Concepts in the Coherence of Radio Cherenkov Emission from Ultra-High Energy Electromagnetic and Hadronic Showers

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Abstract. Radio Cherenkov emission underlies detection of high energy particles via a signal growing like the particle energy-squared. Many physical scales are involved in calculating the radio emission. We find that simple scaling laws exist which determine much of the physics in different regimes. The classic issues of Fraunhofer and Fresnel zones play a crucial role. The calculated electric fields decompose into a "factorized" product of a form factor, characterizing a moving charge distribution, multiplying a general integral depending on the charge evolution. The resulting expressions can be evaluated explicitly in terms of a few parameters obtainable from shower codes, while retaining exact constraints of analyticity.

I A MULTIPLE-SCALE PROBLEM

The most effective known method of detecting ultra-high energy particles, via coherent radio emission [1], exploits a wide range of physical phenomena. The process starts with a primary cosmic ray interaction at (say) 100 TeV or higher energy, leading to electromagnetic and hadronic showers. Shower evolution proceeds over many decades of energy and involves complicated strong interaction and electromagnetic physics. A charge imbalance develops in the shower, creating a net current which radiates in the radio regime, until finally the shower and radio emission die away. High energy physics is needed for the primary interaction, which might come from a neutrino in the Standard Model [2] or some other scenario such as Large Extra Dimensions [3]. Relativistic quantum electrodynamics and strong interaction physics predicts the shower evolution [4], atomic physics and low-energy strong interactions are needed for the charge imbalance, and classical electrodynamics is needed for the radio emission. Coherence of the radio emission involves multiple length scales: these include a distance scale for the shower to evolve, a scale length over which the shower is near maximum, the distance from the shower to the radio detector, the transverse and longitudinal spatial extension of the moving charge packet, and of course the wavelength of the radio emission.

Despite this complexity, Askaryan's original style of estimating the radio emission [1] works remarkably well. Assume that about 10% of the shower particles contribute to the charge excess, and use the "classic" Cherenkov formulas for power radiated–more on this below. Cut off the radio emission above a frequency scale set by the length of the moving charge packet, which is about 10 cm in ice. Then analytic calculations of km-scale radio-frequency fields [5,6] are surprisingly close to the subsequent outputs of immense Monte Carlos [7].

Yet estimating the last several dB of radio power correctly has proven to be amazingly complicated by any method. Accurate calculations are needed for the purposes of quantifying the RICE project [9], for comparison with test-beam experiments [10–12], and for planning future projects. Among the milestones along the way was the important paper of Zas, Stanev and Halzen [7] (ZHS), which predicted a broad oscillating "diffraction pattern" in the angular distribution of the radiation due to the finite length of the evolving shower. This feature does not seem to have been appreciated before. The ZHS Monte Carlo also predicted a charge excess exceeding 20% in some cases, and a frequency dependence for the electric field that is *linear in* ω up to the coherence cutoff. The conceptual development of RICE [13] accepted the Monte Carlo over the previous analytic estimates, while Price [19] helpfully compared radio and optical methods for the Antarctic ice medium. We intensified our own studies to support RICE starting about 1996-1997. It became clear to us that there was more than one way to do the electrodynamics, yielding different answers, apparently in contradiction to the exact formulas cited in the ZHS text. This created a few puzzles which we resolved, so that by now there is general agreement on certain conceptual issues to be briefly described here. Our longer paper [8] gives a great deal of detail, while the recent papers of Alvarez-Muñiz and Zas can also be recommended, and appear to be in accord.

II THE COHERENCE ZONE

Consider a charge moving on an arbitrary straight line not pointing toward the observation point. Let $R(t)$ be the instantaneous distance from the charge to the observation point. Information propagating at speed c_m will arrive simultaneously from a segment of the track if $\partial R/\partial t = c_m$. This is the Cherenkov condition: $\partial R/\partial t = v\cos\theta = c_m$ for velocity $\vec{v}$ oriented at angle θ relative to the direction $\hat{R}$.

