Coherent Radio Radiation Produced by 15-MeV – 30 GeV Electron and Photon Bunches in Thin and Thick Radiators

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1. Introduction.

Review of YerPhI past RFR works

Exp. Following [1] L. G. Lomize, „Comp. char. of ChR, TR and BS in short radiowave region", Zh. Techn. Fiz. 31, 301, 1961.

[2] A. I. Alikhanian, K. A. Ispirian and A. G. Oganesian, Zh. Eksper. Teor. Fiz., (OTR) 56, 1896, 1969
the 50 MeV linac-injector of YerPhI 4,5 GeV Synch. was used for RFR exp-s in waveguids among which one can mention

1) [3]. E. M. Laziev, G. G. Oksuzian, „RFR in a plate in W.G." and „RFR (parametric) in layered medium in WG", Izv. Akad. Nauk. Arm. Fiz., 6, 467, 1971 and Radiotechnik and Electronica 17, 1336, 1972. ($I_{ch+Tr} \sim N^2$)

2) [4]. E. M. Laziev, G. G. Oksuzian, „The determination of Phase Length of Bunches", Izv. Akad. Nauk Arm. Fiz., 10, 185, 1975.

3) [5]. Kh. S. Arutiunian et al, „The Confined RFR in WG", Izv Akad. Fiz. 11, 405, 1976.

Teor. Following [6] G. A. Askarian, „On Coherent RFR", JETP Rus. 30, 584, 1956.
[7]. G. A. Askarian, „Excess neg. charge", JETP, 41, 616, 1961.

1) [8] A. Ts. Amatuni „$I \sim [\sin(\pi L_e N_0/\lambda)/\sin(\pi L_e/\lambda)]^2$", Izv. Akad Arm. Fiz, 15, 109, 1962.

2) [9] A. Ts. Amatuni, G. M. Garibian and S. S. Elbakian, „RFR of varyable (in time) charge in medium with v=const", Izv. Akad Arm. Fiz. 16, 101, 1963.
The shower model $\sim \exp(-\omega_0^2 t^2)$ (Bell form)

Plans were until

[10] P.W. Gorham et al. „RF Measurements...", hep-ex/0004007 Sept. 2000.

[11] D. Saltzberg et al. „Observation of the Askaryan Effect..." hep-ex/0011001. Nov. 2000.

1) To study the stimulated RF TR, theoretically first studied by late V.M. Harutunian (1976) and first observed at SLAC SUNSHINE in FIR region by ★

[12]. H. Wiedemann et al, Phys. Rev. Lett., 76, 4113, 1996 (some words on physics?)

2) To publish our work

[13]. R.O. Avakian et al, „Charge Asymmetry of 1÷1000 GeV E-M showers and Possibility of Its Detection„ hep-ex ......, To be publ. in NIM. January 2001.
in which ...

$(15 \div 50)$ MeV Facility

$\boxed{\text{Fig. 1}}$ The beam/bunch parameters

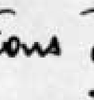
$f = 2856\,\text{MHz}$ ∴ $T = 3.5\cdot10^{-10}\,s$ ∴ $L_b = 10.5\,\text{cm}$ ∴ $l_b = 1\,\text{cm}$

$I_{av} = 1\,\mu A$ ∴ $6.24\cdot10^{12}\,el/s$

$T_p = 1\,\mu s$ ∴ $N_b^{FP} = 2.86\cdot10^3$ 

$\nu = 50\,\text{Hz}$ ∴ $N_b^{s} = 1.43\cdot10^5$

∴ $N_e^{P.b.} = 4.37\cdot10^7\,el$

RFR signal?? Measuring Channeling Radiation under the influence of ultrasonic oscillations at $E_e = 20\,\text{MeV}$ in autumn 1999 we observed signals  we ascribed them to „наводкам„.
of the type μs may be they were RFR?

Our purposes after [10, 11]

1) To help understanding of the production of RFR by independent calculation (not MC)

2) To develop theory (quantitative) for the RFR suppression when $L_{obs} < L_f$

3) To carry out new RFR experiments (without W-G).

2. The Longitudinal Distribution of the Charge Excess

MC simulations have been carried out for sand (SiO_2) with $X_0 = 18$ cm; $\rho = 1.58$ g/cm^3; $n = 1.55$ (β_{thresh}) using the updated EGS4 codes:

[14] I. Kawrakow and D.W.O. Rogers „The EGSnrc Code System, NRCC Rep. PIRS-701 1999–2000

As the shower curves the excess development we fit by gamma distribution

$$f(t) = \frac{\beta^{\alpha} \cdot t^{\alpha-1}\, e^{-\beta t}}{\Gamma(\alpha)} \qquad (1)$$

$\boxed{\text{Fig. 2}}$ shows the results.

