Signal Characteristics from Electromagnetic Cascades in Ice

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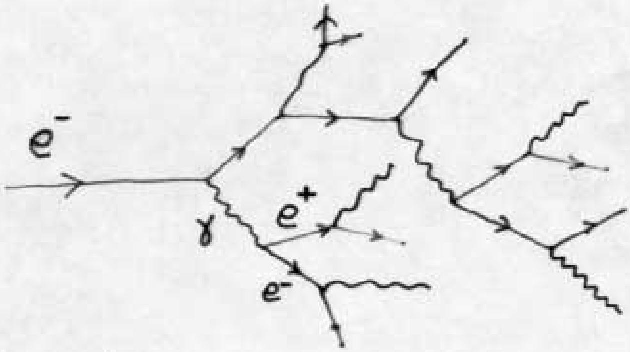
Electromagnetic Cascade

High energy e^- or γ hitting a material target, creates an electromagnetic cascade inside the material.



Features of a Cascade

- Number of particles grows exponentiallly at the beginning due to *Bremsstrahlung* and *Pair Production* processes at high energy.
- Particles lose energy mostly through *Ionization* after reaching a critical energy.
- A net charge imbalance (more e^- than e^+) is created in the cascade (*Askaryan, 1961*) due to the following processes:

  Compton scattering $(\gamma + e^-_{atom} \rightarrow \gamma + e^-)$
  Positron Annihilation $(e^+ + e^-_{atom} \rightarrow 2\gamma)$
  Bhabha scattering $(e^+ + e^-_{atom} \rightarrow e^+ + e^-)$
  Möller Scattering $(e^- + e^-_{atom} \rightarrow e^- + e^-)$

Electromagnetic Pulse

Charged particles in a cascade radiates electromagnetic wave coherently at radio frequency (*Askaryan, 1961*).

Far Field or Fraunhoffer Approximations

- Distance of observation (R) is large compared to the shower length.
- Wavelength of radiation is large compared to the shower dimensions.

Electric field at a distance R due to a single charged track in the medium of refractive index n is

$$\begin{aligned} R\vec{E}(\omega) &= \frac{\mu_r}{\sqrt{2\pi}}\left(\frac{e}{c^2}\right)e^{i\omega\frac{R}{c}}e^{i\omega(t_1-n\vec{\beta}.\vec{r}_1)}\vec{v}_T \\ &\quad\times\frac{(e^{i\omega\delta t(1-\hat{n}.\vec{\beta}n)}-1)}{1-\hat{n}.\vec{\beta}n}. \end{aligned}$$

Cherenkov condition is $1-\hat{n}.\vec{\beta}n=0$ which defines the Cherenkov angle $\theta_c=\cos^{-1}(\frac{1}{n\beta})$.
At or very near θ_c, the field is

$$R\vec{E}(\omega)=\frac{\mu_r i\omega}{\sqrt{2\pi}}\left(\frac{e}{c^2}\right)e^{i\omega\frac{R}{c}}e^{i\omega(t_1-n\vec{\beta}.\vec{r}_1)}\vec{v}_T\delta t.$$

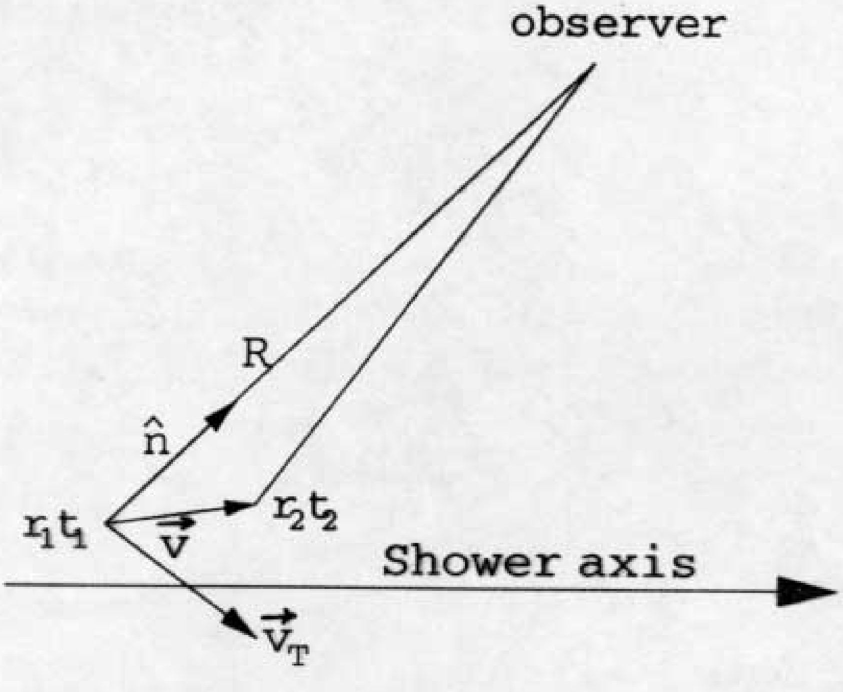


Figure 1: Set up to calculate electric field from a single track

Monte Carlo

- Need Monte Carlo to simulate the cascade.
- Calculate electromagnetic pulse from the cascade:
  - ■ on track by track basis or
  - ■ by parameterizing the cascade.
- Pioneering work has been done by *Zas, Halzen* and *Stanev* (1992).
- *Buniy* and *Ralston* (1999) have also investigated this issue by parametrization method.

We use GEANT Monte Carlo simulation tool to simulate electromagnetic cascades. We calculate electromagnetic pulse on track by track basis.

Features of GEANT *Monte Carlo*

- Well understood and well documented.
- Widely used in high energy accelerator experiments to simulate detectors.
- Better handle on Physical Processes.
- Detailed and flexible output.
- Ultimately want hadronic shower included.

