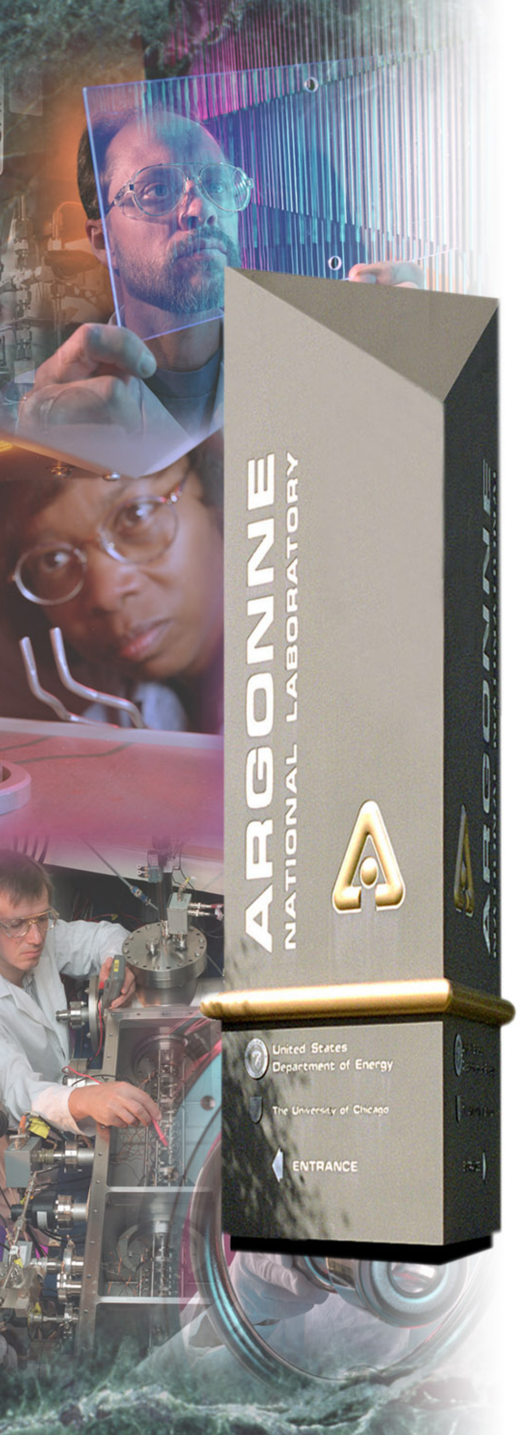


Emittance oscillation in the drift of split photoinjectors

Chun-xi Wang

Accelerator Physics Group/Advanced Photon Source

35th ICFA Workshop on the Physics and Applications of High Brightness Electron Beams
Erice, Sicily, Italy, October 9-14, 2005

Argonne National Laboratory

Advanced Photon Source

Office of Science
U.S. Department of Energy

A U.S. Department of Energy
Office of Science Laboratory
Operated by The University of Chicago

