# WAKE FIELDS EFFECTS
# IN A HIGH BRIGHTNESS PHOTO-INJECTOR

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## Outline

- The Homdyn code: model and improvements

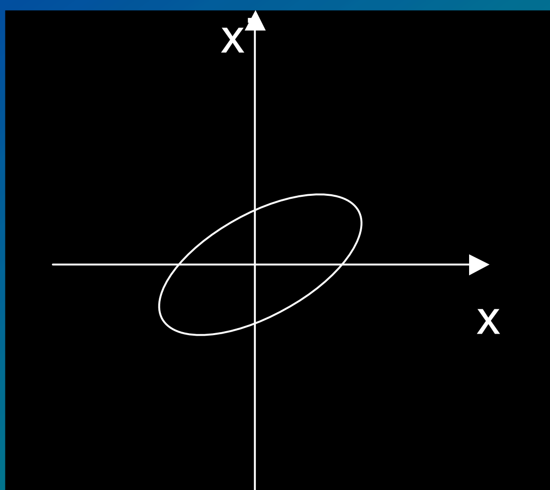
- Applications to the SPARC project:
  - emittance preservation for misaligned structures
  - emittance degradation and energy spread in the emittance meter experiment

$$B_{\perp} = \frac{2I}{\varepsilon_x \varepsilon_y}$$

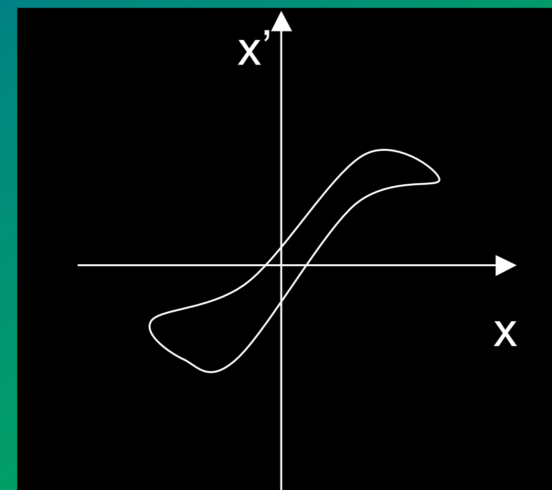
High current

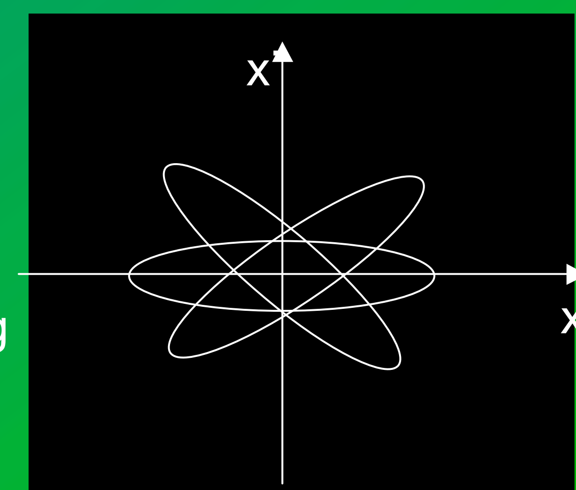
**Low emittance**

Emittance degradation due to




non linear e.m. fields




Longitudinal correlation along the bunch induced by e.m e.m.: RF fields, space charge, WAKE FIELDS