Moliere Radius (R_M)

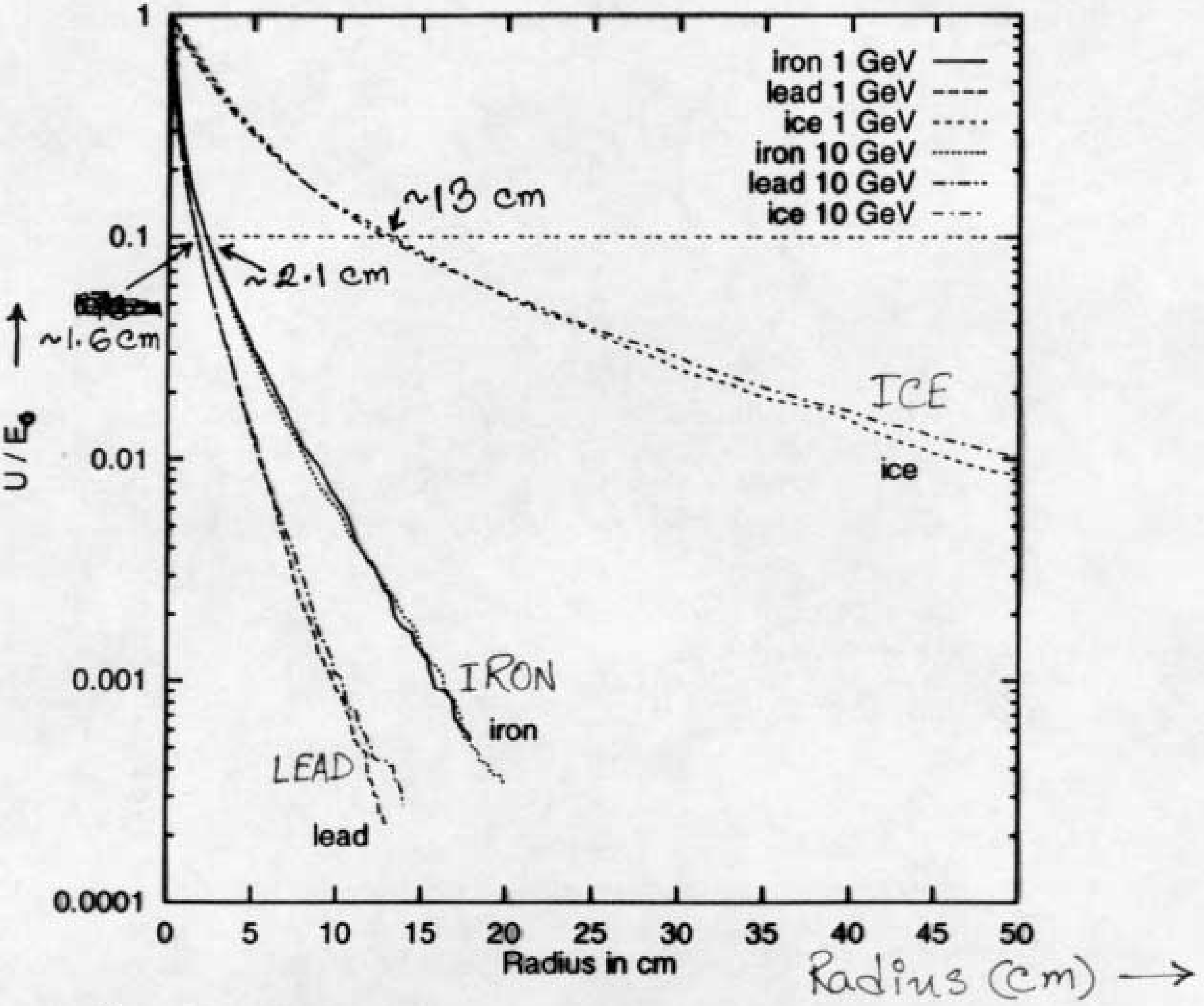


Figure 3: Moliere radius corresponds to the transverse development of the shower. When the fraction U/E (the ratio of total energy inside an imaginary tube along the showers axis to the initial energy of the shower) is 0.1 (the horizontal straight line), the corresponding radius is the Moliere radius for that material. It depends on the material and not on the energy of the shower. Here we use showers of energies 1 GeV and 10 GeV for each material.

GEANT Consistency Checks

Relationship between *Radiation Length* (X_0), *Moliere Radius* (R_M) and *Critical Energy* (E_c):

$$E_c = \frac{X_0 E_s}{R_M}\ ;\ \ E_s \cong 21.2\, MeV \ \ (scale\ energy).$$

Also, $E_c \approx 605/Z$; $Z -$ *atomic number*.

Methods

- Calculate Moliere radius for different materials using GEANT track informations.
- Calculate the *critical energy* using above formulas and compare.

Results

Parameter	Iron	Lead	Ice
$X_0\ (cm)$	1.76	0.56	39.05
$R_M\ (cm)$	2.1	1.6	13
$E_c\ (MeV)$	17.77	7.42	63.7
	(23.3)	(7.4)	

References: *Nelson et al.* (1966) and *Bathow et al.* (1970).