Uniform motion of the charge produces acceleration of $R(t)$. In a time interval Δt near the Cherenkov point, this produces an extra radial displacement of order $\Delta R = 1/2(\partial^2 R/\partial t^2)(\Delta t)^2$. Coherence of modes of wavelength λ is then maintained only over a finite region of $\Delta R < \lambda$. Since $\partial^2 R/\partial t^2 = v^2 \sin^2\theta/R$, we solve to find the condition $\Delta t_{coh} < \sqrt{R\lambda}/(v\sin\theta)$. Equivalently, there is a finite spatial coherence region Δz_{coh} for given R, given wavenumber $k = 2\pi/\lambda$, namely

$$\Delta z_{coh} < \sqrt{R/(k\sin^2\theta)}$$

over which the "sonic boom" of radiation is built coherently. Since $\Delta z_{coh} \sim \sqrt{R}$, the coherence zone grows to infinite size as $R \to \infty$: but this limit cannot be taken prematurely. If one is at *finite distance* (as in, say test-beam experiments or RICE for $R < 300 - 1000m$) then the "bite" of radiating source is *smaller* than the same source observed at infinite radius.

As we show momentarily, knowledge of the coherence zone plus basic dimensional analysis brings a good understanding of the radio emission problem. The coherence zone argument was original with us–we discovered it by working a powerful saddle-point calculation [8] backwards for pedagogical purposes–but we subsequently learned that it is closely related to a classic construction. Our coherence zone is the finite R, Cherenkov-regime generalization of the *coherence length* invoked in ultra-high energy arguments for the LPM effect. None of the Landau, Pomeranchuk, or Migdal papers can be recommended to see this: but we eventually found Galitsky and Guerevich [21], and also benefited from the review by Boris Bolotovsky [22] at the meeting.

III FRAUNHOFER VERSUS FRESNEL

Textbooks routinely (and with little explanation [14]) replace the Helmholz Green function for radiation of frequency ω by the expansion

$$\frac{e^{i\omega|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|} \to \frac{e^{i\omega R - i\vec{k}\cdot\vec{x}'}}{R},$$

where $|\vec{x}| = R$ and $\vec{k} = \omega\hat{x}$; we use $c = 1$ except for emphasis. The expansion of the exponent is valid for $R \to \infty$ and sets up the *Fraunhofer approximation*, as it is termed in optics. Because this approximation is so common, it leads to the notion that it is general, along with the misconception that radiation fields always fall like $1/R$. An early Allen [16] review, as well successors [7,20] cited certain Feynman formulas to calculate the Fraunhofer limit of the radio emission from showers in air and ice, respectively.

The difficulty is that the series expansion of the exponent may fail spectacularly when a source, such as a cosmic ray shower, has finite spatial extent. The evolving shower has a length scale a over which it is near its maximum, and radiating copiously. This length scale is known as the "longitudinal spread" in cosmic ray physics [4]. Then the dimensionless ratio

$$\eta = (a/\Delta z_{coh})^2 = \frac{ka^2}{R}\sin^2\theta$$

controls how the limit $R \to \infty$ is taken. (Indeed the term "large R" is undefined until the limit parameter η is specified.) The Fraunhofer approximation needs $\eta << 1$ to be reliable. In the RICE experiment one optimistically has $a \sim$ meters, $\omega \sim 10cm^{-1}$, and $R \sim 10^3 m$, so $\eta < 1$ holds. Pushing the parameters to $R \sim$

$100-300m$, $\omega \sim 300MHz$, and a much larger a, makes $\eta \gg 1$ possible. Corrections "of fractional order unity" occur in the "Fresnel zone", namely the far-field region where Fraunhofer fails, and these are of some concern. The problem may be severe in the super-high energy regime where LPM corrections may affect the shower length [25].