3. RFR Produced in Thick Radiators

3a. Thick/Thin radiators

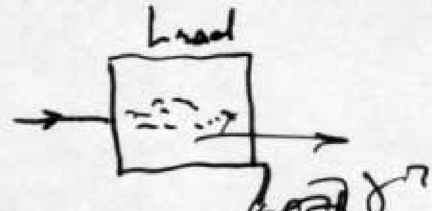
$$L_f = \frac{2\pi\beta c}{|\omega(1-\beta\sqrt{\varepsilon}\cos\theta)|} \qquad (2)$$

where

$$\varepsilon = \varepsilon' + i\varepsilon''; \quad n = \sqrt{\varepsilon'}; \quad L_{abs} = \frac{c}{\beta\omega\varepsilon''}.$$

when $\varepsilon \gtrsim 1$ $L_f \sim \lambda$; for vacuum $\varepsilon = 1$ $L_f \sim \lambda\gamma^2$ $\left(\gamma = E/mc^2 = \frac{1}{\sqrt{1-\beta^2}}\right)$

We call the radiator



i) thick if $L_{rad} \gg L_f \sim \lambda$

ii) thin if $L_{rad} \lesssim L_f \sim \lambda$

Thick metallic radiators are not reasonable due RFR absorption (some exc...

In the case of thick dielectric (ice, SiO_2, NaCl) radiators one can separate ChR from TR. This follows from Fig. 2, since TR is produced on the first few X_0, while ChR at $(5 \div 20)\, X_0$. This follows also from the work

[14] T. Takahashi et al „ChR from a finite trajectory of electrons. Phys. Rev. E 50, 4041, 1994

④

in which the I. Tamms formula is integrated over θ when $(\beta n - 1)\frac{L_{rad}}{\lambda} \gg 1$.

3b. ChR or (ChR-like) RFR in Thick Radiators

We shall use [9 (A.GE)]. The advantages of [9] i) it is not MC i.e. analytical using Maxwell eq-s ii) no sudden, or certain acceleration of the excess is assumed as in [15] I.E. Tamm, J. Phys. M. 1, 439, 1939 as well as [14].
[16] I.M. Frank, Izluchenie V. Chera, Voprosi Teorii. M. 1988.
[17] G.V. Afanasiev and V.M. Shilov, Preprint JINR, Dubna, E2-2000-41, 2000

Using the results and methods of [9], for a gamma distribution form charge excess (1), one can show that the frequency-angular distribution of the RFR is given by

$$\frac{1}{\hbar}\frac{d^2W(\nu,\theta)}{d\nu \cdot d\theta} = 137^{-1}\left(\frac{\nu}{\nu_0}\right)^2 N_0^2 \sqrt{\varepsilon}\beta^2 \frac{\sin^3\theta}{\left[1+\left(\frac{\nu}{\nu_0}\right)^2\left(1-\beta\sqrt{\varepsilon}\cos\theta\right)^2\right]^{4}}, \quad (3)$$

where N_0 is a normalization constant

$$\nu_0 = \frac{c \cdot \beta}{2\pi X_0 (cm)}$$

Expression (3) can be integrated over θ from 0 to $\pi/2$ and derive lengthy expression for $dW/d\nu$, which for $\nu \gg \nu_0$ and Cherenkov condition $\beta\sqrt{\varepsilon} > 1$ gives:

$$\frac{1}{\hbar}\frac{dW}{d\nu} \simeq 137^{-1} N_0^2 \frac{2\pi\nu}{v}\left[1-\frac{1}{\beta^2\varepsilon}\right] \cdot L_{eff} \qquad (4)$$

$$L_{eff} = \frac{5}{24}\frac{v}{2\pi\nu_0} \qquad (4')$$

For electrons with $E = 30$ GeV (see Fig. 2) $N_0 = 14.1$; $\nu_0 = 1.53 \cdot 10^8$; in sand one obtains the following angular distribution for RFR with $\nu = 3 \cdot 10^9$; $1 \cdot 10^9$; $5 \cdot 10^8$ and $1 \cdot 10^8$ s^{-1}.

$\boxed{\text{Fig. 3}}$ no absorption!

