California Physics Standard 2a Send comments to: layton@physics.ucla.edu

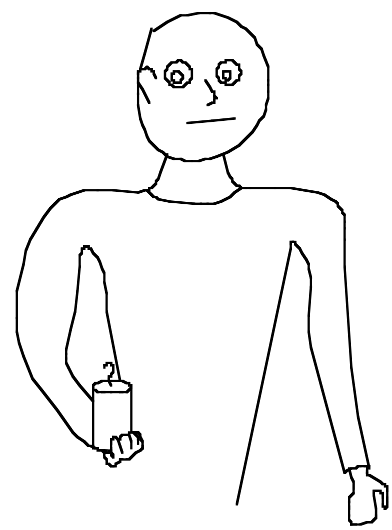
Preliminary note: Students often confuse the concepts inertia, momentum and kinetic energy. We don't encourage discussing the distinction between these concepts at the beginning of this unit, but be prepared to make clear distinctions between them when the unit concludes.

2. The laws of conservation of energy and momentum provide a way to predict and describe the movement of objects. As a basis for understanding this concept:
a. Students know how to calculate kinetic energy using the formula $E = 1/2mv^2$.
Note: We do not know why the Standards have chosen to use "E" for kinetic energy when "PE" is used for potential energy. We will use KE for kinetic energy and PE for potential energy in all the discussions that follow.

It is useful to introduce the idea of energy through the concept of work. Once work is understood, it can be shown that energy is <u>transferred</u> through work.

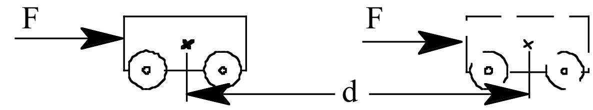
Introducing the concept of work.
Students need to know that force and work are different. Several different examples of applying forces, some when work is being done and others when work is not being done can help. Begin by holding a mass motionless and ask:

"I am exerting a force upward on this mass but am I doing any work?" The discussion that follows might include that although I might get tired doing this, the basic definition of work says no. Next, slowly move the mass upward and ask if you are doing any work. This time, since the force is applied through a distance, work is done. Next hold the mass at a constant height and slowly move it in a horizontal direction and ask if you are doing any work. This time the answer is no since there is no component of force in the direction of motion. Now try to accelerate the mass in a horizontal direction and ask if while you were accelerating were you doing any work. All of these examples should help the students to come to an understanding that there must be a component of force in the direction of motion if work is done.



Finally, hold the mass at arm's length and move it in a circle at constant speed. Here again no work is done since there is no force in the direction of motion. (A useful observation when trying to explain why satellites require no energy to keep orbiting.) In all of these examples the force has remained constant in value and this can lead to a basic definition of work. **Work equals force parallel times the distance the force moves.** In more advanced presentations you can include the possibility that the force could change in size while the work was done. Then give the more general definition: **Work equals the area under a force parallel vs. distance curve.** Students may have experienced the difference between simply pushing on something *vs*. pushing with the same force as the thing moves. (Such as pushing a car to start it.) In the second case you will get more tired since the energy in your body is being transferred to the moving object as you do work. End this introduction with a discussion of the units of work. In the mks system, one Joule equals the work done when one Newton acts through a distance of one meter. The Standard seems to suggest that the important skill is knowing how to use the formula $KE = 1/2mv^2$. As well as plugging into formulas we would also hope students would be brought to understand that kinetic energy is the work done by an unbalanced force. This might be demonstrated with a mathematical derivation as follows*:

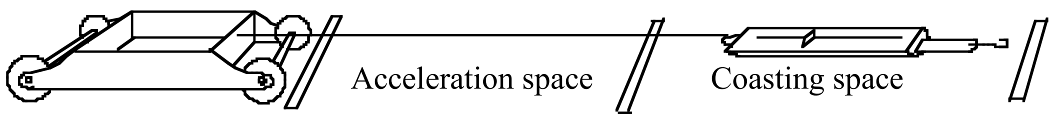
A constant force F is applied to a frictionless cart of mass m while it moves a distance d. The work done on the mass is F d. From Newton's 2nd law, the work becomes m a d. Since when an object



uniformly accelerates from rest it can be shown (see * below) that ad =1/2v^{2,} it follows that the total work done in accelerating the mass is 1/2mv^2. This very useful result says that the work of an unbalanced force = 1/2mv^2 and this "energy of motion" is called kinetic energy or KE.

The above derivation suggests a simple and direct experiment to measure kinetic energy.

The work of an unbalanced force turns into KE.
The experiment involves a small fairly frictionless cart, a spring balance, a stopwatch and a level surface long enough to allow for some acceleration followed by a measured distance at constant speed. Measure the mass of the cart (perhaps including additional



masses), and measure the distance between the two "accelerating distance tapes" and the distance between the two "coast velocity tapes". The string attached to the cart is long enough to allow dropping the force to zero when the cart passes the second tape. The cart will be timed as it coasts at constant velocity between the second and third tape. The experiment can be repeated with different mass configurations but it is important to supply a constant force to the cart during the acceleration phase. This experiment can test to see if the work done in accelerating the cart equals the final kinetic energy of the cart. (Even though the results may not be great, students get to experience doing work and observing the resulting kinetic energy.)

The work of an unbalanced force equals change in kinetic energy. F_U d = 1/2mv^2

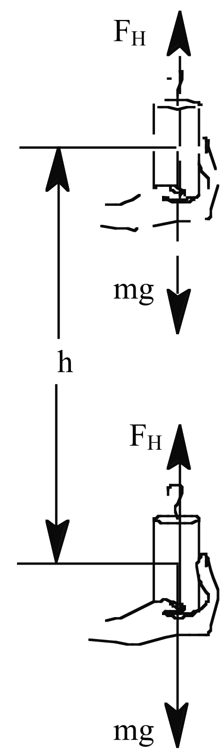
* This derivation assumes that when the formulas for kinematics were developed, the following was done:
Starting from rest, we have shown that v = at and d = ½ at^2
Eliminating t between these two equations yields: $v^2 = 2ad$
(or, for the above, ad = 1/2v^2)

2. The laws of conservation of energy and momentum provide a way to predict and describe the movement of objects. As a basis for understanding this concept:

b. Students know how to calculate changes in gravitational potential energy near the Earth using the formula (change in potential energy) =mgh (h is the change in the elevation).

(Again the Standard seems to be stressing how to use a "formula". If students understand the relationship between work and change in energy, they might better appreciate why ΔPE near the surface of the earth = mgh).

Work and Potential Energy near the surface of the earth.



