Coherent Radio Radiation Produced by 15-MeV - 30 GeV Electron and Photon Bunches in Thin and Thick Radiators

Yerevan Physics Institute, Brothers Alitkhanian, Armenia

1. Introduction.


the 50 MeV linac-injector of YePPI 4.5 GeV Synch. was used for RFR exp. in waveguides among which one can mention


The shower model ~ exp(\mu e^{-t^2}) (Bell form)
1) To study the stimulated RF TR, theoretically first studied by late V.M. Kuzmin and first observed at SLAC SunSHINE in FIR region by #

2) To publish our work

3) To help understanding of the production of RFR by independent calculations.

4) To develop theory (quantitative) for the RFR suppression when $\theta_{obs} < \theta$.

5) To carry out new RFR experiments (without WIG).
2. The Longitudinal Distribution of the Charge Excess

MC simulations have been carried out for sand (SiO₂) with \( x_0 = 18 \text{ cm} \);
\( \rho = 2.58 \text{ g/cm}^3 \); \( n = 1.55 \) (\( \beta_{\text{iso}} \)) using the updated EGS4 codes.


As the shower evolves the excess development we fit by gamma distribution

\[ f(t) = \frac{\beta^t \cdot t^{\alpha-1} \cdot e^{-\beta t}}{\Gamma(\alpha)} \]  \hspace{1cm} (1)

Fig. 2 shows the results.

3. RFR Produced in Thick Radiators

3a. Thick/Thin radiators

\[ L_f = \frac{2\pi p c}{\mu (1 - \beta \sqrt{2} \cos \theta)} \] \hspace{1cm} (2)

where

\[ \varepsilon = \varepsilon' + i \varepsilon'' ; \quad \mu = \sqrt{\varepsilon} ; \quad \tan \delta = \frac{\varepsilon''}{\varepsilon'} \]

When \( \varepsilon \approx 1 \), \( L_f \approx \lambda \); for vacuum \( \varepsilon = 1 \), \( L_f \approx \lambda \beta \) \hspace{1cm} (\( \beta = \frac{c}{v} \))

We call the radiator

i) thick if \( \mu \text{adm} \gg L_f \approx \lambda \\ ii) thin if \( \mu \text{adm} \ll L_f \approx \lambda \\

Thick metallic radiators are not reasonable due RFR absorption [27].

In the case of thick dielectric (ice, SiO₂, wall) radiators one can separate CHR from TR. This follows from Fig. 3, since TR is produced on the first few \( x_0 \), while CHR at \( (5 - 10) x_0 \). This follows also from the work


in which thejwt formula is integrated over $\Theta$ when $(p_{\text{lab}} - \text{jet})^2 \gg m^2$.

3b. CHR or (CHF-like) RFR in Thick Radiators

We shall use [3 (A-6E)]. The advantage of [3] is that it is not HC: analytically using Maxwell eqs. ii) no sudden or certain acceleration of the excess is assumed, as in [15]. I. E. Tommas, J. Phys. M.L, 439, 1939 as well as [14].


Using the results and methods of [3], for a gamma distribution from charge excess (1), one can show that the frequency-angular distribution of the RFR is given by

$$\frac{1}{\hbar} \frac{d^2W_{1,2}(\nu, \Theta)}{d\nu \cdot d\Theta} \sim \left( \frac{\nu}{V_0} \right)^2 N_0^2 \sqrt{\nu} \xi \frac{\sin^2 \Theta}{\left[ 1 + \left( \frac{\nu}{V_0} \right)^2 (1 - \beta \sqrt{\nu} \cos \Theta)^2 \right]^{1/2}},$$

