California Physics Standard 1a  Send comments to: layton@physics.ucla.edu

Please understand that the first two California Standards in Grade 8 are “Motion and Force” and although some review will certainly be necessary, make use of the fact that students have been introduced to such ideas as: Position, Speed, Average Speed, Velocity, Change in Velocity, and Force.

1. Newton's laws predict the motion of most objects.
   As a basis for understanding this concept:
   a. Students know how to solve problems involving constant speed and average speed.

Pedagogical Note: Don’t simply give formulas and show students how to plug into them. Work on having students appreciate what speed means in terms of change in position per interval of time.

Making graphs of motion might help.

Apparatus for this demonstration
is an uninhibited teacher and some sort of reference point.

The teacher will identify the agreed point of reference (the ring stand in the middle of the table above) and agree that all motion will be in a straight line (probably parallel to the edge of the table top) and which direction will be plus (probably to the right.)

First suggest that a graph of position vs. time will be sketched for assorted “trips” the teacher takes while running back and forth in front of the table. Agree that the graph should begin when the teacher says: “start”, and ends when the teacher says: “end”. Begin by running to the right at constant speed and yell “start” when the zero reference is crossed and “end” a short time later—always running a constant speed. Have the students sketch a position vs. time for this trip. Repeat only start at a different position but still move at the same speed. Try running in the opposite direction, etc. The student’s sketches can be checked and discussed and perhaps finally sketched by the teacher on the board. This simple procedure can be extended to velocity vs. time graphs plotted directly below the position vs. time graphs and at a later time, acceleration vs. time graphs can be introduced. Those who have computers with motion detectors might find this quite crude, however, even those who are so fortunately equipped may find having the teacher move around and sketch graphs will make things clearer.

Some suggested sketches of graphs resulting from the above exercise. All graphs have
position on the vertical axis and time on the horizontal axis. (It is advised that the teacher begin with very simple motions and later extend to more complex trips.)

A. “Start” at the reference point and move to the right at constant velocity.
B. “Start” at the reference point and move to the right at a higher velocity.
C. “Start” to the left of the reference point and move to the right at a constant velocity.
D. “Start” to the right of the reference point and move to the left at a constant velocity.

After helping students to understand that the definition of velocity will lead to the conclusion that the slope of a position vs. time graph will give a corresponding velocity vs. time graph, graphing these both from the point of the slopes involved and by referring to the actual motions should be instructive.

It should be interesting to start as in graph A but after moving a short distance, quickly stop but don’t shout “end” for a second or two. Students are often confused when they learn that the resulting graph does not drop to zero when motion is stopped but continues on in a level straight line.

After the concept of acceleration is introduced and students come to appreciate that the slope of a velocity vs. time graph yields acceleration, more elaborate motions can be illustrated.

The graphs on the right are position, velocity and acceleration vs. time for a fairly complex “trip”. All curved sections on the position vs. time graph are assumed to be uniform accelerations. It is fun to select a student or two to see if they can run out this graph. Even if they find it difficult, it can be done and the students will enjoy watching the teacher (if this is required) go through the necessary motions to produce these graphs. Stress that they are all related by slope and perhaps it is easiest to look at the velocity graph first to see what must be done.
Making and analyzing graphs from actual motions can help students to better understand these motions and can also lead to simple ways of deriving the traditional kinematics formulas. Stress that the definitions of velocity and acceleration directly lead to slope and area concepts. (Even through students with calculus may know these ideas from their experience with derivatives and integrals, all students, even those with less mathematics can be helped in their understanding of kinematics with a careful use of graphs.

When position vs. time graphs are plotted above velocity vs. time graphs and in this turn, above acceleration vs. time graphs, the following general rules can be demonstrated:

<table>
<thead>
<tr>
<th><strong>Slope rule:</strong></th>
<th>The slope on any graph will equal the value on the graph directly below it.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Area rule:</strong></td>
<td>The area under any graph between two stated times (from the horizontal axis to the graph) will equal the change in value during the same time interval on the graph immediately above it.</td>
</tr>
</tbody>
</table>

An exercise for the students is to supply them with a carefully made graph of position vs. time for an object beginning with some initial displacement from the origin and an initial velocity all with a constant positive acceleration. That is, a position vs. time graph of

\[ s = \frac{1}{2} at^2 + vt + s_0 \]

Make the graph carefully and assign units and values on the horizontal and vertical axis. Place this graph on the top of a sheet of otherwise blank paper and instruct your students to construct velocity and acceleration vs. time graphs directly below this graph you have supplied. (They will have to construct slopes at assorted places on the position vs. time graph to generate the velocity vs. time graph.) If things have been done correctly, this will lead to a velocity vs. time graph with a positive intercept and a constant slope. This graph can then be used to construct the corresponding acceleration vs. time graph (which will be a simple horizontal line.)

Using the graphs the students have constructed, they will be able to work upward from the lower graphs with the Area rule. If this is carefully done, an analysis of their work will lead to a verification of \( \Delta v = a \Delta t \) and ultimately, \( s = \frac{1}{2} at^2 + vt + s_0 \)

Learning to make and use graphs of motion is not only a valuable skill, it can lead to a much better understanding of the kinematics formulas and will help many students to appreciate related concepts they are learning in their mathematics classes.