IV "FACTORIZATION"

Under broad conditions, we find [8] that the radio emission amplitudes allow a "factorization" into a product of a form factor describing the spatial extent of the net charge, and a term representing the evolution of the shower as a whole. This is a non-trivial result, reminiscent of the scale-separation between hard and soft physics of the parton model, and due to a separation between the scale of the form factor and the scale of shower evolution. While we cannot expand around $\vec{x}\,' = 0$ (the Fraunhofer approximation), the conditions of the problem do permit an expansion around $\vec{\rho}\,' = 0$ (the transverse distance from the shower axis), namely for $R(z') = [(z-z')^2 + \rho^2]^{1/2}$, so that

$$\begin{aligned} |\vec{x} - \vec{x}\,'| &= [(z-z')^2 + (\vec{\rho} - \vec{\rho}\,')^2]^{1/2}, \\ &= R(z') - \frac{\vec{\rho}\cdot\vec{\rho}\,'}{R^2} + \mathcal{O}(\frac{\rho'^2}{R^2}). \end{aligned}$$

For typical values in applications, the second term is ~ 10 times smaller than the first, and the third is $\sim 10^3$ smaller than the second. For the expansion of the exponent, the third term does not contribute if $k\Delta\rho'^2/R \ll 1$, that is $\omega \ll 250GHz$.

The shower evolution generates a net charge $en(z)$ whose evolution in z must be taken into account. The vector potential $\vec{A}_\omega$ in (medium-adapted) Lorentz gauge then takes the form [8]

$$\begin{aligned} cR\vec{A}_\omega &= \vec{v}\int dz'\, n(z')\exp{[i(\frac{\omega}{v}z' + kR)]} \\ \times \int\int dt'\, d^2\rho'\, &\exp{\{-i[\frac{\omega}{v}(z'-vt') + \vec{q}\cdot\vec{\rho}\,']\}} f(z'-vt', \vec{\rho}\,'). \end{aligned}$$

We have shifted the t' integral which produces the translational phase in the z' integral. This gives the factorization:

$$\vec{A}_\omega \approx F(\vec{q})\vec{A}_\omega^{FF}(\eta),$$

where

$$F(\vec{q}) = \int d^3x'\, e^{-i\vec{q}\cdot\vec{x}\,'} f(\vec{x}\,'),$$

and $\vec{A}_\omega^{FF}$ is a certain integral [8] that does not depend on the details of the charge distribution. $F(\vec{q})$ is the form-factor of the charge distribution, which happens to be defined, just as in the rest of physics, in terms of the Fourier transform of the snapshot of the distribution. From our definitions $F(0) = 1$. Dependence on $\vec{q}$ may be observable in Giant Air Showers [16,17], where charge separation might cause an azimuthal asymmetry about the shower axis. As a consequence of separating out the form-factor, the integrations have become effectively one-dimensional.

V SCALING LAWS

The disagreement of the Monte Carlo frequency dependence with the classic Cherenkov formulas can be traced to use of the Fraunhofer approximation. The Fraunhofer field assumes complete coherence over the region of radiation, namely $\Delta z_{coh} >> a$. It follows that $E_{Fraunhofer} \sim Qa/R$, where we inserted the $1/R$ of the approximation explicitly. But the dimensions of the electric field $E(\omega) \sim \int dt \epsilon^{i\omega t} E(t) \sim m^1$, where m is a mass scale. (The basic field dimension follows from the action $\sim \int d^4x E^2$ being dimensionless in our units.) There being no other dimensionful scale besides ω, dimensional analysis requires

$$E_{Fraunhofer}(\omega) \sim Q\omega a/R.$$

This is a very general rule, and reliable up to the frequency where another scale (the coherence cutoff) can modify the argument.