“double-minimum” emittance oscillation

HOMDYN STUDY FOR THE LCLS RF PHOTO-INJECTOR†

LNF-00/004 (P)
3 Marzo 2000

M. Ferrario[1], J. E. Clendenin[2], D. T. Palmer[2], J. B. Rosenzweig[3], L. Serafini[4]

SLAC-PUB 8400

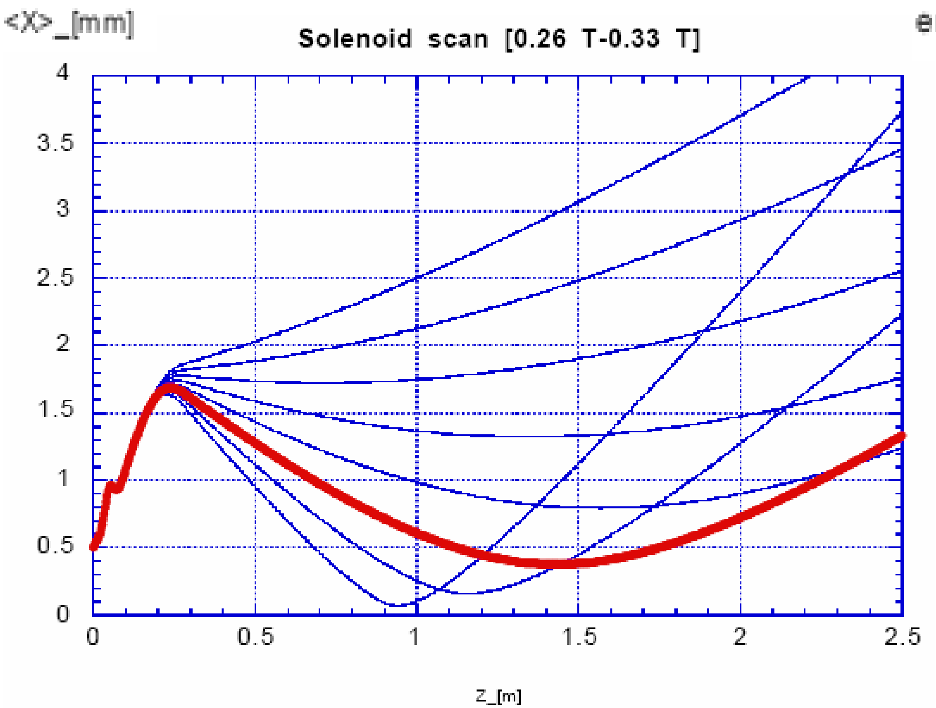


FIG. 10: Beam envelope versus z for different solenoid strengths.

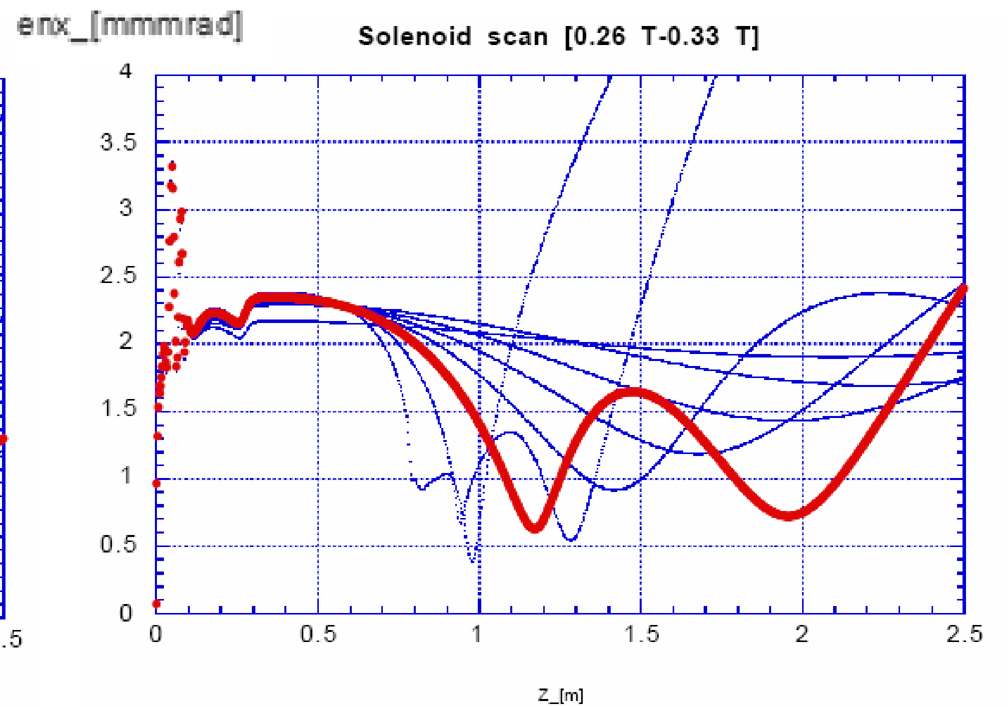


FIG. 11: Beam emittance versus z for different solenoid strengths.