The spectral distribution of RFR under the Cherenkov angle ($= 49.85°$) is shown in $\boxed{\text{Fig. 4}}$

The spectral distribution of RFR integrated over θ from 0 up to $\pi/2$ is shown in $\boxed{\text{Fig. 5}}$

Note, the total energy of RFR from single 30 GeV electron emitted under angular interval $\theta = 0 \div \pi/2$ in wave length interval $\lambda = 10 \div 20$ cm is 10^{-22} J.

3c. TR produced at the Boundaries of Thick Radiator

Let N_b electron bunches, each containing N_e electrons with distance between the bunches L_b much larger than the length l_b of each bunch, $L_b >> l_b$ enter thick radiator without absorption with $\varepsilon_2 = \varepsilon$ from vacuum with $\varepsilon_1 = 1$: Then TR spectral distribution from the N_b bunches in forward direction (not integrated over θ) is given by:

$$\frac{dW_{N_e}(\lambda)}{d\lambda} = \frac{dW_1(\lambda)}{d\lambda} F_f(N_e, L_e, \lambda) \cdot F_b(N_b, L_b, \lambda)\, F_2(L_{obs}, L_f) \qquad (5)$$

where

$$\frac{dW_1(\lambda)}{d\lambda} = \int_0^{\phi}\int_0^{\Theta} \frac{dW_1(\lambda,\theta)}{d\lambda\, d\theta}, \qquad (6)$$

$$\frac{dW_1(\lambda,\theta)}{d\lambda\, d\Omega} = \frac{2\alpha(\hbar c)\beta^2 \varepsilon^{1/2} \sin^2\theta \cos^2\theta}{\lambda^2} \,|\xi|^2, \qquad (7)$$

$$\xi = \frac{(\varepsilon_1 - \varepsilon_2)\left(1 - \beta^2\varepsilon_2 - \beta\sqrt{\varepsilon_1 - \varepsilon_2\sin^2\theta}\right)}{(1-\beta^2\varepsilon_2\cos^2\theta)\left(1-\beta\sqrt{\varepsilon_1-\varepsilon_2\sin^2\theta}\right)\left(\varepsilon_1\cos\theta + \sqrt{\varepsilon_1\varepsilon_2 - \varepsilon_2^2\sin^2\theta}\right)} \qquad (8)$$

is TR from single particle.

$$F_b(\quad) = N_e\left[1 + N_e f(\lambda)\right] \;(9) = N_e^2 \quad \text{for } \lambda \gg L_b$$

is a factor taking into account the bunch form factor $f(\lambda)$

$$F_b(N_b, L_b, \lambda) = \left[\frac{\sin(\pi L_b N_b/\lambda)}{\sin(\pi L_b/\lambda)}\right]^2 (10) = \begin{cases} N_b^2 & \text{if the detector } \Delta\lambda/\lambda \ll 1 \\ N_b & \text{if } \quad \Delta\lambda/\lambda \gg 1 \end{cases}$$

is a factor taking into account the interference between TR from various bunches

$$F_2(L_{obs}, L_f) \qquad (11) = \begin{cases} \approx 1 & \text{if } L_{obs} > L_f \sim \lambda \\ < 1 & \text{if } L_{obs} < L_f \sim \lambda \end{cases}$$

is a factor depending on the distance L_{obs} between the int... and detector and on L_f.

$\boxed{\text{Fig.6}}$ shows $\int\limits_{\lambda_1=10\,cm} \frac{d^2W}{d\lambda d\Omega}$ (eV/st) and $\int\limits_{\lambda_1}\int\limits_{0}\int\limits_{0} \frac{d^2W}{d\lambda d\Omega} d\lambda d\theta d\varphi$ vs θ (rad) for $E_e = 15$ MeV and 30 GeV. (since without absorption $L_{abs} = \frac{c}{\beta\omega\varepsilon''} \to \infty$ we don't calculate the $\int$ after θ_{ch} due as the knowledge of ε'' is required)

Note, the total energy of RF TR from single 30 GeV electron emitted in angular interval $\theta = 0 \div \theta_{ch}$ in wave band $\lambda = 10 \div 20$ cm is 10^{-26} J.