Compare the opposite extreme of classic Cherenkov radiation, by which we mean the Frank-Tamm formulas, as cited by textbooks [14,15] and universally applied in particle physics. In this regime the track of the moving charge is assumed infinite in length. Now one cannot take $a \to \infty$, $R \to \infty$ and calculate this in the Fraunhofer approximation at all, leading to great confusion. We may obtain the correct scaling law by replacing $a \to \Delta z_{coh} \sim \sqrt{R/\omega}$, taking into account the finite-sized coherence zone cited earlier. Immediately the Cherenkov electric field scales like

$$E_{Cherenkov}(\omega) \sim Q\Delta\omega z_{coh}/R \sim Q\sqrt{\frac{\omega}{R}}.$$

It is rather unfamiliar to have a radiation field falling like $1/\sqrt{R}$. Yet this behavior also follows from geometry and dimensional analysis: observe that the *radiated power* has *cylindrical symmetry* concentric with the track. In this "cylindrical region", the power $P(\omega) \sim 2\pi R|E_{Cherenkov}(\omega)|^2 \sim$ *const*, which requires $|E_{Cherenkov}(\omega)| \sim 1/\sqrt{R}$ after all. The proportionality of the field to $\sqrt{\omega}$ is also the origin of the well-know rule that the classic Cherenkov power spectrum (not the electric field!) is linear in ω.

The accomplishment of our longer paper [8] was to understand this and to develop a systematic procedure for *interpolating between* the different regimes from cylindrical waves (classic Cherenkov behavior) to spherical waves (Fraunhofer approximation). We exploited the factorization cited earlier, and introduced an elegant saddle-point expansion which automatically takes into account the important coherence regions. Taking the Frauhnhofer limit of the interpolating formulas, we still needed a parameter for the shower longitudinal spread a. Running the ZHS code many times and fitting the output of a $1TeV$ shower with a particle threshold of $611MeV$ gives $a \sim 1.5m$. It is rather beautiful that the complicated numerical calculations can be summarized [8] by the parameter a, the charge at shower maximum, and a single length scale for the form factor. After extracting these quantities, we were then able to extend the prediction throughout the entire Fresnel-zone, revealing an intricate complexity in the interplay of frequency, distance, and angular distribution. The analytic formulas are very accurate, and the differences of parameters between different Monte Carlo calculations currently dominates the discussion: see the report by Razzaque at the meeting [26].

VI ANALYTIC STRUCTURE IN THE COMPLEX PLANE

Experiments on Cherenkov radiation generally see a sharp pulse in an oscilloscope (say) and are done in the time domain. Meanwhile, calculations are done in the frequency domain. The relation between the two is just a Fourier transform, but again there are subtleties. Causality requires [14,15] that any singularities of the electric field $E(\omega)$ lie in the lower half of the complex plane. Among the different methods to parametrize the field, at least two ($E(\omega) \sim 1/(1 + (\omega/\omega_0)^2$; $E(\omega) \sim (\omega/\omega_0)^\delta$) violate the basic requirements of causality. This has been another source of confusion, because the fitting of a Monte Carlo output appears to be a straightforward numerical task, where any naive function that fits "well enough" in a region was previously thought ok. However the time-domain pulses, which are the observable trigger of experiments, depend quite sensitively on proper analytic structure. Wrong-analytic structure lacks the causal front-edge that experiments look for. The explanation is that the Fourier transform is highly *non-local*, so that the correct time-structure demands attention to the niceties of the complex plane.

We resolved this by choosing a set of analytic functions to describe the form factor. A Gaussian for the form-factor works just fine [8]. The saddlepoint calculation also generates proper analytic structure, so that its time-domain dependence is physical: we found there is an effect distinguishing the arrival of the "front" and "back" ends of the evolving shower.

VII NUMERICAL STUDIES

We compared [8] the saddle-point approximation to direct numerical integration for numerous regions of frequency, angle, radius R, and shower energy for radio emission in circumstances of interest to RICE. In all cases the agreement was extremely good, at the level of 1% or less relative error. Our group has been in the process of evaluating Fresnel-zone changes in the effective volumes of radio receivers for neutrino detection. We believe that the next step will be the resolution of discrepancies among the different Monte Carlos that generate the net current: after the current is known, the electrodynamics seems under good control for the entire far-field regime relevant to RICE and similar experiments.

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