RADIO DETECTION OF HIGH ENERGY PARTICLES:

COHERENCE vs. MULTIPLE SCALES

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INTRODUCTION

Electromagnetic shower:

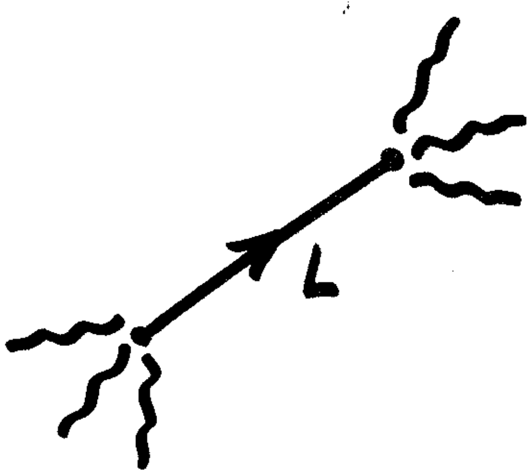
a pancake of charge with

- <u>finite</u> width
- <u>finite</u> thickness
- <u>finite</u> wavelength of radiation
- <u>finite</u> longitudinal spread

\+ Long history of coherent radio Cherenkov emission studies

\- NO careful treatment of evolving charge distributions !

- FRANK–TAMM (1937) infinite track
- TAMM (1939) finite track



inteference of radiation from acceleration at end points

↓

Strong oscillations in angular distributions

TAMM model is not reliable!

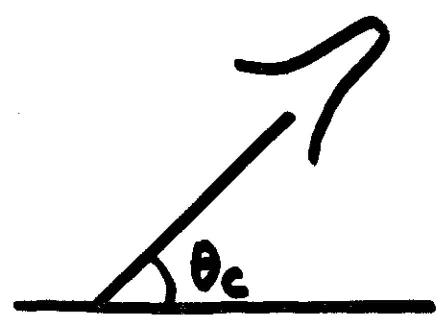
Another problem :

$$\frac{d^2 P}{d\omega d\Omega} \sim (\omega L)^2 \frac{\sin^2 X}{X^2}$$

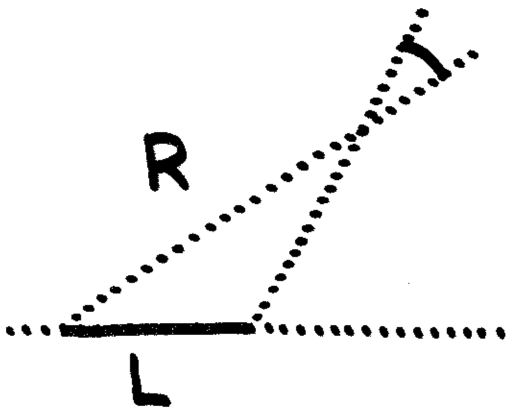
$$X \sim \omega L (\cos\theta - \cos\theta_c)$$

$L \to \infty$: is TAMM good?

$$\frac{\sin^2 X}{X^2} \to \frac{1}{\omega L} \delta(\cos\theta - \cos\theta_c)$$



$$\frac{d^2 P}{d\omega dL} \sim \omega$$



$\Delta\theta \sim \frac{L}{R}$

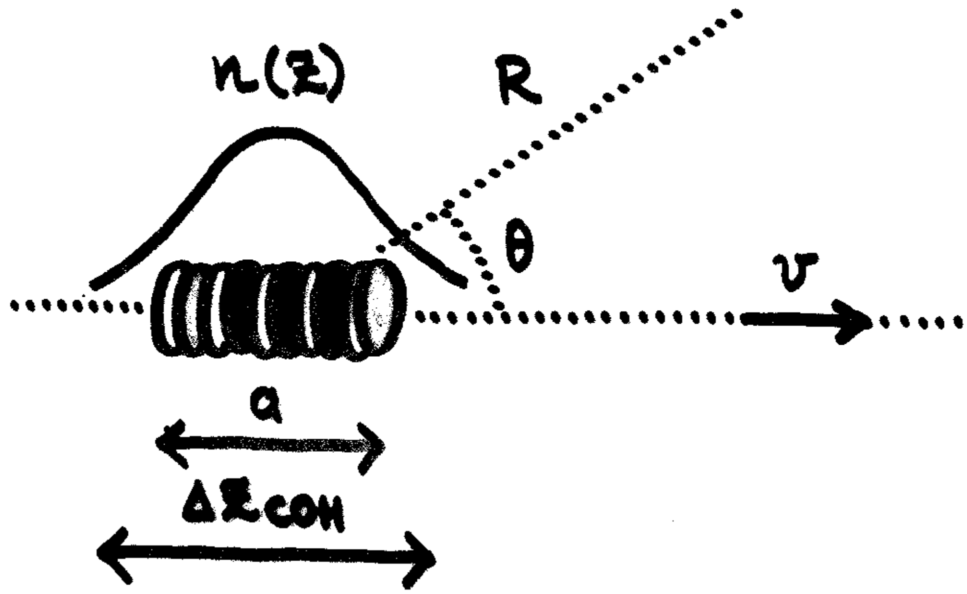
NO R dependence in TAMM !