**California Physics Standard 1b**

Send comments to: layton@physics.ucla.edu

1. **Newton's laws predict the motion of most objects.**
   As a basis for understanding this concept:
2. **Students know when forces are balanced no acceleration occurs, and thus an object continues to move at a constant speed or stays at rest (Newton's First Law).**
Conduct a straightforward demonstration of this statement with a low friction cart or an air track. If you lack both, a skateboard works fairly well. First show that a force is needed to get the cart going but when you stop pushing (or pulling), it continues in uniform motion.

A very useful demonstration device to use frequently when discussing Newton laws is a low friction cart with a large spring balance attached. The wheels should be fairly frictionless and the entire cart should be able to support several kilogram masses. (A modified skateboard might work.) This cart can be used for quick demonstrations of the effect of changing mass and force on the acceleration of objects. With the cart loaded with several kilograms, a large force can be shown to accelerate it but when the force drops to zero, the cart will continue at about the same speed as when the force dropped to zero. If the approximate mass of the cart is determined (easy to do by using its spring balance to weigh itself and divide by “g”) quick qualitative demonstrations of Newton’s second law can be demonstrated.

A very simple demonstration to use with the “spring balance cart” that should help students to begin thinking about the meaning of balanced and unbalanced forces is to hold the cart as a hanging mass extends the scale giving a reading as illustrated below:

The students may realize that the scale reads the weight of the hanging mass but the important question is to ask is: “What will the scale read just after you release the cart?”

Many will conclude that now that the cart is accelerating, more force is required without appreciating that the forces were originally in opposite directions and the spring balance appeared to be measuring only the force supplied by the hanging mass.

When the cart is released, only the force supplied by the “hanging” mass is applied but since this mass is now accelerating downward, the tension in the string (as measured by the spring balance) must necessarily be reduced.

The next step in the discussion might be to have the students analyze the forces acting on the hanging mass before and after it begins accelerating downward.
What happens in elevators when they are accelerated up and down?

Popular sets of questions that illustrate Newton’s second law involve what happens to your “weight” as you accelerate up and down in an elevator. A nice simple set of demonstrations that can be done to help students understand the related concepts involves nothing more than a large demonstration spring scale attached through a string to a kilogram mass.

The idea is to move the scale and mass up and down in constant velocity and with acceleration and have the students anticipate what the scale will read in several different situations. To make it possible for the readings to be easily seen, it will be necessary to lower the mass all the way to the floor and to keep it moving for as long a distance as possible. Constant velocity is fairly easy but acceleration can be challenging. You can look at the scale while you are making the desired motion and this feedback will help you to achieve the desired result. Make sure the students understand that starting and stopping motion is always an acceleration and will give a reading that may different from what you desire them to see after the motion is initiated.

If you use a kilogram mass and the scale is calibrated in Newtons, you can suggest that the reading on the scale will be approximately equal to the numerical value of the acceleration of gravity and anything less or more than “10” will indicate an upward or downward acceleration and the amount the reading differs from “10” will be the acceleration in m/s².

California Physics Standard 1c

Send comments to: layton@physics.ucla.edu

1. Newton's laws predict the motion of most objects.

As a basis for understanding this concept:

- Students know to apply the law F=ma to solve one-dimensional motion problems involving constant forces (Newton’s Second Law).

By the time students reach High School, they should be quite familiar with how to plug into formulas. However, any strong advocate of the beauty of physics should try to bring his/her students to appreciate that Newton’s second law is not “just another formula”. Nevertheless, using Newton’s second law requires that the students understand where the units come from for each term in the expression and how to keep track of these units when solving problems. Some teachers prefer a “unit cancellation” or “factor label” method and others prefer a “systems approach”. In the former the units are carried along with the numerical values and treated as algebraic quantities. In the latter, each numerical value is placed in the same system of units at the outset and the final answer
will therefore be in the appropriate unit of that system. Each method has its advocates and arguments can be made in favor of each.

The following should be carefully discussed with students:

**Mass** is a basic quantity and cannot be defined in terms of other quantities. Mass has two properties. One is its property of inertia, called its “inertial mass” and the other is its gravitational property called its “gravitational mass”. Inertial mass gives objects their tendency to resist acceleration and is the “m” in Newton’s second law. Gravitational mass gives objects their tendency to gravitationally attract other objects, and is the “m” and “M” in Newton’s law of Universal Gravitation. The unit of mass we will use in this class is the kilogram and a kilogram of inertial mass equals a kilogram of gravitational mass.

**Acceleration** is the time rate of change in velocity or, \( a = \frac{\Delta v}{\Delta t} \). Velocity is the time rate of change of position or, \( v = \frac{\Delta s}{\Delta t} \), and the units of velocity are meters per second (m/s). It follows that acceleration is measured in units of meters per second per second or meters per second squared (m/s²).

**Force** is a push or a pull and the unit of force we will use in this class is the Newton (N). One Newton will accelerate one kilogram at the rate of one meter per second squared. That is, we can use Newton’s second law to define the unit of force and one Newton equals one kilogram meter per second squared or, \( N = kg \ m/s^2 \).

All of the above will be very clear to someone who has taught physics but it will take a little time for students new to these concepts to appreciate all they represent. However, it is important to help students to have a clear understanding of all of these basics discussed above. Finally, it might help to have the students know that on the surface of the earth a kilogram mass weighs 9.8 N and this equals 2.2 pounds. This can become the essential conversion factor between Newtons and pounds.