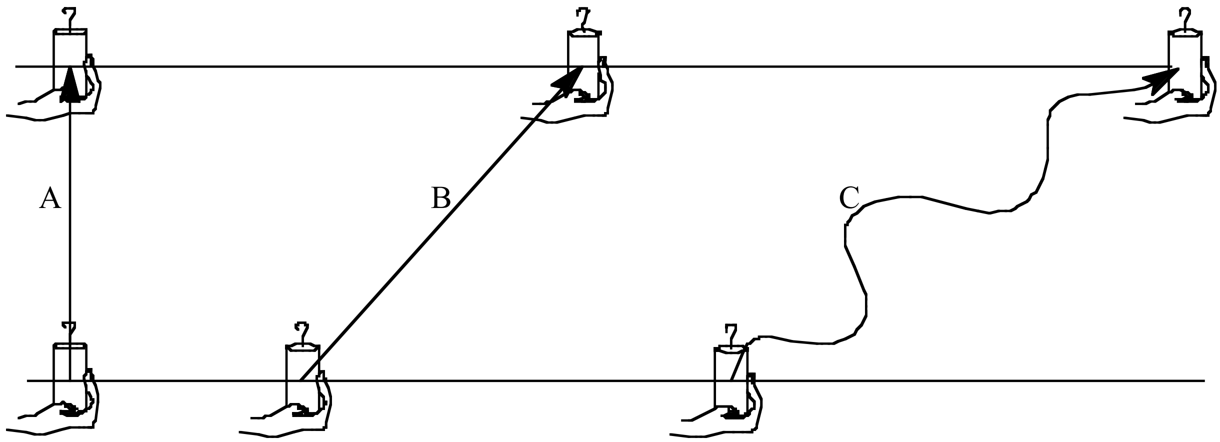
The illustration on the right shows a hand lifting a mass a vertical height, h. The hand exerts an upward force; F_H on the mass and gravity exerts a downward force mg on the mass. It is assumed that the mass is moving upward at constant speed, hence the upward force on the mass, F_H equals in magnitude the downward force of gravity, mg. Since work equals force in the direction of motion times, the distance the force moves, the work done by the hand is: $F_H h$. Since the magnitude of $F_H = mg$, it follows that the work done in lifting the mass is mgh. The work done in lifting the mass is stored and would be released later if the mass were dropped. Such stored energy is called potential energy and in this case, gravitational potential energy. Compressing a spring, or stretching a rubber band can also store energy. In these situations, the stored energy can always be related to the work done in storing the energy. However, when work is done against friction, the energy changes form and turns into heat, hence no potential energy results. In general, the work done against a "conservative force" is stored as potential energy. A crude way to define a "conservative force" is to say that it is a force that "remembers". That is, the force will remain the same over time. The weight of a mass at a particular point in space will always remain the same, no matter how long you leave it there.

A compressed spring will "remember" the work done in compressing it long after it was compressed, as stored potential energy. The force of friction, on the other hand, dissipates the energy expended when working against it and is therefore a "non-conservative" force.

The work done against a conservative force is stored as potential energy. When objects are moved near the surface of the earth, the vertical height lifted times the weight of the object will equal its change in gravitational potential energy. $\Delta PE = mgh$

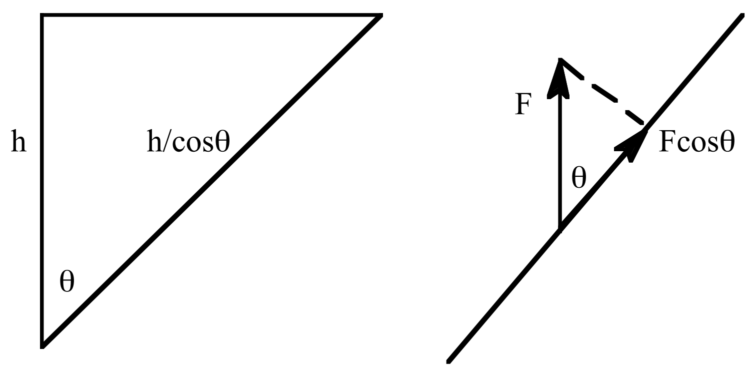
One very interesting consequence of how work is defined and how objects move in gravitational fields, is that the amount of work done, hence the potential energy stored, only depends upon the difference in height and not upon the distance moved. The following discussion should clarify that only the vertical height lifted influences gravitational potential energy.

Work is defined as the component of force in the direction of motion times the distance the force moves. This means moving in a direction different from the direction of the force of gravity could involve a greater distance but since the component of the force in the direction of motion is less, the resulting work done is the same.



In the illustrations "A, B, C" above, the mass is always moved the same height h. In "A" the force is in the same direction as the motion so the work done is Fh. In "B", however, the distance moved is greater yet the work is still Fh. This is easy to show using trig:

As illustrated on the right, if the vertical height is h and the angle of the distance moved is θ with respect to h, then the distance moved is h divided by cosθ. On the other hand, the component of the force in the direction moved is Fcosθ. Hence the product still comes out to be Fh. The work is the same.



Another way of showing the same result without trig is to recall the definition of work and assert that moving in a direction perpendicular to the force does no work. Hence in case B, we could have initially moved directly under the intended final location <u>horizontally</u>, doing no work, then moved vertically upward a distance h applying a force F and we would have done the same amount of work. This is also true for the complex path illustrated in C. The fact that the work done is independent of path in a conservative force field will be very useful in the next Standard.

California Physics Standard 2c Send comments to: layton@physics.ucla.edu

2. The laws of conservation of energy and momentum provide a way to predict and describe the movement of objects. As a basis for understanding this concept:
c. How to solve problems involving conservation of energy in simple systems such as falling objects.

The discussion in the Framework at this point only considers kinetic energy and potential energy. We will also follow this plan, however, a more general discussion of energy conservation would also include such things as the energy dissipated by the force of friction, etc. The Framework uses "T" for total energy and we will revise this slightly and call total energy "TE". The big idea with energy conservation is that whatever amount of energy you have at one time **in a closed system,** the total energy will be exactly the same at a later time. The big yet simple equation you will always write at the beginning of any energy problem is that the total energy at one time will equal the total energy at some later time, or, TE = TE'. Next, plug in the initial and final values of PE and KE and solve.

Lets start by solving a problem using energy conservation that could just as easily have been done using our earlier kinematics formulas:

A rock is dropped from a high bridge 100 meters above the water. How fast is the rock going after it falls 60 meters from the bridge? Ignore air friction.

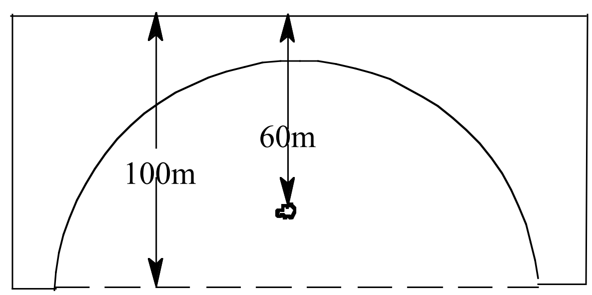
The total energy when dropped must equal the energy at any later time:

TE =TE'

Considering potential and kinetic energy:

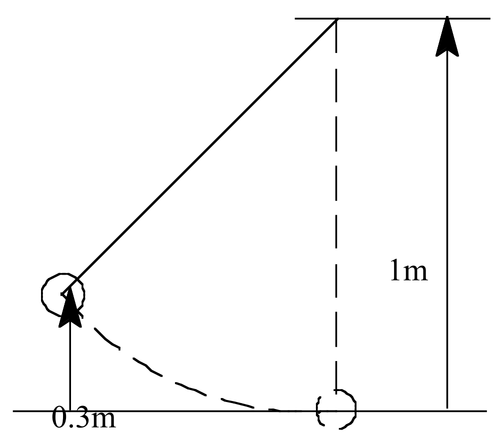
PE + KE = PE' + KE'

The next step is to pick a zero of PE. We select the water surface. Since the rock was dropped, the initial velocity was 0.



