Coherent Radio Radiation of 15MeV ÷ 30GeV Electron and Photon Bunches in Thin and Thick Radiators

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Abstract. To understand the recent experimental results on coherent radiation in the radio wave region produced by bunches of $15MeV$ electrons [1] and $30GeV$ bremsstrahlung photons [2] as well as to carry out similar experiments with the help of $20MeV$, $50MeV$ and $4.5GeV$ electron and photon beams at Yerevan Synchrotron, the angular and spectral distributions of the radiation produced in thin and thick radiators are calculated using the theories of Cherenkov (ChR) and transition radiation (TR) and of the radiation of variable charges [3] as well as of the radiation at distances comparable with the formation zone [4]. The comparison of the obtained results with those of the Monte Carlo calculations of the type [5] and experimental results [1,2] is important for the projects and works on the detection of super high energy neutrinos with the help of radio radiation of the electromagnetic showers produced in Antarctic ice.

INTRODUCTION

The experiments [1,2] carried out at Argon and SLAC almost 40 years after the pioneering works of G. Askarian [6] have shown the importance of the accelerator investigations of the problems on the way to the high energy neutrino astrophysics. The Yerevan Physics Institute (YerPhI) physicists, who in the past have published theoretical works devoted to the charge excess and coherent radio radiation (CRR) of high energy electromagnetic showers [7,8] as well as have obtained experimental results on CRR production in wave guides (see [9]), have plans to continue studies to answer the following questions: 1)What are the real michanisms of production of CRR ? 2) What is the number of detected photons when the distance z between the detectors and radiation sources is less or of the order of the formation length L_f ? The first question concerns the role of sudden acceleration of the produced charges [10,11] and analytic calculation of CRR which usually is performed by Monte Carlo simulations [5], while the second question requires more efforts since the existing theoretical [12,13] and experimental [1] results are not sufficient.

It is convenient to call the radiator thick (or thin) if its thickness $L_{rad} \geq L_f$ and $L_{rad} \gg L_{abs}$ (or $L_{rad} \ll L_f$ and $L_{rad} \ll L_{abs}$) where L_{abs} is the radiator's absorption length. As it is well known in solid radiators $L_f \simeq \lambda$ where λ is the CRR wave length. Of course, it is not reasonable to prepare thick metallic radiator because of strong absorption of the produced CRR. If the photons are detected inside the radiator or at its lateral sides and/or the radiator is sufficiently thick one can neglect the contribution of the output interface. Such assumption corresponds to the many experimental situations, in particular, to the experiments [1,2]. Such a differentiation allows one to consider separately the radiation produced at the boundary and the radiation produced in the thick radiator and make concrete conclusions in the case of thin radiators without using complicated expressions.

CRR PRODUCED IN THICK RADIATORS

Longitudinal Distribution of the Charge Excess

All the Monte Carlo simulations are carried out for sand (SiO_2) radiators with radiation length unit $X_0 = 18$cm and refraction index n = 1.55 using the modified EGS4 code EGSnrc [14] for a cut energy corresponding to the Cherenkov threshold energy as it is described in [8]. In the case of calculation of the radiation of the

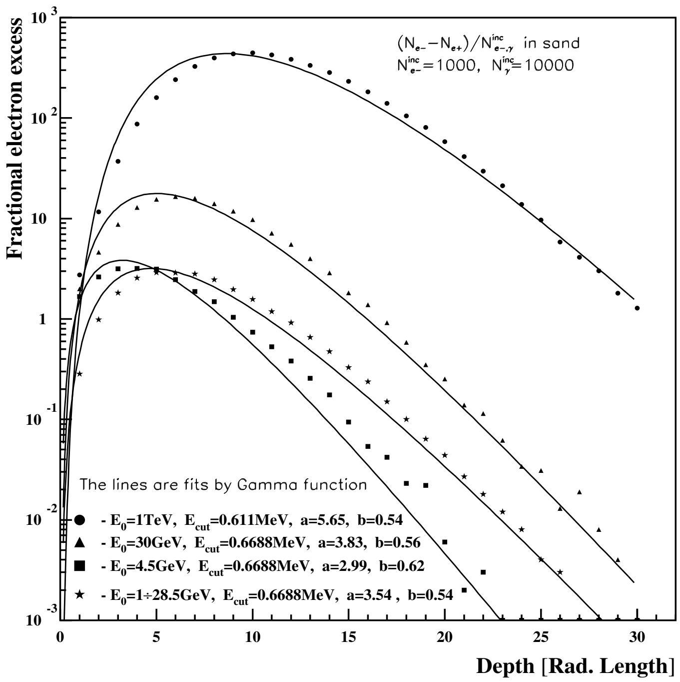

FIGURE 1. The longitudinal distribution of the charge excess. The excess points for the bremsstrahlung photons ($\star$) are multiplied by factor 100.