OLd song again:
LIMITS INTERCHANGE PROBLEM !

if $R \to \infty$
then cannot recover $\frac{L}{R}$ = finite

- ALLAN (1971) wrong use of one of FEYNMAN'S formulas in electrodynamics

COHERENCE ZONE



Waves arrive simultaneously:
$$\dot{R} = c_m$$
$$\dot{R} = v\cos\theta$$
$$\cos\theta = \frac{c_m}{v}$$

Second variation:
$$\Delta R \sim \frac{1}{2}\ddot{R}(\Delta t)^2 = \frac{(\Delta z)^2}{2R}\sin^2\theta$$

Extra change of phase:
$$k\Delta R \sim \frac{k(\Delta z)^2}{2R}\sin^2\theta$$

Coherence is maintained over

$$|\Delta z| \lesssim \Delta z_{COH}$$

$$\Delta z_{COH} = \left(\frac{R}{k \sin^2\theta} \right)^{\frac{1}{2}}$$

$$\eta = \left(\frac{a}{\Delta z_{COH}} \right)^2 = \frac{ka^2}{R} \sin^2\theta$$

FRESNEL-FRAUNHOFER PARAMETER

FRAUNHOFER AND FRESNEL

TWO limits:

$a \ll \Delta z_{COH}$: Coherence is over the whole region

$$E_\omega \sim \omega \frac{1}{R}$$

FRAUNHOFER (SPHERICAL) regime: $\eta \ll 1$

$a \gg \Delta z_{COH}$: Coherence is over only a small part of the track

$$E_\omega \sim \omega \, \Delta z_{COH} \frac{1}{R} \sim \sqrt{\frac{\omega}{R}}$$

FRESNEL (CYLINDRICAL) regime: $\eta \gg 1$

Both are far fields: $kR \gg 1$

CHERENKOV radiation is fundamentally FRESNEL zone effect

WHEN IS FRESNEL IMPORTANT?

RICE:

$$\left.\begin{array}{l} a \sim 1.5\,\text{m} \\ R \sim 100\,\text{m} \\ \omega \sim 1\,\text{GHz} \end{array}\right\} \rightarrow \eta \sim 1$$

corrections are of order 1

Air showers:

$\eta \gg 1$

FACTORIZATION

Exact solution to Maxwell's equations:

$$c\vec{A}_\omega(\vec{x}) = \int d\vec{x}' \, \frac{\exp\left(i \frac{\omega}{c}\sqrt{\epsilon}\,|\vec{x}-\vec{x}'|\right)}{|\vec{x}-\vec{x}'|} \, \vec{j}_\omega(\vec{x}')$$

Very general model for the current:

$$\vec{j}(t,\vec{x}) = \vec{v}\, \underbrace{n(z)}_{\text{charge evolution}}\, \underbrace{f(z-vt,\vec{\rho})}_{\text{charge density}}$$

in all cosmic ray applications the characteristic size of the moving charge distribution is much smaller than the scale over which the charge develops.

$$\vec{A}_\omega \simeq \underbrace{F(\vec{q})}\,\underbrace{\vec{A}^{FF}_\omega(\eta)}$$

form factor

shower evolution

$$\boxed{F(\vec{q}) = \int d\vec{x}\, e^{-i\vec{q}\cdot\vec{x}} f(\vec{x})}$$

$$\boxed{\vec{A}^{FF}_\omega(\eta) = \frac{\vec{v}}{vcR}\, I^{FF}(\eta)}$$

$$\boxed{I^{FF}(\eta) = \int dz'\, n(z') \exp\left\{ i\frac{\omega}{c}\sqrt{\epsilon}\left[z'\cos\theta_c + R(z',\rho)\right]\right\}}$$

RESULTS BY THE SADDLE POINT METHOD

$$\vec{E}_\omega = \frac{i\omega}{Rc^2} F(\vec{q})\, I^{FF} \Big[(\cos\theta - \cos\theta_c)\, \vec{e}_R$$

$$- \Big(1 - i\eta \frac{\cos\theta_c}{\sin^2\theta} \frac{\cos\theta - \cos\theta_c}{1 - i\eta}\Big) \sin\theta\ \vec{e}_\theta \Big]$$