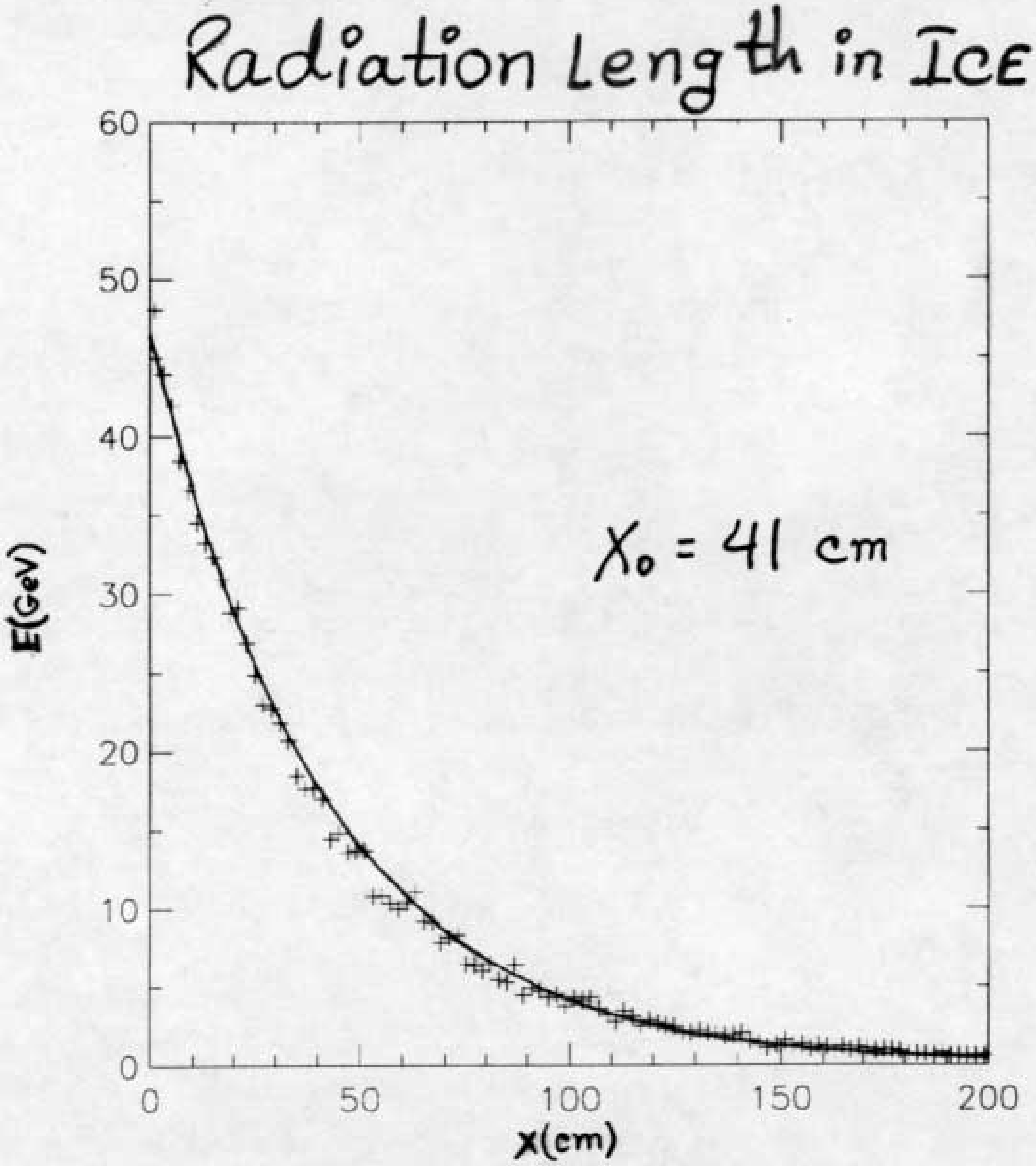
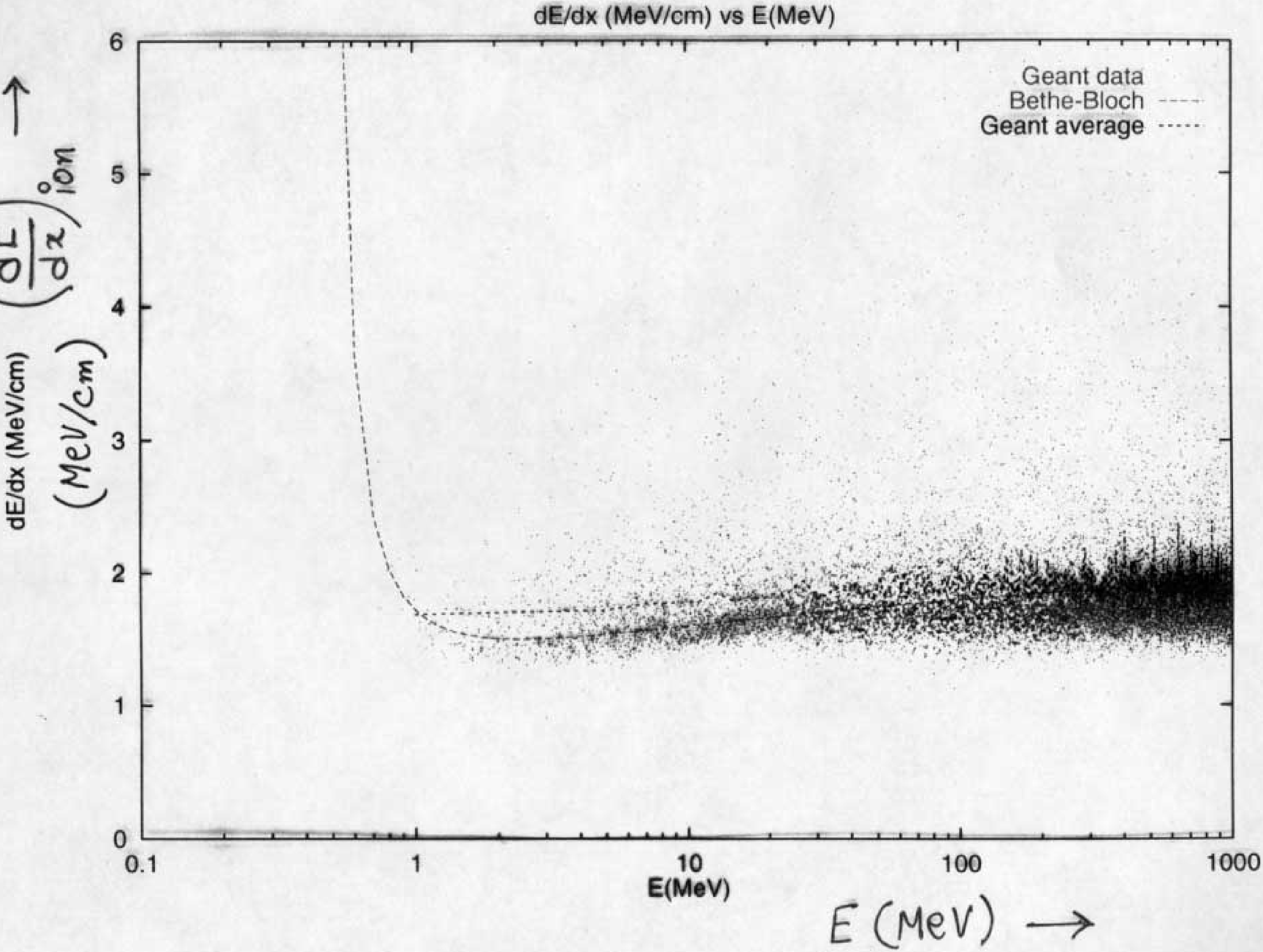
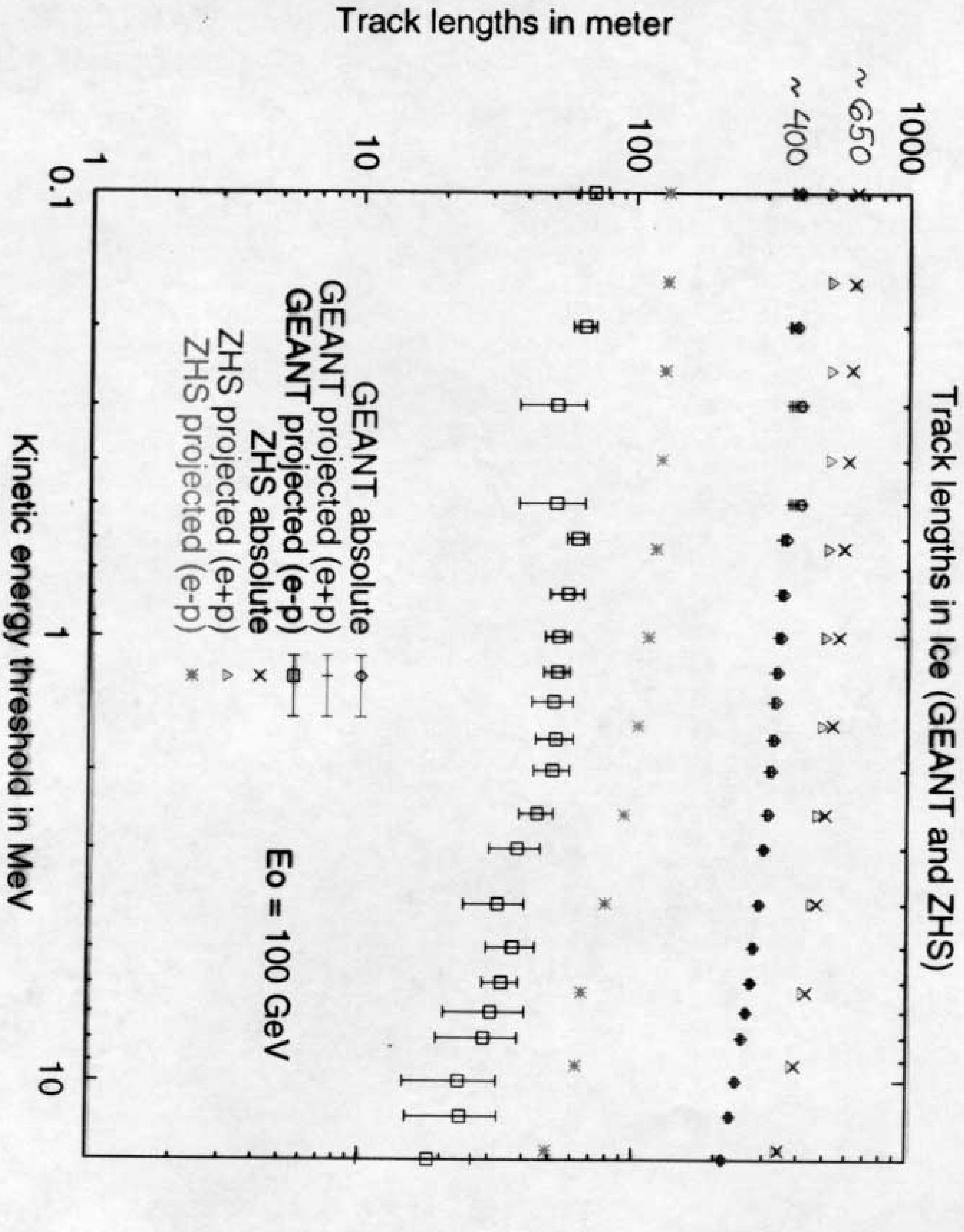