4. RFR Produced in Thin Radiators

4a. TR

Since for thin radiators TR dominates, we shall consider only TR. For $|\varepsilon - 1| \gtrsim 1$ and, of course, $\gamma \gg 1$ according to $(\theta \ll 1)$

[18]. A.I. Alikhanian, K.A. Ispirian and A.G. Oganesian, Zh. Exp. Teor. Fiz. 56, 1796, 1969.

$$\frac{d^2W_1(\lambda,\theta)}{d\lambda\, d\theta} = 2\frac{\alpha\,(\hbar c)}{\lambda^2}\frac{\theta^3}{(\gamma^{-2}+\theta^2)}. \quad (12)$$

Integrating this exp. over θ in the RFR detection interval $0 \div \Theta$.

$$\frac{dW_1(\lambda)}{d\lambda} = \frac{2\alpha\,(\hbar c)}{\lambda^2}\left[\ln\left(\Theta^2\gamma^2+1\right) - \frac{\Theta^2\gamma^2}{1+\Theta^2\gamma^2}\right]. \quad (13)$$

Note (12) and (13) do not depend on ε i.e the TR yield is the same for metals, dielectrics $\varepsilon \gtrsim 1$

$\boxed{\text{Fig.7}}$ shows... no problem, no Cherenkov

The TR yield is slightly higher than in Fig.6

4b. The Diffraction-like factor $F_b = \left[\frac{\sin \pi N_b L_b/\lambda}{\sin \pi L_b/\lambda}\right]^2$

The consequent N_b bunches with distance between them $= L_b$ make the continuous RFR spectra "discretes" with m-th harmonic wavelength $\lambda_m = L_b/m$ $(m=1,2..)$, width $\sim 1/N_b$ and amplitude $\sim N_b^2$. If the detector resolution is low $N_b < F_b < N_b^2$. The calculations show that for our beam parameters $F_b \simeq 5N_b$.

4c. Formation length (near field) suppression factor $F_L(L_{obs}, L_f)$ is essential for $\xrightarrow{\varepsilon=1}$ when $L_f \sim \lambda\gamma^2$ (in the case $\varepsilon \neq 1$ $\rightarrow$ $L_f \sim \lambda$). There is no quantitative theory (excluding the Verzilov work for backward TR). The experimental data [10] show that for $E = 15$ MeV $\lambda \cong 15$ cm when $L_f \gtrsim 100$ m $F_L \simeq 1/30$. We want to explain this and trying to construct a theory. For this purpos we calculate the Pointing vectors ~~which~~ which consist of 3 part (For the simplest case $\varepsilon_1 = \infty$ $\varepsilon_2 = 1$) we have

$$\frac{d^2W}{d\omega d\Omega} = \frac{d^2W_{rad}}{d\omega d\Omega} + \frac{d^2W_{charge}}{d\omega d\Omega} + \frac{d^2W_{int}}{d\omega d\Omega} = \qquad (14)$$

$$= \frac{e^2}{\pi^2 c}\frac{\theta^2}{\left(\frac{1}{\gamma^2}+\theta^2\right)^2}\left\{\overset{I_{rad}}{1} + \overset{I_{charge}}{\cos\theta} - \overbrace{(1+\beta\cos\theta)\cos\left[2\pi\frac{z}{\lambda}(1-\beta\cos\theta)\right]}^{I_{int}}\right\}$$

Only I_{int} depend on z.

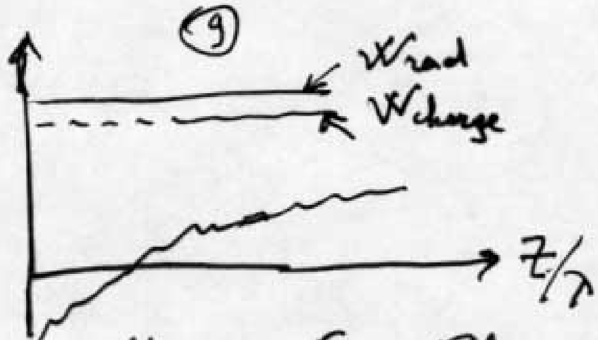
In all TR works it is considered TR in wave zone $z \gg L_f$ when $I_{int} \to 0$ oscillating after averaging in a small λ interval. We have studied the behavior of these 3 components when $z \lesssim L_f$. One can integrate (14) over angles

$$\frac{dW_{rad}}{d\lambda} = \frac{4\alpha(\hbar c)}{\lambda^2}\left\{\ln\gamma + \ln 2 - \frac{1}{2}\right\} \qquad \frac{dW_{int}}{d\lambda} = -\frac{\alpha(\hbar c)}{\lambda^2} I_0 \text{ where}$$

$$\frac{dW_{charge}}{d\lambda} = \frac{4\alpha(\hbar c)}{\lambda^2}\left\{\ln\gamma - \frac{1}{2}\right\} \qquad I_0 \cong 4\left\{ci\left(\frac{2\pi z}{\lambda}\right) - ci\left[\frac{2\pi z}{\lambda}\frac{1}{2\gamma^2}\right]\right\}$$