Beam-envelope equation in drift space

$$\sigma'' + \frac{\gamma'}{\beta^2\gamma}\sigma' + K_r\,\sigma - \frac{\kappa_s}{\beta^3\gamma^3\,\sigma} - \frac{\epsilon_n^2}{\beta^2\gamma^2\,\sigma^3} = 0$$

no acceleration, no focusing, quasi-laminar

no invariant envelope, thus not compensated oscillation

assuming negligible change in perveance, let $\tau = \frac{\sigma}{\sqrt{\kappa_s/\beta^3\gamma^3}}$

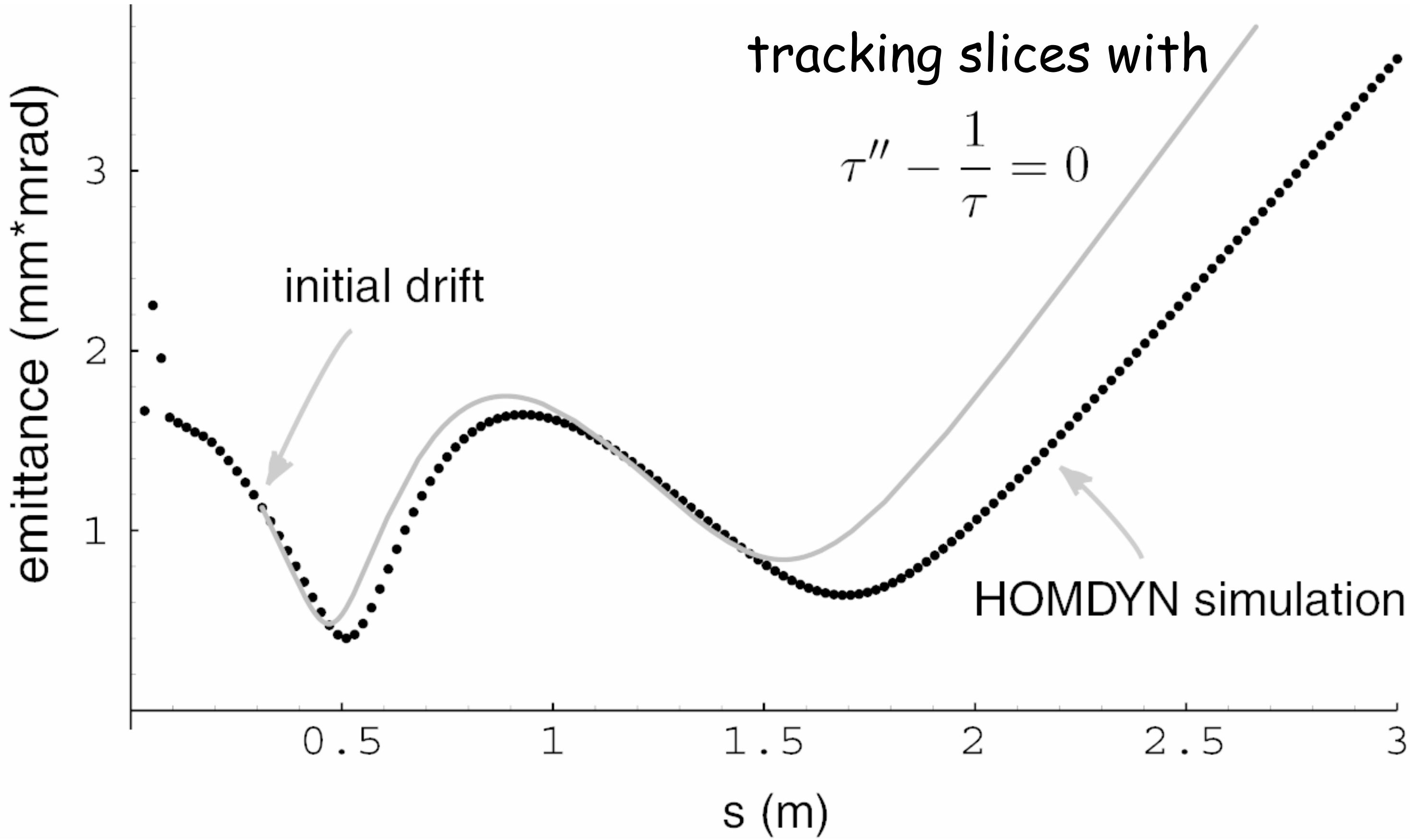
$$\tau'' - \frac{1}{\tau} = 0$$

a well-known universal equation for beam spreading under space charge

$$H = \frac{p_\tau^2}{2} - \ln\tau$$

Oscillations due to beam spreading

Example: an optimized SPARC design courtesy of M. Ferrario



