arxiv:0805.4199)

$$\langle T_{\mu\nu}\rangle_{\Sigma} dx^{\mu} dx^{\nu} = h_{\Sigma} \left( ds_{AdS_3}^2 - 3 d\psi^2 \right)$$

with

$$h_{\Sigma} = \frac{N^2}{2\pi^2} \left[ \frac{1}{16} - \frac{1}{3} \left( m_{220} + m_{202} + \frac{1 - m_{400}}{2} \right) \right]$$

- Calculate  $\langle \mathcal{O} \rangle_{\Sigma}$  on  $AdS_3 \times S^1$  by

$$\langle \mathcal{O}_{\Sigma} \rangle = \exp \left[ - (I - I_{(0)}) \right]$$

- Problem: Self-duality of  $F_5$  we use (Giddings et al. hep-th/0105097)

$$h_{\text{bulk}} = -\frac{4}{\kappa^2} \int F_5^{\text{el.}} \wedge \star F_5^{\text{el.}}$$

# LM formula for holographic surface defects

- Total result for on shell action

$$I = I_{bulk} + I_{GH} = \frac{\pi}{2\kappa^2} \text{Vol}(AdS_3) \text{Vol}(S^3) \text{Vol}(S^1) \left[ \frac{5}{\epsilon^4} + \frac{2}{\epsilon^2} + \frac{3}{8} - m_{400} - \mathcal{F} \right]$$

- vacuum subtraction for expectation value removes  $1/\epsilon$  divergences

$$\Delta \log \langle \mathcal{O}_\Sigma \rangle = \frac{N^2}{(2\pi)^2} (m_{400} - 1 + \mathcal{F}) \text{Vol}(AdS_3) \text{Vol}(S^1)$$

- $\mathcal{F}$  is a complicated expression in terms of the moments  $m_{abc}$

$$\mathcal{F} = \frac{3}{32} \left[ 1 + 4m_{220} + 4m_{202} - 2m_{400} + 10(m_{220}^2 + m_{202}^2) \right. \\ \left. + 24m_{211}^2 - 4(m_{220} + m_{202})m_{400} + m_{400}^2 - 4m_{220}m_{202} \right]$$

# LM formula for holographic surface defects

- Map results from  $AdS_3 \times S^1$  to  $S^1 \times H_3$  via Weyl rescaling.
- After careful about identification of cutoffs the final result is

$$\begin{aligned}
 \log \langle \mathcal{O}_\Sigma \rangle &= \int_{S^1 \times H^3} d^d x \sqrt{g} \Delta \langle T_{\tau\tau} \rangle_\Sigma \\
 &= \frac{2N^2}{3} \left( 1 - m_{220} - m_{202} - m_{400} - \frac{3}{2} \mathcal{F} \right) \log \left( \frac{2R}{\eta} \right) \\
 &= \Delta S_{\mathcal{A}} - N^2 \mathcal{F} \log \left( \frac{2R}{\eta} \right)
 \end{aligned}$$

- Mismatch between holographic entanglement entropy and LM formula. Express  $\mathcal{F}$  in terms of field theory parameters

$$\begin{aligned}
 \frac{\mathcal{F}}{6144} &= \left[ \frac{1}{2} - \frac{1}{2N^2} \sum_{i=1}^M N_i^2 + \frac{4\pi^2}{g_{YM}^2 N^2} \sum_{i=1}^M N_i (\beta_i^2 + \gamma_i^2) \right]^2 \\
 &\quad + \frac{24\pi^4}{g_{YM}^4 N^4} \sum_{i=1}^M N_i (\beta_i + i\gamma_i)^2 \sum_{j=1}^M N_j (\beta_j - i\gamma_j)^2
 \end{aligned}$$

# Comments on mismatch

- For Wilson loop (1 dim defect) we found exact agreement of RT and LM entanglement entropy
- In arxiv:1506.00052 the bubbling solution in M-theory corresponding to Wilson-surface defect in 6dim  $(0, 2)$  theory was used to calculate the RT and LM entanglement entropies. A similar mismatch was found.
- Possible reasons for mismatch:
  - ① More careful treatment of cutoffs
  - ② Expectation value from on shell supergravity action
  - ③ 2dim surface: Graham-Witten anomalies
  - ④  $\beta$  variation of defect operator
- Probe branes: We can calculate entanglement entropy and expectation values. Does a similar mismatch appear ?