(9)

We see



We have no result on F_2 still

5. Conclusion

The following

	N_e	N_f	N_b	l_f	L_B	W	pictures
Gorham et al [10]	10^{10}	single	1	0.7 cm	–	$\sim N_e^2 \frac{dW}{d\lambda} \Delta\lambda$	10^{-9} s
YerphI	$4 \cdot 10^7$	50 Hz	$2.8 \cdot 10^3$	1 cm	10.5 cm	$N_e^2 \frac{dW}{d\lambda} F_B$	10^{-6} se

since $W \simeq \frac{4V^2}{R}$ $\therefore$ $V \sim \sqrt{W}$ $\therefore$

$$\frac{W^{Yerphi}}{W^{Gor}} = 2.2 \cdot 10^{-2} \qquad \frac{V^{Yerphi}}{V^{Gor}} \simeq 0.15$$

shows that

Result on RFR can be obtained in YerPhI.

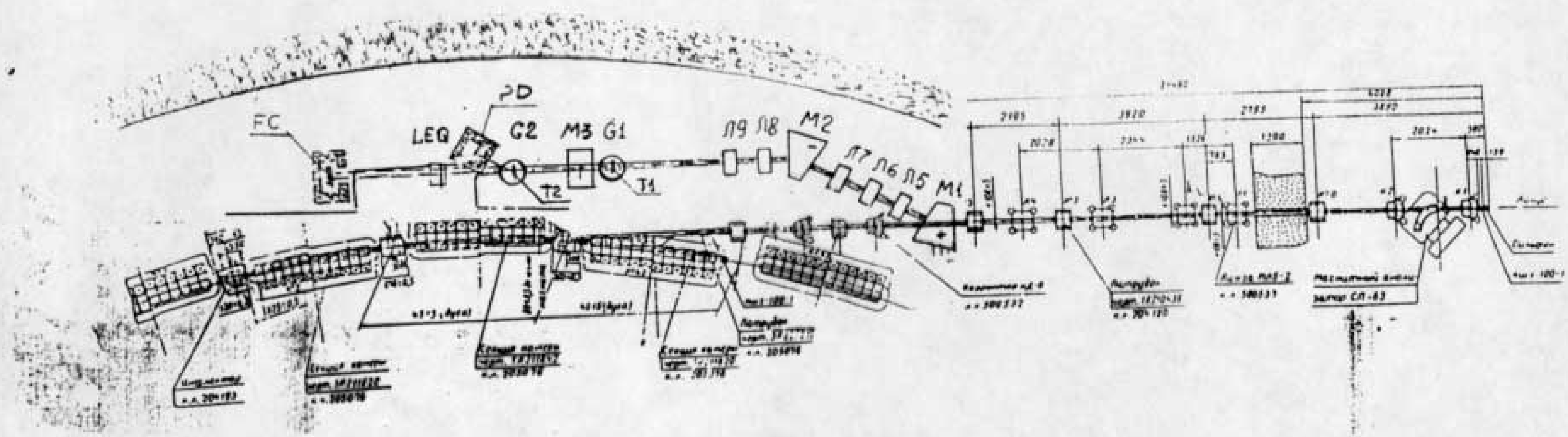
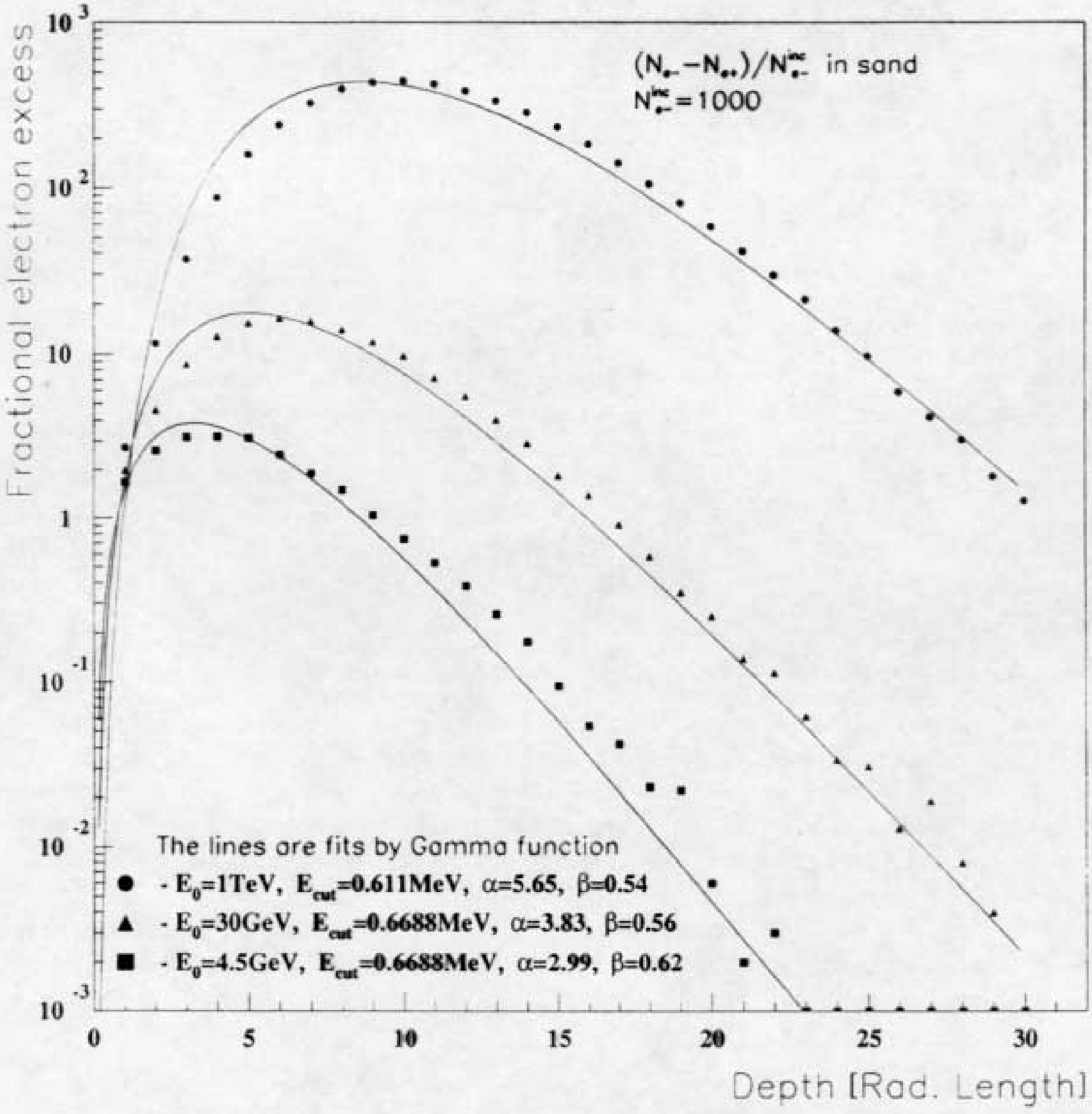