$$\vec{B}_\omega = - \frac{i\omega}{v c R \cos\theta_c} F(\vec{q})\, I^{FF}$$

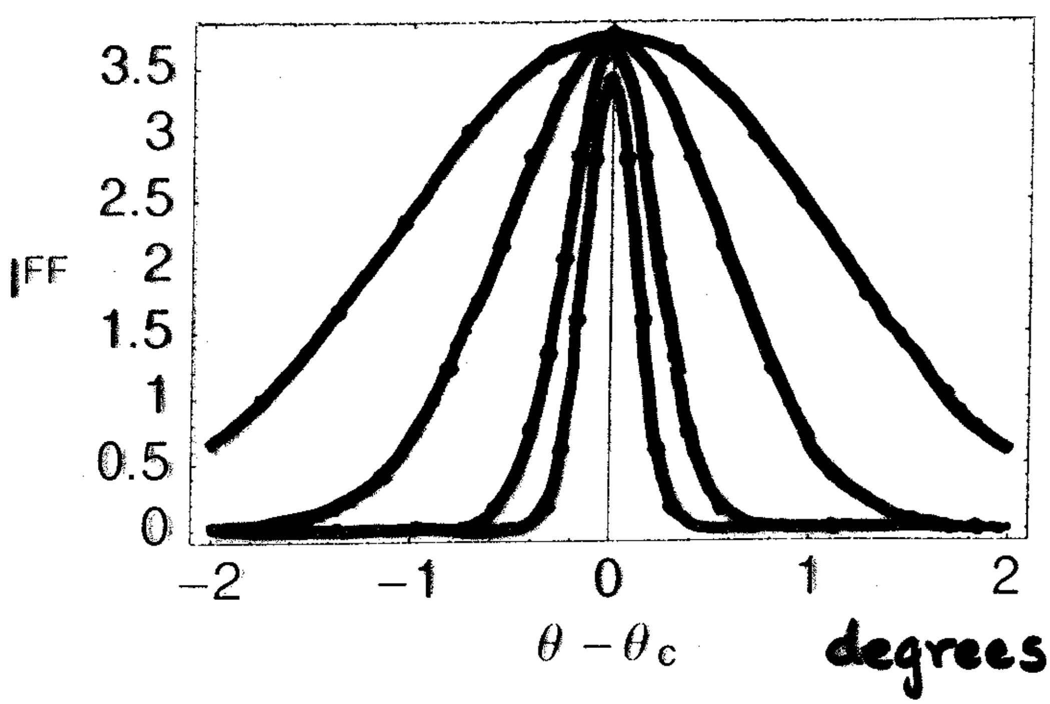
$$\times \Big[1 + i\eta \frac{\cos\theta}{\sin^2\theta} \frac{\cos\theta - \cos\theta_c}{1 - i\eta} \Big] \sin\theta\ \vec{e}_\phi$$

$$I^{FF} = e^{i\frac{\omega}{c}\sqrt{\epsilon}R}\, a\sqrt{2\pi}$$

$$\times \Big[1 - i\eta \Big(1 - 3i\eta \frac{\cos\theta}{\sin^2\theta} \frac{\cos\theta - \cos\theta_c}{1 - i\eta}\Big) \Big]^{-\frac{1}{2}}$$

$$\times \exp\Big[-\frac{1}{2} \Big(\frac{\omega}{c}\sqrt{\epsilon}\, a\Big)^2 \frac{(\cos\theta - \cos\theta_c)^2}{1 - i\eta} \Big]$$

FRESNEL - FRAUNHOFER INTEGRAL



POINTS: numerical integration
LINES: analytic fit
PARAMETERS: $a = 1.5$m, $R = 1$km
$\nu = 1, 2, 5, 10$ GHz

AGREEMENT IS SPECTACULAR:
ERROR $\ll 1\%$

FRAUNHOFER

$$\frac{R|\vec{E}_\omega^{\eta=0}(\theta)|}{F(\omega)} = (1.2 \pm 0.1) \times 10^{-7} \frac{\nu}{\mathrm{GHz}}$$

$$\times \exp\left[-\frac{1}{2}\left(\frac{\cos\theta - \cos\theta_c}{0.032 \frac{\mathrm{GHz}}{\nu}}\right)^2\right] \quad \left[\frac{\mathrm{V}}{\mathrm{MHz}}\right]$$

$$\simeq (1.2 \pm 0.1) \times 10^{-7} \frac{\nu}{\mathrm{GHz}}$$

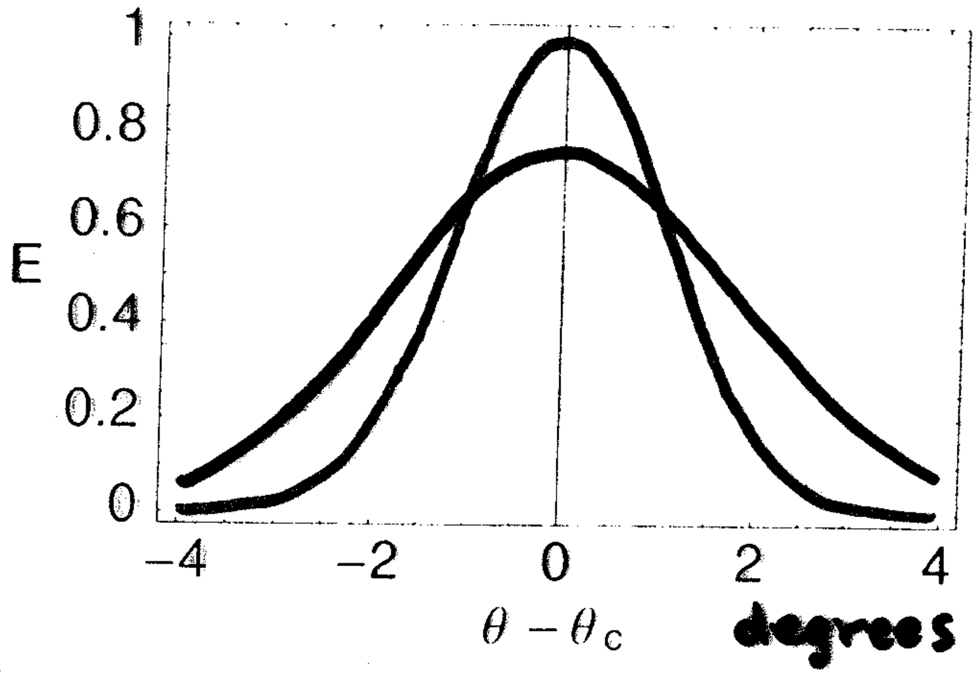
$$\times \exp\left[-\left(\frac{\theta - \theta_c}{1.1^\circ \frac{\mathrm{GHz}}{\nu}}\right)^2\right] \quad \left[\frac{\mathrm{V}}{\mathrm{MHz}}\right]$$

