Beam Shaping and Permanent Magnet Quadrupole Focusing with Applications to the Plasma Wakefield Accelerator

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The Plasma Wakefield Accelerator (PWFA)

**Overdense Plasma**

\[ n_b < n_p ; k_p \sigma_z > 1 \]  current neutralized

\[ n_b < n_p ; k_p \sigma_r << 1 \]  self-focused

**Underdense Plasma**

\[ \frac{n_b}{n_p} > \gamma^2 \]  unfocused

\[ \gamma^2 > \frac{n_b}{n_p} > 1 \]  ion focused (blowout)

Wake Fields in Blowout Regime

Blowout Regime Mechanism:

- Plasma electrons disturbed by drive beam.
- Longitudinal space-charge wave generated.
- Ion channel created, pulls electrons back.
- Ion focusing is linear in r.
- Accelerating electric field, \( E_{acc} \approx n_0^{1/2} \) [V/cm]

Rosenzweig, et al. PRA, 44, R6189 (1991)

Overview of Drive Beam Issues for Blowout Regime of PWFA

- Longitudinal bunch shape
- High beam brightness
  - High charge
  - Strong Focusing
  - Betatron matching
- Creation of a Witness Bunch

The UCLA Neptune Laboratory

Beam Charge: 100-300 pC
Beam energy: up to 15 MeV
Emittance: $\varepsilon_N = 6 \text{ mm mrad}$
Power Source: 18 MW Klystron
RF Frequency: 2.856 GHz
Cathode laser: 60-80 $\mu$J at $\lambda = 266$ nm
Laser pulse length: 5 ps RMS

1.6-Cell Photoinjector

7&2/2 Cell PWT Linac
Neptune Dogleg Compressor
S-Bahn Compressor

- S-Bahn is a "dogleg" or dispersionless translating section.
- Half-chicane with focusing elements between the bends.
- Can be operated in a nondispersive mode with symmetric beta function and 2" betatron advance.
- Like a chicane, may be used as a bunch-length compressor.
- Nominal first order temporal dispersion ($R_{56} = -5\text{cm}$) is suitable for beam-shaping.
Neptune Dogleg Compressor
PARMELA Simulation Results: 1000 particles, 300pC

Initial:

Final: Sextupoles Off

Final: Sextupoles On

- 2D PIC Simulation
- 5 GeV/m gradients
- 6 nC drive beam w/ $n_0=2\times10^{16}$ cm$^{-3}$
Neptune Dogleg Compressor
ELEGANT: Simulated Witness Beam

For PWFA application, drive beam needs a witness beam to accelerate.

Region of high dispersion in x
Strong correlation b/w x and z
Insert mask in x to sever beam in z

No mask inserted
Undercorrected with sextupoles to elongate profile

With 1cm mask inserted at above location

witness beam
ramped drive beam
Temporal Bunch Shaping: Diagnostic Deflecting Mode Cavity

Lowest dipole mode is TM$_{110}$
Zero electric field on-axis (in pillbox approx.)
Deflection is purely magnetic
Polarization selection requires asymmetry

\[
E_z = E_0 J_1(\kappa r)e^{i\phi}; \\
B_r = B_0 \frac{J_1(\kappa r)}{\kappa r} e^{i\phi}; \\
B_{\phi} = iB_0 J_1'(\kappa r)e^{i\phi}; \\
\]

on axis $\kappa r = 0$

\[
E_z = 0; \\
B_x = \frac{B_0}{2}; \\
B_y = i\frac{B_0}{2}; \\
\]

Pillbox Fields

Courtesy of D. Alesini

J.D. Fuersi, et. al., DESY Report CDR98, 1998
Deflecting Cavity: Power & Resolution

screen deflection: \( \sigma_{x,f} = \sqrt{\sigma_{x,0}^2 + \sigma_{\text{def}}^2} \)

\( \sigma_{\text{def}} = 2\sigma_z L \frac{\pi V_\perp f}{cU/e} \)

\[ V_\perp \gg V_{\text{min}} = \frac{\sigma_{x,0} U / e}{L\pi \sigma_{t,f}} \]

\( \sigma_{t,\text{min}} = \frac{\sigma_{x,0} U / e}{L\pi V_\perp f} \)

\[ V_{\perp,\text{design}} = 3V_{\text{min}} = 545\,kV \]

\( \sigma_{t,\text{min}} = 545\,fs \)

\( \Delta t = \frac{\Delta x U / e}{L\pi fR_\perp^{1/2} \sqrt{nP_{\text{in}}}} \)

\( \sigma_{x,f} \) = beam size at screen with deflector on;

\( \sigma_{x,0} = 0.3\,mm \) = beam size at screen with deflector off;