**A student activity to introduce Newton’s second law:**
The following activity involves a minimum of equipment and will introduce students to the essential idea behind Newton’s second law and give some idea of the influence of the force of friction. The equipment required is several books (it’s best if they are all the same), a piece of string and a spring balance (or “force meter”), a meterstick and a stopwatch.

The basic idea behind the experiment is to measure the force necessary to pull a book across a tabletop at constant velocity and then increase this force by a measured amount and measure the resulting acceleration. More books can be piled on top of the first book to investigate how this increases the friction force and how the increased mass influences the force required to accelerate this mass.

Students should learn that a larger force is required to overcome standing friction than is required to move the book at constant velocity. They should also come to appreciate that the unbalanced force required to accelerate the
book is the difference between the force required to move the book at constant velocity and the larger force required to accelerate it. Calculating the acceleration can be made from measuring the time for the book to move from rest while accelerating a measured distance. (Simply applying \( s = \frac{1}{2} at^2 \) will give an average acceleration). The results from this activity are not too accurate but the experience with such simple equipment can give rise to lots of discussions about the basics of friction and how an unbalanced force is required to cause a mass to accelerate.

California Physics Standard 1d  
Send comments to: layton@physics.ucla.edu
1. Newton's laws predict the motion of most objects.
   As a basis for understanding this concept:
   d. Students know when one object exerts a force on a second object, the second object always exerts a force of equal magnitude and opposite direction. (Newton's Third Law).
   The main misconception that prevents students from understanding Newton’s Third Law is that objects can push even if they are not alive. If you push down on a table with your hand, the table pushes back on your hand exactly as hard only in the opposite direction. It takes experience and some convincing to bring students to understand that tables (and other inanimate objects) can push. The following simple activity has been used in elementary and high school with some success in helping to overcome this difficulty.

Activity to show that for every action force, there is an equal and opposite reaction force.
It is probably best if students work in pairs. The only equipment required for each pair is a binder clip and a rubber band. When you open a binder clip you always push in one direction with your thumb and the opposite direction with your finger. These are equal and opposite forces. Now try to open the binder clip having your partner push on one side of the clip and you on the other. (Carefull—don’t let the clip slip and pop into someone’s face!) If you are careful, you can do it. You and your partner are supplying equal and opposite forces and the clip will open.

How could just one person open the clip by only pushing on one side of the clip?
If you placed one side of the clip against a wall or down on the surface of a table, you could open the clip by pushing only on the other side. But what must the wall be doing to the other side of the clip? Just as your partner had to push on the other side of the clip to open it, so must the wall be pushing on the clip to open it. The amazing thing about Newton’s third law is that on the one hand it is so simple but on the other if you don’t appreciate that it is always true, you might not understand how walls and tables can push. The law also works for pulling (after all, a force is a push or a pull) and you can verify this by repeating the above experiments except by nulling on a rubber band.
1. Newton's laws predict the motion of most objects.
   As a basis for understanding this concept:
   e. The relationship between the universal law of gravitation and the effect of gravity
      on an object at the surface of the Earth. (Note: Framework is not concerned with the
      law of universal gravitation in this section. The only discussion is of objects falling with
      constant acceleration near the surface of the earth. If you are interested in the law of
      universal gravitation, see 1 m*.)

When introducing how objects fall near the surface of the earth, don’t overlook doing the
simple demonstration of dropping two objects of different weight at the same time to
compare the time it takes for them to reach the floor. I have a 1.5” steel sphere and a 0.5”
steel sphere and start the discussion by asking: “how many of the smaller spheres will it
take to weigh the same as the larger sphere?”

   I have been amazed to learn how few students can give the correct answer.
   For dramatic effect I set up a simple balance and with a large number of the
   smaller steel spheres and establish that about 27 of the smaller spheres are
   required to weigh as much as the larger sphere.

After establishing the ratio of the masses of the two objects you plan to drop (27 in the
case above) discuss with your students if they think the larger object will fall “27” times
faster than the smaller object. They probably will disagree but this was one of the
arguments Galileo suggests in his famous cannon ball vs. musket ball dialogue. Galileo
even suggests the cannon ball will fall slightly faster but certainly not by a difference
suggested by the ratio of their weights. When you perform the simple dropping
experiment, they should strike the floor at almost the same time (repeat for effect and be
aware that letting go at even a slightly different time will influence the results.)

When all students are convinced that large objects and small objects fall at nearly the
same rate of acceleration, have them discuss in groups why they think this is true.

An intuitive argument might suggest a large object would be expected to fall faster. But
the large object also has more inertia hence would be expected to require more force to
accelerate it. The amazing truth that whatever gives a larger object a larger force of
attraction to the earth, also gives it a larger inertia, hence a larger resistance to
acceleration. Newton found it truly amazing that the weight and inertia were always in
the same ratio. It may be a little early to discuss with your students the difference
between inertial mass and gravitational mass but if you do, be sure they understand that
gravitational mass is not weight. It remained for Albert Einstein and the general theory
of relativity to help the rest of us to come to a better understanding of all of this.
1. Newton’s laws predict the motion of most objects.

As a basis for understanding this concept:

f. Students know applying a force to an object perpendicular to the direction of its motion causes the object to change direction but not speed (for example, the Earth’s gravitational force causes a satellite in a circular orbit to change direction but not speed). (See discussion at the bottom of this page.)