Fig. 1. 15-50 MeV Electron Beams

Fig. 9 The longitudinal distribution of the charge excess.



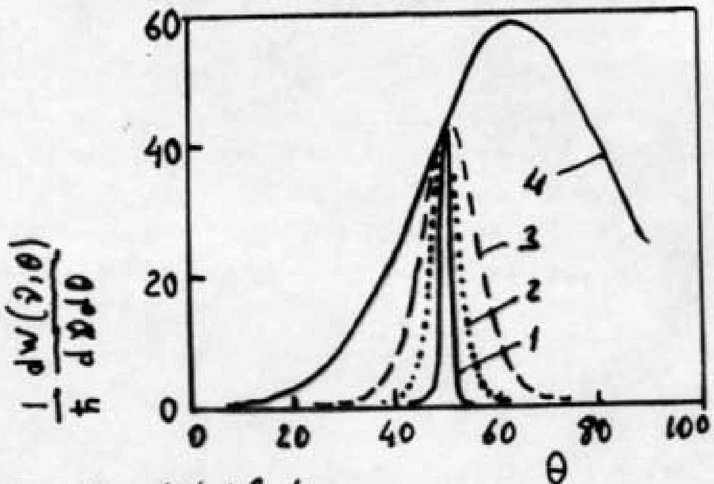


Fig. 3. Angular distributions; curves 1, 2, 3 and 4 are for $\nu = 3\cdot10^9$ ($\times\frac{1}{5}$), $1\cdot10^9$ ($\times 1$), $5\cdot10^8$ ($\times 4$) and $1\cdot10^8$ ($\times 100$), respectively