Now the energy equation becomes: $mg(100) + 1/2m(0)^2 = mg(40) + 1/2m(v')^2$ Notice that the m drops out of the equation. Solving for v' gives: $v' = (2g60)^{1/2}$. Your students might complain: why didn't we just plug into a kinematics formula in the first place! Give them this second problem:

A pendulum of length 1 meter is pulled back along its path and held a height of 0.3 meters above its lowest position and is then released. How fast will the bob be moving when it swings to the lowest point?
(Remember, our kinematics formulas will only work for uniform acceleration! The pendulum will start with maximum acceleration but will have zero acceleration in the direction of motion when it reaches its lowest point.)

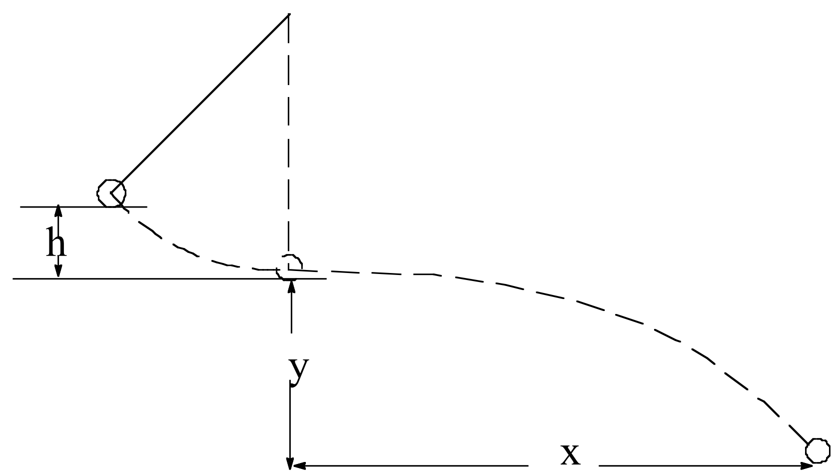


We choose the zero of potential energy to be the lowest point of the swing. The fact that the pendulum was held at the start, the initial kinetic energy was zero.

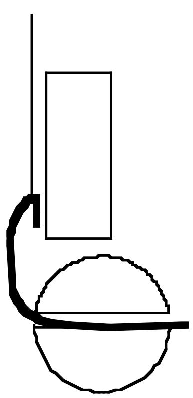
TE =TE' or, $mgh + 1/2mv^2 = mgh' + 1/2m(v')^2$ or, $mg(0.3) + 0 = 0 + 1/2m(v')^2$

Again the masses drop out and we can solve for v'. Point out to your students that while the pendulum was swinging downward, the string was always exerting a force on the bob, but since this force was always at right angles to the direction of motion, it never did any work on the bob.

An experiment to show Potential Energy being converted to Kinetic Energy.



The idea behind this experiment is that a pendulum is elevated to a measured height and is released. When the bob reaches the lowest part of its swing, it is released and allowed to move as a projectile to strikes the floor some distance away. The distance it freely falls vertically can be used to find the time of free fall and the distance it moves horizontally can be use to find its horizontal velocity. With this data the initial potential energy can be compared with the kinetic energy of the bob just as it separates from the string and begins to move as a projectile.



One way to have the pendulum bob release at the lowest point of its swing is to bend a paper clip and attach it to the lower end of the string. Bend the clip so that it will have a long section pointing in a horizontal direction as illustrated on the left. Using a drilled pendulum bob, pass the horizontal section of the paper clip through the bob. This will support the bob during its downward swing but place a stop block that is firmly attached to the table exactly at the lowest point of the swing. This block will stop the paper clip and pendulum string but allow the pendulum bob to continue on in freely falling motion. (Some have used a sharp razor blade to cut the string at its lowest point but this will take some energy from the bob and requires retying after each run.

Experiment to measure work to potential energy.

A suggested experiment is to arrange a way to use a spring balance to measure the force required to pull a pendulum away from its rest position and record this force at several measured distances along its curved path. A graph of force vs. distance is made. The area of this graph could be determined by "square counting" (observing the proper unit value of each square). This area in joules should equal the value of mgh, where "h" is the vertical height of the bob for each position along the curved path.

California Physics Standard 2d Send comments to: layton@physics.ucla.edu

Note: The Framework does not introduce Impulse in this Standard but chooses to do so later in Standard 2f. We believe that just as work should be introduced with energy, impulse should be introduced with momentum. This helps students understand that **Work transfers energy** and **Impulse transfers momentum.** It also helps students to focus on how force and distance relate to energy and force and time relate to momentum.

2. The laws of conservation of energy and momentum provide a way to predict and describe the movement of objects. As a basis for understanding this concept:

d. How to calculate momentum as product mv.

Students often confuse the concepts inertia, momentum and kinetic energy. (So did many of the leading scientists in world up to about the mid 19th century!) Perhaps at this time a discussion with your students that stresses and clarifies following might be useful:

Inertia is the tendency of an object to resist acceleration. Inertia is measured in mass units and in the mks system; the unit of inertia is the kilogram. We use "m" to symbolize inertia (also to symbolize gravitational mass). Inertia is a scalar.

Kinetic energy is energy of motion. The work done in changing an object's speed equals its change in kinetic energy. The mks unit of kinetic energy is the joule. It's easy to show that the work done in speeding up an object from zero to v is $1/2mv^2$. Kinetic energy is a scalar.

Momentum is the product of inertia (m) and velocity (v). That is, momentum = mv. Momentum, like energy is conserved. Energy can be confusing since it has so many different forms (mechanical, electrical, heat, biological, etc.) but momentum never changes form and can be found by computing mv. There is no given mks unit of momentum but it is usually stated as: kilogram meters/sec. Momentum is a vector and its direction is determined by the direction of the velocity vector.

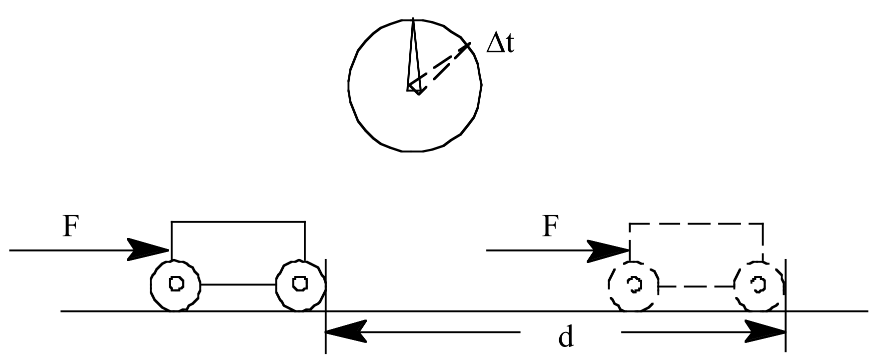
Discussing how work transfers energy and impulse transfers momentum.

