Signal Characteristics from Electromagnetic Cascades in Ice

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Electromagnetic Cascade

High energy $e^-$ or $\gamma$ hitting a material target, creates an electromagnetic cascade inside the material.

![Diagram](image)

**Features of a Cascade**

- Number of particles grows exponentially at the beginning due to Bremsstrahlung and Pair Production processes at high energy.

- Particles lose energy mostly through Ionization after reaching a critical energy.

- A net charge imbalance (more $e^-$ than $e^+$) is created in the cascade (Askaryan, 1961) due to the following processes:

  - *Compton scattering* ($\gamma + e^-_{\text{atom}} \rightarrow \gamma + e^-$)
  - *Positron Annihilation* ($e^+ + e^-_{\text{atom}} \rightarrow 2\gamma$)
  - *Bhabha scattering* ($e^+ + e^-_{\text{atom}} \rightarrow e^+ + e^-$)
  - *Möller Scattering* ($e^- + e^-_{\text{atom}} \rightarrow e^- + e^-$)
Electromagnetic Pulse

Charged particles in a cascade radiates electromagnetic wave coherently at radio frequency (Askaryan, 1961).

Far Field or Fraunhofer Approximations

- Distance of observation ($R$) is large compared to the shower length.
- Wavelength of radiation is large compared to the shower dimensions.

Electric field at a distance $R$ due to a single charged track in the medium of refractive index $n$ is

$$R \vec{E}(\omega) = \frac{\mu_r}{\sqrt{2\pi}} \left( \frac{e}{c^2} \right) e^{i \omega \frac{R}{c} e^{i \omega (t_1 - n \beta \cdot \vec{r}_1)} \vec{v}_T \left( \frac{e^{i \omega \delta t (1 - \hat{n} \cdot \beta n)} - 1}{1 - \hat{n} \cdot \beta n} \right).$$

Cherenkov condition is $1 - \hat{n} \cdot \beta n = 0$ which defines the Cherenkov angle $\theta_c = \cos^{-1} (\frac{1}{n\beta}).$

At or very near $\theta_c$, the field is

$$R \vec{E}(\omega) = \frac{\mu_r i \omega}{\sqrt{2\pi}} \left( \frac{e}{c^2} \right) e^{i \omega \frac{R}{c} e^{i \omega (t_1 - n \beta \cdot \vec{r}_1)} \vec{v}_T \delta t.$$
Figure 1: Set up to calculate electric field from a single track
Monte Carlo

- Need Monte Carlo to simulate the cascade.
- Calculate electromagnetic pulse from the cascade:
  - on track by track basis or
  - by parameterizing the cascade.
- Pioneering work has been done by Zas, Halzen and Stanev (1992).
- Buniy and Ralston (1999) have also investigated this issue by parametrization method.

We use GEANT Monte Carlo simulation tool to simulate electromagnetic cascades. We calculate electromagnetic pulse on track by track basis.

Features of GEANT Monte Carlo

- Well understood and well documented.
- Widely used in high energy accelerator experiments to simulate detectors.
- Better handle on Physical Processes.
- Detailed and flexible output.
- Ultimately want hadronic shower included.
Figure 3: Moliere radius corresponds to the transverse development of the shower. When the fraction $U/E$ (the ratio of total energy inside an imaginary tube along the showers axis to the initial energy of the shower) is 0.1 (the horizontal straight line), the corresponding radius is the Moliere radius for that material. It depends on the material and not on the energy of the shower. Here we use showers of energies 1 GeV and 10 GeV for each material.
GEANT Consistency Checks

Relationship between Radiation Length ($X_0$), Moliere Radius ($R_M$) and Critical Energy ($E_c$):

$$E_c = \frac{X_0 E_s}{R_M} ; \quad E_s \approx 21.2 \text{ MeV (scale energy)}.$$

Also, $E_c \approx 605/Z ; \quad Z$ – atomic number.

Methods

- Calculate Moliere radius for different materials using GEANT track informations.
- Calculate the critical energy using above formulas and compare.

Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Iron</th>
<th>Lead</th>
<th>Ice</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_0$ (cm)</td>
<td>1.76</td>
<td>0.56</td>
<td>39.05</td>
</tr>
<tr>
<td>$R_M$ (cm)</td>
<td>2.1</td>
<td>1.6</td>
<td>13</td>
</tr>
<tr>
<td>$E_c$ (MeV)</td>
<td>17.77</td>
<td>7.42</td>
<td>63.7</td>
</tr>
<tr>
<td></td>
<td>(23.3)</td>
<td>(7.4)</td>
<td></td>
</tr>
</tbody>
</table>

References: Nelson et al. (1966) and Bathow et al. (1970).
Figure 2: Average energy of 500 GeV electrons vs distance in cm. Crosses indicate Monte Carlo data points and the solid line is the exponential fit. This fit gives a radiation length of $41.5 \pm 3.2$ cm. 0.611 MeV kinetic energy threshold was used in all 500 cases.
$\text{Depth in Radiation Length}$

$\text{Number of particles (e+p)}$

$E_{\text{th}} = 1.5 \text{ MeV}$

$(e+p) \text{ ZHS}$

$(e+p) \text{ GEANT}$

Griesen

$(e+p) \text{ PGG}$

$30 \text{ GeV Electron Shower in Iron}$
Comparison Table for GEANT and ZHS

100 GeV cascades in Ice. $E_{th} = 0.611$ MeV.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>GEANT</th>
<th>ZHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total track length ($m$)</td>
<td>400 ± 5</td>
<td>642</td>
</tr>
<tr>
<td>Track length ($e + p$) ($m$)</td>
<td>375 ± 4</td>
<td>519</td>
</tr>
<tr>
<td>Track length ($e - p$) ($m$)</td>
<td>70 ± 8</td>
<td>131</td>
</tr>
<tr>
<td>Number of ($e + p$)</td>
<td>111 ± 26</td>
<td>155 ± 45</td>
</tr>
<tr>
<td>Number of ($e - p$)</td>
<td>20 ± 11</td>
<td>37 ± 14</td>
</tr>
<tr>
<td>Charge excess</td>
<td>$\sim 18%$</td>
<td>$\sim 24%$</td>
</tr>
<tr>
<td>Cherenkov peak at 1 GHz</td>
<td>$7.5 \times 10^{-9}$</td>
<td>$1.4 \times 10^{-8}$</td>
</tr>
</tbody>
</table>
Excess Charge in Ice (GEANT)

$E_0 = 100 \text{ GeV}$

$E_{th} = 0.611 \text{ MeV}$
Conclusions

- GEANT Monte Carlo simulation tools has been adapted for the RICE experiment.
- Many different consistency checks have been done and obtained reasonable agreements.
- Electromagnetic Pulse calculations have been done for cascades of energy ranging from 100 GeV to 1 TeV. Results can be scaled up to ~ 100 TeV.

- Initial setup has been done for Hadronic cascades.
- GEANT can be easily adapted for other materials and is ideal to simulate accelerator experiments.