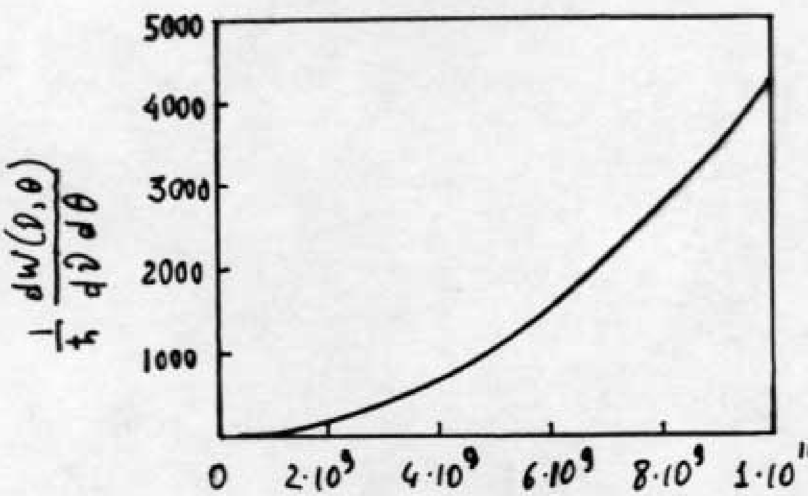


Fig. 4. Differential spectral distribution for $\theta = \theta_{Ch}$

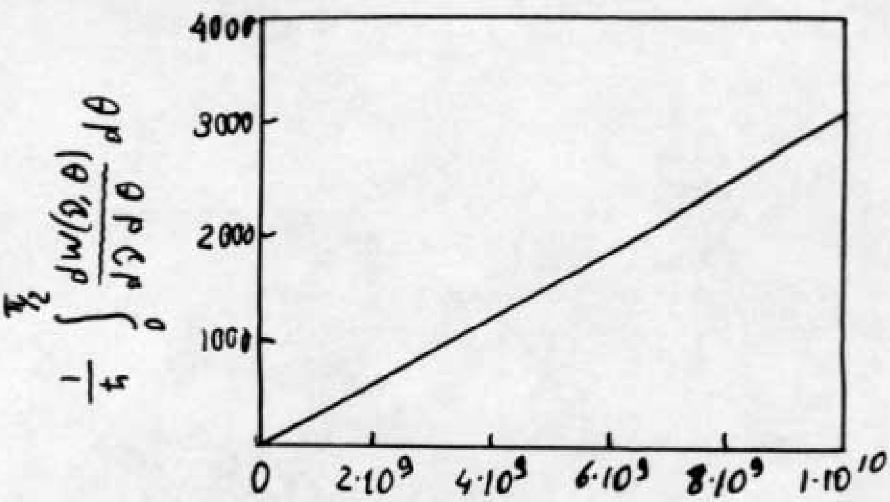


Fig. 5. Integral angular distribution

