Signal Characteristics from Electromagnetic Cascades in Ice

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Abstract. We investigate the development of electromagnetic cascades in ice using a GEANT Monte Carlo simulation. We examine the Cherenkov pulse that is generated by the charge excess that develops and propagates with the shower. This study is important for the RICE experiment at the South Pole, as well as any test beam experiment which seeks to measure coherent Cherenkov radiation from an electromagnetic shower.

I INTRODUCTION

Ultra high energy neutrinos can travel a long distance without scattering. With these sources of particles, we can probe the physics of standard model and beyond at energies un-obtainable at current accelerators. An ultra high energy neutrino can interact via a charge current interaction giving most of its energy to the secondary electron, which can then initiate an electromagnetic cascade or shower. Askaryan [1] predicted a negative charge imbalance in the cascade which gives rise to coherent Cherenkov radiation at radio frequencies. The predicted flux of ultra high energy neutrinos is small and model dependent [2–4]. An experiment to detect ultra high energy neutrinos using radio antennas requires a large volume because of the small flux. A dense, radio-transparent target is needed for the small shower size needed for coherence and small signal attenuaton. Antarctic ice is suitable for this purpose. A detailed analysis of such an experiment was done [5] which concluded that a modest array of antennas can detect many events per year. Radio Ice Cherenkov Experiment (RICE) at the south pole [6] is a prototype designed to detect neutrinos with energy $\geq$ PeV using this method. A reliable Monte Carlo simulation tool is needed to study the shower development, Cherenkov radiation, detector, and data acquisiton system. One can also test the idea of coherent Cherenkov emission at accelerator facilities by dumping bunches of electrons or photons in a dense target like sand or salt or any other suitable medium [7]. Zas, Halzen and Stanev (ZHS) [8] developed a Monte Carlo simulation to study electromagnetic showers and

Cherenkov emission in ice. Buniy and Ralston [9] have also developed a method to estimate Cherenkov signal by parametrizing the cascades.

We have developed a GEANT Monte Carlo simulation primarily to study the coherent Cherenkov emmission in ice but it can easily be modified for other similar studies with different materials. GEANT [10] is a well known and widely used detector simulation package in particle physics. It allows access to all details of the simulation such as controls of various processes, definition of target and detector media, and a complete history of all events simulated. It can be used to simulate all dominant processes in 10 keV – 10 TeV energy range. Cross sections of electromagnetic processes are reproduced in GEANT within a few percent up to a hundred GeV. We use GEANT to simulate electromagnetic cascades inside materials from which we extract track information. We take this track information and determine the resulting radio pulse using standard electrodynamics calculations. We also calculate other shower parameters using the track and energy information from GEANT.

II SHOWER DESCRIPTION

When a high energy electron (e^-) or photon (γ) hits a material target, an electromagnetic cascade is created inside the material. *Bremsstrahlung* and *pair production* are the dominant high energy processes at the beginning of shower development. Due to bremsstrahlung, an e^- loses $1/e$ of its energy on average over a distance X_0, the *radiation length*. The secondary γ can then produce an e^+ e^- pair. The number of particles thus grows exponentially. The e^- and e^+ lose energy due to *ionization* as they travel inside the material. After reaching a *critical energy*(E_c), when the energy loss due to bremsstrahlung is equal the energy loss due to ionization, e^-, e^+ lose their energy mostly due to ionization and the cascade eventually stops. A rough estimate of the critical energy is $E_c \approx 605/Z\ MeV$ which can be obtained by equating radiation and ionization loss formulae. Z is the atomic number of the medium.

All models of an electromagnetic cascade, including a simplified model developed by Heitler [11], and a more realistic model developed by Carlson and Oppenheimer [12], show basic features which include linear scaling of the track lengths and logarithmic scaling of the maximum position with primary energy.

The cascade is concentrated near an axis along the direction of the primary. *Multiple Coulomb scattering* is responsible for transverse spread of the cascade. A quantity called *Moliere radius* (R_M) is determined by the average angular deflection per radiation length at the critical energy (E_c) and is used to estimate the transverse spread. One can also look at the fraction of energy that escapes transverse to the shower axis [13,14] to get a good estimate of R_M. About 90% of the primary energy is contained inside a tube of radius R_M along the shower axis. A relationship between the Moliere radius, the critical energy and the radiation length is $R_M = X_0 E_s/E_c$ where $E_s \approx 21.2\ MeV$ is the scale energy.