the double-minimum feature is mainly due to beam-spreading

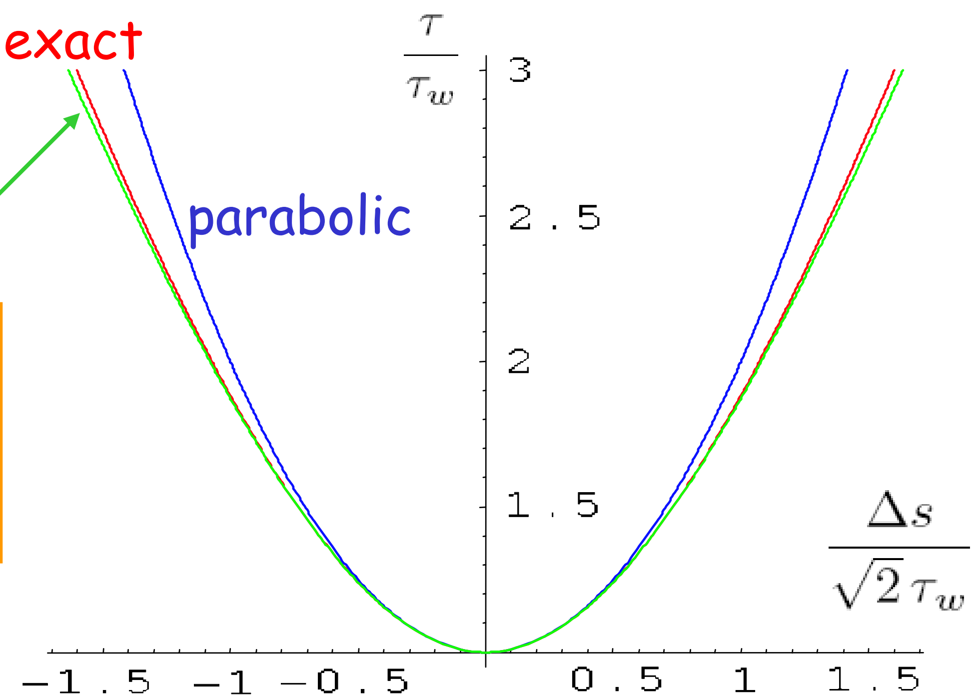
Universal beam-spreading curve

$$H = \frac{p_\tau^2}{2} - \ln\tau \;\Rightarrow\; \tau'^2 = \tau_0'^2 + 2\ln\frac{\tau}{\tau_0} \;\Rightarrow\; \tau = \tau_0 \exp\left(\frac{\tau'^2 - \tau_0'^2}{2}\right) = \tau_w\, e^{\tau'^2/2}$$

$$\frac{\Delta s}{\sqrt{2}\,\tau_w} = \pm\int_1^{\frac{\tau}{\tau_w}} \frac{dx}{2\sqrt{\ln x}} = \sqrt{\tau/\tau_w - 1} + (1/12)\,(\tau/\tau_w - 1)^{3/2} + \cdots$$

$$\frac{\tau}{\tau_w} \simeq 1 + \left(\frac{\Delta s}{\sqrt{2}\,\tau_w}\right)^2$$

$$\frac{\tau}{\tau_w} \simeq 3\sqrt{1 + \left(\frac{\Delta s}{\sqrt{3}\,\tau_w}\right)^2} - 2$$