photon bunches it is assumed that an additional amorphous converter and sweeping magnet are placed before the radiator providing a pure photon Bethe-Heitler bremsstrahlung beam incident on the radiator. The thickness dependence of the charge excess $\delta N = N_{e^-} - N_{e^+}$ per incident electron for 4.5, 30 and 1000 GeV (the later energy is for comparison with [5]) electrons and 28.5 GeV bremsstrahlung beam [2] are shown in Fig. 1.

As in the case of shower curves one can approximate the excess results with the help of Gamma distributions

$$f(t) = b^a t^{a-1} exp(-bt)/\Gamma(a), \tag{1}$$

where a and b are fitting parameters given in Fig. 1, and $\Gamma(x)$ is the Gamma function. We do not show the results concerning the increase of the longitudinal and transversal beam sizes because even at lower energies [1] these sizes are small and give no essential contribution at relatively large thicknesses.

ChR Like CRR Produced in Thick Radiators

For simplicity we shall assume that the thick radiator has no absorption, and besides the above condition $L_{rad} \gg \lambda$ the radiator thickness is so large that very small quantity of charged particles come out from the radiator. Using the results and methods of the work [3], for a Gamma distribution form charge excess (1) one can derive the following frequency-angular distribution for the ChR type CRR:

$$\frac{d^2W(\nu,\theta)}{d\nu d\Omega} = \alpha(\frac{\nu}{\nu_0})^2 N_0^2 \hbar\sqrt{\varepsilon}\beta^2 \frac{sin^3\theta}{\left[1+(\frac{\nu}{\nu_0})^2(1-\beta\sqrt{\varepsilon}cos\theta)^2\right]^a}, \tag{2}$$

where $\nu_0 = cb/2\pi X_0(cm)$, $\beta = v/c$, and N_0 is a normalization constant determined by the charge excess distributions.

The expression (2) can be integrated over θ from 0 up to $\pi/2$ giving a lengthy expression for the frequency distribution, which for $\nu \gg \nu_0$ and Cherenkov condition $\beta\sqrt{\varepsilon} \geq 1$ fulfillment gives:

$$\frac{dW(\nu)}{d\nu} = \alpha N_0^2 \frac{2\pi\hbar\nu}{v}\left[1-\frac{1}{\beta^2\varepsilon}\right] L_{eff}, \tag{3}$$

Note that without the effective length $L_{eff} = 5v/28\pi\nu_0$ the CRR intensity is given by the usual formula of Cherenkov intensity per cm.

For 30 GeV electrons and 28.5 GeV bremsstrahlung bunches (see Fig. 1) $N_0 =$ 84.54 and 12.57; $\nu_0 = 1.53 \cdot 10^8 s^{-1}$ and $1.43 \cdot 10^8 s^{-1}$, respectively. For $E_0 = 30$GeV electrons the angular distributions of CRR at various ν, the spectral distribution of CRR under the Cherenkov angle and the spectral distribution of CRR integrated over θ from 0 up to $\pi/2$ are shown in Fig.s 2 a), b), and c), respectively. Note that the total energy of CRR from single 30GeV electron and 28.5GeV equivalent quanta from single electron emitted under angles from 0 up to $\pi/2$ in the wave length interval $\lambda = 10 \div 20$cm is $1.9 \cdot 10^{-23}$J and $1.19 \cdot 10^{-25}$J, respectively.