(3)

where $N_0$ is a normalization constant

$$V_0 = \frac{c \cdot \beta}{2 \pi \times 0(\text{cm})}$$

Expression (3) can be integrated over $\Theta$ from 0 to $\pi/2$ and derive lengthy expression for $dW/d\nu$, which for $\nu \gg V_0$ and Chengkaus condition $\beta \sqrt{\nu} > 1$ gives:

$$\frac{1}{\hbar} \frac{dW}{d\nu} \approx 13 \times \frac{2}{4} N_0^2 \frac{\sqrt{\nu} \xi}{V_0} \frac{1}{\left[ 1 - \frac{1}{\beta \sqrt{\nu}} \right]} \cdot \text{left}$$

(4)

$$\text{left} = \frac{5}{32} \frac{2V_0}{2 \pi V_0}$$

(4')
For electrons with $E = 30 \text{ GeV}$ (see Fig. 2) $N_0 = 14.1$; $\gamma_0 = 1.5 \times 10^8$; in sand one obtains the following angular distribution for RFR with $\theta = 3 \times 10^4$, $4 \times 10^3$, $5 \times 10^3$ and $3 \times 10^4$ sr.

no absorption!

The spectral distribution of RFR under the Cherenkov angle $\theta = 45 \text{.} 85^\circ$ is shown in Fig. 4.

The spectral distribution of RFR integrated over $\theta$ from 0 up to $\pi/2$ is shown in Fig. 5.

Note: the total energy of RFR from single electron emitted under angular interval $\theta = 0 \div \pi/2$ in wave length interval $\lambda = 10 \div 20 \text{ cm}$ in $10^{-13}$.  

30 GeV

3c. TR Produced at the Boundaries of Thick Radiator

Let $N_0$ electron bunches, each containing $N_e$ electrons with distance between the bunches $L_b$ much larger than the length $L_e$ of each bunch.

$L_e > L_b$ enter thick radiator without absorption with $E_e = E$ from vacuum with $E_0 = E$. Then TR spectral distribution from the $N_0$ bunches in forward direction (but integrated over $\theta$) is given by:

...
\[
\frac{dN_e}{d\lambda} = \frac{dW_p}{d\lambda} F_e(N, e, \lambda) \cdot F_e(N, e, \lambda) F_e(L_e, L) \tag{5}
\]

where

\[
\frac{dW_p}{d\tau} = \int_0^\Theta \int_0^\Theta \frac{dW_\perp(\tau, \theta)}{d\tau d\theta} \tag{6}
\]

\[
\frac{dW_\perp(\tau, \theta)}{d\tau d\omega} = \frac{2 \lambda^2 (\pi \sigma_\perp)^2 \frac{\sin^2 \theta \cos^2 \theta}{2}}{\lambda_\perp} \tag{7}
\]

\[
\lambda_\perp = \left( \lambda^2 - \lambda^2 \right) \left( 1 - \beta^2 \lambda^2 - \beta \sqrt{\lambda^2 - \lambda^2 \sin^2 \theta} \right) \tag{8}
\]

\[
\lambda_\perp = \frac{(\lambda^2 - \lambda^2) \left( 1 - \beta^2 \lambda^2 - \beta \sqrt{\lambda^2 - \lambda^2 \sin^2 \theta} \right)}{(1 - \beta^2 \lambda^2) \left( 1 - \beta \sqrt{\lambda^2 - \lambda^2 \sin^2 \theta} \right)} \tag{9}
\]

\[
F_e(N, e, \lambda) = N_e \left[ 1 + N_e \phi(\lambda) \right] \tag{10}
\]

\[
F_e(N, e, \lambda) = \left[ \frac{\sin(\phi L_e N_e / \lambda)}{\sin(\phi L_e / \lambda)} \right]^2 \text{ if detector } \delta \lambda \text{ and }
\]

\[
F_e(N, e, \lambda) = \left\{ \begin{array}{ll}
N_e & \text{if } \delta \lambda < 1 \\
N_e & \text{if } \delta \lambda \geq 1
\end{array} \right. \tag{11}
\]

is a factor depending on the distance \( L_e \) between the target and the detector.
Fig. 6 shows \( \int \frac{dW}{d\nu d\Omega}(\nu/\nu_0) \) and \( \int \left( \frac{dW}{d\nu} \right) d\nu d\Omega d\theta \) for \( \theta = \theta_0 \) and \( 1 \leq \theta \leq 7 \).