Emittance calculation formulas (1)

rms emittance $\epsilon = \sqrt{\overline{X^2}\,\overline{P^2} - \overline{XP}^2}$, an expression difficult to manipulate

assuming $\begin{pmatrix} X \\ P \end{pmatrix} = \begin{pmatrix} \bar{X} \\ \bar{P} \end{pmatrix} + \sum_\alpha \begin{pmatrix} \partial_{q^\alpha} X \\ \partial_{q^\alpha} P \end{pmatrix} q^\alpha$ with $\bar{q}^\alpha = 0$ and $\overline{q^\alpha q^\beta} = 0$ for $\alpha \neq \beta$

$$\hat{X} = (\bar{X}, (\partial_{q^1} X) q^1_{\mathrm{rms}}, \cdots)$$

$$\begin{aligned}
\epsilon^2 &= \left| \overline{\begin{pmatrix} X \\ P \end{pmatrix} (X, P)} \right| \\
&= \left| \begin{pmatrix} \bar{X} \\ \bar{P} \end{pmatrix} (\bar{X}, \bar{P}) + \sum_{\alpha,\beta} \begin{pmatrix} \partial_{q^\alpha} X \\ \partial_{q^\alpha} P \end{pmatrix} (\partial_{q^\alpha} X, \partial_{q^\alpha} P)\, \overline{q^\alpha q^\beta} \right| \\
&= \left| \begin{pmatrix} \bar{X} \\ \bar{P} \end{pmatrix} (\bar{X}, \bar{P}) + \sum_{\alpha} \begin{pmatrix} \partial_{q^\alpha} X \\ \partial_{q^\alpha} P \end{pmatrix} (\partial_{q^\alpha} X, \partial_{q^\alpha} P)\, \overline{(q^\alpha)^2} \right| \\
&= \begin{vmatrix} \hat{X} \cdot \hat{X} & \hat{X} \cdot \hat{P} \\ \hat{X} \cdot \hat{P} & \hat{P} \cdot \hat{P} \end{vmatrix}
\end{aligned}$$

> 0 Cauchy-Schwarz inequality

$$\begin{aligned}
&= \left\| \hat{X} \times \hat{P} \right\| = \sum_{\alpha<\beta} \left(\hat{X}_\alpha \hat{P}_\beta - \hat{X}_\beta \hat{P}_\alpha \right)^2 \\
&= \sum_\alpha \left(\bar{X} \partial_{q^\alpha} P - \bar{P} \partial_{q^\alpha} X \right)^2 \overline{(q^\alpha)^2} + O\left(\overline{(q^\alpha)^2}\, \overline{(q^\beta)^2} \right),
\end{aligned}$$

Lagrange's Identity

Emittance calculation formulas (2)

➢ emittance due to current variation among slices

$$\epsilon = \left|\hat{\sigma}\partial_I\hat{\sigma}' - \hat{\sigma}'\partial_I\hat{\sigma}\right|_{I_p}\widehat{\delta I} = \hat{\sigma}(I_p)^2\left|\frac{\partial}{\partial I}\left(\frac{\hat{\sigma}'}{\hat{\sigma}}\right)\right|_{I_p}\widehat{\delta I}$$

$$\epsilon(z) \cong \frac{1}{\sqrt{2}}\,\sigma_0(\delta I_{\mathrm{rms}})\left|\frac{\partial}{\partial I}\left(\frac{\sigma'}{\sigma}\right)\right|_{I=I_p}$$ (Serafini & Rosenzweig, **PRE55**)