A very useful device for a teacher to always have on hand when discussing circular motion is a rubber stopper on the end of a string. This simple device is constantly useful when illustrating assorted concepts in circular motion. Since it is rubber, if you are careful, you can release it at an appropriate time and demonstrate that it will move off in a straight-line tangent to the curve at the point of release. It is advised that you arrange to have the stopper directly in front of you facing the class when you release it. Students who feel that the “outward centrifugal force” will cause the stopper to move at the class will be surprised to see it move off at a tangent, continuing in the same direction as it was going when released. As you rotate the stopper at constant speed, ask the class:

“What must be the direction of the force the string is pulling on the stopper as I pull it into this circular path?” As you swing it around, emphasize how you are pulling on the string to keep the stopper turning in the circle and should you stop pulling at any time, the stopper will move off in a straight line.

Another simple demonstration that is lots of fun is to place a marble or a small steel ball inside of a metal can and race the marble around in the can by rapidly shaking you hand. Ask the students: “What is the direction of the force that the can must exert on the marble to keep it moving in a circle inside of the can?”

The Standard suggests discussing how the gravitational force acts at right angles to a satellite’s motion as it moves about the earth. However, this is true only if the satellite moves in a circular path. It would be very instructive to discuss what happens to the satellite when the gravitational force is not at right angles to its motion and, say, has a component of force tangent to its direction of motion. The illustration shows a satellite moving around the earth in top view. The motion of the satellite is counter clockwise.

When the satellite is in the “top” part of its orbit and moving to the left, the force of gravity has a component in its direction of motion and thus speeds the satellite up. Likewise when the satellite is in the “bottom” part of its orbit and moving to the right, a component of the force of gravity is against its direction of motion, slowing it down.
1. Newton's laws predict the motion of most objects.

As a basis for understanding this concept:

g. Students know circular motion requires application of a constant force directed toward the center of the circle. (Note: Here the Standard means “constant” in magnitude only since a change in direction obviously changes the force.)

Since students are usually introduced to acceleration in the context of straight line motion, they frequently only think of acceleration as “speeding up” or “slowing down.” A discussion of centripetal acceleration provides an excellent opportunity to stress that velocity and acceleration are vectors and should they stay at constant magnitude, they are changing even if they only change direction. Also, since Newton’s second law is a vector equation, the vector force is always in the same direction as the vector acceleration. These centrally directed quantities are called “centripetal force” and “centripetal acceleration”. Finally, when an object moves in a circle, the velocity is always changing direction such that the change is directed toward the center of the circle. Hence, the acceleration and the force are therefore directed toward the center of the circle.

The conical pendulum lab to study centripetal force.

A simple lab that gives reasonably good results involves the use of a “conical pendulum.” The essential equipment is a pendulum about 1 meter long, a meter stick, a clock with a sweep second hand or a stop watch. If basic trigonometry is a challenge, a spring balance can be used to eliminate the need for trig. The basic idea behind the lab is to set the pendulum moving in a circle so that the string of the pendulum sweeps out a cone. (Several launches may be necessary to have the pendulum move in a circle.) If the pendulum is near the floor, the radius of the circle can be measured and the time it takes for the pendulum to make a recorded number of cycles can be measured. Since the friction is so low, the orbits will repeat many times without appreciable decrease in the radius of the orbit. The experiment can be repeated using several different radii of the launch orbit. (Warning, students will not expect the period to come out the same no matter the radius!)

At this point, with a little trig, there is enough data to test the basic expression for centripetal acceleration. Classes without trig can use a spring balance to weigh the pendulum bob (dividing by “g” will yield its mass) and then use the spring balance to pull the bob aside to the assorted radii used earlier. This will give the inward directed force hence making it possible to compare with $F = m \frac{V^2}{r}$.\[\text{fig:conical_pendulum}\]
There are many interesting problems and applications involving circular motion. The following is simply a discussion of some of these that seem to be the most interesting and perhaps confounding:

1. A car can accelerate either by speeding up, slowing down or turning a corner. Which of these three do you think can produce the greatest acceleration in the average automobile? What about the average bicycle?

The answer can be argued, but it is interesting to suggest that students look at the way the tread on the tires are grooved. Particularly with bicycle tires, the grooves are usually around the circumference of the tire (not across the tire) suggesting that the suspected larger forces will come from turning rather than breaking or speeding up.

2. If you are in a freely falling elevator you will seem weightless since the floor of the elevator accelerates downward at the acceleration of gravity, as do you. Space stations circle the earth and the astronauts seem to be weightless for the same reason. What must be the speed of a satellite in circular orbit?

First, it must be appreciated that most space stations orbit very near the earth so the radius of the orbit is essentially the radius of the earth. A plug into \( g = a_c = \frac{V_T^2}{r} \) and solving for \( V_T \) will yield the answer.

A great demonstration is to place water into a bucket, or can you have fixed with a bail and attached to a string, and whirl it in a vertical circle at a fast enough speed to prevent the water from falling out. However, it is best to discuss with the class before you do the demonstration the rate you must accelerate the bucket downward in a straight line so that the bucket and the water fall at the same rate. (Students need to know that linear acceleration and centripetal acceleration are the same thing.)

3. After the discussion and demonstration, measure the radius of the bucket’s swing and have the students calculate the minimum tangential velocity at the top of the swing.