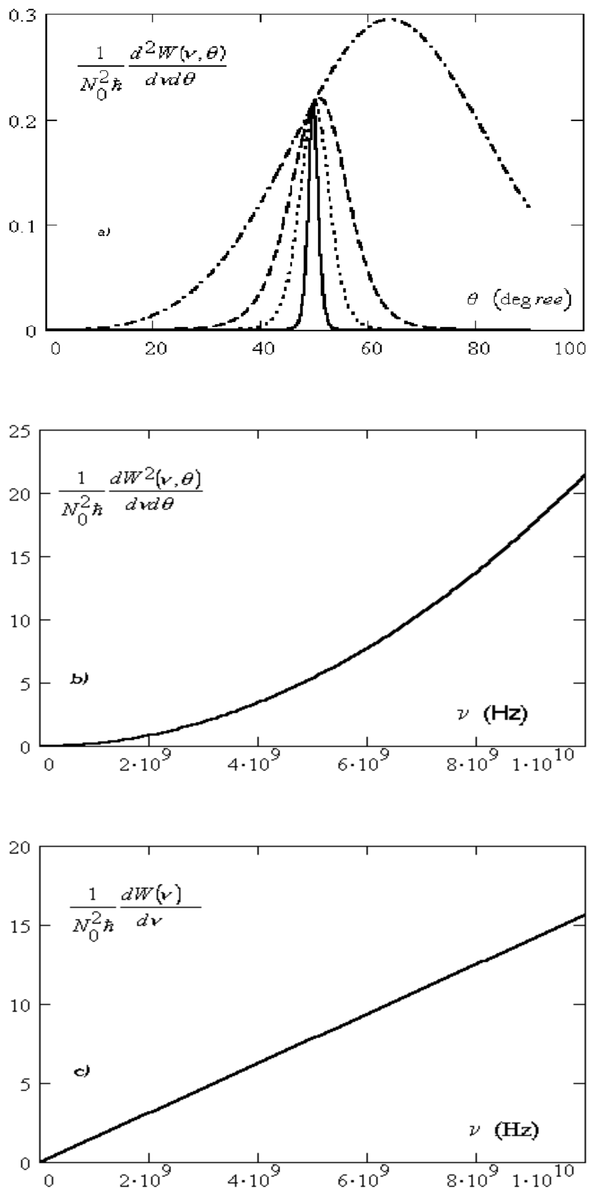

FIGURE 2. Angular distributions for $a = 4$. Solid, dotted, dashed and dash-dotted curves are for $\nu = 3 \cdot 10^9(\times 1/9)$, $1 \cdot 10^9(\times 1)$, $5 \cdot 10^8(\times 4)$, and $1 \cdot 10^8(\times 100)$, respectively. -a); Differential spectral distribution for $\theta = \theta_{Ch}$. -b); Spectral distribution integrated over all the angles. -c).

TR Produced at the Boundaries of Thick Radiators

Let N_b electron bunches, each containing N_e electrons with distances between the bunches L_b much larger than the length l_b of the bunches, $L_b \gg l_b$, enter a thick radiator without absorption from vacuum. Then the spectral distribution of the forward TR integrated over θ is given by:

$$\frac{dW_{N_e}(\lambda)}{d\lambda} = \frac{dW_1(\lambda)}{d\lambda} F_f(N_b, L_b, \lambda) F_b(N_b, L_b, \lambda) F_L(z, L_f), \tag{4}$$

where $dW_1(\lambda)/d\lambda$ is the spectral distribution of usual TR in wave zone;

$$F_f(N_b, L_b, \lambda) = N_e[1 + N_e f(\lambda)] \tag{5}$$

is a factor taking into account the bunch form factor $f(\lambda)$, equal to N_e^2 for $\lambda \gg l_b$;

$$F_b(N_b, L_b, \lambda) = \left[\frac{sin(\pi L_b N_b/\lambda)}{sin(\pi L_b/\lambda)}\right]^2 \tag{6}$$

is a factor taking into account the interference between the radiation from various bunches. It is equal to N_b^2 if the detector resolution is good, $\Delta\lambda/\lambda \ll 1$, and is equal to N_b if $\Delta\lambda/\lambda \gg 1$;

$F_L(z, L_f)$ is a suppression factor depending on the distance z between the interface and the detector and on the formation length L_f. This factor is of the order of 1 in the case of thick radiators when $z \geq L_f \sim \lambda$, and will be considered in next section when $L_f \simeq \lambda\gamma^2$.

The calculated (not shown) angular dependences of TR differential and integral intensities emitted in the wave length interval $10 \div 20$cm by a single E = 15MeV and 30GeV electrons are similar to those given in [1] and give total energy of TR from single 30 GeV electron emitted in the angular interval $\theta = 0 \div \theta_{Ch}$ equal to 10^{-26} J. Since this TR intensity is less than the those corresponding to Cherenkov like radiation of 30 GeV electrons and is emitted under very small angles not available for the antenna, the SLAC experiment could be carried out with pure electron bunches having much larger CRR yield.