Low energy processes like *Compton*, *Moller* and *Bhabha* scatterings and *positron annihilation* build up a net charge (more e^- than e^+) in the cascade, as atomic electrons in the target medium are swept up into the forward moving shower.

III ELECTROMAGNETIC PULSE

High energy e^- and e^+ in the cascade can travel faster than the speed of light in the medium and give rise to Cherenkov radiation. The electric field due to a single charge moving uniformly from position $\vec{r}_1$ to $\vec{r}_2 = \vec{r}_1 + \vec{v}\delta t$ in *Fraunhoffer* limit, is given by

$$R\vec{E}(\omega) = \frac{\mu_r}{\sqrt{2\pi}} \left(\frac{e}{c^2}\right) e^{i\omega \frac{R}{c}} e^{i\omega(t_1 - n\vec{\beta}\cdot\vec{r}_1)} \vec{v}_T \frac{(e^{i\omega\delta t(1-\hat{n}\cdot\vec{\beta}n)} - 1)}{1 - \hat{n}\cdot\vec{\beta}n} \tag{1}$$

where μ_r is the relative permeability and n is the refractive index of the medium. The condition $1 - \hat{n}\cdot\vec{\beta}n = 0$ defines the Cherenkov angle θ_c as $\cos\theta_c = 1/n\beta$. At or very close to the Cherenkov angle or at low frequency, the equation (1) can be reduced to the form

$$R\vec{E}(\omega) = \frac{\mu_r i\omega}{\sqrt{2\pi}} \left(\frac{e}{c^2}\right) e^{i\omega \frac{R}{c}} e^{i\omega(t_1 - n\vec{\beta}\cdot\vec{r}_1)} \vec{v}_T \delta t. \tag{2}$$

See Fig. 1 for a description of various quantities in equations (1, 2).

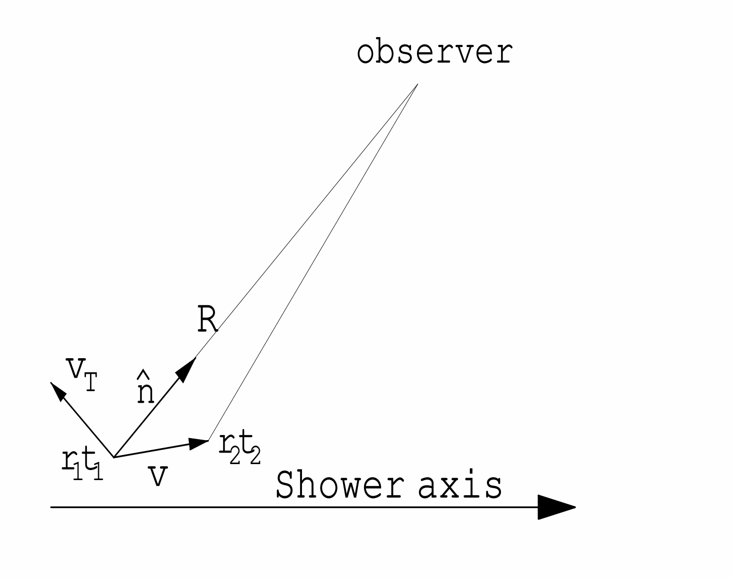

FIGURE 1. Set-up to calculate electromagnetic pulse from a single track.

IV MONTE CARLO RESULTS

Since on average, the electrons lose $1/e$ of their energy in each radiation length due to bremsstrahlung, one can fit an exponential to the energy loss data generated by the Monte Carlo and find a value for the radiation length (X_0). This will serve as an internal consistency check to ensure that we are tracking all the particles along with their energies. We generated 500 showers of 50 GeV electrons and recorded the energy loss due to bremsstrahlung. A fit to this gives a radiation length of 41.5 ± 3.2 cm. Given the molecular composition, GEANT also calculates the medium's radiation length. For ice it is 38.8 cm, which is in agreement with the value we calculated.