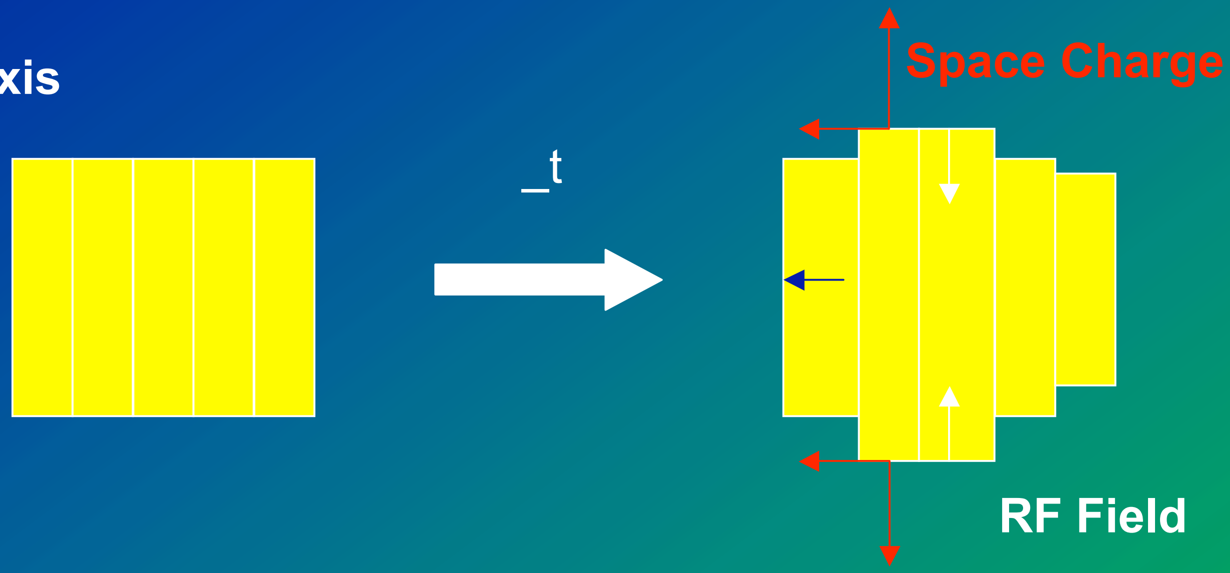
↓

WAKE FIELDS have been inserted in the HOMDYN code

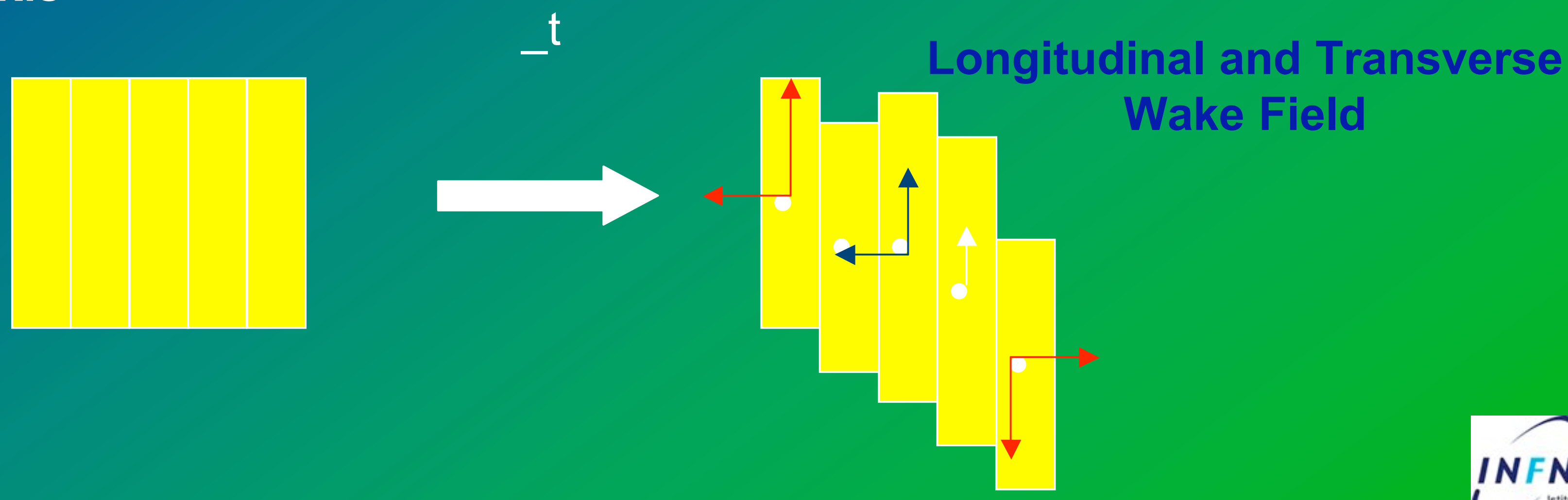
↓

# •BEAM DYNAMICS MODELING IN HOMDYN

## On Axis




## Off Axis



## On axis ⟶ Envelope equations

**RF field**

**Solenoid field**

**Space charge**

$$\ddot{x} + \beta\gamma^2\dot{\beta}x + (k^{rf} + k^{sol})^2 x = \frac{e}{\gamma^3 m} E_x^{sc}(\xi_s, A_{xs}, x) + \left(\frac{4\varepsilon_n^{th}}{\gamma}\right)^2 \frac{1}{x^3}$$

$$\ddot{y} + \beta\gamma^2\dot{\beta}y + (k^{rf} + k^{sol})^2 y = \frac{e}{\gamma^3 m} E_x^{sc}(\xi_s, A_{xs}, y) + \left(\frac{4\varepsilon_n^{th}}{\gamma}\right)^2 \frac{1}{y^3}$$

## Off axis → Centroid equations

**RF field** **Space charge** **Wake field** **Solenoid Field**

$$\ddot{x}_c + \beta\gamma^2\dot{\beta}\dot{x}_c + k^{rf} x_c = \frac{e}{\gamma^3 m} E_x^{sc}(\xi_s, A_{xs}, d_{xc}) + \frac{e}{\gamma m}\left\{ E_\perp^{w}(x_c^1, \xi_s) + \dot{y}B_z + \frac{1}{2}\dot{z}\left(yB'_z - \sum_i y_{i,off} B'_{z,i}\right)\right\}$$

$$\ddot{y}_c + \beta\gamma^2\dot{\beta}\dot{y}_c + k^{rf} y_c = \frac{e}{\gamma^3 m} E_y^{sc}(\xi_s, A_{ys}, d_{yc}) + \frac{e}{\gamma m}\left\{ E_\perp^{w}(y_c^1, \xi_s) - \dot{x}B_z - \frac{1}{2}\dot{z}\left(xB'_z - \sum_i x_{i,off} B'_{z,i}\right)\right\}$$

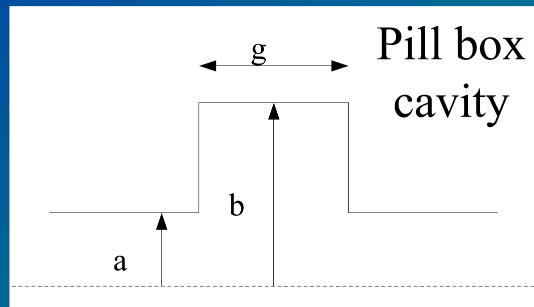
$$\ddot{z}_c = \frac{e}{\gamma^3 m} E_z^{sc}(\xi_s, A_s) + \frac{e}{\gamma m}\left\{ E_z^{rf}(z) + E_\parallel^{w}(\xi_s) - \frac{1}{2}\left[\dot{x}\left(yB'_z - \sum_i y_{i,off} B'_{z,i}\right) - \dot{y}\left(xB'_z - \sum_i x_{i,off} B'_{z,i}\right)\right]\right\}$$

# Wake fields  diffraction model:

**hp: L<< a**

**_>>c/a**

**pill-box cavity**




→ **Green function**

By a convolution of the Green function with the uniform distribution

$$W_{||}(s) = \begin{cases} 0 & s < 0 \\ \frac{2}{\sqrt{2}} \frac{Z_0 c}{\pi^2 a L} \sqrt{g s} & 0 < s < L \\ \frac{2}{\sqrt{2}} \frac{Z_0 c}{\pi^2 a L} \sqrt{g} \left( \sqrt{s} - \sqrt{s - L} \right) & s > L \end{cases}$$