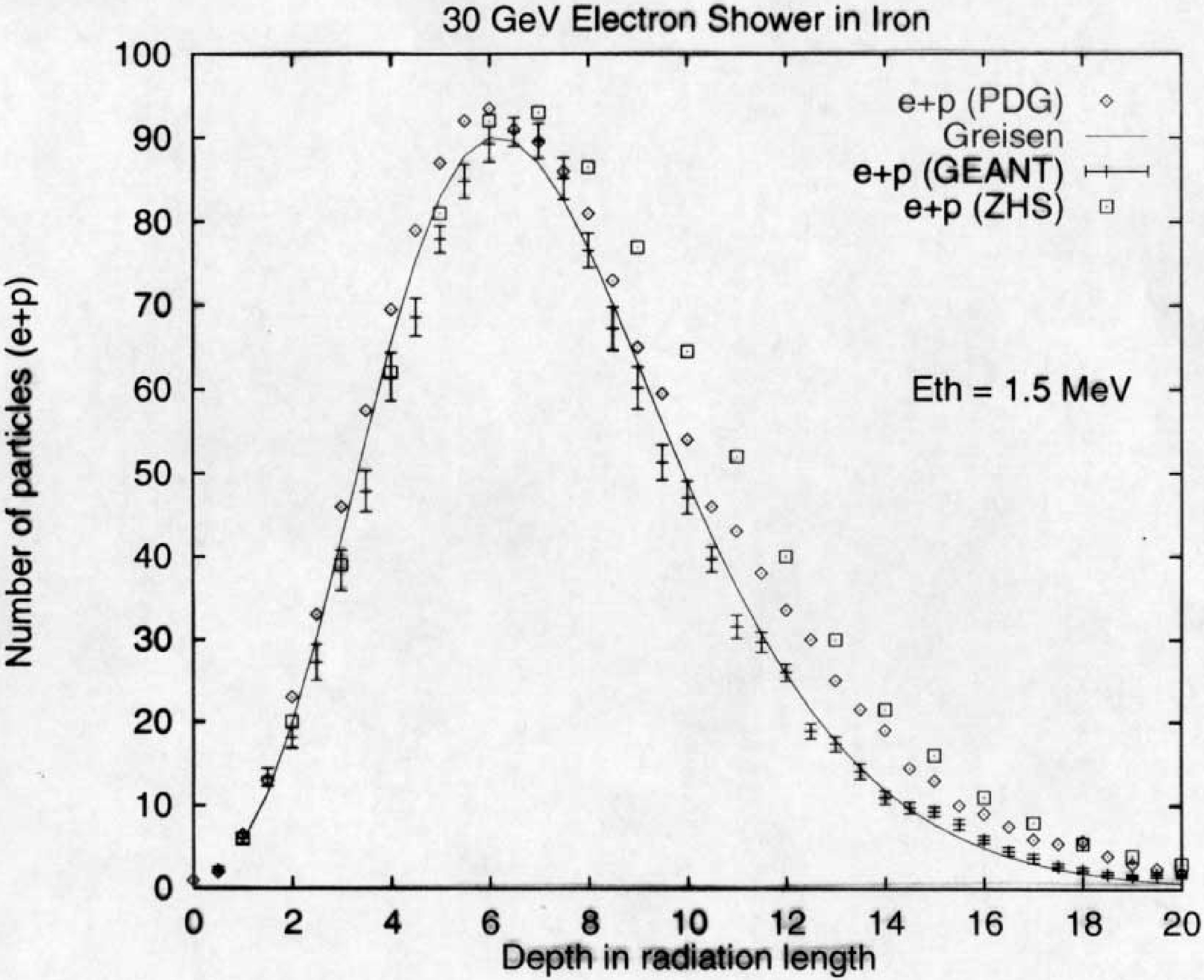
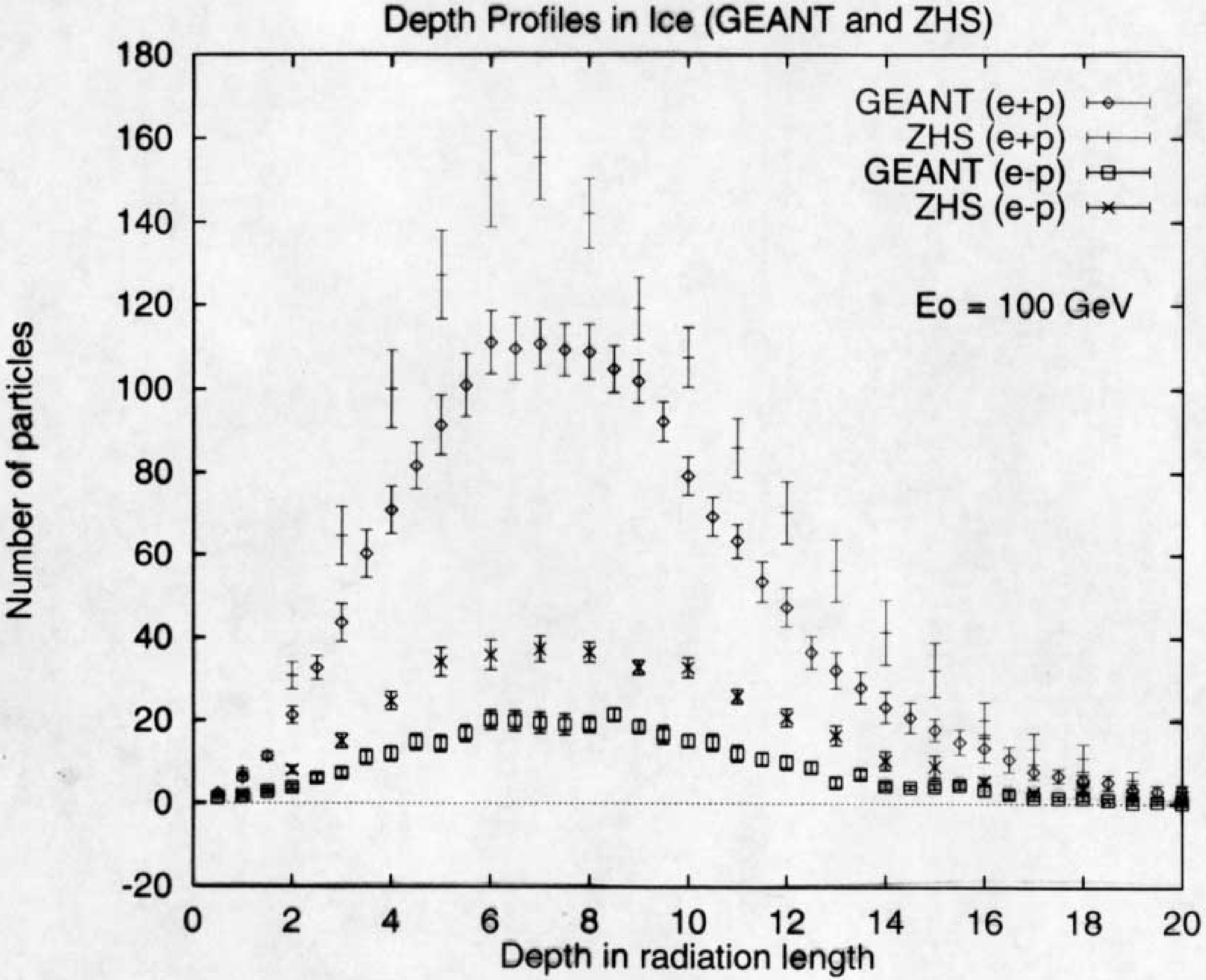