ZHS (1992)

$$\frac{R|\vec{E}_\omega^{ZHS}(\theta)|}{F^{ZHS}(\nu)} = 1.1 \times 10^{-7} \frac{\nu}{\mathrm{GHz}}$$

$$\times \exp\left[-\left(\frac{\theta - \theta_c}{1.2^\circ \frac{\mathrm{GHz}}{\nu}}\right)^2\right] \quad \left[\frac{\mathrm{V}}{\mathrm{MHz}}\right]$$

MAGNITUDE OF THE RESCALED ELECTRIC FIELD AS A FUNCTION OF ANGLE



RED LINE: EXACT RESULT

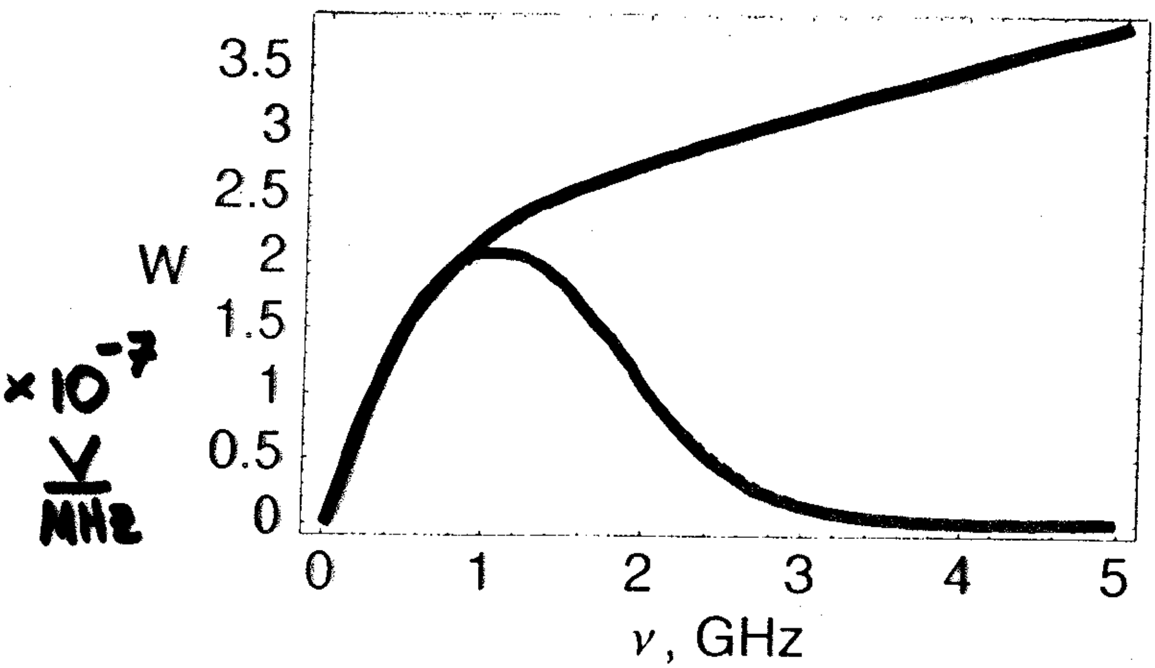
BLUE LINE: FRAUNHOFER APPROXIMATIO

PARAMETERS: $a = 1.5$m, $R = 50$m, $\nu = 1$GHz