➢ two-slice emittance ($\hat{\sigma} = \sqrt{\beta\gamma}\,\sigma$)

$$\epsilon = \frac{1}{2}\left|\hat{\sigma}_+\hat{\sigma}'_- - \hat{\sigma}_-\hat{\sigma}'_+\right| = \frac{\beta\gamma}{2}\left|\sigma_+\sigma'_- - \sigma_-\sigma'_+\right|$$

provided that we let $\hat{\sigma}_+ = \hat{\sigma}_{I_p}$ and $\hat{\sigma}_- = \hat{\sigma}_{I_p} + \partial_I\hat{\sigma}\,\Delta I$, i.e., $\partial_I\hat{\sigma} = (\hat{\sigma}_- - \hat{\sigma}_+)/\Delta I$ and $\widehat{\delta I} = \Delta I/2$.

Oscillations due to initial spreads in τ_0, τ_0'

$$\epsilon = \sqrt{W_\tau^2 \frac{(\Delta\tau_0)^2_{\mathrm{std.}}}{(\tau_0)^2_{\mathrm{avg.}}} + W_{\tau'}^2 \frac{(\Delta\tau_0')^2_{\mathrm{std.}}}{(\tau_0')^2_{\mathrm{avg.}}}}$$

$$W_\tau = \left(\tau\partial_{\tau_0}\tau' - \tau'\partial_{\tau_0}\tau\right)\tau_0\,, \qquad W_{\tau'} = \left(\tau\partial_{\tau_0'}\tau' - \tau'\partial_{\tau_0'}\tau\right)\tau_0'$$

$$\partial_{\tau_0}\tau = \frac{\tau - \tau' s}{\tau_0} \qquad \partial_{\tau_0'}\tau = \tau_0\,\tau' - \tau_0'(\tau - \tau' s)$$

$$\partial_{\tau_0}\tau' = -\frac{s}{\tau_0\,\tau} \qquad \partial_{\tau_0'}\tau' = \frac{\tau_0 + \tau_0'\,s}{\tau}$$