\( L = 43\,cm \) = drift from deflector to screen;

\( f = 9.6\,GHz \) = RF frequency;

\( V_\perp \) = deflecting voltage;

\( R_\perp = 820\,k\Omega \) = transverse shunt impedance per cell;

\( P_{\text{in}} \) = input RF power;

\( U = 12\,MeV \) = electron beam energy;

\( \varphi_0 \) = deflector injection phase = 0;

\( \sigma_{t,\text{min}} \) = minimum resolvable rms bunch length;

\( \Delta x = 30\,\mu m \) = spatial resolution of screen & optics;

\( \Delta t \) = effective temporal resolution of deflector;

9 cells; 50 kW; 50 fs resolution
ELEGANT Simulation Results

- Using RFDF element with 9 cells
- 10,000 macroparticles
- Shunt Impedance: $R_T = 6.12 \text{ M}\Omega$
- Power: $P = \frac{V_0^2}{R_T}$

Initial Current Profile

- $V_0 = 0 \ ; P = 0$
- $V_0 = 272 \text{ kV} \ ; P = 12 \text{ kW}$
- $V_0 = 545 \text{ kV} \ ; P = 48 \text{ kW}$
- $V_0 = 609 \text{ kV} \ ; P = 61 \text{ kW}$
Deflecting Cavity: HFSS Design

HFSS Geometry of 1/2 structure

Resonance of the $\pi$-Mode out of phase by 90°

$E$-field

$H$-field

• X-Band, 9-cell design.
• Collaboration with INFL Frascatti.
• Will be built at UCLA; diffusion bonded at SLAC.
• Powered by a VA-24G X-Band klystron @ 50 kW.
• Frequency: 9.59616GHz
Deflecting Cavity: Polarization Separation

- Rods give greater mode separation but shift the desired mode too much
- Holes give less mode separation (5 MHz) but only perturb desired mode by 2 MHz (within range of temperature tuning).
- Holes look like better option: 5 MHz is large compared to the resonance width

Undesired: +1358 MHz  Desired: +53 MHz  Undesired: -7 MHz  Desired: -2 MHz
Prototype Cavity
Constraints on Brightness & Emittance

blowout regime        transformer ratio        betatron matching

\[ n_0 > \frac{mc^2}{\pi e^2 L} \quad \sigma_r < \sigma_{max} = \sqrt{\frac{Q}{e} 4\pi n_0 \sigma_z} \quad \varepsilon_N < \varepsilon_{Nmax} = \gamma \beta \frac{\sigma_{max}^2}{\beta_{eq}} \quad B > B_{min} = \frac{2I}{\varepsilon_N^2 \varepsilon_{Nmax}^2 \sigma_z} \]

Q=300 pC ; L=2mm, \( \sigma_z = 0.5 \) mm

\[ B_{min} = \frac{2cQ}{\varepsilon_N^2 \varepsilon_{Nmax}^2} \]

\[ n_0 \approx 2.8 \times 10^{13} \text{ cm}^{-3} \]

\[ n_b \approx \pi \sigma_r^2 \]

\[ Q = 0.3 \text{ nC}; \sigma_z = 0.05 \text{ cm} \]

\[ n_0(10^{13} \text{ cm}^{-3}) \]

\begin{array}{c}
\sigma_{max} \\
\varepsilon_{N, max} \\
(\mu m)
\end{array}

\begin{array}{c}
\sim 110 \mu m \\
\sim 50 \text{ mm mrad}
\end{array}

\begin{array}{c}
\sim 250 \text{ mA/\mu m}^2
\end{array}

\begin{array}{c}
Q = 0.3 \text{ nC}; \sigma_z = 0.05 \text{ cm}
\end{array}

\begin{array}{c}
(\text{A/\mu m}^2)
\end{array}

\begin{array}{c}
\varepsilon_{N, max} \\
(\text{cm}^{-3})
\end{array}

\begin{array}{c}
\sim 2.8 \times 10^{13} \text{ cm}^{-3}
\end{array}
Permanent Magnet Quad Focusing

- Hybrid Permanent Magnet and Iron
- Grey cubes are Alnico; $M = 1.175 \, \text{T}$
- Field gradient: $B' = 110 \, \text{T/m}; B'' = -0.002 \, \text{T/m}$
- Bore diameter: 8 mm
- Benefits: cheaper, better field profile
- Downsides: small bore; in-vacuum