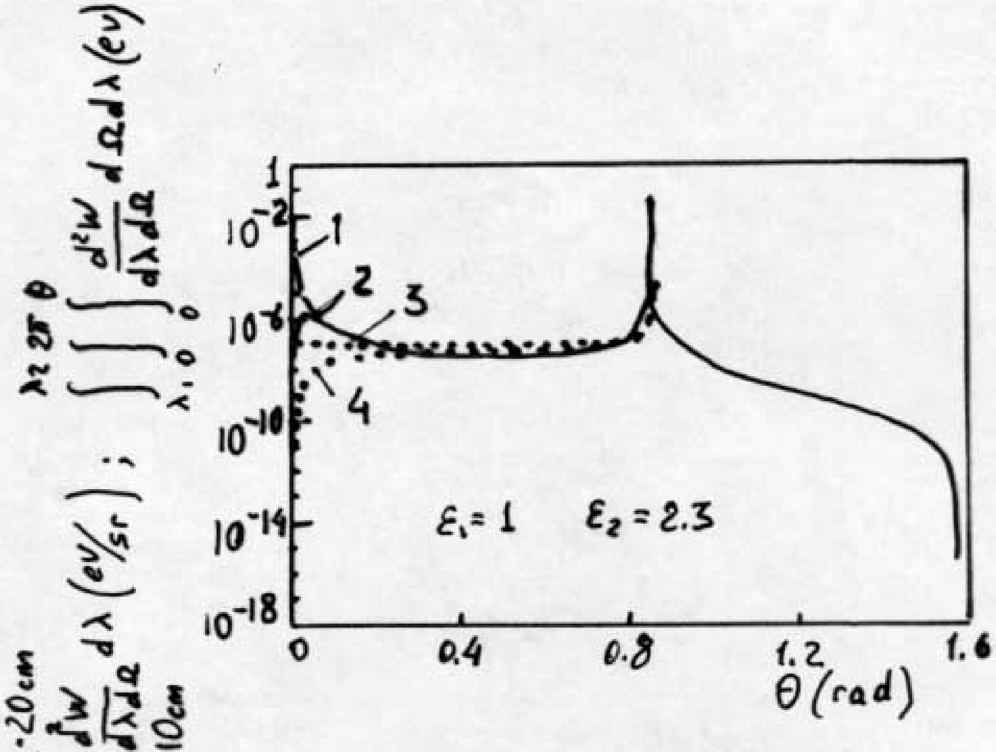


Fig. 6. 1, 3 for E=30 GeV; 2, 4 for E=15 MeV angular distributions

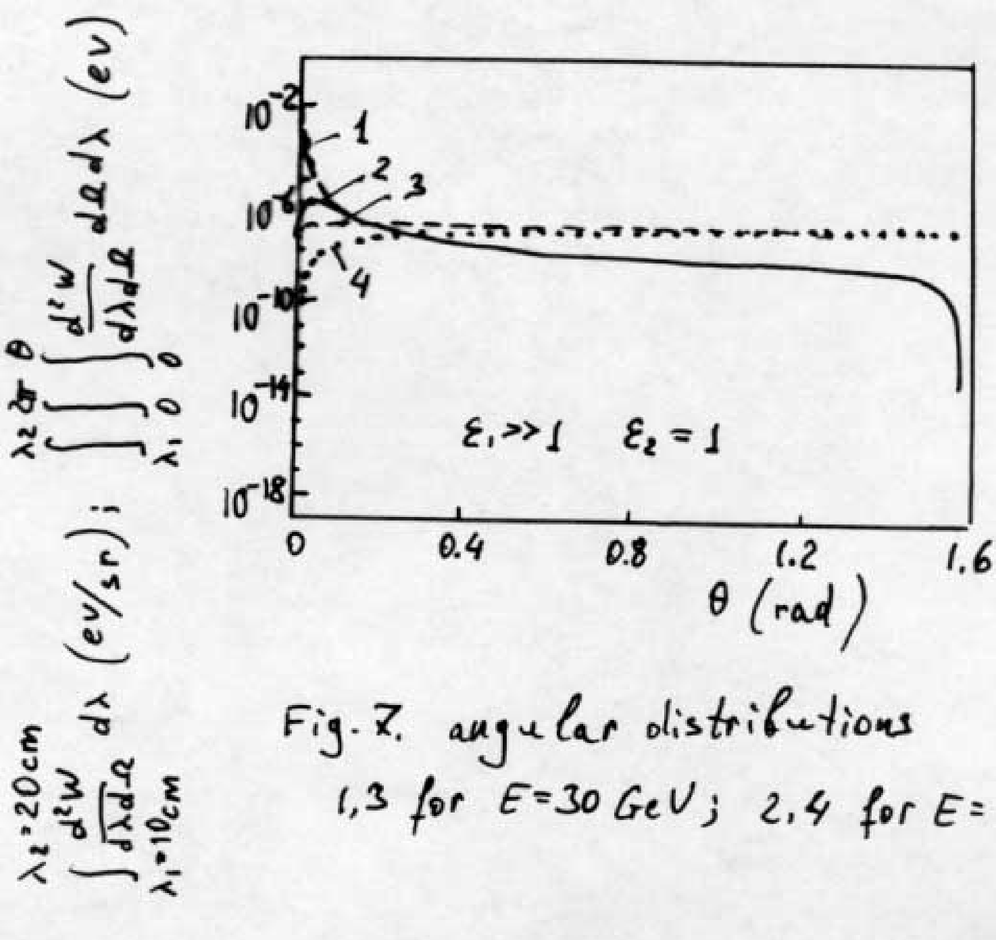


Fig. 7. angular distributions
1,3 for E=30 GeV; 2,4 for E=15 MeV