Table I GEANT consistency checks

Parameters	Iron	Lead	Ice
X_0 (cm) (from ref [15])	1.76	0.56	39.05
R_M (cm)	2.1	1.6	13
$E_c = X_0 E_s / R_M$ (MeV)	17.77	7.42	63.7
$E_c = 605/Z$ (MeV)	23.3	7.4	83.8

To calculate the Moliere radius (R_M) we construct an imaginary cylinder centered on the shower axis (up to the physical length of the shower). We add the energy (U), of all the tracks that leave the cylinder without re-entering. By varying the radius of the cylinder we arrive at R_M which is the radius of the cylinder when the fraction U/E is equal to 0.1, E being the initial energy of the cascade. We have checked R_M for lead, iron and ice (see table I). One can also calculate the critical energy (E_c) using two different formulae quoted earlier. Shower depth (t) is measured in units of radiation length (X_0) as $t = x/X_0$. We have simulated 30 GeV electron-induced cascades in iron. The longitudinal profile was obtained by adding the number of particles with total energy greater than 1.5 MeV crossing planes at every half radiation length perpendicular to the shower axis as shown in Fig. 2a. The number of particles agree reasonably with EGS4 simulation of the same shower [15]. The Greisen-Rossi distribution for total number of particles (electrons and positrons) is shown by the dashed line. A least squares fit to the GEANT data with Greisen-Rossi distribution gives about 90% confidence level.

We used the same method to study the longitudinal profile of showers in ice. Simulation shows the correct scaling behavior of the number of particles with the initial energy of the shower. The position of the shower maximum also shows the correct logarithmic scaling with the initial energy. The charge excess (ΔQ) is defined as $\Delta Q = \frac{N(e)-N(p)}{N(e)+N(p)}$ where $N(e)$ and $N(p)$ are the number of electrons and positrons respectively as functions of shower depth. According to our simulations, the net charge imbalance is $15\% - 18\%$ at the shower max. Note that there is no direct experimental data on this.

A comparison of 100 GeV shower (averaged over 20 showers) with 611 keV threshold from GEANT to the same from ZHS Monte Carlo (Fig. 2b) shows about a $25\% - 35\%$ discrepancy for the total number of particles at the shower max. The discrepancy between the two simulations remains the same at higher energies.

In Fig. 3, we show the charge excess in a 500 GeV shower from GEANT with kinetic energy threshold 1.5 MeV. This is an average of 20 showers and the charge excess is broken down to different energy bins. It shows that most of the contributions to charge excess come from the energy range 1.5 to 30 MeV. The Cherenkov pulse from a cascade is proportional to the total track length (energy deposition) of the cascade. Our results show the correct scaling behavior of the track lengths with the initial energy (E_o). If we consider all processes to be elastic except ionization, then an upper bound for the total track length is $L = E_o / \left(\frac{dE}{dx}\right)_{ion}^{avg}$ where $(\frac{dE}{dx})_{ion}^{avg}$ is the average ionization energy loss per unit length. To determine $(\frac{dE}{dx})$ we generated 500 of 1 GeV tracks and kept a record of the rate at which energy was lost due to ionization. Fig.4a shows the average $(\frac{dE}{dx})$ for the 500 tracks. It can be seen that the average value matches theoretical Bethe-Bloch result reasonably well. The average ionization loss in the relativistic rise region is approximately 1.9 MeV/cm.

V PULSE CALCULATION

To calculate the electric pulse from the cascade, we added up the contributions in Eq. (1) from all charged tracks. There are two assumptions we made to evaluate pulse described by Eq. (1) and Eq. (2) from GEANT track information. First, we