**Longitudinal wake field**

$$W_{\perp}(s) = \begin{cases} 0 & s < 0 \\ \frac{2^{5/2}}{3} \frac{Z_0 c}{\pi^2 a^3 L} \sqrt{g} s^{3/2} & 0 < s < L \\ \frac{2^{5/2}}{3} \frac{Z_0 c}{\pi^2 a^3 L} \sqrt{g} \left( s^{3/2} - (s - L)^{3/2} \right) & s > L \end{cases}$$

**Transverse wake field**

## Periodic structure

$$W_{||}(s) = \begin{cases} 0 & s < 0 \\ \frac{2Z_0 c s_1}{\pi a^2 L}\left[1 - e^{-\sqrt{s/s_1}}\left(1 + \sqrt{\frac{s}{s_1}}\right)\right] & 0 < s < L \\ \frac{2Z_0 c s_1}{\pi a^2 L}\left[e^{-\sqrt{\frac{s-L}{s_1}}}\left(1 + \sqrt{\frac{s-L}{s_1}}\right) - e^{\sqrt{\frac{s}{s_1}}}\left(1 + \sqrt{\frac{s}{s_1}}\right)\right] & s > L \end{cases}$$

**Longitudinal wake field**

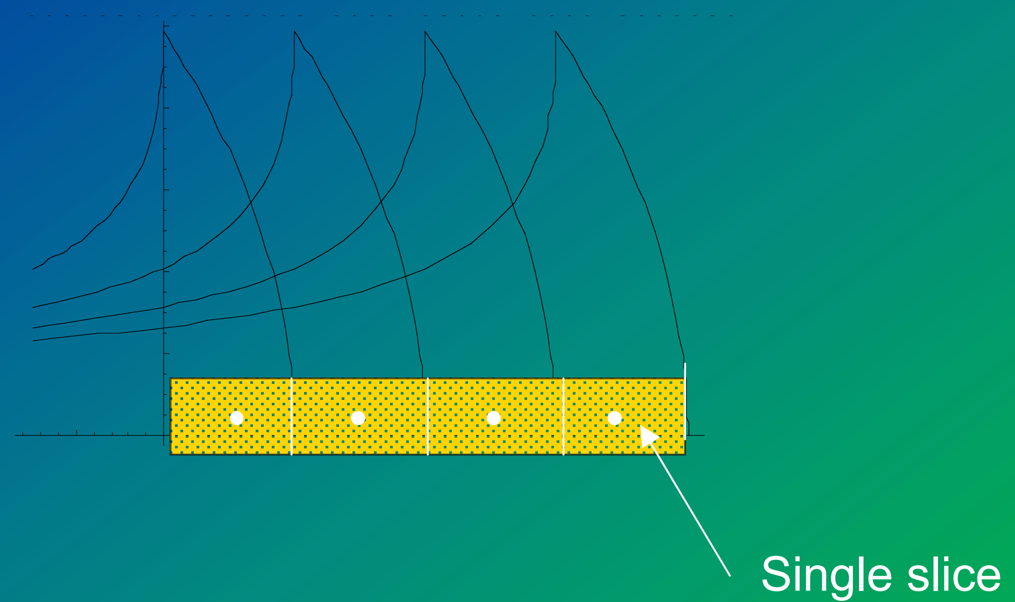
$$W_{\perp}(s) = \begin{cases} 0 & s < 0 \\ \frac{4Z_0 c {s_2}^2}{\pi a^4 L}\left[-6 + \frac{s}{s_2} + 2e^{-\sqrt{\frac{s}{s_2}}}\left(3 + 3\sqrt{\frac{s}{s_2}} + \frac{s}{s_2}\right)\right] & 0 < s < L \\ \frac{4Z_0 c {s_2}^2}{\pi a^4 L}\left\{\frac{L}{s_2} + 2\left[e^{-\sqrt{\frac{s}{s_2}}}\left(3 + 3\sqrt{\frac{s}{s_2}} + \frac{s}{s_2}\right) + \right.\right. & \\ \left.\left. +e^{-\sqrt{\frac{s-L}{s_2}}}\left(-3 - 3\sqrt{\frac{s-L}{s_2}} - \frac{s-L}{s_2}\right)\right]\right\} & s > L \end{cases}$$

**Transverse wake field**

Asymptotic wake fields obtained numerically and fitted to a simple function

K. Bane

The single slices generate wake fields:




$$E_{\perp}^{W}(x_{cs},\xi_s) = \sum_{s=i}^{N} q_s x_{cs} W_{\perp}(\xi_s)$$

$$E_{\perp}^{W}(y_{cs},\xi_s) = \sum_{s=i}^{N} q_s y_{cs} W_{\perp}(\xi_s)$$

$$E_{\parallel}(\xi_s) = \sum_{s=i}^{N} q_s W_{\parallel}(\xi_s)$$

$X_{cs}$, $y_{cs}$ is the leading slice offset

_s is the position of the test slice respect to the leading slice

## Emittance computation

$$\varepsilon_{nx} = \sqrt{<(x-<x>)^2><(\beta\gamma x'-<\beta\gamma x'>)^2> - <(x-<x>)(\beta\gamma x'-<\beta\gamma x'>)>^2}$$

$$<> = \frac{1}{N}\sum_{n=1}^{N} = \frac{1}{S\cdot M}\sum_{s=1}^{S}\sum_{m=1}^{M} = \frac{1}{S}\sum_{s=1}^{S}<>$$

$$\varepsilon_n^{e^2} = <\frac{X^2}{4}><\frac{p_X{}^2}{4}> - <\frac{Xp_X}{4}>^2$$

envelope

$$(\varepsilon_n^c)^2 = \left\langle (x_c - <x_c>)^2 \right\rangle \left\langle (p_{x_c} - <p_{x_c}>)^2 \right\rangle - \left\langle (x_c - <x_c>)(p_{x_c} - <p_{x_c}>) \right\rangle^2$$

centroid

$$(\varepsilon_n^{cross})^2 = \left\langle \frac{X^2}{4} \right\rangle \left\langle (p_{x_c} - <p_{x_c}>)^2 \right\rangle + \left\langle \frac{p_X{}^2}{4} \right\rangle \left\langle (x_c - <x_c>)^2 \right\rangle - 2\left\langle \frac{Xp_X}{4} \right\rangle \left\langle (x_c - <x_c>)(p_{x_c} - <p_{x_c}>) \right\rangle$$