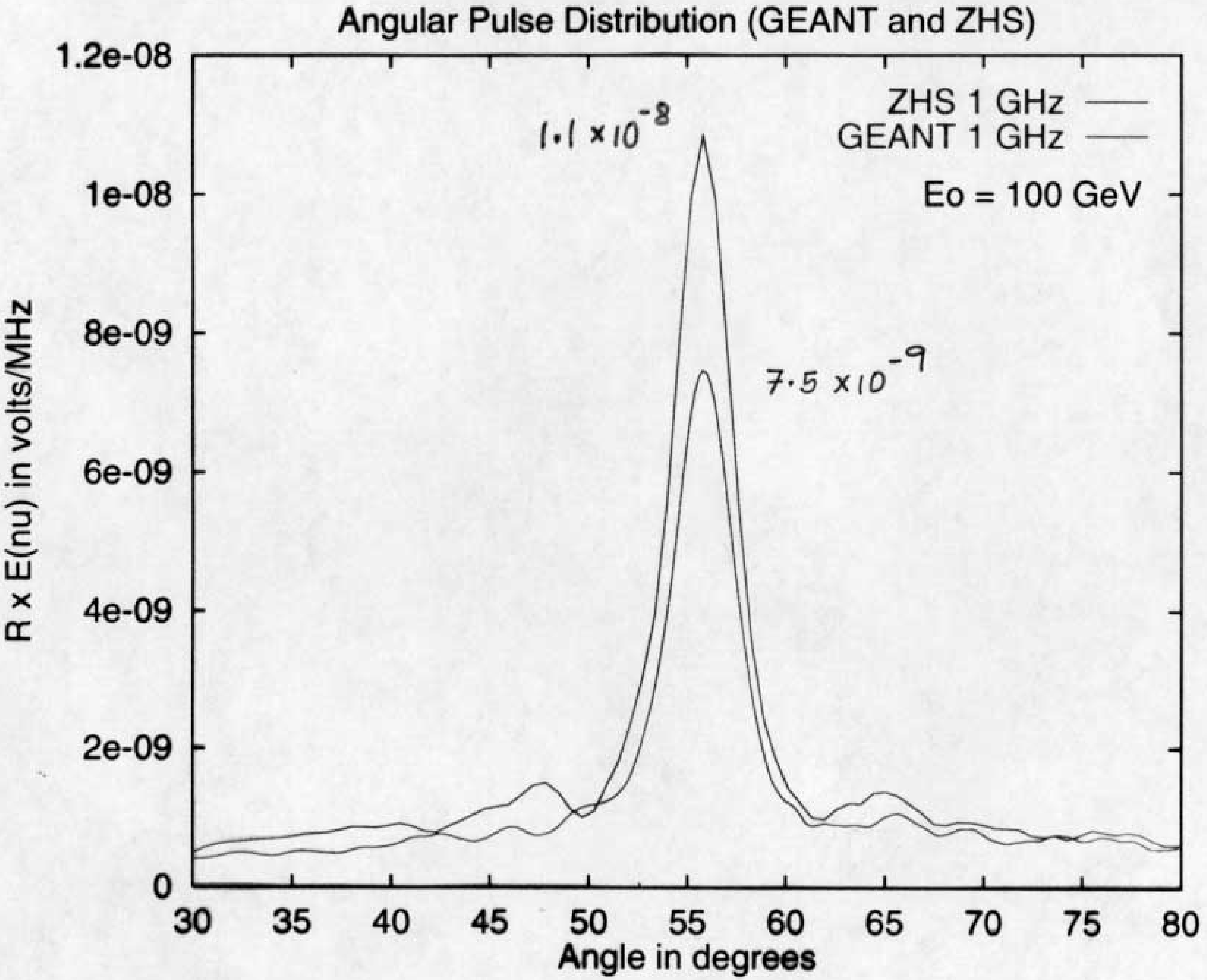
Figure 2: Average energy of 500 50 *GeV* electrons vs distance in *cm*. Crosses indicate Monte Carlo data points and the solid line is the exponential fit. This fit gives a radiation length of 41.5 ± 3.2 cm. 0.611 *MeV* kinetic energy threshold was used in all 500 cases.