ELEGANT Simulation Result:
- $x = 130 \, \text{mm}; y = 41 \, \text{mm} \, \text{mrad}$
- $z = 0.51 \, \text{mm}$
- $\epsilon = 1.84\%$

\[ B \frac{2Q}{\sigma_i} = \frac{\epsilon_{N,x} \epsilon_{N,y}}{\epsilon_{N,x} \epsilon_{N,y}} = 412 \, \text{mA} / \mu\text{m}^2 \]
Scaling to Higher Charge: 4 nC

$$n_0 > \frac{mc^2}{\pi e^2 L} \quad , \quad \sigma_r < \sigma_{\text{max}} = \sqrt{\frac{Q/e}{4\pi n_0 \sigma_z}} \quad , \quad \varepsilon_N < \varepsilon_{N\text{max}} = \gamma \beta \frac{\sigma_{\text{max}}^2}{\beta_{eq}} \quad , \quad \mathcal{B} > \mathcal{B}_{\text{min}} = \frac{2cQ}{\varepsilon_{N\text{max}}^2 \sigma_z}$$

Q=4 nC ; L=4mm; $\sigma_z = 1$ mm

$n_0_{\text{min}} \sim 0.8 \times 10^{13}$ cm$^{-3}$

~500 $\mu$m

~420 mm mrad

~12 mA/μm$^2$
Scaling to Higher Charge: 4 nC

scaling applied to laser at cathode: \( \sigma_x, \sigma_y, \sigma_t \propto Q^{1/3} \) \( (Q: 300 \ pC \rightarrow 4 nC) \)

Q = 4 nC; pt-to-pt space charge

UCLA-Parmela 2.0, 1000 particles

\( \varepsilon_N \sim 25 \text{ mm mrad} \)

simulation of gun and linac w/solenoid for emittance compensation

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*** BEAM PARAMETERS AT END - SAVECOR6000 ***

Beam Energy:

\( \gamma = 25.4691, \ V = 13.0147 \text{ MeV}, \ T = 12.5037 \text{ MeV}, \ p_T/\text{mc} = 25.4494 \)

X Twiss Parameters:

\( \sigma_x = 1.695 \text{ mm}, \ \alpha_x = -0.915558, \ \beta_x = 3.01018 \text{ m}, \ \varepsilon_x = 24.2613 \text{ mm mrad} \)

Y Twiss Parameters:

\( \sigma_y = 1.69297 \text{ mm}, \ \alpha_y = -0.875431, \ \beta_y = 2.95505 \text{ m}, \ \varepsilon_y = 24.6839 \text{ mm mrad} \)

Z Phase Space Parameters:

\( \sigma_z = 2.70076 \text{ mm}, \ \eta_z = 9.011155 \text{ ps}, \ \alpha_z = 4.4692 \text{ %} \)

N_final = 964 particles
Scaling to Higher Charge: 4 nC

scaling applied to laser at cathode: $\sigma_x, \sigma_y, \sigma_t \propto Q^{1/3}$

$(Q : 300 \text{ pC} \rightarrow 4 \text{nC})$

dispersion killed to 1st Order

longitudinal ph. space and profile at end

ELEGANT, 1000 particles

simulation of sbahn and final focus

horizontal dispersion killed to first order ($R_{16}, R_{26}$)

2nd order longitudinal dispersion $T_{566}$ killed

tail is 3rd order ($U_{5666}$)

octupoles needed?

$\epsilon_{N,x} = 742 \text{ mm mrad}$

$\epsilon_{N,y} = 456 \text{ mm mrad}$
Scaling to Higher Charge: 4 nC

scaling applied to laser at cathode: $\sigma_x, \sigma_y, \sigma_z \propto Q^{1/3}$ \quad (Q : 300 pC $\rightarrow$ 4 nC)

dispersion killed to 2nd Order

longitudinal ph. space and profile at end

ELEGANT, 1000 particles

simulation of sbahn and final focus

horizontal dispersion killed to second order

2nd order longitudinal dispersion not killed

tail is 2nd and 3rd order \quad (T_{566}, U_{5666})

$\epsilon_{N,x} = 96 \text{ mm mrad}$

$\epsilon_{N,y} = 141 \text{ mm mrad}$
Conclusions

• PWFA drive issues: ramped profile, strong focus, high charge

• Ramped profile:
  - improved transformer ratio (R > 2)
  - feasible using dogleg compression with sextupoles
  - deflecting cavity diagnostic (50 fs resolution)

• Strong focus:
  - traditional EM quads + permanent magnet quadrupoles
  - adequate emittance and brightness (~ 100 μm, 450 mA/μm² @ 300 pC)

• High Charge:
  - scaling to high charge (~4 nC) at Neptune has some dilemmas
  - tradeoff between optimal profile and good emittance
  - extra sextupoles and octupoles may be required
  - beam sizes become bigger than the beam pipes
  - implies complete or partial redesign of the compressor