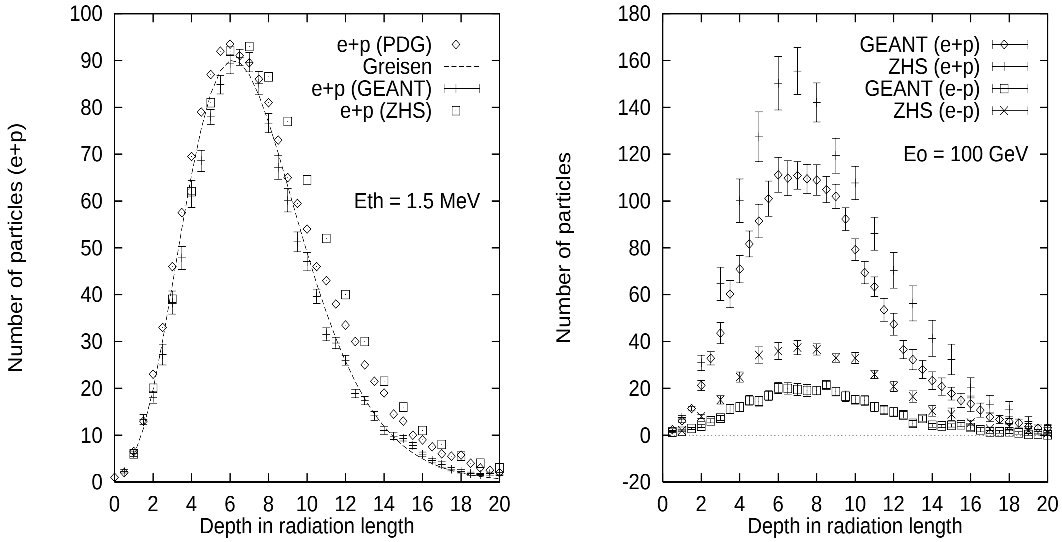

FIGURE 2. (a) The longitudinal profile of a 30 GeV cascade in iron using GEANT (average of 30 showers) and comparison with the same produced by EGS4 (from particle data book) and by ZHS Monte Carlos. (b) Comparisons between the profiles of a 100 GeV cascade (averaged over 20 showers each) from GEANT and from ZHS Monte Carlos. The total energy threshold is 0.611 MeV in both cases.

assume an azimuthal symmetry about the shower axis, which allows us to evaluate pulse equations in 2-dimensions. This is a good approximation as long as we have many tracks in the shower. A typical 500 GeV shower has thousands of tracks and the number goes up as we increase the energy. We checked this approximation on a shower-by-shower basis and did not find any noticeable difference between the pulses evaluated in the $x-z$ plane and in the $y-z$ plane (z - is the shower axis). Second, we evaluate the times t_1 and δt in (1) and (2) assuming that the particles travel at the speed of light. This is a good approximation if the frequency is not too high as can be seen from (1). In Fig. 4b, we calculate pulses[1] from 1 TeV and 500 GeV showers at 1 GHz and 300 MHz frequencies. The pulse height at the Cherenkov peak ($\approx 55.8^o$) scales with the energy of the shower, and the Gaussian half width is inversely proportional to the frequency which is analogous to a slit diffraction pattern.

VI CONCLUSION

We find distinctive, coherent, Cherenkov radio frequency emission from simulated electromagnetic cascades in ice with energies in the 100 GeV to 1 TeV range.

[1] we use different normalization here for the field, which requiring that one multiply (1) and (2) by $2\sqrt{2\pi}$

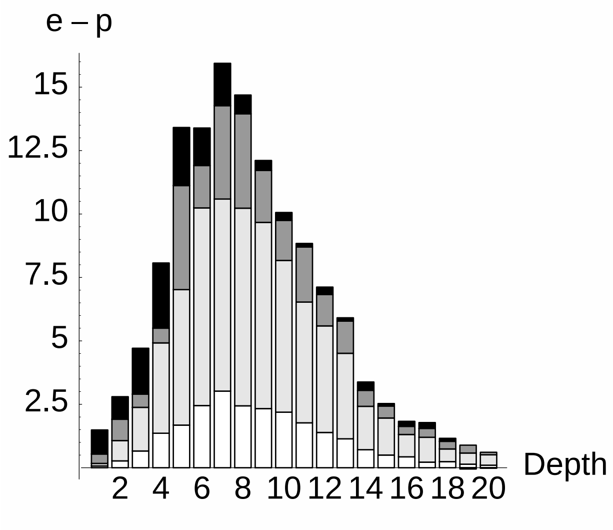

FIGURE 3. Excess charge plot for a 500 GeV cascade (averaged over 20 showers) in ice using GEANT. The excess charge number is shown in energy bins (from bottom to top) 1.5-5 MeV, 5-30 MeV, 30-100 MeV and above 100 MeV.

The multi-purpose, detector simulation package GEANT provides a suitable modelleing of electromagnetic shower details for this purpose, with the flexibility to include hadronic cascades in future studies. This work also serves as an independent treatment of the problem, which has been studied previously [8]. The GEANT generated showers are qualitatively in agreement with those in [8]. The showers differ in several details; however our far-field calculation of the pulse from a shower gives the same result up to a GHz frequency as that of [8] when both are applied to the same shower simulation. This has been checked explicitly by calculating the field using our field code and the track informtions from ZHS code with the direct field output from ZHS code for the same shower. The differences between a GEANT and a ZHS shower of initial energy 100 GeV is summarized in Table II. The origin of differences between the two Monte Carlos is under study.