Help students to see that force times distance is one concept, and force times time is a different concept. Illustrate this with a drawing like the one below:

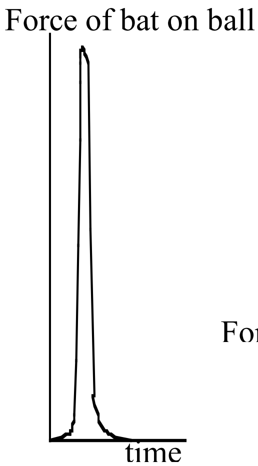
The force accelerating the cart acts through a distance d, but while the force acts, a time interval Δt also passes. We have shown that $Fd = 1/2mv^2$. Using Newton's 2nd law it is easy to show: $F = ma = m\,\Delta v/\Delta t$ or,

$\mathbf{F\Delta t = m\Delta v.}$ That is,

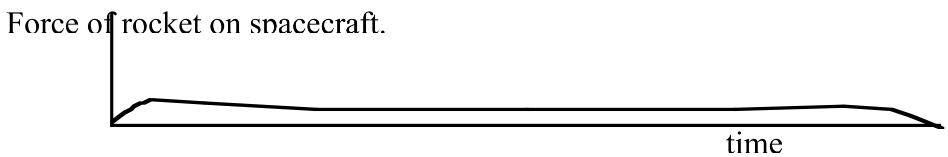
Impulse equals change in momentum.



The area under a graph of force vs. time will give impulse just as the area under a graph of force vs. distance will give work. Impulses can have a large force for a very short time such as the force a baseball bat exerts on a baseball or a small force over a long period of time such as the force a small rocket exerts on a spacecraft to make a course correction.

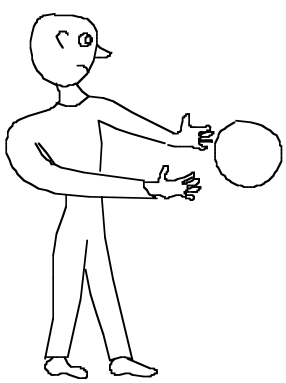


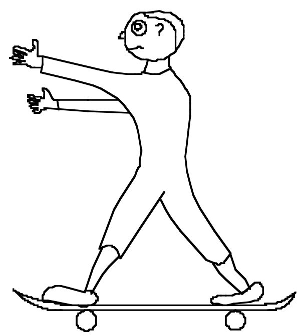
The graph on the left shows the force exerted on a baseball by a bat during a hit. The force will be very great but for a very short time. The change of momentum of the baseball as it stops, turns around and moves away at great speed will equal the area under the graph. The graph below shows the small force exerted by a course correction rocket on a spacecraft. The force is small but is exerted for a long time. Again the area under the graph will be the impulse and it will equal the change in momentum of the spacecraft.



Agile student on skateboard demonstration.

A well-coordinated and accomplished skateboarder can be a big help when introducing momentum. You will also need a bag of sand or a medicine ball like object.





A little practice may be required to make this effective. Have the skateboarder toss the ball away at assorted speeds and observe the resulting momentum. Toss the ball to the stationary skateboarder and observe the resulting transfer of momentum. Experiment with different speeds and different direction of motion of the skateboarder.

In the discussion that follows, concentrate on how momentum was transferred from the ball to the skateboarder and skateboarder to ball, both when tossed and received. Discuss what might have happened with different mass ratios of the ball and skateboarder. End the discussion by pointing out that the person standing on the ground at rest who tossed the ball (in most cases this will be the teacher) did not seem to move. Isn't this a violation of the conservation of momentum principle?!! The ball seemed to gain momentum from nowhere. Stress this point again by running across the room and then abruptly come to a stop. Where did the momentum come from to get moving? Where did the momentum go when you stopped? This should be a good time to point out that the earth is a "giant skateboard" and in every case the momentum was transferred to and from the earth. However, since its mass is so large the momentum was conserved with a very small velocity of the earth. An interesting problem is to have the students compute the velocity of the earth as a result of you running in one direction at a given speed.

2. The laws of conservation of energy and momentum provide a way to predict and describe the movement of objects. As a basis for understanding this concept:

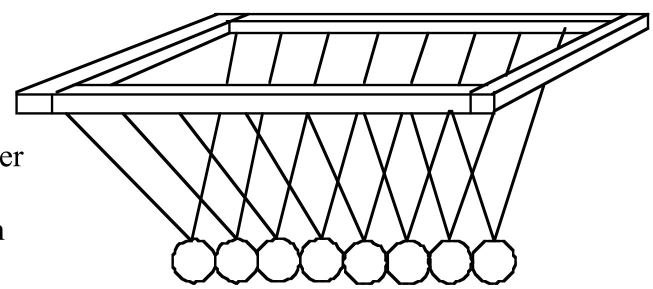
e. Momentum is a separately conserved quantity, different from energy.

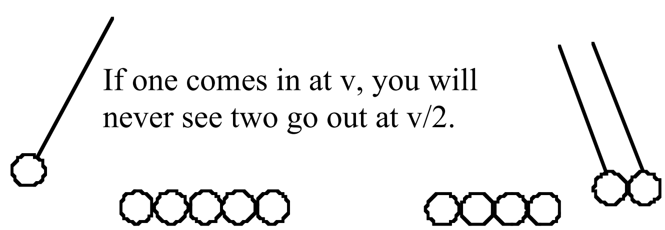
Momentum and energy are both conserved quantities, meaning that they can be neither created nor destroyed. These conservation properties are not always obvious and the fact that momentum and energy are different and are conserved separately needs to be stressed. Emphasize that it takes force times time to change momentum and that force acting through a distance changes energy. Another difference is that momentum is a vector and energy is a scalar. The following familiar situation can be discussed to help get an intuitive feel about the transfer of momentum and energy:

A car racing down the road at a high speed slams on its breaks and comes to a stop. What happened to the car's kinetic energy? What happened to the car's momentum? This straightforward question can begin a discussion of how energy can change form yet still be conserved and how momentum can be transferred to the earth and still be conserved. Students can be helped to understand where the energy went if they are asked what do they think they would feel if they jumped out of the car and felt the breaks. The momentum transfer to the earth may be harder to appreciate but suggest what might have happened if the car had been racing on a huge skateboard.