dE/dx (MeV/cm) vs E(MeV)
Geant data
Bethe-Bloch
Geant average
dE/dx (MeV/cm)
$\left(\frac{dE}{dx}\right)_{ion} \rightarrow$
(MeV/cm)
0
1
2
3
4
5
6
0.1
1
10
100
1000
E(MeV)
E (MeV) →


Track lengths in Ice (GEANT and ZHS)
Track lengths in meter
1000
~ 650
~ 400
100
10
1
0.1
1
10
Kinetic energy threshold in MeV
GEANT absolute
GEANT projected (e+p)
GEANT projected (e-p)
ZHS absolute
ZHS projected (e+p)
ZHS projected (e-p)
Eo = 100 GeV


30 GeV Electron Shower in Iron
Number of particles (e+p)
Depth in radiation length
e+p (PDG) ◇
Greisen
e+p (GEANT)
e+p (ZHS) □
Eth = 1.5 MeV
0
10
20
30
40
50
60
70
80
90
100
0
2
4
6
8
10
12
14
16
18
20


Depth Profiles in Ice (GEANT and ZHS)
Number of particles
Depth in radiation length
GEANT (e+p)
ZHS (e+p)
GEANT (e-p)
ZHS (e-p)
Eo = 100 GeV
180
160
140
120
100
80
60
40
20
0
-20
0
2
4
6
8
10
12
14
16
18
20