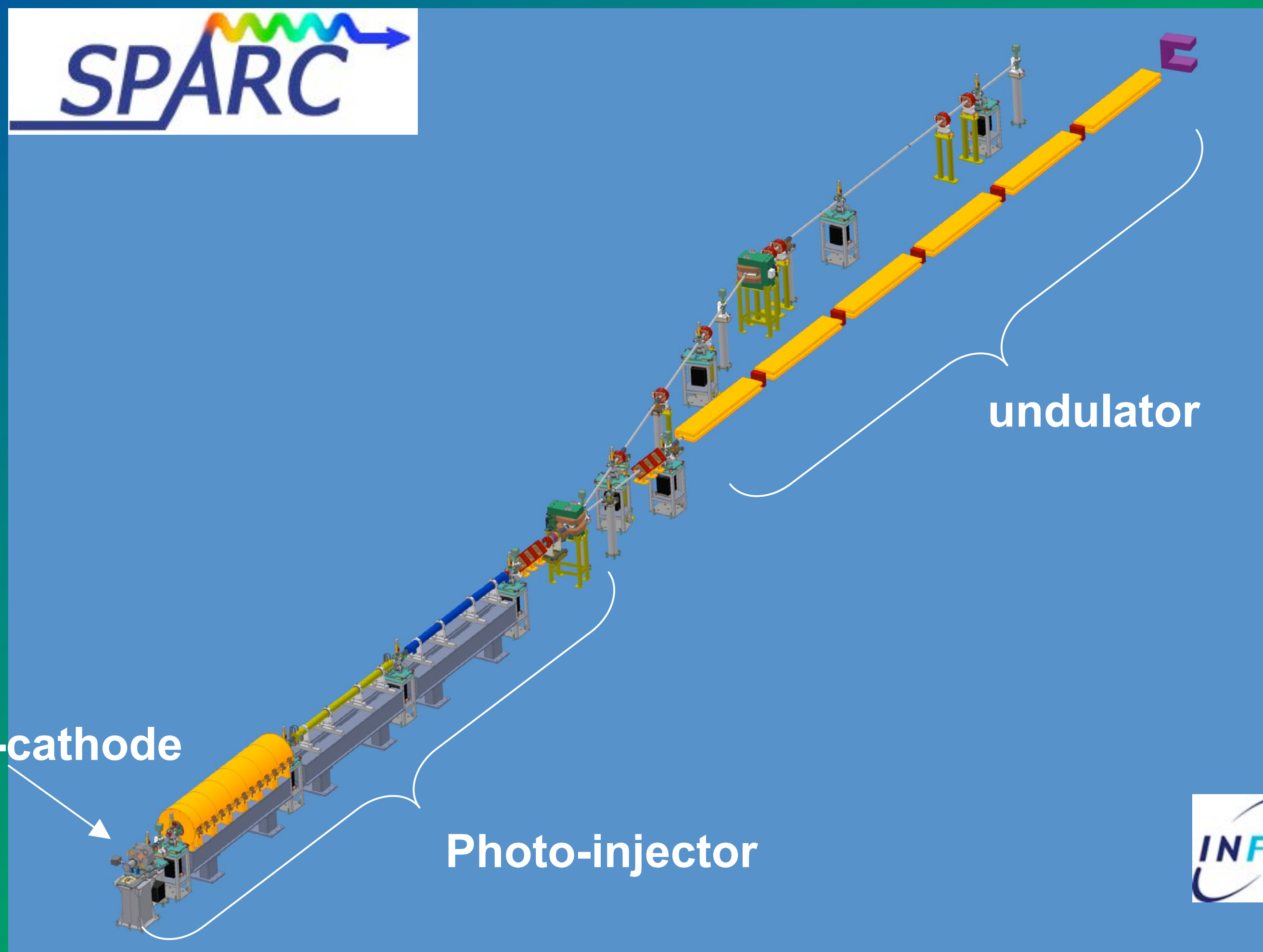
cross

$$\varepsilon_{ntot} = \sqrt{\varepsilon_n^{e^2} + \varepsilon_n^{c^2} + \varepsilon_n^{cross^2}}$$