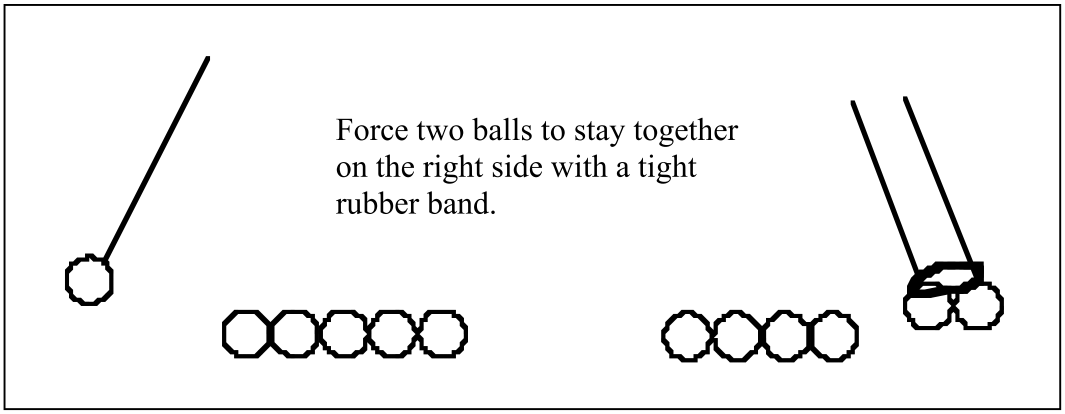
Newton's cradle or the "click clack device".

This device can be purchased at novelty stores as well as scientific equipment supply houses. It is a wonderful demonstration of the conservation of momentum and kinetic energy. After you experiment with lifting and releasing a given number of balls and observing the same number rise on the opposite side, drop one from one side and two from the other and observe the result. Lift one high and one low on the opposite side and observe the result.



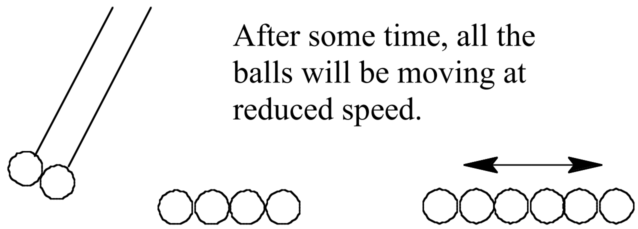


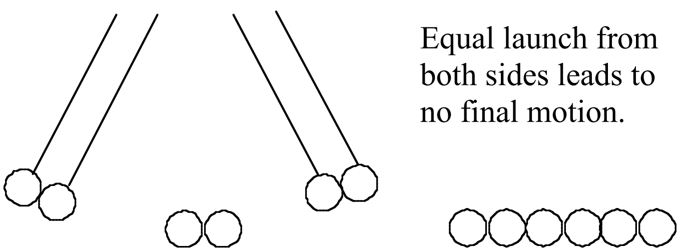
After demonstrating how the device always mirrors the initial conditions on the other side ask why, when you lift one on the left side, two never come out the right side at half the speed? This surely would conserve momentum. Hopefully the realization that this would not conserve kinetic energy will be discovered.



An interesting demonstration is to use a small rubber band and hold two balls together on the right side and only lift one ball on the left side. It is easy to hear the rapid clacking of the two balls as they use a different method to get rid of the extra kinetic energy.

Also show that if you start with say two balls on the left side drop it on to the main collection and then let the action continue without interaction for a long time, all the balls will be swinging together with a fraction of the motion of the original two balls. Momentum has been conserved but kinetic energy has been converted to heat and sound.





However, if you lift the balls equally from each side making the original vector sum of momentum zero, when the full collection of balls finally come together, there will be no motion. Demonstrating that momentum is conserved as a vector.

Be sure to set this device at a place where students can easily play with it. This play will teach them much.

California Physics Standard 2f Send comments to: layton@physics.ucla.edu E11

2. The laws of conservation of energy and momentum provide a way to predict and describe the movement of objects. As a basis for understanding this concept:

f. Students know how an unbalanced force on an object produces a change in its momentum.

Activity to show that FΔt = Δ(mv), that is, impulse equals change in momentum.

This activity uses the same equipment as described in Standard 2a above, only times rather than distances are measured. Probably two stopwatches are required. One to measure the time of acceleration and the other to measure the time of coasting.

As before, the string must be long enough to allow acceleration with a constant force but should also allow this force to drop to zero just as the cart enters coasting space. This experiment will definitely be a team effort with one student providing the force and two other students measuring the acceleration and coasting time. (If available, a split stopwatch might require only one student.) If the friction is fairly low, the coasting distance divided by the coasting time will give the final velocity. Different masses can be used and the total mass of the cart plus included masses can be measured with the spring balance and dividing by g. This experiment should help students to see how the product of force and time equals the change in momentum just as the previous experiment should have illustrated how the product of force and distance equaled the change in kinetic energy.

Bottle Rockets (or Water Rockets)

Students can have a lot of fun building, testing and competing with one another with bottle rockets. The best way to find out about these is to engage in a web search for "bottle rockets" or "water rockets". Often the URLs for related pages do not hold up but one that describe building the rocket and even includes a parachute, if it is still up, is:
http://www.lnhs.org/hayhurst/rockets/ wrbook.htm
And a NASA PDF download of how to build the launcher: http:// www.nasa.gov /audience/foreducators/topnav/materials/listbytype/Bottle_Rocket_Launcher.html

The essential idea behind the bottle rocket is that an empty plastic soft drink bottle is fixed with fins to make a rocket. It is half filled with water and using a special launcher, it is pumped with air to provide the energy to expel the water through the inverted top of the bottle. The resulting application of energy and momentum makes a spectacular display as the rocket rises into the air. Many variables can be tested such as the amount of water, the amount of energy stored by pumping, the shape of the rocket and how all of this influences the height the rocket rises. Be sure to observe the safety measures suggested in many of the articles on the web as well as in the NASA launch download.

California Physics Standard 2g Send comments to: layton#physics.ucla.edu E12

2. The laws of conservation of energy and momentum provide a way to predict and describe the movement of objects. As a basis for understanding this concept:

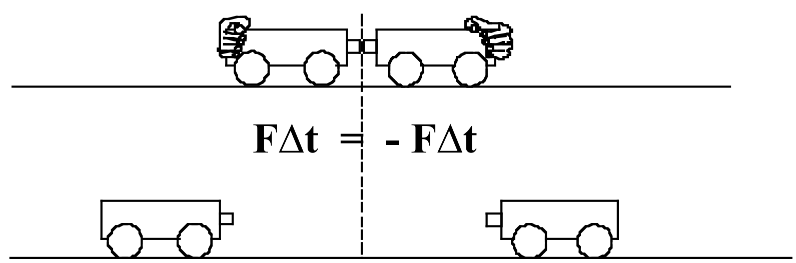
g. Students know how to solve problems involving elastic and inelastic collisions in one dimension using the principles of conservation of momentum and energy.

Momentum-Energy Demonstration Carts and Track.