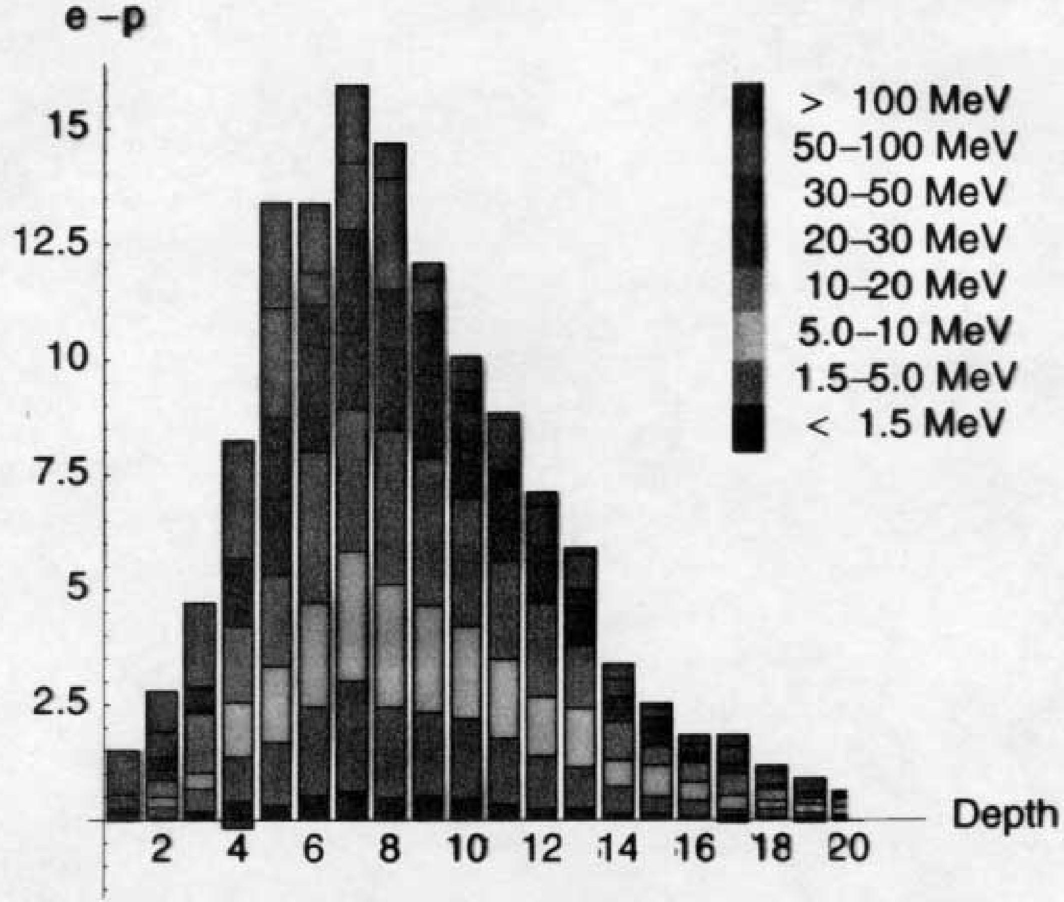

Angular Pulse Distribution (GEANT and ZHS)
R x E(nu) in volts/MHz
1.2e-08
1e-08
8e-09
6e-09
4e-09
2e-09
0
30
35
40
45
50
55
60
65
70
75
80
Angle in degrees
ZHS 1 GHz
GEANT 1 GHz
Eo = 100 GeV
1.1 x10^-8
7.5 x10^-9

Comparison Table for GEANT and ZHS

100 *GeV cascades in Ice.* $E_{th} = 0.611\ MeV$.

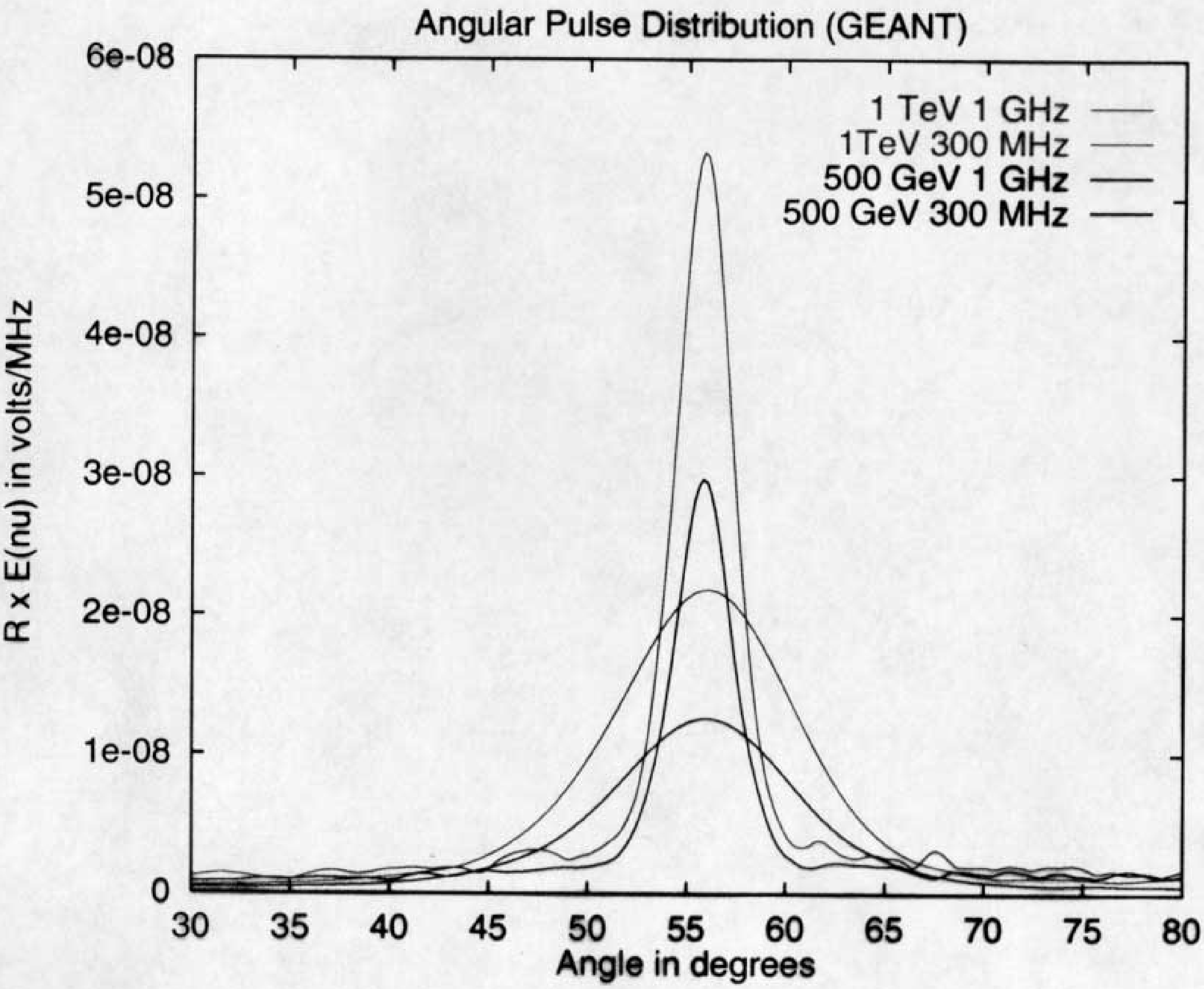
Quantity	GEANT	ZHS
Total track length (m)	400 ± 5	642
Track length ($e+p$) (m)	375 ± 4	519
Track length ($e-p$) (m)	70 ± 8	131
Number of ($e+p$)	111 ± 26	155 ± 45
Number of ($e-p$)	20 ± 11	37 ± 14
Charge excess	$\sim 18\%$	$\sim 24\%$
Cherenkov peak at 1 GHz ($Volts/MHz$)	7.5×10^{-9}	1.1×10^{-8}