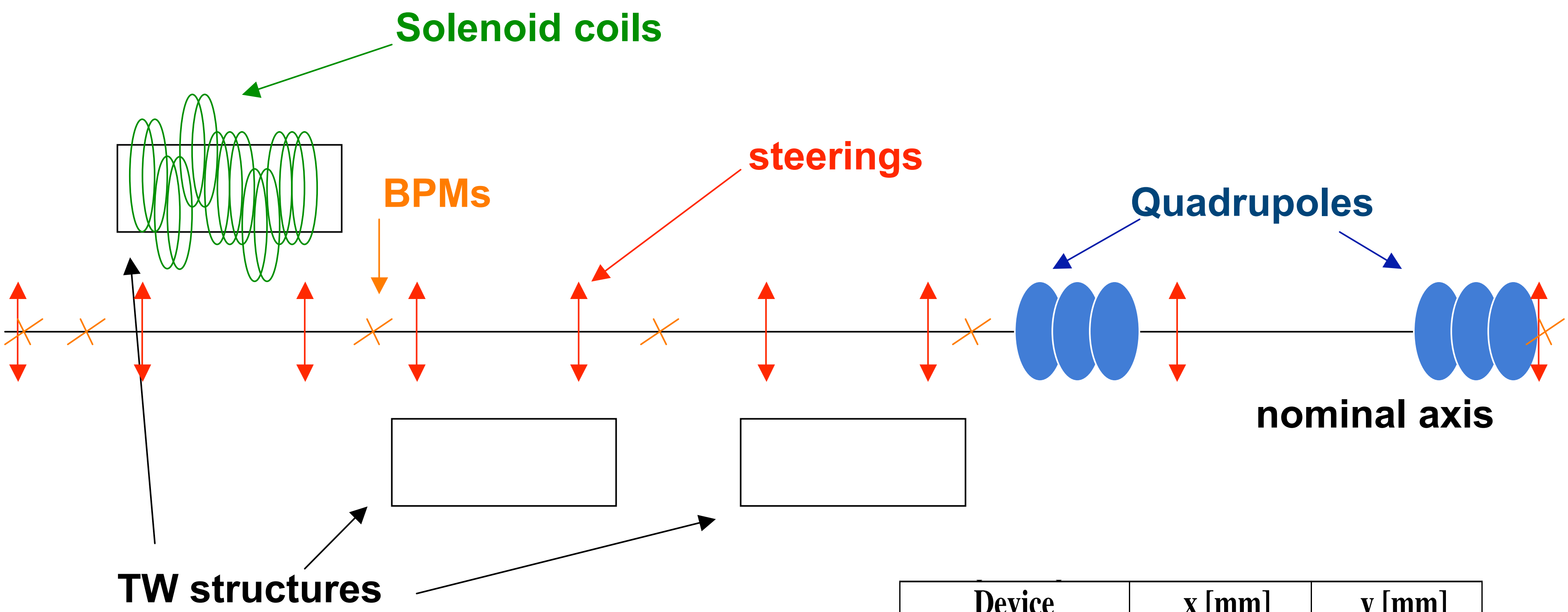
Application to the SPARC photo-injector

- emittance preservetion for misaligned structures

- emittance degradation and energy spread
  in the emittance meter experiment

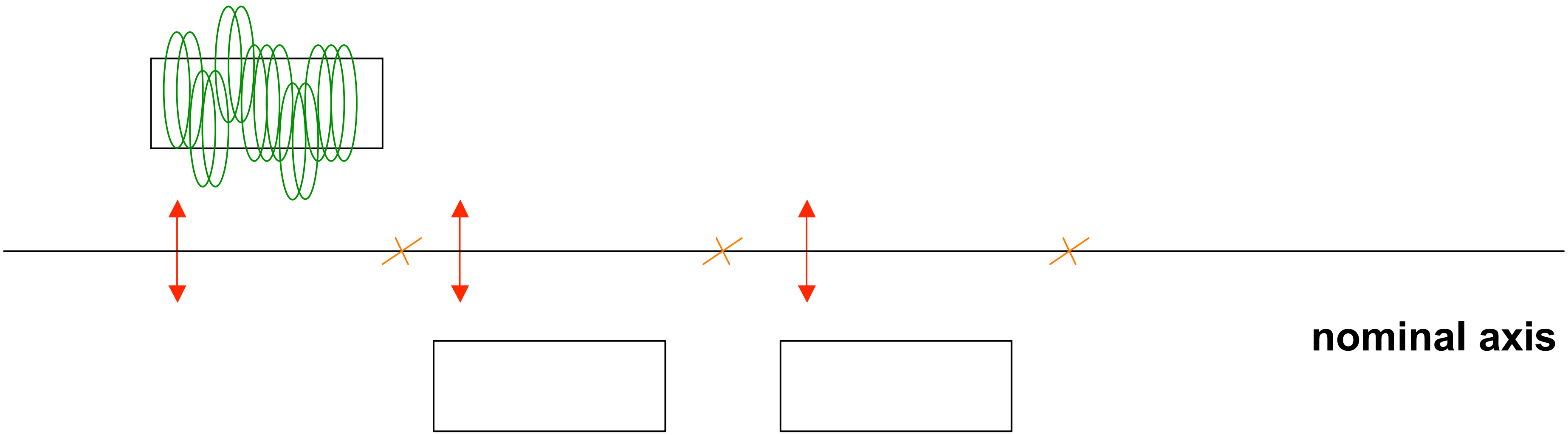


# Correction of a Misaligned Configuration

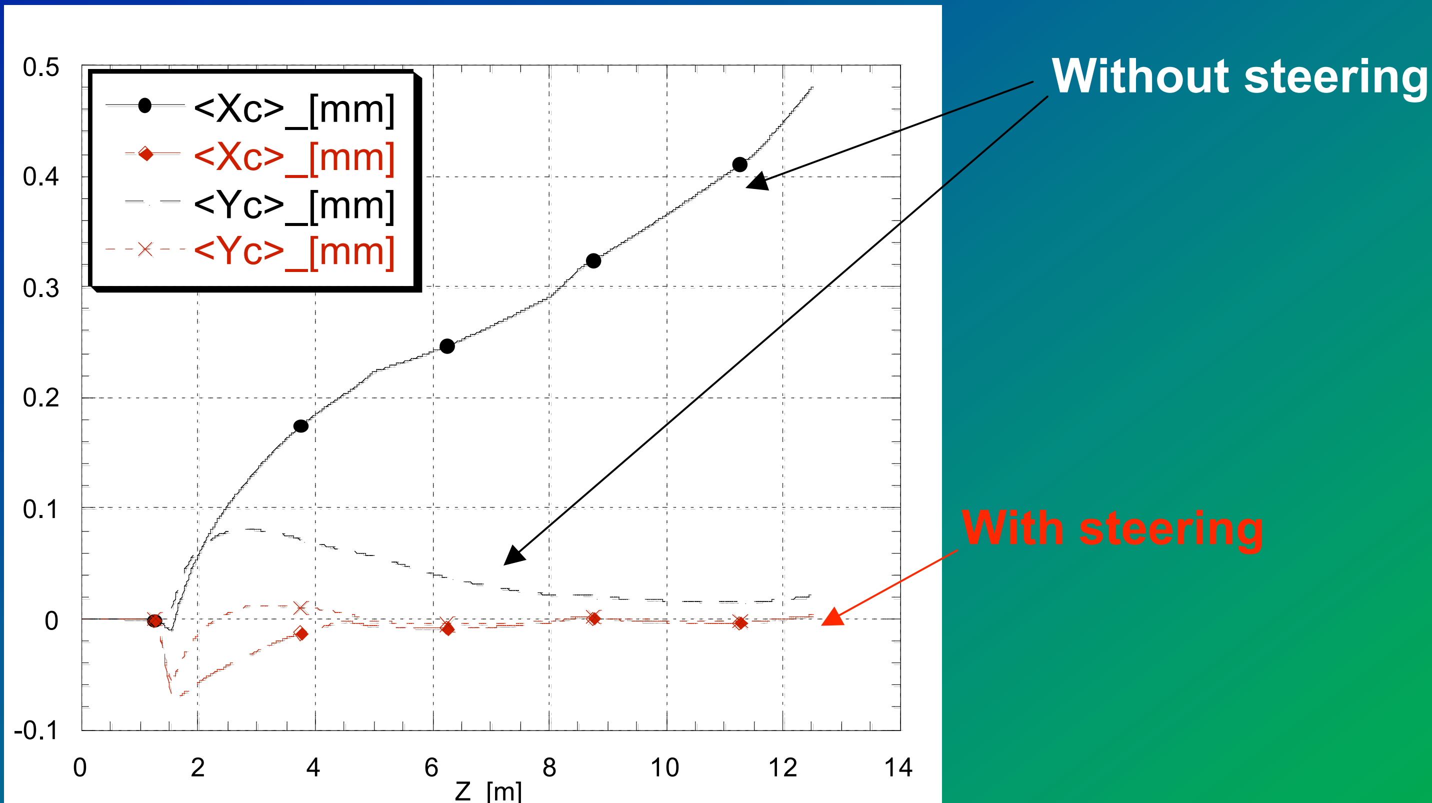



| Device | x [mm] | y [mm] |
|---|---|---|
| Solenoid coil 1 | 0. | +0.1 |
| Solenoid coils 2-3-4-5-6 | +0.1 | 0. |