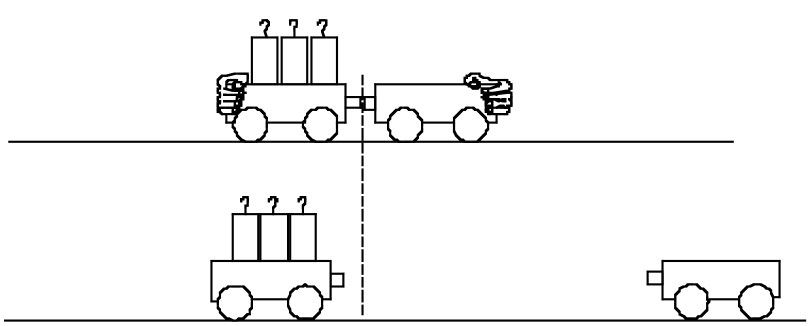
Explaining the basics of momentum and energy conservation during elastic and inelastic collisions can be much enhanced with two carts on a low friction track. The carts can be fitted with repelling magnets on one end and Velcro on the other to help with the demonstrations. Pasco sells excellent examples of tracks and carts for this purpose but they are not cheap. Cynmar has several carts that could be modified for this purpose such as their "General Purpose Cart" (095 1851) but it would still require constructing a track. Pitsco sells wheels at a very reasonable price but this would require construction of both the carts and the track. However, the educational value of having a track with two carts that can be weighted differently and that can be made to interact elastically or inelastically is so valuable, every effort should be made to obtain such equipment. The essential specification for this equipment is that the two carts should have low friction wheels, they should be able to be weighted differently, they should have elastic and inelastic "bumpers" and they should be confined to move in one direction with a linear track. It is useful if the track can be easily elevated and leveled so the entire class can see it.

Explosions and rocket basics.

Using two carts of the same mass with magnets repelling, press the carts together and release them at the same time. Discuss how the repulsive forces must be equal (Newton's 3rd Law) and the times the carts act on each cart must be equal, hence each must receive the same impulse in opposite directions.



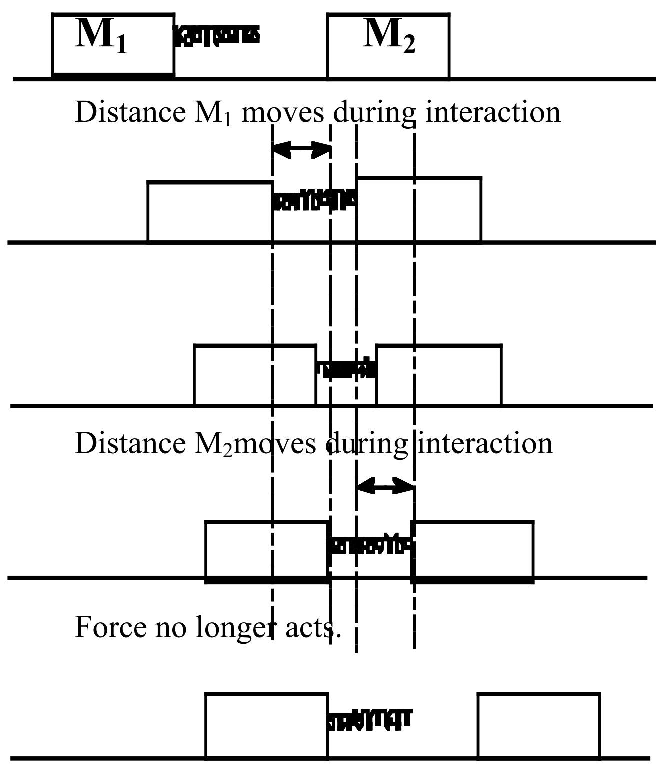
Since the magnitude of the impulse on each cart is the same, they will each receive the same momentum in opposite directions and will move at the same speed. Now repeat this "explosion" only with one of the carts weighted. This time the impulse will still be the same on each cart hence the change in momentum will be the same but the speeds of each will be different, in inverse proportion to the masses.



A very interesting demonstration can be performed with this apparatus by carefully balancing the carts and track on a fulcrum before the explosion and then releasing the carts. If carefully done, they will remain balanced for a short while showing that the center of mass is unchanged—an important concept when discussing rocket propulsion.

It is important to stress that in elastic collisions, kinetic energy as well as momentum is conserved. The forces that act between the colliding bodies are conservative and will return all of the energy that was temporarily stored when the bodies were at their closest distance separation. (Another way of saying a spring or magnet interaction is "perfectly elastic" is to say the force depends only upon separation. Springs are elastic since their restoring force depends only on the distance they are compressed or stretched. Balls of putty exert one force when they are compressed but do not return this force and do not elastically expand again.) Students might appreciate the following argument involving work to show that the kinetic energy that is temporarily stored as potential energy when the objects are at minimum separation will all be returned if the bumpers are perfectly elastic.

The important point to realize is that if the spring is perfectly elastic, the force applied on M_1 while it slows down will always be equal and opposite to the force that speeds up M_2 for the entire distance of the complete interaction. Since force and distance represent work, the work "done" in slowing M_1 exactly equals the work done in speeding up M_2. The kinetic energy lost by M_1 is transferred to M_2. If, on the other hand, the spring had been inelastic (say a coil of soft solder) it would not have returned the energy after compression and more energy would have been lost by M_1 than would have been gained by M_2.



Momentum is always conserved since the force and time are the same no matter how far each object moves during the interaction. The elastic collision transfers both momentum and kinetic energy. The inelastic collision also observes momentum conservation but not kinetic energy conservation. It is also interesting to note that during the heart of the elastic collision, even kinetic energy is not conserved since some energy is temporarily stored in the spring as potential energy.

Solving elastic and inelastic collision problems.

Solving inelastic collision problems in one dimension is fairly straightforward since the two objects stick together after the collision, therefore move at the same velocity. A simple example would be:

Find the final velocity if a mass m_1 moving at velocity v_1 collides inelastically with a mass m_2 moving at velocity v_2. Since they stick together, we apply momentum conservation:

$m_1v_1 + m_2v_2 = (m_1 + m_2)\, v'$ and solving for v' is simple algebra.

However, if the same problem is said to be a perfectly elastic collision, the masses will be moving at different velocities after the collision and will involve solving two equations in two unknowns. Although this is not impossible, it will involve a little messy algebra. We will do this solution here because the final result is interesting and useful. Again, mass m_1 moving at velocity v_1 collides with mass m_2 moving at velocity v_2 and the collision is perfectly elastic. The problem is to find the final velocity v_1' of mass m_1 and the velocity v_2' of mass m_2 after the collision.