In Standards 1g and 1l* the expression for centripetal force and centripetal acceleration are given without derivation. Your class might appreciate a derivation of these expressions.
Here is the result for centripetal acceleration:
\[ a_C = \frac{v_T^2}{r} \] where \( a_C \) is centripetal acceleration, \( v_T \) is tangential speed and \( r \) is the radius of the circle.

What follows is a derivation of the very important relationship for centripetal acceleration:

Derivations of this very useful “formula” can be found in any college physics text but they usually involve some calculus. The following derivation might be useful to help good students to an appreciation where the “formula” comes from and only uses “calculus” to the extent that students recognize “instantaneous” velocity and acceleration are the ratios of \( \Delta s/\Delta t \) and \( \Delta v/\Delta t \) in an “instant”. That is, when \( \Delta t \) goes to zero.

The illustration on the left represents an object moving in a circle at constant speed \( V \). The radius of the circle is \( S \), and \( S_0 \) is the initial position of the object, and \( S_1 \) is the position of the object at some small time later, \( \Delta t \). The change in position of the object is therefore, \( \Delta S \). The velocity \( V \) is always at right angles to the radius \( S \) as it changes in direction from the original velocity \( V_0 \) to \( V_1 \). Since the object moves at constant speed, the length of the \( V \) vectors remain the same. Now let us move the velocity vectors so their tails come together making it is possible to measure \( \Delta V \).

\[ V_0 \]

The illustration on the right shows the original velocity \( V_0 \) and \( V_1 \) tail to tail, hence the change in velocity is \( \Delta V \). Since the velocities are always perpendicular to the radius vectors, they sweep out the same angle and therefore the triangle \( S_0 \Delta S S_1 \) is similar to the triangle \( V_0 \Delta V V_1 \). Since corresponding parts of similar triangles are in the same ratio, it follows that \( \Delta V/V = \Delta S/S \).

Solving this for \( \Delta V \) gives \( \Delta V = V \Delta S/S \). Dividing both sides of this relationship by \( \Delta t \) gives \( \Delta V/\Delta t = V \Delta S/\Delta t /S \). But, when we make \( \Delta t \) very small, this equation becomes \( a = \frac{VV}{S} \) (from the definition of acceleration and velocity. Since \( S \) is the radius of the original circle we can write the equation \( a_C = \frac{V_T^2}{r} \) where \( a_C \) is the centripetal acceleration and \( V_T \) is the tangential speed.

It is also easy to show that the units of this equation are correct.

Finally, since \( F = ma \), it naturally follows that centripetal force \( F_C = ma_C = m(v_T)^2/r \).
California Physics Standard 1h*  Send comments to: layton@physics.ucla.edu  F13

1. Newton’s laws predict the motion of most objects.
   As a basis for understanding this concept:
   h* Students know Newton's Laws are not exact but they provide very good approximations unless an object is moving close to the speed of light or is small enough that the quantum effects are important.

This purpose of this Standard is probably only to let students know that Newton’s laws are not all powerful. However, it also wouldn’t hurt for the teacher to point out that they worked perfectly well, without exception for almost four centuries and even with the high speed and size limitations, they work perfectly well in all but the most extreme speeds and at the smallest dimensions. Newton’s laws of motion work without measurable error at even the greatest speeds that humans have ever experienced. They also work without measurable error down to the smallest dimensions that humans can visualize. On the other hand, at the speeds regularly experienced by sub atomic particles in high-energy accelerators and at small sizes approaching nanometers, Newton’s laws start to show significant errors.

Probably a good way to explore these limitations is to show students one of the many video presentations on special relativity (for example from the Mechanical Universe) and then show the amount of error that develops for speeds even as high as 0.1 c. Some appreciation of the small size limitations of Newton’s laws will require ideas related to waves, interference and diffraction which will be taken up in another section of the Standards.

Note: The discussion in the following Standard, 1i*, requires knowledge of vectors and their components. For this reason, it might be best to do Standard 1j* before doing this Standard.

California Physics Standard 1i*  Send comments to: layton@physics.ucla.edu

1. Newton's laws predict the motion of most objects.
   As a basis for understanding this concept, students know:
   i. * How to solve two-dimensional trajectory problems.

The key to solving projectile problems is to resolve the initial velocity into components perpendicular and parallel to the direction of the force of gravity. The motion perpendicular will keep its initial velocity component and continue to move at this constant velocity. The motion parallel (or anti parallel) to the force of gravity will be accelerated.

In the illustration below, the initial velocity, $V_0$ has been resolved into horizontal and vertical components $V_{0x}$ and $V_{0y}$. At any later time the horizontal component of the velocity will be the same as it was originally. However, the vertical component will be accelerated downward at rate $g$, hence the vertical component of velocity will be given by $V_y = g(t)$. To find the instantaneous velocity of the projectile at a later time, simply find the Pythagorean sum of the horizontal and vertical components of velocity.
To find the horizontal distance the object has moved some time \((t)\) after being released, simply multiply the original horizontal velocity, \(V_{0X}\) by the time \((t)\). To find the vertical height of the object after some time \((t)\), plug the time into \(h = \frac{1}{2} gt^2 + V_{0Y} + h_0\). Projectile motion thus becomes an exercise of using our knowledge of kinematics with two vector components—one that keeps a constant velocity and the other that is accelerated.