note

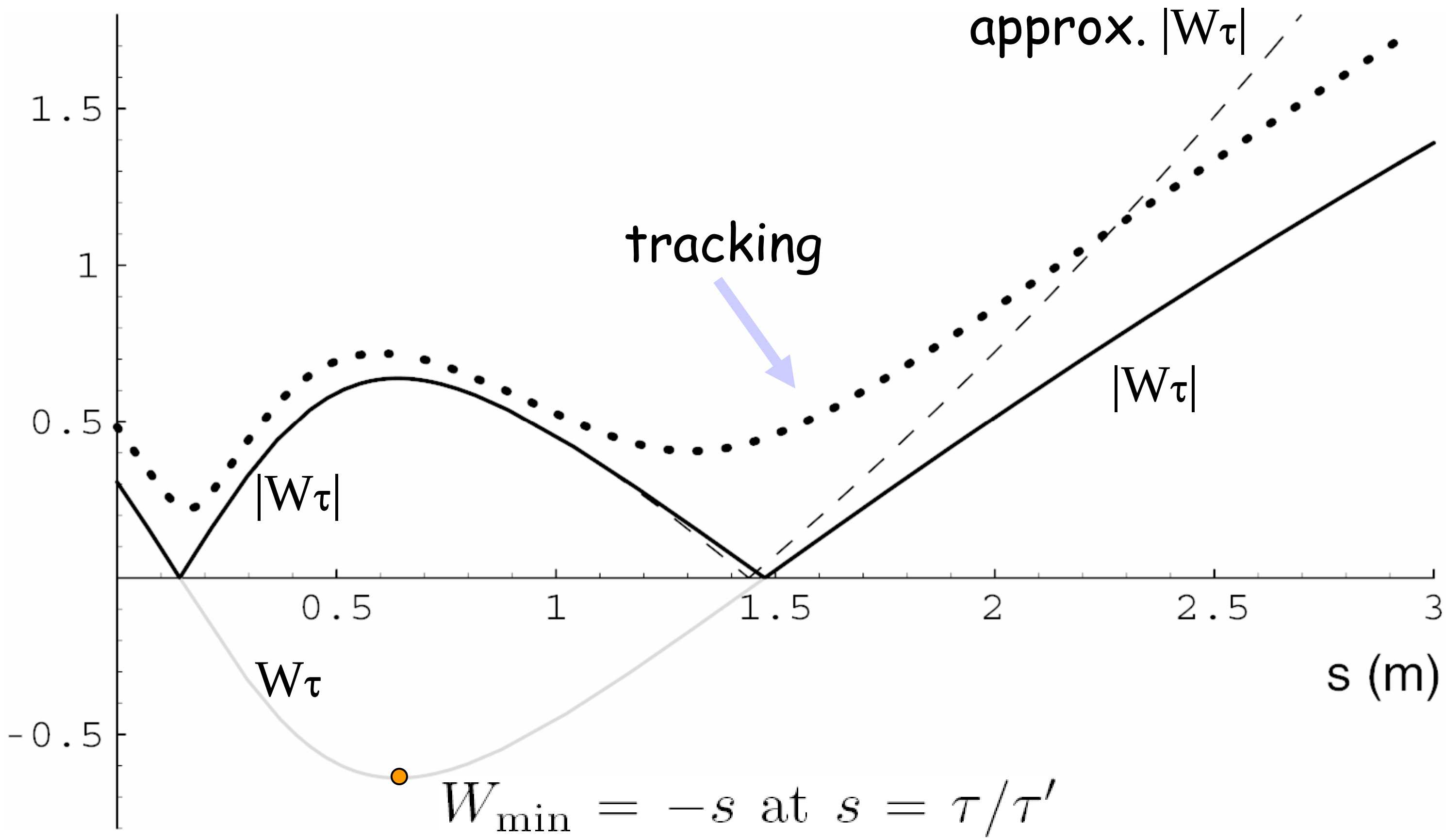
$$\frac{s - s_w}{\sqrt{2}\,\tau_w} = -\mathrm{sgn}[\tau'] \int_1^{\tau/\tau_{\mathrm{w}}} \frac{\mathrm{dx}}{2\sqrt{\ln \mathrm{x}}}\,, \qquad \frac{\mathrm{s_w}}{\tau_{\mathrm{w}}} = -\mathrm{sgn}[\tau'] \int_1^{\tau_0/\tau_{\mathrm{w}}} \frac{\mathrm{dx}}{2\sqrt{\ln \mathrm{x}}}$$

$$\tau'^2 = \tau_0'^2 + 2\ln\frac{\tau}{\tau_0}\,, \qquad \tau = \tau_0 \exp\left(\frac{\tau'^2 - \tau_0'^2}{2}\right) = \tau_w\, e^{\tau'^2/2}$$