CRR PRODUCED IN THIN RADIATORS

TR Produced in Thin Radiators by Single Particles

In this case TR dominates since very small ChR comes out. For $\mid \varepsilon - 1 \mid \geq 1$ TR do not depend on ε, i.e. the TR yield is the same for metallic and dielelectric radiators [15]. The calculated differential and integral angular distributions of TR produced by a single 15MeV and 30GeV electron coming out from the metallic radiators into vacuum are slightly higher than for thick radiators considered above.

The Diffraction-Like Factor F_b

Fig. 3 shows the spectral distribution of TR taking into account the factor F_b for the beam parameters of the YerPhI 50 MeV injector in the wavelength region $\lambda \sim L_b$ ($N_e = 4.37 \cdot 10^7$, $N_b = 2860$ and $L_b = 0.105m$). In this case as expected the

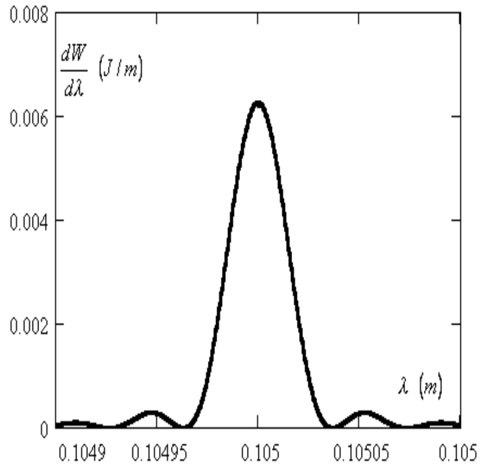

FIGURE 3. Spectral distribution of coherent TR of Yerevan Synchrotron injection beam.

coherence factor $F_f = N_e^2$, and it is assumed that the measurements are carried out in far wave zone when the formation zone suppression factor $F_L = 1$. As it is seen from Fig. 3 as a consequence of the interference of TR from N_b bunches the continuous spectrum of TR becomes a monochromatic spectrum with $\Delta\lambda$ / $\lambda \sim 1/N_b$. If the detector resolution is low, as in the case of the experiment [1], $N_b \leq F_b \leq N_b^2$. The calculations show that for the given values of the beam parameters $F_b \sim 5N_b$. Total TR energy is about $2 \cdot 10^{-7}$J which gives pulses with amplitudes about $3 \cdot 10^{-3}$.

Formation Length Suppression Factor F_L

The experimental results [1] show that at near field distances about 1m for $E_e =$ 15MeV, $\lambda \sim 15$cm, the measured value $F_L \sim 1/30$. With a purpose to understand this fact and trying to construct a quantitative theory [4] we have calculated and studied the z-dependence of the flux of the Pointing vector which consists of three components due to the radiation field, particle charge, and their interference. For the simplest case of passing an interface between metal and vacuum one obtains:

$$\frac{dW^2(\lambda,\theta)}{d\omega d\theta} = \frac{2e^2}{\pi\lambda^2}\frac{sin^3\theta}{(1-\beta^2cos^2\theta)^2}\left[1+cos\theta-(1-\beta cos\theta)cos[\frac{2\pi z}{\lambda}(1-\beta cos\theta)]\right], \tag{7}$$

where the first, second and third terms are due to the radiation field, charge field and their interference, and only the last term depends on the distance z from the interface.

In all publications TR is investigated in the wave zone $z \gg L_f$ when after averaging in a small λ interval the last term oscillates and vanishes. We have

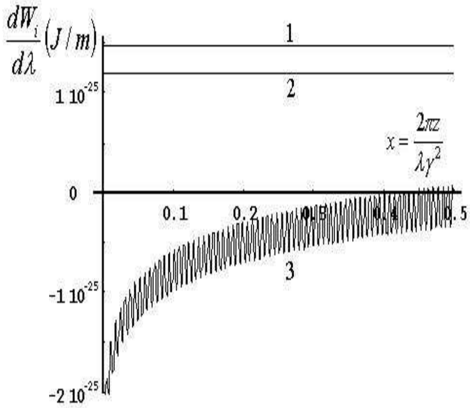

FIGURE 4. The dependence of $dW_{rad}/d\lambda$, $dW_{ch}/d\lambda$, and $dW_{int}/d\lambda$ (curves i = 1, 2, and 3, respectively) on $x = 2\pi z/\lambda\gamma^2$ for $\gamma = 30$ and $\lambda = 0.15m$.