From KE conservation: $1/2m_1(v_1)^2 + 1/2m_2(v_2)^2 = 1/2m_1(v_1')^2 + 1/2m_2(v_2')^2$ Eq. (1)

and from momentum conservation $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$ Eq. (2)

Rearrange equation (1) so all terms in m_1 are on one side of the equation and m_2 on the other:

$1/2m_1(v_1)^2 - 1/2m_1(v_1')^2 = 1/2m_2(v_2')^2 - 1/2m_2(v_2)^2$

Dividing both sides by ½ and factoring m_1 and m_2 : $m_1[v_1^2 - (v_1')^2] = m_2[(v_2')^2 - v_2^2]$

From the difference of two squares: $m_1 (v_1 + v_1')(v_1 - v_1') = m_2(v_2' + v_2)(v_2' - v_2)$ Eq. (3)

Rearranging Eq. (2) $m_1v_1 - m_1v_1' = m_2v_2' - m_2v_2$ or: $m_1(v_1 - v_1') = m_2(v_2' - v_2)$ Eq. (4)

Dividing equation (3) by equation (4) yields: $v_1 + v_1' = v_2' + v_2$

Rearranging this equation gives an interesting result: $\boxed{\mathbf{v_1 - v_2 = v_2' - v_1'}}$ Eq. (5)

This is not yet the solution to the problem but it says something very important. This equation says the relative velocity before the collision equals minus the relative velocity after the collision. This is always true in all perfectly elastic collisions. If the baseball is perfectly elastic, the velocity the ball and bat approach one another before collision will always equal the velocity of separation after the collision. A "super" ball should bounce up with the same speed it had just before striking the floor. This equation also makes it easy to solve the original problem. Now with equation (2) and equation (5) we have two simple linear equations that can be easily solved simultaneously for v_1' and v_2'

Perfectly elastic problem can be a mess to solve using simultaneous solutions of kinetic energy and momentum equations but the result that the velocity of approach before the collision must equal the velocity of separation after the collision can make these problems much easier to solve.

California Physics Standard 2h* Send comments to: layton#physics.ucla.edu E15

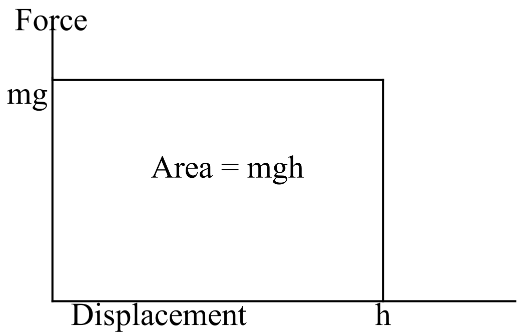
The laws of conservation of energy and momentum provide a way to predict and describe the movement of objects. As a basis for understanding this concept:
h.* how to solve problems involving conservation of energy in simple systems with various sources of potential energy, such as capacitors and springs.

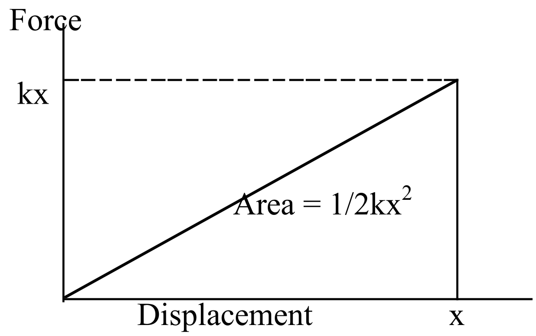
Storing potential energy when the force is not constant.

In our previous considerations of work, the force remained constant hence it was sufficient to say Work = Force X distance in the direction the force acts. However, often the force changes as the work is done. A more general definition of work is:

Work = The area under a force parallel vs. distance curve

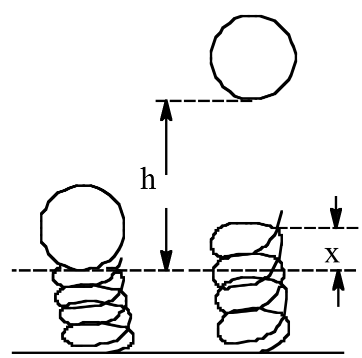
When lifting an abject near the surface of the earth, the force remains constant so it is sufficient to say the potential energy gained = mgh. However, when you stretch or compress a spring, the force can start at zero and rise linearly to some higher value at the end. The equation that describes the force vs. displacement for a spring (Hooke's Law) is: $F = -kx$ where k is the spring constant and is defined as $\Delta F/\Delta x$ for a particular spring. The graphs below illustrate the work done in lifting a mass vs. the work done in compressing a spring.





In both cases the force is the applied force doing the work and it can be seen that the area under the graph in lifting the mass is mgh and the area under the graph in compressing the spring is $1/2kx^2$. Finding the area under a force vs. displacement graph, if the displacement is in the direction of the applied force, will always give the work done. Students who know calculus will recognize this as: Work = $\int F\,dx$. A problem that illustrates how to apply these ideas is as follows:

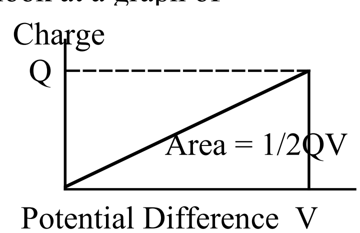
A sphere of mass m is pushed down on a spring of force constant k a distance x below its normal uncompressed length. The ball is released and rises into the air. If the spring is anchored to the floor and delivers all of its energy to the sphere, how high does the sphere rise above its initial position? You are given m, g, x and k.
The solution involves simply equating the spring potential energy when compressed to the sphere's gravitational potential energy at its highest position.



A problem which involves the same setup as the above but asks: "What is the position of the bottom of the sphere when it has its maximum kinetic energy?, requires a little more thought. Students may think they should simply equate 1/2 mv^2 to $1/2kx^2$ but this assumes the sphere reaches its maxim speed just as it leaves the spring. However, the sphere will reach its maxim speed when the upward force of the spring equals the downward force of gravity (somewhat less than x) and it will be required to find this distance by equating ky to mg and solving for y.

The above problem suggests an interesting activity that could be performed with a properly chosen compressible spring, a ball, a spring balance and a meterstick. The students would determine the spring constant using the meterstick and the spring balance. After determining the mass of the ball they could press it down a measured distance on the spring and see if it rises to the predicted height when released.

Energy in a charged capacitor. (Included here only because energy storage in capacitors is briefly mentioned in the Framework. More details are included below.) The Framework suggests that this would be an appropriate time to discuss storing energy in a capacitor. Although details about electrical energy and capacitors will be presented later, here is an attempt to address the basic concepts: A capacitor stores charge. Although electrical charge is not energy, (electrical charge is one of the fundamental quantities of physics like mass and time) it does take energy to store charge in a capacitor. Filling a capacitor with charge is not like filling a glass with water. Storing charge in a capacitor is more like filling a steel tank with air. At first it takes only a little pressure to put the air in the tank but as more air is forced into the tank, more pressure is required to store even more air. In this analogy charge is like the air and voltage is like the pressure. The capacitance of a capacitor, C, is defined as the ratio of the charge Q stored, to the voltage V required to store this charge. That is, C = Q/V or, Q = CV. The unit of capacitance is the Farad and a one Farad capacitor will store one Coulomb of charge if a potential difference of one Volt is placed across it. In order to appreciate how this relates to energy, it is necessary to know that voltage (or potential difference) is a measure of energy per charge. That is, V=Energy/Charge or W/Q. The unit of potential difference is the volt and one volt is one joule per coulomb. Let's look at a graph of charge vs. voltage when charging a capacitor:



Since from V = Energy/Charge, Energy = VQ, it follows that the area under a Charge vs. Voltage graph would equal the energy required to charge the capacitor to a final voltage V. Hence, the energy in a charged capacitor = 1/2 QV and since Q = CV, it follows that the energy of a charged capacitor = $1/2CV^2$

Suggested Activity to observe and measure energy in a charged capacitor.
The following has yet to be developed but might have promise: Using one of the "new" low voltage yet large value capacitors (Jameco 42957CB 1 Farad @ 5.5V--$4.29 each) charged to 4 V, experiment with placing this across a small 3 V motor rigged to lift a small weight of, say 100 grams. Lifting this weight 1 meter would be about one joule. Repeating the lifting would cause the voltage to drop and should be discontinued when the motor no longer turns. Record the voltage during the experiment. (The final voltage should not be zero.) Analysis should allow computation of the energy stored in the capacitor and the percentage that was delivered to the motor lifting the weight. Different values of weight, etc. could be tried.