| Solenoid coils 7-8-9-10-11-12-13 | 0. | -0.1 |
| TW1 | 0.1 | 0.1 |
| TW2 | -0.1 | -0.1 |
| TW3 | -0.1 | -0.1 |

# Steerings and BPMs: centroid offset minimization respect to the nominal axis



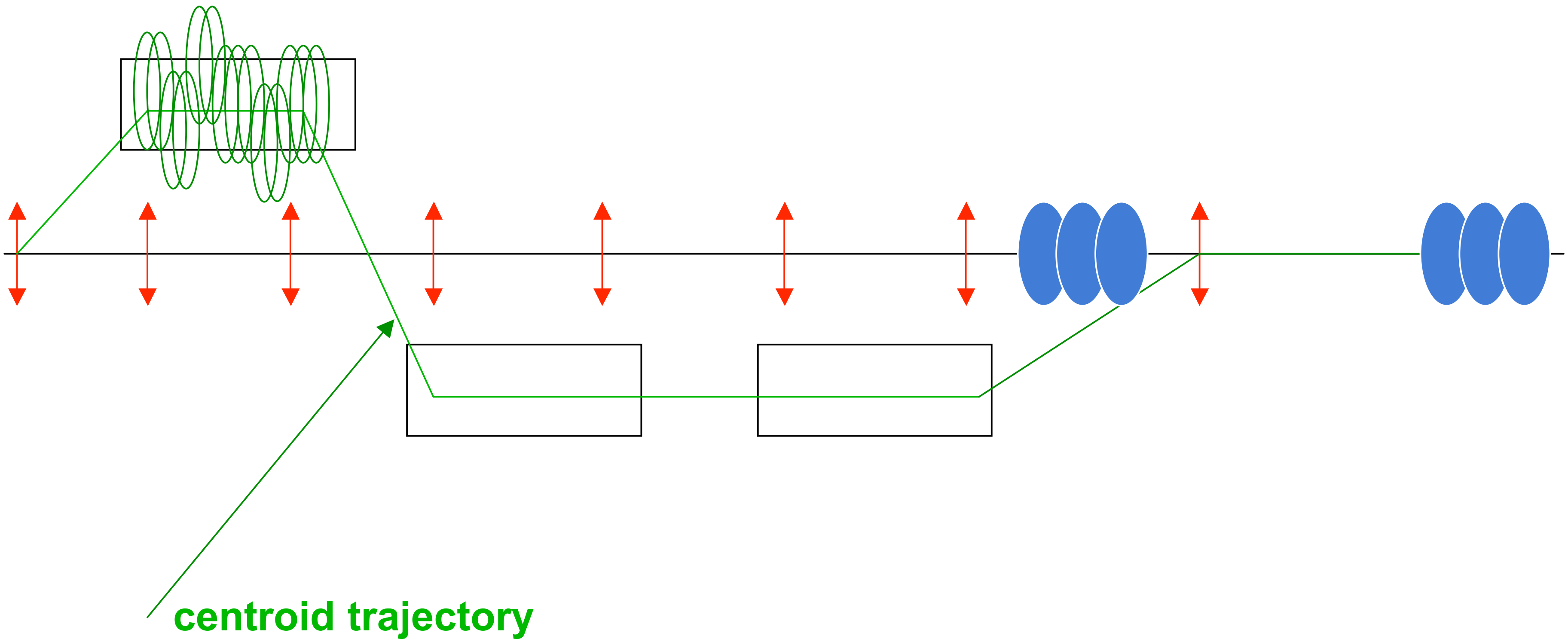
# Centroid position along the structure with and without correction




| ex nominal | ex steer off | ex steer on |
|---|---|---|
| 0.79_m | 2.95_m | 1.08_m |

| ey nominal | ey steer off | ey steer on |
|---|---|---|
| 0.79_m | 1.12_m | 1.06_m |