CONSERVATION OF ENERGY

THE FORM-FACTOR IS DEVIDED OUT

MAGNITUDE OF THE RESCALED ELECTRIC FIELD AS A FUNCTION OF FREQUENCY



RED LINE: EXACT RESULT

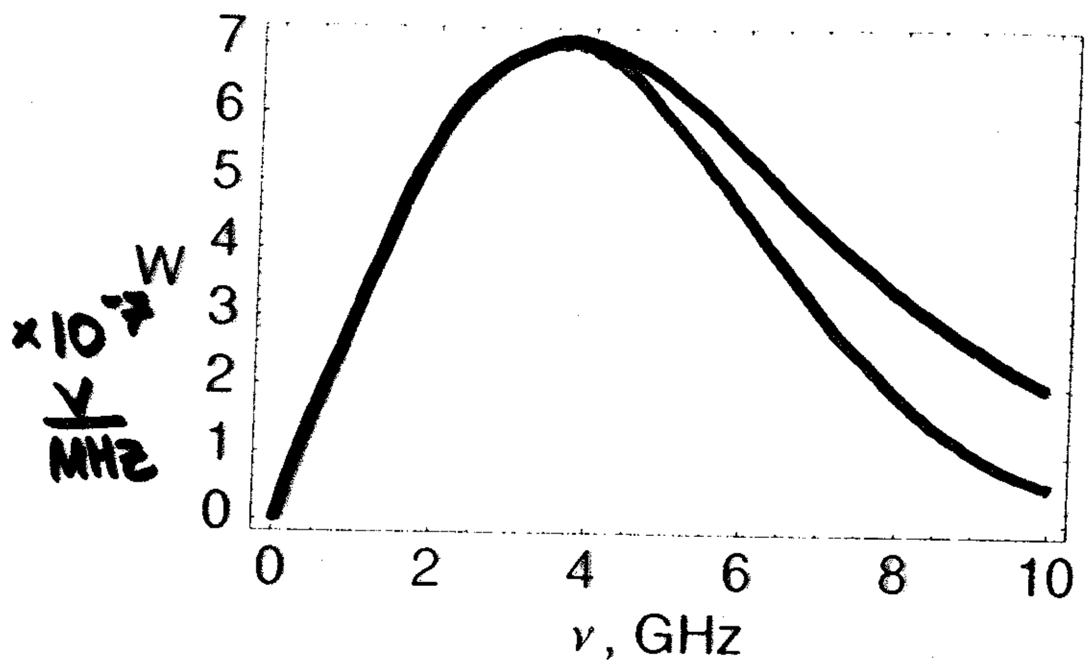
BLUE LINE: FRAUNHOFER APPROXIMATION

PARAMETERS: $a = 1.5\,m$, $R = 100\,m$, $\theta = 1°$

ARTIFICIAL HIGH FREQUENCY CUT-OFF (NOT A FORM-FACTOR EFFECT) FOR THE FRAUNHOFER APPROXIMATION

THE FORM-FACTOR IS DEVIDED OUT

MAGNITUDE OF THE RESCALED ELECTRIC FIELD AS A FUNCTION OF FREQUENCY