studied the z-dependence of these 3 components. After integrating (7) over all the angles we obtain:

$$\frac{dW_{rad}(\lambda)}{d\lambda} = \frac{4\alpha(\hbar c)}{\lambda^2}[ln\gamma + ln2 - 1/2], \tag{8}$$

$$\frac{dW_{Charge}(\lambda)}{d\lambda} = \frac{4\alpha(\hbar c)}{\lambda^2}[ln\gamma - 1/2], \tag{9}$$

$$\frac{dW_{int}(\lambda)}{d\lambda} = -\frac{4\alpha(\hbar c)}{\lambda^2}I_0, \tag{10}$$

where

$$I_0 \simeq ci(\frac{2\pi z}{\lambda}) - ci(\frac{2piz}{\lambda}\frac{1}{2\gamma^2}) - sin^2\pi z/\lambda. \tag{11}$$

Fig. 4 shows the dependence of each of three components on $x = 2\pi z/\lambda\gamma^2$.

To comprehend the z-dependence we propose to define the TR suppression factor in the form:

$$F_L(z,\lambda) = \left[\frac{dW_{rad}(\lambda)}{d\lambda} - k\frac{dW_{int}(\lambda)}{d\lambda}\right] / \left[\frac{dW_{rad}(\lambda)}{d\lambda}\right], \tag{12}$$

where the coefficient $0 < k < 1$ shows the fraction of $dW_{rad}(\lambda)/d\lambda$ which goes to $dW_{int}(\lambda)/d\lambda$. Fig. 5 shows the dependence of $F_L(z,\lambda)$ vs $x = 2\pi z/\lambda\gamma^2$. The observed [1] TR intensity suppression $F_L \sim 1/30$ can be in part be conditioned also by the effect considered in [16,17], according to which TR should be suppressed if the transversal sizes of the radiator is less than the transversal sizes of TR source $\lambda\gamma$.

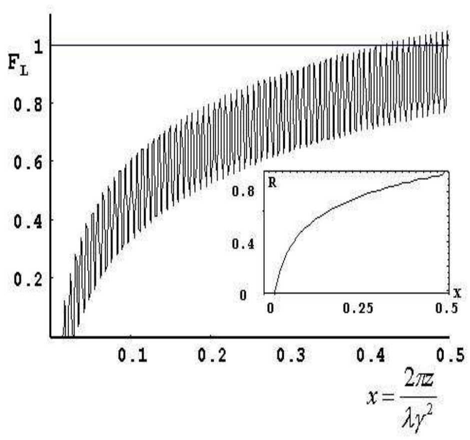

FIGURE 5. The dependence of the suppression factor $F_L(z, \lambda)$ upon $x = 2\pi z/\lambda\gamma^2$ for $\gamma = 30$ and $k = 1$. The dependence of R upon x when $dW_{rad}/d\lambda$ and $dW_{int}/d\lambda$ are integrated over λ from 0.1 to 0.2m is shown inserted.

CONCLUSIONS

The following accelerator experiments can be carried out with

a) 15 - 50 MeV electron beams;

Since at such low energies the number and range of the particles in the region close to the maximum of the shower curves of the charge excess is small the TR intensity dominates, and it is impossible to study ChR. It is of interest to study theoretically and experimentally the dependence of the TR intensity on the distance from the radiator interface and on the radiator transversal sizes.

Since for the single electron bunches of the experiment [1] $W^{SLAC} \simeq N_e^2(dW/d\lambda)\Delta\lambda$, while for bunch consequence of the YerPhI beam $W^{YerPhI} \simeq N_e^2(dW/d\lambda)F_b$, for the above given beam parameters and parameters of [1] one obtains $W^{YerPhI}/W^{SLAC} \simeq 2 \cdot 10^{-2}$ or taking into account that $W \simeq 4V^2/R$ where V is the measured voltage and R is the cable resistance one expects $V^{YerPhI}/V^{SLAC} \simeq 0.15$.

b) 4.5 GeV electron and photon beams fast extracted from YerPhI Synchrotron;

In this case the great number and large range of the particles near the maximum of the shower curve of the charge excess as well as the large difference between the TR and ChR radiation angles allows to study separately TR and ChR. A $3 \div 5$ radiation length thick converter can be used to enhance TR.

However, since the bunch structure of the 1-2 μs electron and photon beams fast extracted from Yerevan Synchrotron consists of about 0.7ns 400 bunches CRR measurements must be carried out at longer wave lengths, longer than 1m;

c) 30 GeV electron and bremsstrahlung beams at SLAC.

In this case also TR and ChR can be experimentally investigated separately because of the above discussed reasons. Moreover, together to the single bunch regime pulses with consequences of bunches can be used to study CRR at other wave length.

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