Our direct calculation of Moliere radius, critical energy and shower population as a function of shower depth using GEANT all agree within a few percent with measurements or with published EGS4 simulations for iron and lead [15]. The extension to ice is straightforward and we expect the same level of reliability here.

Our results agree with the theoretically expected [16] linear scaling of number of particles at shower maximum and logarithimic scaling of depth at shower maximum with energy. These are not straightforward results, since the theoretical predictions are based on inclusion of bremsstrahlung and pair production only, while the physics at the shower maximum is not dominated by these processes. It includes important contributions from Compton, Bhabha and Moller processes as well, which give rise

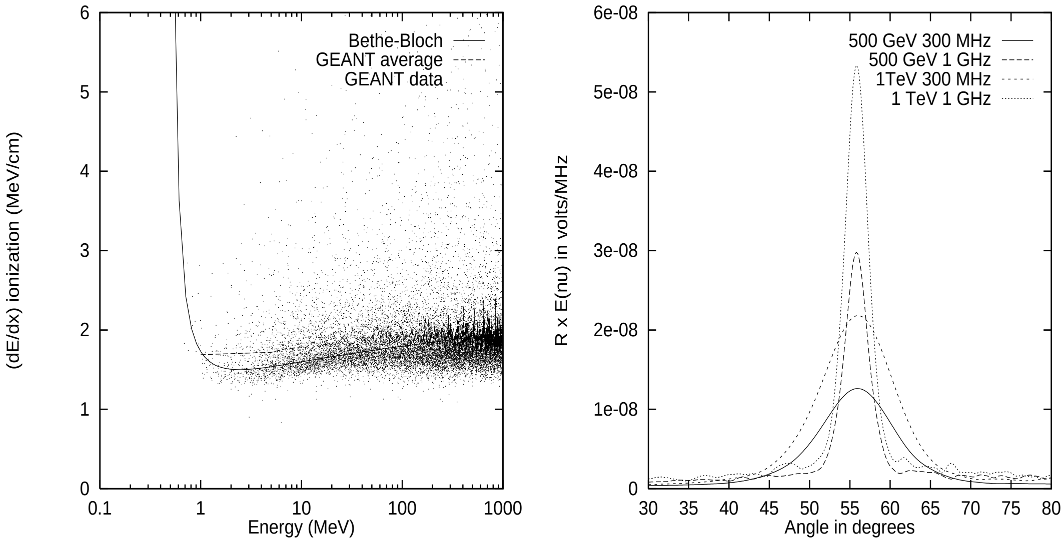

FIGURE 4. (a) $(dE/dx)_{ionization}$ calculated from GEANT output. The average is over 500 electron tracks with 0.1 MeV kinetic energy threshold. We also show theoretical Bethe-Bloch curve. (b) Electromagnetic pulses from cascades in ice generated by GEANT. Pulse height scales linearly and pulse width scales inversely with primary energy. Fraunhoffer limit has been taken to calculate field.

to charge imbalance and, consequently, coherent radio Cherenkov field emission.

Table II Comparisons between GEANT and ZHS Monte Carlos
(averaged over 20 showers each)

Quantity	GEANT	ZHS
Primary Energy (GeV)	100	100
Total Energy Threshold (MeV)	0.611	0.611
Total absolute track length ($meter$)	398.8 ± 4.7	642
Total projected ($e+p$) track length ($meter$) (*sum of electron and positron track lengths projected along the shower axis*)	374.4 ± 4.3	518.7
Total projected ($e-p$) track length ($meter$) (*difference of electron and positron track lengths projected along the shower axis*)	70.0 ± 8.4	131.2
Position of the shower max. (radiation length)	7	7
Number of particles ($e+p$) at shower max.	111 ± 26	155 ± 45
Excess electrons ($e-p$) at shower max.	20 ± 11	37 ± 14
Fractional charge excess at the shower max	$\sim 18\%$	$\sim 24\%$
Cherenkov peak at 1 GHz ($Volts/MHz$)	7.46×10^{-9}	1.08×10^{-8}

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