\( E_0 = 15 \text{ HeV} \) and 30 GeV. (since without absorption \( L < e_n \approx 0 \).)

we don't calculate this after \( \mathcal{F} \) due to the knowledge of \( \varepsilon \) is required.

Note, the total energy of RF TR from single 30 GeV electron emitted in angular interval \( \theta = 0 \), \( \theta \) in wave band \( \nu = 1 \div 20 \) is \( 10^{-26} \).}

4. RF Produced in Thin Radiators

Since for thin radiators TR dominates, we shall consider only TR. For \( \varepsilon_2 - 1 \geq 1 \) and of course \( \gamma \gg 1 \) according to \( \mathcal{F} \approx 1 \).

\[ \frac{dW}{d\nu d\theta} = \frac{2 \lambda}{\pi c} \frac{\theta^3}{(\gamma^2 + \theta^2)} \]  \( 12 \)

Integrating this exp. over \( \theta \) in the RFR detection interval \( 0 \div \theta_0 \).

\[ \frac{dW}{d\nu} = \frac{2 \lambda}{\pi c} \left[ \ln \left( \frac{\theta^2 + 1}{\theta^2} \right) - \frac{\theta^2 \gamma^2}{1 + \theta^2 \gamma^2} \right] \]  \( 13 \)

Note (12) and (13) do not depend on \( \varepsilon \) i.e. the TR yield is the same for metals, dielectric.

Fig. 4 shows... no problem, no Cherenkov

The TR yield is slightly higher than in Fig. 6.
4b. The diffraction-like factor \( F_0 = \left( \frac{\sin \pi T_0 \frac{L_f}{L_0}}{\sin \pi T_0 \frac{L_f}{L_0}} \right)^2 \)

The consequent \( \theta_f \) breaks with distance between them \( = L_f \) make the continuous XRF spectra "discrete" \( \ldots \) with harmonic wavenumber
\( \nu = \frac{L_f}{m} \) \((m = 1, 2, \ldots)\), width \( \sim 1/N_0 \), and amplitude \( \sim 1/N_0 \). If the detector resolution is low, \( N_0 - F_0 = N_0 \). The calculations show that for our beam parameters \( F_0 \approx 5N_0 \).

4c. Formation length \( (\text{near-field}) \) suppression factor \( F_L = \left( \frac{\log \frac{L_f}{L_0}}{\log \frac{L_f}{L_0}} \right) \)

is essential for \( \frac{\log \frac{L_0}{L_f}}{\log \frac{L_0}{L_f}} \) when \( L_f > 38^{\star} \). \( \ldots \) \( \text{in the case} \quad \frac{\log \frac{L_0}{L_f}}{\log \frac{L_0}{L_f}} \approx \frac{1}{2} \).

There is no quantitative theory (excluding the Vergilow work for backward TR). The experimental data \( [10] \) shows that for \( E = 15 \text{MeV} \), \( 2 \lesssim \frac{L_f}{L_0} \) when \( L_f \gtrsim 100 \), \( F_L \approx 1/30 \). We want to explain this and trying to construct a theory. For this purpose we calculate the pointing vectors \( \epsilon_1 \) which consist of 3 parts. \( \text{For the simplest case} \quad \epsilon_1 = \epsilon_0 \quad \epsilon_1 = 1 \) we have

\[
\frac{dW}{d\epsilon_1} = \frac{dW_{\text{rad}}}{d\epsilon_1} + \frac{dW_{\text{shape}}}{d\epsilon_1} + \frac{dW_{\text{int}}}{d\epsilon_1} = T \implies \quad \epsilon_1 = \epsilon_0 \quad \epsilon_1 = 1
\]

\[
\frac{dW_{\text{int}}}{d\epsilon_1} = \frac{\epsilon_1^2}{1 - \epsilon_0^2} \left[ \frac{1}{2} \cos \theta \left( 1 + \frac{1}{2} \beta \cos \theta \right) \right]
\]

Only \( T \) depend on \( \theta \).