Oscillations due to τ_0

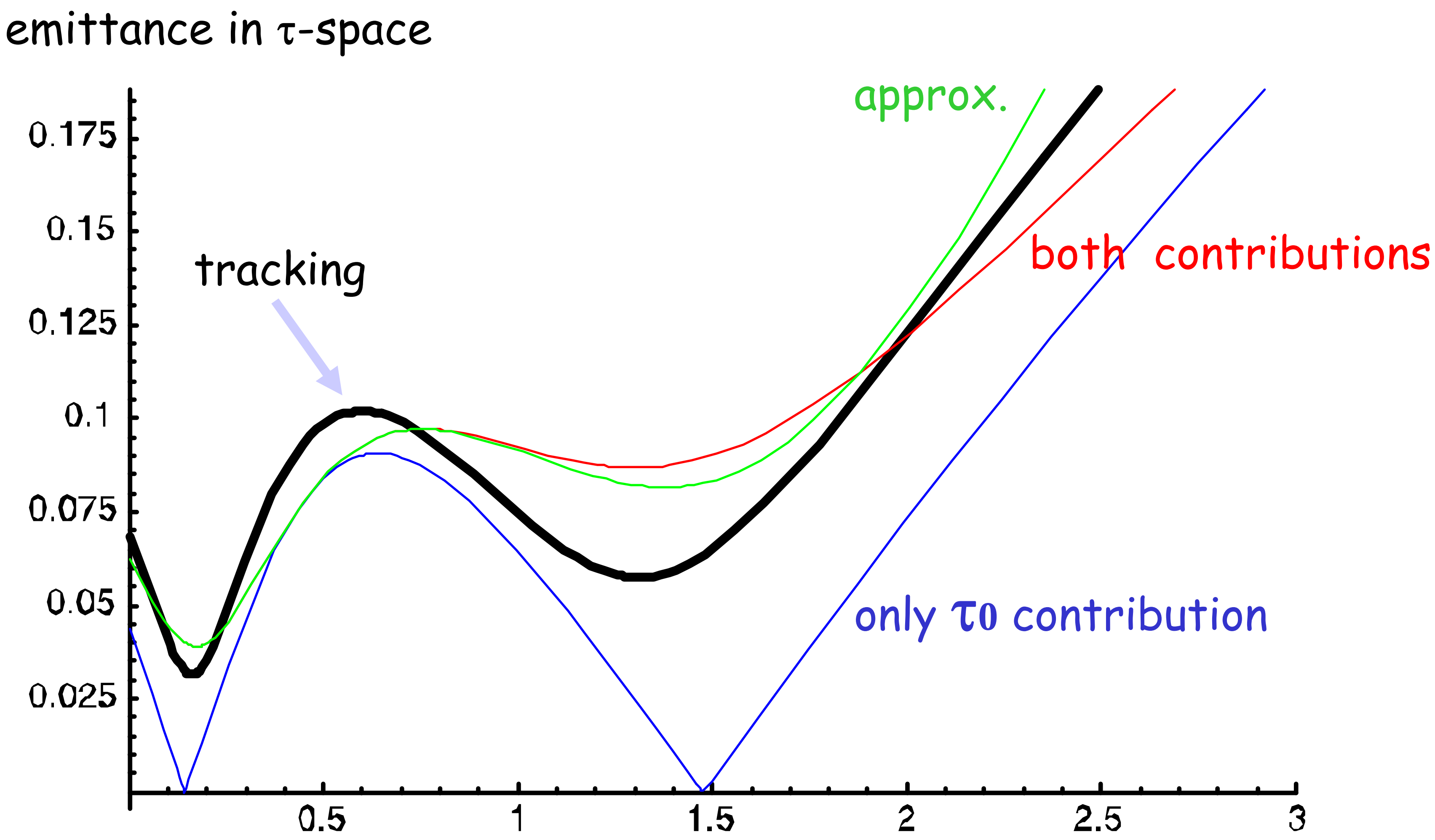
$$\epsilon = |W_\tau| \frac{(\Delta\tau_0)_{\rm rms}}{(\tau_0)_{\rm avg}} \;, \quad W_\tau = (\tau'^2 - 1)s - \tau\tau', \quad W_\tau' = 2[(\tau'/\tau)s - 1] = 0$$

normalized emittance in τ-space



not at the waist location

Oscillations due to τ_0, τ_0'



τ_0, τ_0' Correlation ignored, correlation matrix is $\begin{pmatrix} 1. & 0.232488 \\ 0.232488 & 1. \end{pmatrix}$

Acknowledgement

Special thanks to M. Ferrario for informative communications and more importantly for providing his code HOMDYN with the SPARC design example, which was used in this work.

Work supported by U.S. Department of Energy, Office of Basic Energy Sciences, under Contract No. W-31-109-ENG-38.