**Projectile motion into a “target can”, activity.**

A very popular activity to engage students at the beginning of a presentation of projectile motion is to set up a way to measure the horizontal velocity of a steel ball as it leaves the edge of a table and, knowing the height of the table above the floor, have the students predict where it will land. Provide some kind of launch ramp that can be repeated at a later time. The horizontal track can be two rods pressed together and taped to the surface of the table. The length of this horizontal track, \(X\), as well as the height of the center of the ball above the floor at launch, \(H\), should be carefully measured and provided as data. Make several runs and perhaps have several students time how long it takes for the ball
to cross the distance X, but make sure to catch the ball so they cannot see where it strikes the floor. Finally, have the students calculate the distance from the table that the can must be placed to score a hit. (If the can is tall, its height must also be included.)

**Monkey and Hunter Demonstration:**

If you do not already own the apparatus for this spectacular demonstration, you should obtain it. It can be either purchased from an equipment supplier or if you are strapped for cash, it is a worthwhile project to construct yourself or to encourage talented students to construct. The idea behind the demonstration is that a gun is carefully aimed at a “monkey” that will drop from a “tree” exactly when the gun is fired. The bullet will always intercept the falling “monkey” no matter the initial speed of the bullet—as long as the bullet reaches the monkey while in flight.

The gun is a blowgun consisting of a cylindrical tube with a close fitting steel ball. The end of the gun has an electrical contact that will break the circuit to an electromagnet when the steel ball leaves the barrel. The gun must be rigidly clamped and aimed precisely at the “monkey” while it hangs from the electromagnet. Since the time of flight of the bullet is exactly the same as the “monkey”, the bullet will fall away from its straight line path exactly the same distance that the “monkey” falls, insuring a hit. The apparatus can be challenging to get working properly but it is always rewarding.

Details on the monkey and hunter demonstrations can easily be found by Googling: “Monkey and Hunter Demonstration”. Many pages will come up. One that this author wrote many years ago can be found at:

http://physics.usc.edu/~shaas/workshop/Monkey_and_Hunter.htm

**California Physics Standard 1j* Send comments to: layton@physics.ucla.edu**

**1. Newton's laws predict the motion of most objects.**

As a basis for understanding this concept:

**j * Students know how to resolve two-dimensional vectors into their components and calculate the magnitude and direction of a vector from its components.**

Vector math can be approached at many different levels. A good way to start in high school physics would be with a “graphical” method involving drawing vectors. As the students come to understand the graphical method, they can be introduced to methods involving basic trigonometry. An interesting way to begin a discussion of vector math is to ask students: “What is the sum of 3 plus 4?” Their knowledge of scalar math will
suggest “7” but when you say no, it could be 1, or 3 or 5 or 7 or anything between 1 and 7, they may be surprised. (To be realistic, however, I hope they have been introduced to vectors in their math classes and will only need to be reminded that assumptions were being made as to what “3” and “4” meant.)

Vectors are different from scalars and the mathematics of vectors is different from scalar math. While scalars only require one number to be specified, vectors require two. A vector can be described with a magnitude and an angle or it can be described in terms of its X and Y components.

The vector sum of several vectors is found by placing the vectors point to tail. The resultant is a vector that points from the tail of the first to the head of the last.

An interesting exercise to introduce students to the graphical method of vector addition is to give them a “treasure map” exercise with all sorts of colorful directions that are essentially angles and distances you must pace to find the treasure. Using protractors and rulers, the students plot out these pacing instructions on the map. When they finish, suggest it might have been easier for them to simply sum all the components in one direction (say, North) and then to sum all the components at right angles to this (say South) and use the Pythagorean theorem to solve for the final distance. This should introduce the “component method” of vector addition.

An important formula is this: given a rectangular coordinate system and resolve the vectors into components in this coordinate system. Take the algebraic sum of these components in each of the two rectangular directions. Use the Pythagorean theorem to find the magnitude of the final resultant. The angle of the resultant is found using the arctangent.

\[ \vec{R} = \sqrt{X^2 + Y^2} \]
\[ \theta = \arctan \frac{Y}{X} \]

Essential approach to Trigonometry in Physics.

The trig needed in High School physics is minimal and essentially amounts to knowing the definitions of the sine, cosine and tangent and the meaning of arc functions. However, these ideas are used so frequently, students will profit from learning to apply...
the trig functions directly and quickly without extra algebraic steps. Given a right triangle in any orientation with one of the acute angles specified, a student should immediately be able to point to the adjacent side and know it is the hypotenuse times the cosine of the angle and point to the opposite side and know this is the hypotenuse times the sine of the angle.

Much time will be saved if students can immediately recognize the correct trig function and appropriate side related to the location of the angle. This is particularly useful when finding components of vectors.

California Physics Standard 1k*  Send comments to: layton@physics.ucla.edu

1. Newton's laws predict the motion of most objects.

As a basis for understanding this concept:

k* Students know how to solve two-dimensional problems involving balanced forces (statics).

Statics is an excellent application of vectors and helps bring students to understand that walls and tables can push since all the forces acting on an object at rest must add to zero. Although the standard does not specify having a discussion of friction, this concept seems so important to physics we will discuss it here.

Demonstrating static friction and kinetic friction:

A simple demonstration of most aspects of friction can be made using a small cardboard box large enough to hold 4 cylindrical kilogram masses. Place a wire hook on one end of the box so a demonstration spring balance can easily pull it.