In all TR walks it is considered TR in wave zone \( \epsilon_1 > > L_f \),

when \( T \rightarrow 0 \) oscillating after averaging in a small \( \epsilon_1 \) interval. We have studied the behavior of these 3 components. \( \text{When} \quad \epsilon_1 \lesssim L_f \).

One can integrate (14) over angles

\[
\frac{dW_{\text{rad}}}{d\Omega} = \frac{dW}{d\epsilon_1} \left[ \cos \theta_0 + \cos \theta_0 \right] - \frac{dW_{\text{int}}}{d\epsilon_1} = \frac{dW_{\text{int}}}{d\epsilon_1}
\]

\[
\frac{dW_{\text{rad}}}{d\Omega} = \frac{dW_{\text{int}}}{d\epsilon_1} \left[ \cos \theta_0 + \cos \theta_0 \right] - \frac{dW_{\text{int}}}{d\epsilon_1}
\]

Where

\[
T \approx 4 \left[ \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \left[ \cos \theta_0 \right] - \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \left[ \cos \theta_0 \right] \right]
\]
We see

\[ W = \text{Wind} \quad W_{\text{change}} \]

We have no result on \( F_2 \) still

5. Conclusion

The following

<table>
<thead>
<tr>
<th>Gradient of CLO</th>
<th>( N_e )</th>
<th>( N_f )</th>
<th>( N_0 )</th>
<th>( L_B )</th>
<th>( W )</th>
<th>pictures</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^4</td>
<td>single</td>
<td>1</td>
<td>0.4cm</td>
<td>-</td>
<td>( \frac{e^{-1/4}}{A} )</td>
<td>( 10^{-3} )</td>
</tr>
<tr>
<td>Yerphi</td>
<td>4 \times 10^4</td>
<td>50m</td>
<td>2.8 \times 10^3</td>
<td>1cm</td>
<td>10.5cm</td>
<td>( \frac{N_e^c}{4} \frac{V}{B} )</td>
</tr>
</tbody>
</table>

since \( W \approx \frac{4V^2}{R} \) \( \Rightarrow V \approx \sqrt{W} \)

\[
\frac{W_{\text{Yerphi}}}{W_{\text{Grad}}} = 2.1 \times 10^{-2} \quad \frac{V_{\text{Yerphi}}}{V_{\text{Grad}}} \approx 0.15
\]

shows that

Results on RFR can be obtained in Yerphi.
Fig. 9  The longitudinal distribution of the charge excess.

\[
\frac{(N_e - N_{0e})}{N_{0e}^c} \text{ in sand, } N_{0e}^c = 1000
\]

The lines are fits by Gamma function:
- \(E_0 = 1\) TeV, \(E_{cut} = 0.611\) MeV, \(\alpha = 5.65\), \(\beta = 0.54\)
- \(E_0 = 30\) GeV, \(E_{cut} = 0.6688\) MeV, \(\alpha = 3.83\), \(\beta = 0.56\)
- \(E_0 = 4.5\) GeV, \(E_{cut} = 0.6688\) MeV, \(\alpha = 2.99\), \(\beta = 0.62\)
Fig. 3. Angular distributions: curves 1, 2, 3 and 4 are for \( \Theta = 3 \times 10^3 (x \frac{1}{3}) \), \( 1 \times 10^3 (x 1) \), \( 5 \times 10^3 (x 4) \) and \( 1 \times 10^8 (x 100) \), respectively.

Fig. 4. Differentiated spectral distribution for \( \Theta = \Theta_{cb} \).

Fig. 5. Integral angular distribution.
Fig. 6. 1, 3 for $E=30$ GeV; 2, 4 for $E=15$ MeV

angular distributions

Fig. 7. angular distributions
1, 3 for $E=30$ GeV; 2, 4 for $E=15$ MeV