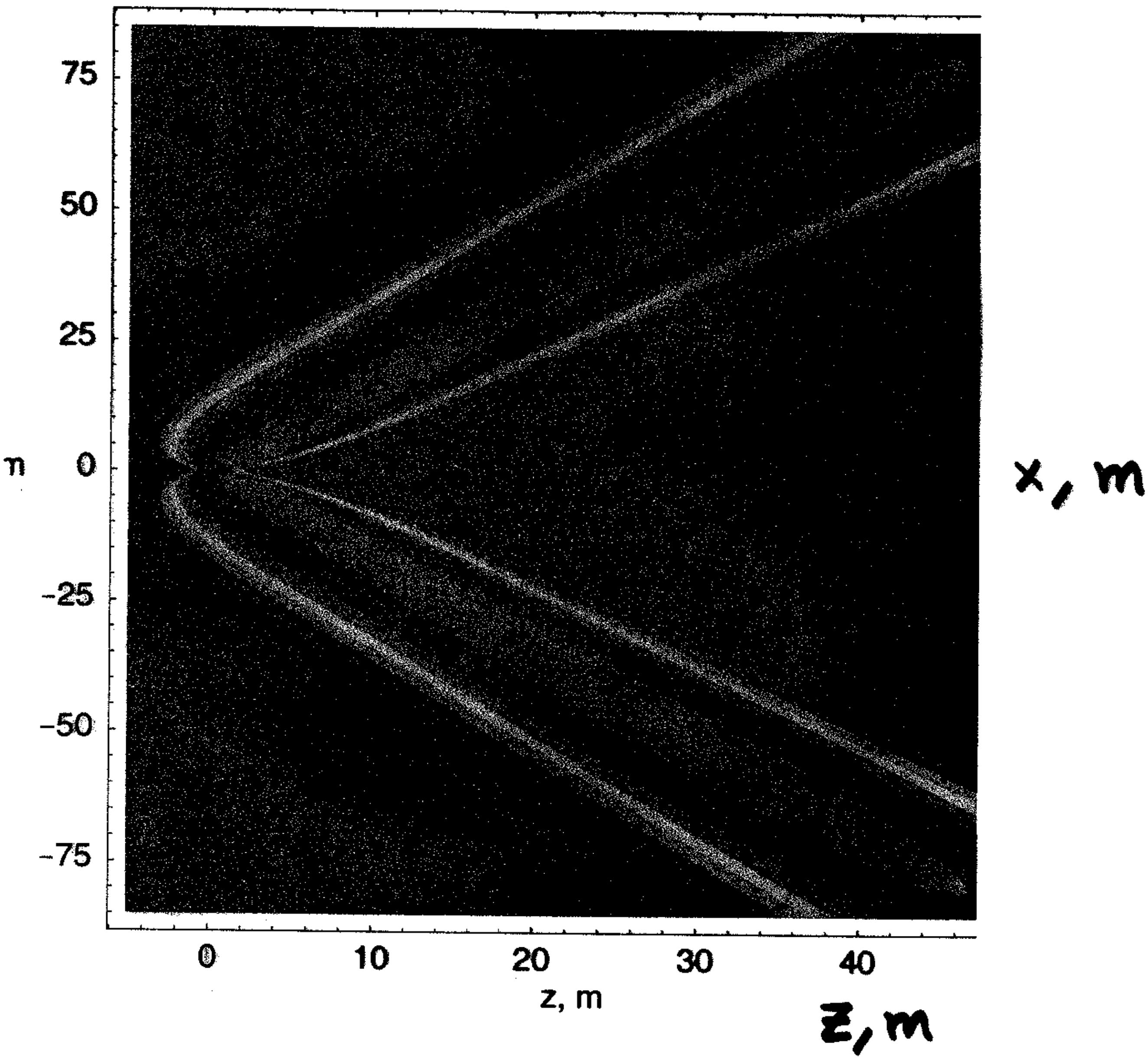
RED LINE : EXACT RESULT

BLUE LINE : FRAUNHOFER APPROXIMATION

PARAMETERS : $a = 1.5$m, $R = 1000$m, $\theta = 0.3°$

FRAUNHOFER APPROXIMATION BEGINS TO APPLY

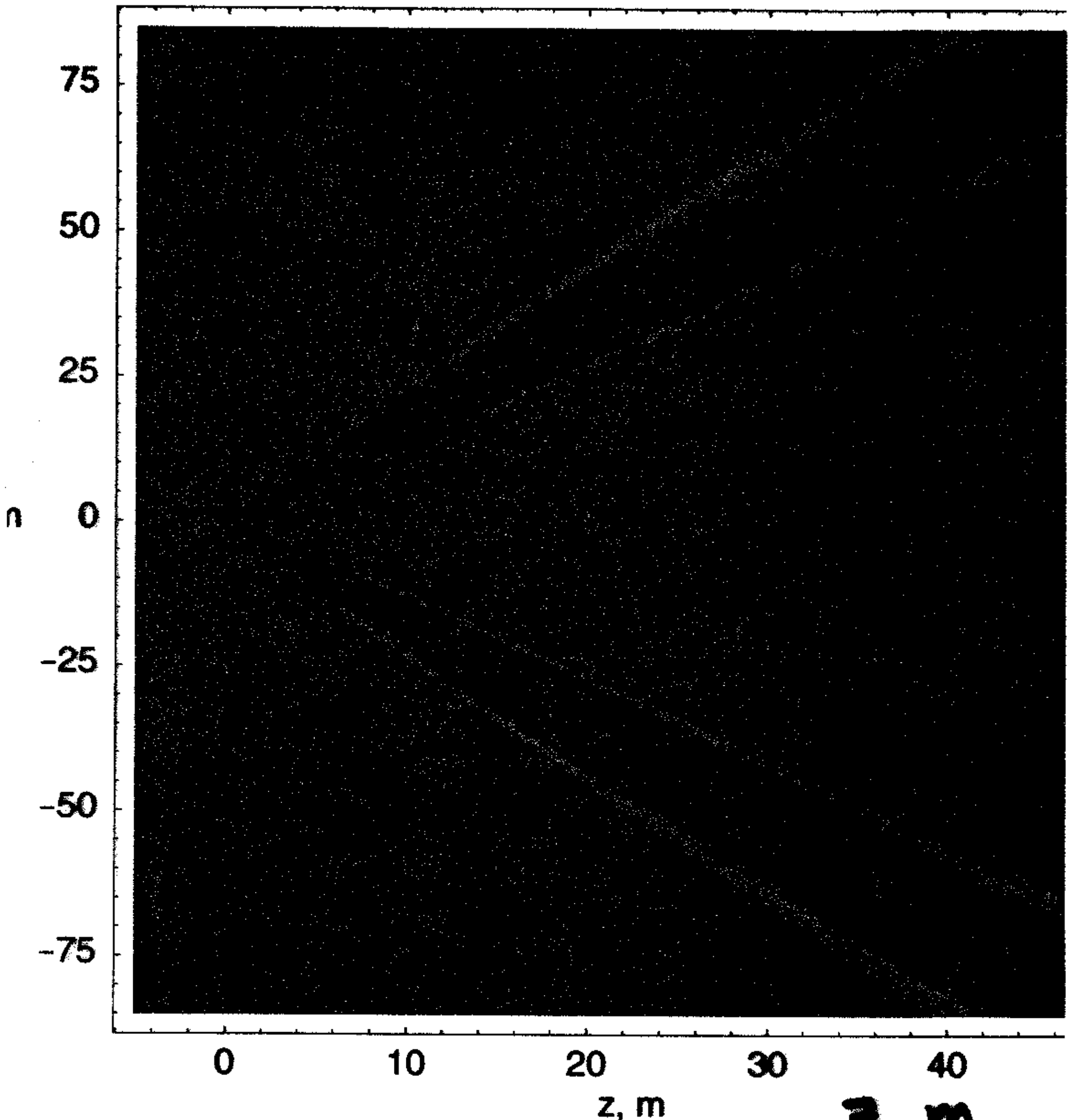
MAGNITUDE OF THE ELECTRIC FIELD



PARAMETERS: $a = 5m$, $\nu = 100MHz$

EVOLUTION FROM CYLINDRICAL TO SPHERICAL BEHAVIOUR

MAGNITUDE OF THE ELECTRIC FIELD IN THE FRAUNHOFER APPROXIMATION



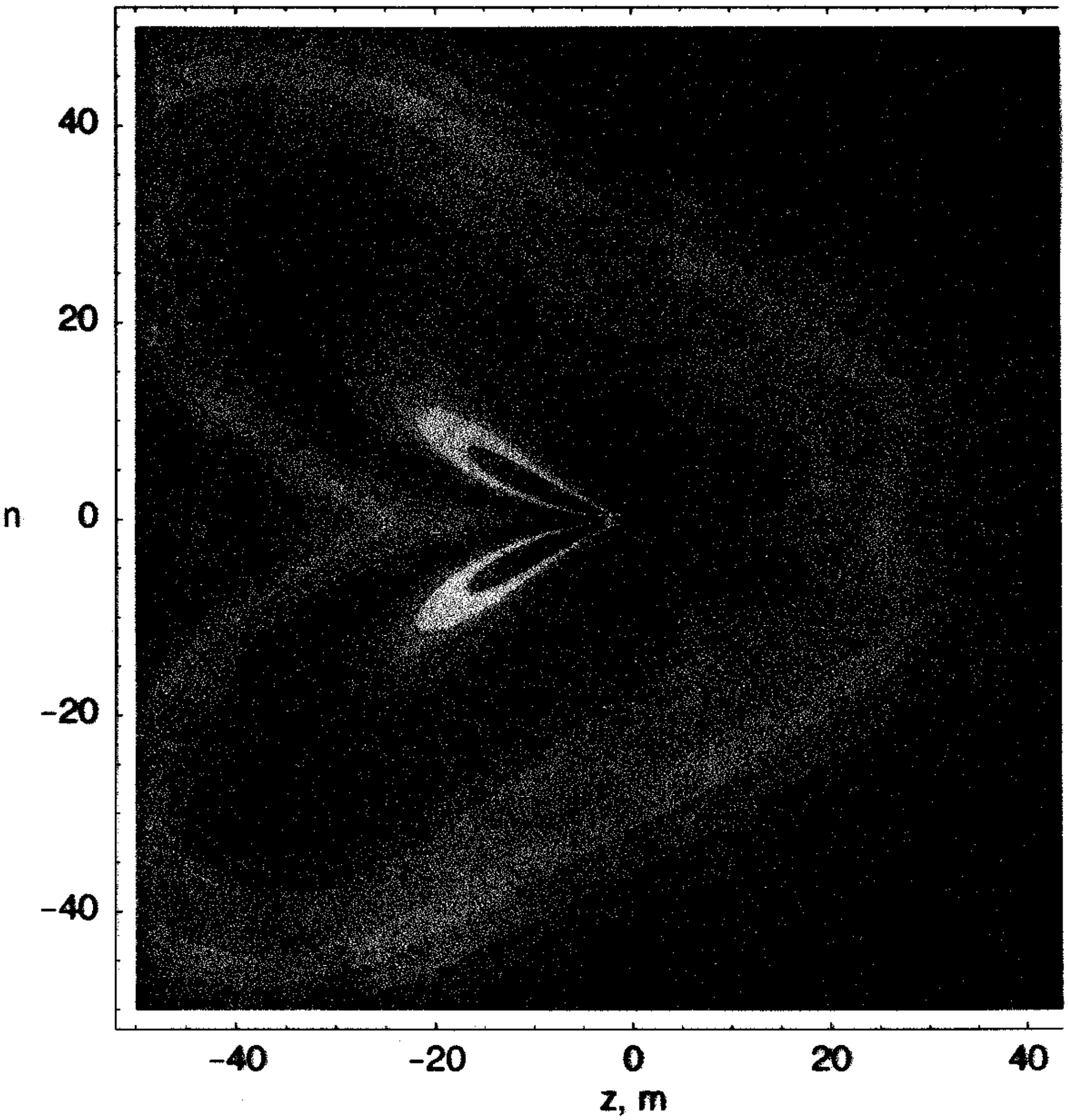
x, m

z, m