# Beam Based Alignment technique: emittance minimization



**Transfer matrix to determine the steerings' angle to be inserted in Homdyn**

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_2 = \begin{pmatrix} a & b & e & f \\ c & d & g & h \\ i & l & o & p \\ m & n & q & b \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_1$$

**First step->** determine the matrix element

**Second step->** determine the angle (horizontal and vertical) for each steering

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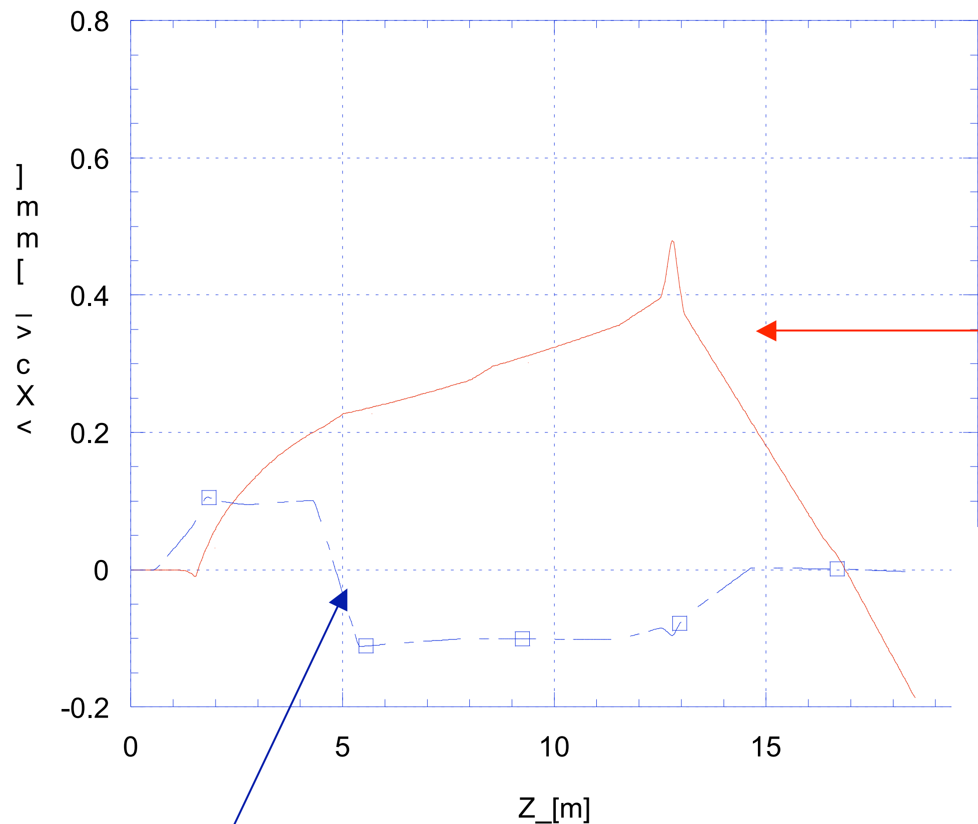
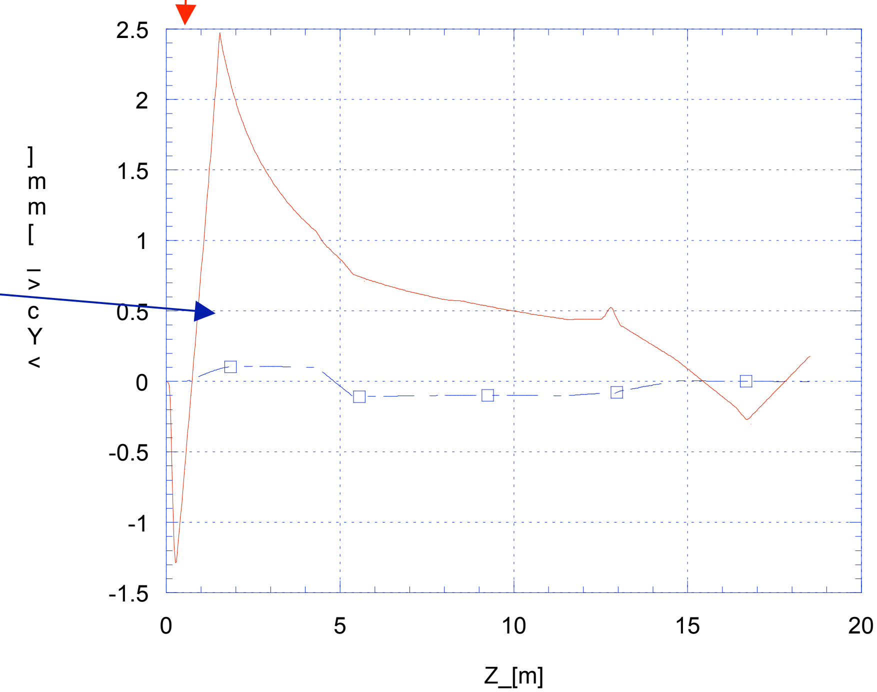
## Centroid postion along the structure