With this device you can demonstrate how an increased normal force influences the force of sliding friction by simply loading more kilogram masses (that weigh “10 N”) into the box and observing the linear increase in the force required to pull the box. It also clearly shows the difference between static and sliding friction since as long as the box is not moving, you can apply any force. However, after it starts to move, the force will drop to a lower value and remain essentially the same no matter how fast you move it at constant velocity. A student activity can also be done using spring balances and books. (See details of this activity discussed earlier in Forces and Motion 1c.)
**Forces on an inclined plane:**
Discussing friction on an inclined plane provides an excellent way to use vectors as well as helping students to understand the meaning of free body diagrams and the meaning of normal force.

A small cart can be loaded with a mass that, ideally, makes the cart plus the mass weigh 10 N. When this cart is placed on an adjustable inclined plane, a demonstration spring balance can be used to show how the force parallel to the plane and the force normal to the plane will add to give the weight of the cart. Measuring the angle of the plane with the horizontal will give the angle between the normal and the weight. These values can then be used to show the vector sum of the force parallel and the normal force will always add to give the weight.

**Tension:**
The concept of tension can be a little confusing when first encountered. To appreciate that this, “force of a string”, always pulls away from the object it is attached to yet at any point on the string it is equal and in opposite directions, requires a little thought and familiarity. The following demonstration might help students to come to a better understanding of tension.

With two low friction pulleys held apart using ring stand supports as illustrated, hook a string to either side of a large spring balance and pass these strings over the pulleys and have them support a kilogram mass on either end. Turn the spring balance so the students can’t see what it reads and ask them for their conclusion as to the reading on the scale. (Before doing this demonstration, it is well to show the students what each mass weighs.)

Have the students vote as to their individual feelings about what the scale will read. Past experience suggests that you will get three answers: twice the weight of a single mass, the weight of a single mass and zero. Other answers can appear but these three are the most common. As you discuss why the reading is the weight of the mass, remind them of all the forces on the spring balance when you simply weigh the mass. (It is easy to forget that you are pulling up on the spring balance as the mass pulls down.)
Problems involving tension:

A typical problem involving tension (discussed in the California Science Framework) involves the tension in the support strings on a hanging picture.

There are several ways to approach this problem, two of which are illustrated on the right. One is to look at the forces acting on the nail supporting the picture (illustrated above the picture.) The upward force must be the weight of the picture and assuming the tension in the string is the same on both sides and the angles are equal, it is easy to show: \(2T \sin \theta = W\). Another way to look at this problem is illustrated to the right of the picture. Assuming that the right support holds half the weight of the picture, the vertical component of the tension is given by: \(T \sin \theta = \frac{1}{2}W\).

The impossibility of pulling a string perfectly horizontal:

The hanging picture example can lead to a question: “Would it be possible to support the picture with the string straight and parallel to the floor?” That is, what happens to the tension in the string if we try to make \(\theta\) zero?

This question quickly leads to the inverted question: Is it possible, by pulling outward to pull a string supporting a weight into a straight horizontal line. Finally, there is a popular demonstration involving a long strong rope held on either end by two strong students. Then invite a smaller less robust student to try to press the center of the rope to the floor as the two strong students attempt to hold the rope from touching the floor. If the rope is long enough, almost anyone will be able to pull the center of the rope down to the floor.

A practical consequence of this is that telephone lines are always stretched with some droop between poles. Apparently, was this not the case, in winter, accumulations of ice could weight down the wires to an extent that the wires could break.
A simple classroom experiment in statics involving vector addition:

A very nice experiment can be done in small groups with several spring balances, some string, a washer and a piece of paper. In the illustration three students pull outward on the washer carefully holding a constant value while a forth student sketches the position of the center of the washer, the direction the strings are pulling and writes on each string drawing the reading on each of the spring balances. This “data sketch” can now be used to construct the force vectors involved and test to see if they sum to zero. Other string and spring balance configurations can be easily repeated on a separate piece of paper.

An important topic in statics that the Framework and Standards omit:

No mention is made of applications involving torque. The Framework even suggests discussing ladders leaning against walls, which, without considering torque, can only be solved in the simplest cases. Torque will not be discussed here, with regrets.

California Physics Standard 1 l*

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1. Newton's laws predict the motion of most objects.

As a basis for understanding this concept:

i.* Students know how to solve problems in circular motion, using the formula for centripetal acceleration in the following form: \( a = \frac{v^2}{r} \). (Note. Standard 1f relates to this and seems more concerned with the direction of centripetal force than how to compute it. In our discussion of Standard 1g in these papers, experiments to measure centripetal force (the conical pendulum experiment) and a derivation of the expression \( a_c = \frac{v^2}{r} \) were given.)

As was discussed in 1f previously, students need to understand that an acceleration resulting in changing direction is the same dynamically as an acceleration that requires speeding up or slowing down. Swinging a bucket containing water in a vertical circle might help to make this clear. It would be possible to accelerate the bucket downward at or greater than the acceleration of gravity, with the same result. However, swinging the bucket in a vertical circle is an easier way to maintain the necessary downward acceleration at the top of the circle. The following problem might help explain this concept:
A bucket containing a 1 kg mass is accelerated downward at 12 m/s². What force must the bottom of the bucket supply to the mass to cause this downward acceleration?