Note: For some reason the California Standards do not discuss power in the context of mechanics. When they get to electrical circuits, power is considered (Standard 5c), but students should realize that power is a concept that has more applications than just in the study of electricity. For this reason, we have added the following standard here.

I Work, power and efficiency.**
Students should know that power is the rate of doing work or expending energy. Students should not confuse the concepts of power and energy and should be able to compute the efficiency of various processes.

Note the following quotation from a widely distributed publication from a large US warehouse store, made in July 2008:

"Operating nearly 400 warehouses in the United States requires a tremendous amount of energy--1.9 billion kilowatts last year, to be exact."

Energy would be measured in kilowatt-hours and power would be measured in kilowatts. You could guess what they might have meant but it's hard to know for sure. Why be "exact" when the units are wrong?

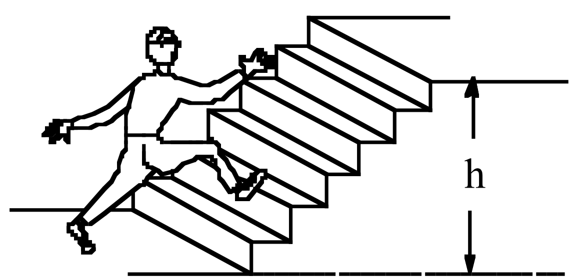
Power = energy or **work/time** **Power is measured in watts. 1watt = 1joule/second.**

The power of automobile engines is usually measured in horsepower, or HP. Horse power is a British unit originally developed by James Watt to help people understand how the power of the newly developing steam engines compared with the old standard, the horse. The British unit of work is the foot pound and Watt's measurements suggested that the average horse could lift 550 pounds one foot per second. Today the definition of the horsepower is 550 ft lbs/sec and this equals 756 watts (or about ¾ kilowatt).

Measuring a student's power.

A simple exercise that students seem to have a lot of fun doing is to have them run up a flight of stairs of known vertical height while being timed to measure their power.

The student will start from rest and run up the stairs while another student measures the time. If the weight of the running student is W, the height of the stairs, h, the time to run the stairs, t, then the power developed will be $P = Wh/t$. If W is in pounds, h in ft. and t in seconds, then the power will be in foot pounds/sec. Dividing this by 550 will give the power in horsepower.



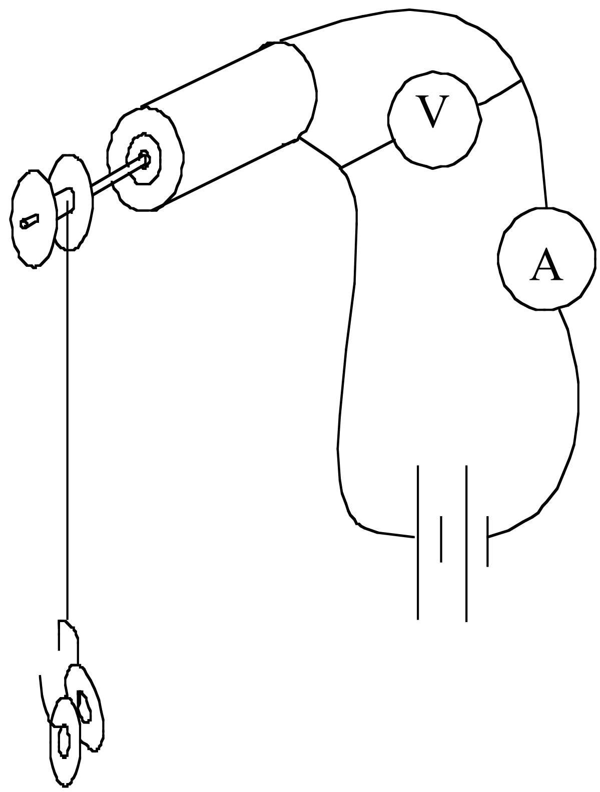
Students usually prefer finding out their power in units of HP but these can be easily converted to kilowatts. Almost always the heavier students develop the most power much to the disappointment of the smaller, faster students. Finally, make sure you choose the time and place to do this activity so it will not disturb other classes.

Efficiency.

The essential definition of efficiency is the ratio of what you get out of a system over what you put into the system. For example, the efficiency of an automobile engine could be the ratio of the mechanical energy output of the engine over the corresponding energy stored in the fuel that was burned in the engine during the same length of time. Another example would be the efficiency of an electric motor. This could be the ratio of the power the motor delivers in lifting a weight to the electrical power required to make the engine lift this weight. Obviously, the units of the output and input must be the same. Students often confuse "power" with "efficiency" and this can easily lead to confusion about what it takes to be "energy efficient", etc. Although the following lab exercise would more appropriately performed after an introduction to electricity, it could also be used at this time to start students to thinking about the meaning of electrical and mechanical power.

Measuring the efficiency of a small electric motor.

A small electric motor that operates on around 3V DC can be used to measure its power efficiency. A small pulley can be made using a short section of dowel with two circles of cardboard glued to either end. This pulley can be drilled along its axis to enable a tight fit to the motor shaft. Using a piece of thread with a bent paperclip on the lower end, a few washers can be hooked to the paperclip and the weight adjusted to enable the motor to pull the washers up at a constant speed. By measuring the time and distance the washers are lifted in a single run, together with the voltage and current through the motor windings, the power into the motor can be computed (P = VI)* as well as the output power (P = Wh/t). The weight of each washer will have to measured and adjustments will have to be made to find the correct load to have a constant power in. Students can experiment with different loads perhaps to find an optimum efficiency. *[Since Voltage = Work/Charge and Current(I) = Charge /time, it follows that their product = power.]



Power related to force and velocity.

From the definition of power it is easy to show that an object moving at constant speed against a constant opposing force will require a power equal to the product of the force and the speed. [Since P = W/t = Fd/t = F (d/t) = F(Velocity)]. This simple relationship will help students to appreciate why a car uses more fuel at higher speeds, particularly since the force of air drag increases with velocity. Having students solve problems that relate to this simple fact might encourage them to consider the fuel saving advantage of traveling at lower speeds.