PARAMETERS : $a = 5\,m$, $\nu = 100\,MHz$

LACKS THE RICHNESS OF STRUCTURE OF PREVIOUS FIGURE

PHASE OF THE ELECTRIC FIELD

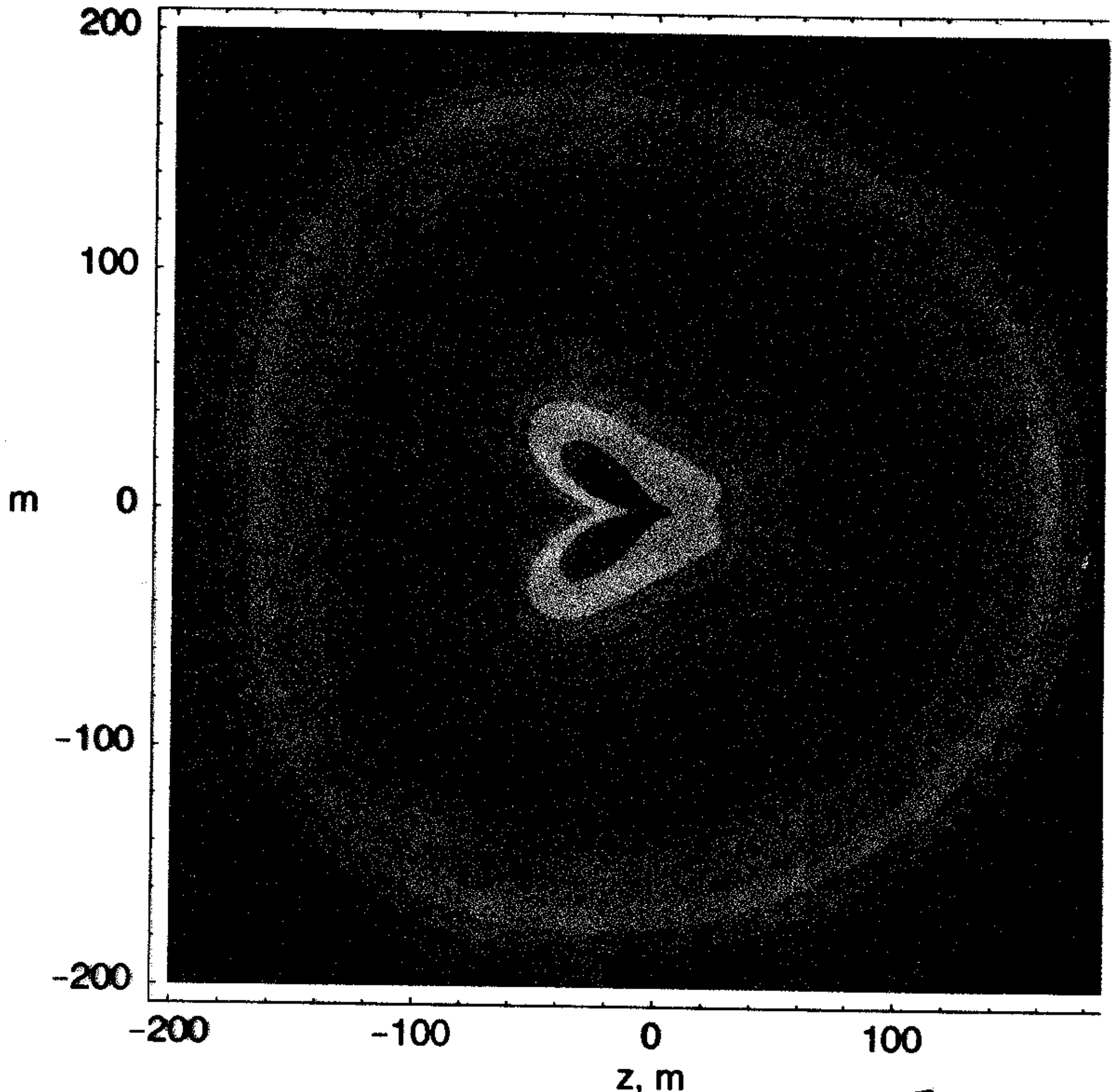


x, m

z, m

PARAMETERS: $a = 5m$, $\nu = 100 MHz$

PHASE OF ELECTRIC FIELD



x, m

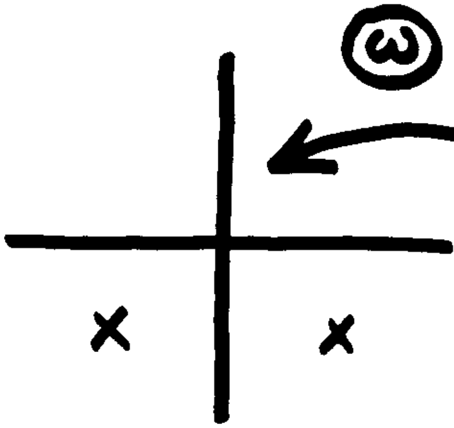
z, m

PARAMETERS: $a = 5m$, $\nu = 100 MHz$

IN THE FRAUNHOFER APPROXIMATION WAVEFRONTS BECOME SPHERICAL

CAUSAL FEATURES

$$E(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega \, e^{-i\omega t} E_\omega$$



NO POLES IN THE UPPER HALF-PLANE OF ω!

Of course, implemented EXACTLY in our calculations

CONCLUSIONS

PROVIDES FRAMEWORK FOR THEORETICAL STUDY

- FIRST PRINCIPLE CALCULATIONS
- SYMMETRIES AND DIMENSIONAL ANALYSIS

NEW FEATURES

- ADDITIONAL CONTROLLING VARIABLES
- FASCINATING STRUCTURES AT $R \not\to \infty$ NOT ACCESSIBLE BY MC

ROLE COMPLEMENTARY TO NUMERICAL CALCULATIONS

- MC IS STILL IMPORTANT
- COMBINATION OF BOTH IS IDEAL

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