The illustration on the left shows the inverted bucket with the mass “resting” on the bottom while it accelerates downward. The total force downward on the mass must be its weight plus the downward force of the bottom of the bucket. Or, \( F_T = W + F_B \). Solving for \( F_B \):

\[
F_B = F_T - W = ma - mg
\]

In this case \( F_B = 1(12 - 10) \) N (assuming \( g = 10 \) m/s²). On the right is illustrated the same situation only the bucket is being swung in a vertical circle of a particular radius.

A second problem might be to ask for the necessary tangential velocity of the bottom of the bucket to maintain this downward force if swung in a circle of given radius. The idea of these two problems is to stress that a “linear” acceleration has the same effect as a “centripetal” acceleration.

**Experiments to show that** \( a_c = \frac{v_T^2}{r} \).

The conical pendulum experiment discussed in 1 f previously is easy to do and gives fairly good results. Another popular experiment involves swinging a rubber stopper around using a fire polished glass cylinder (ask your Chemistry teacher for help) with a nylon string passing through the cylinder and, either known weights, or a spring balance providing the centripetal force. A PSSC lab manual will describe this experiment in detail but, lacking that, the essential idea is illustrated below:

The section of glass cylinder is fire polished on either end and the nylon string passes through it. Using a hand on the glass cylinder, the rubber stopper can be made to move in a circle and the other hand pulls down on the spring balance providing the centripetal force. The timing of several rotations and then pressing a thumb over the top of the glass cylinder will freeze the length of the string in order to measure the radius. The reading on the spring balance will give the centripetal force and the weight of the rubber stopper will give the rotating mass. With the time of a single revolution and the circumference of the circle, the tangential velocity can be calculated.
This experiment takes lots of room and might best be done as a demonstration. The friction of the string as it passes over the fire polished glass cylinder will give a range of possible values for the tangential velocity. A nice feature of this experiment is that the students can clearly experience the centripetal force necessary to keep the rubber stopper in orbit.

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1. Newton's laws predict the motion of most objects.

As a basis for understanding this concept:

m* Students know how to solve problems involving the forces between two electric charges at a distance (Coulomb's Law) or the forces between two masses at a distance (Universal gravitation).

It seems that the objective here is to have students understand that satellites orbiting the earth and electrons orbiting atoms involve applying an inverse square law force to supply the centripetal force. The main idea to impress upon your students is that any force acting at right angles to the direction of motion will cause a centripetal acceleration. This Standard deals with gravitational force and electrostatic force. By using Coulomb’s Law or Newton’s Law of Universal gravitation to describe the centripetal force, and equating these forces to the expression: $F_c = \frac{mv^2}{r}$, assorted facts about orbiting objects can be revealed.

Helping students understand the inverse square law.

Any physical “essence” that spreads out uniformly into three-dimensional space will display an inverse square law. Coulomb’s law, Universal gravitation, light and sound from small sources, all display an inverse square law. Students can often be helped to understand this law by describing its basis with simple three-dimensional geometry.

Consider a quick burst of energy (like the sound from a gunshot) that spreads out into three-dimensional space. As the energy moves outward into space, it spreads thinner and thinner. The total amount of energy is confined to the surface of a sphere, hence the energy per unit area (intensity) must vary inversely as the surface of area of a sphere. The surface area of a sphere varies as the square of its radius, so it follows that the energy per unit area must vary inversely as the square of the radius. In the example of a gunshot, a microphone placed at different distances from the gun would measure the sound of the shot dropping off in intensity inversely as the square of the distance from the gun. The microphone has a constant area so it really is measuring the energy per unit area.
A demonstration experiment that illustrates the inverse square law:

(This experiment is probably best done as a demonstration since it involves a dark room and a single source of light.)

The equipment required for this experiment is a clear bright light bulb with a small filament, a light meter, a meterstick and a dark room. The smaller the filament of the bulb, the better. One of the best would be a 12V automobile tail light bulb but this would require a 12-volt power supply. Clear bulbs that use 120 Volts often have extended filaments but such bulbs can give fair results if the distance is not too small.

A light meter is nice but any photovoltaic device with a linear output can be used with an appropriate voltmeter. Simply place the meter at different distances from the bulb and measure the intensity. Make sure there are no reflections from light objects near the bulb as well as reflections from the tabletop.

If you have several light meters, a single light bulb in the center of the room with each group of students using a light meter might work. However, even reflections from the students could compromise the results. A demonstration usually works best.

Problems using Coulomb’s Law, Universal Gravitation and circular motion.

1. Assuming an electron moves in a circular path around the proton in a hydrogen atom, how much time does it take to make one revolution? (You can either have the students find the radius of the first Bohr orbit, or give them an approximate value. Unless you have discussed Coulomb’s law and related electrostatics, they will need to know the charge and mass of the electron, the Coulomb force constant and that the charge of the electron and proton are equal in magnitude.) This problem is fairly “formula pluggy” but should give students an appreciation of how to use Coulomb’s law and how to compute a basic fact about atoms.) As well as the time (period) of a single revolution, you might have them find the reciprocal or, orbital frequency.

2. What is the distance from the earth of a geosynchronous satellite? (This problem takes a little more math effort than the first but should involve ideas that the students know about. It gives you a chance to discuss what it must mean to have a satellite that always appears to stay at the same place in the sky, why this is useful for communication satellites, etc. A common minor error is to compute the distance from the center of the earth and fail to convert this to the distance from the surface of the earth, as requested.)