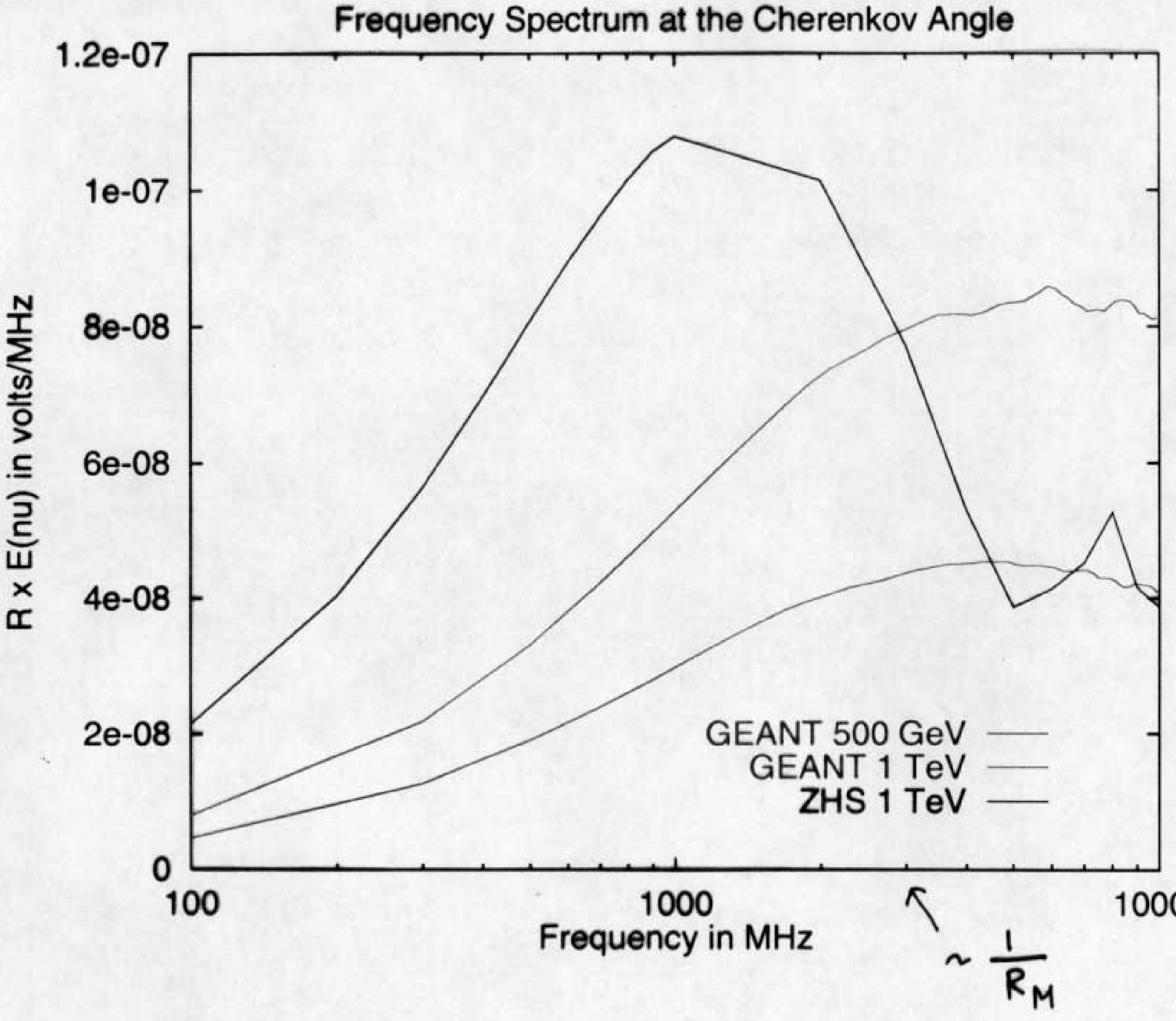
Excess Charge in Ice (GEANT)



E_0 = 100 GeV

E_{th} = 0.611 MeV


Angular Pulse Distribution (GEANT)
R x E(nu) in volts/MHz
6e-08
5e-08
4e-08
3e-08
2e-08
1e-08
0
30
35
40
45
50
55
60
65
70
75
80
Angle in degrees
1 TeV 1 GHz
1TeV 300 MHz
500 GeV 1 GHz
500 GeV 300 MHz


Frequency Spectrum at the Cherenkov Angle
R x E(nu) in volts/MHz
1.2e-07
1e-07
8e-08
6e-08
4e-08
2e-08
0
100
1000
10000
Frequency in MHz
GEANT 500 GeV
GEANT 1 TeV
ZHS 1 TeV
~ $\frac{1}{R_M}$

Conclusions

- GEANT Monte Carlo simulation tools has been adapted for the RICE experiment.
- Many different consistency checks have been done and obtained reasonable agreements.
- Electromagnetic Pulse calculations have been done for cascades of energy ranging from 100 GeV to 1 TeV. Results can be scaled up to $\sim 100\,TeV$.

[redacted]

- Initial setup has been done for Hadronic cascades.
- GEANT can be easily adapted for other materials and is ideal to simulate accelerator experiments.