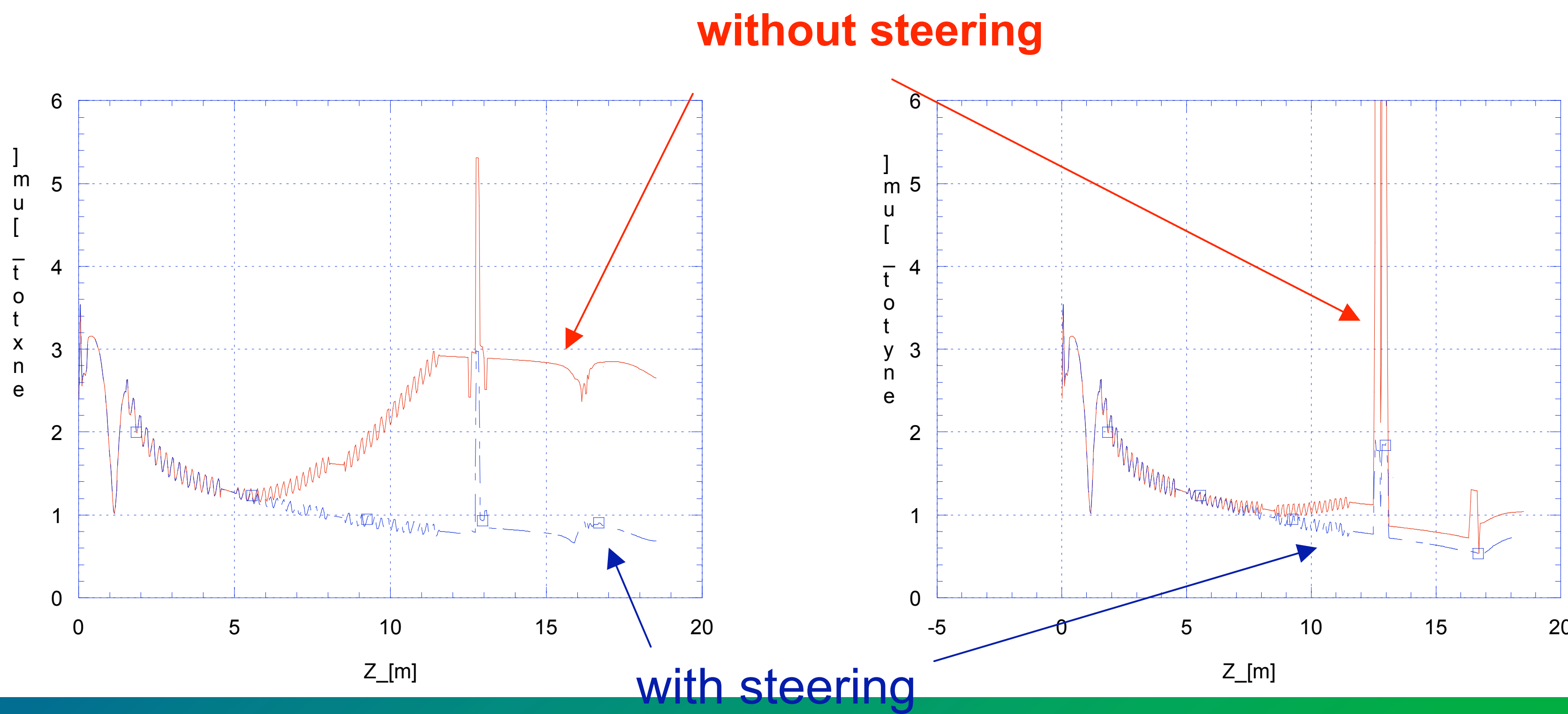
without steering




with steering



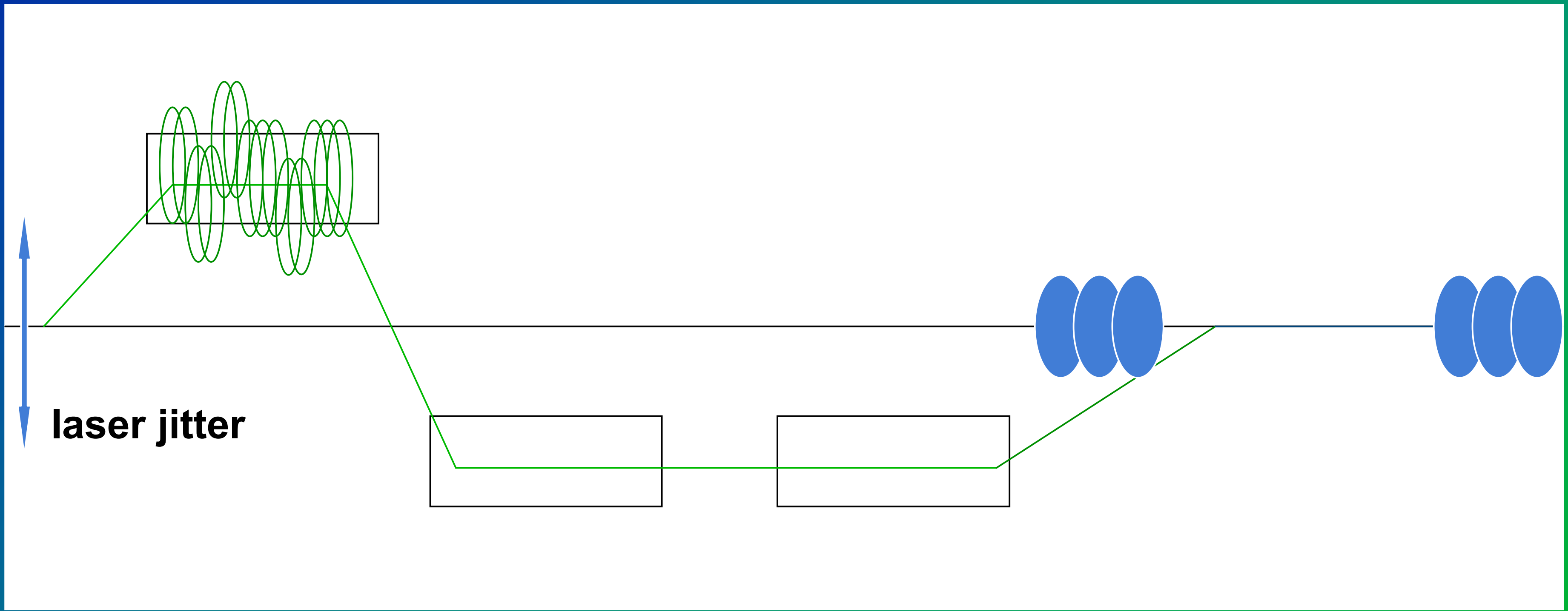
# Emittance along the structure



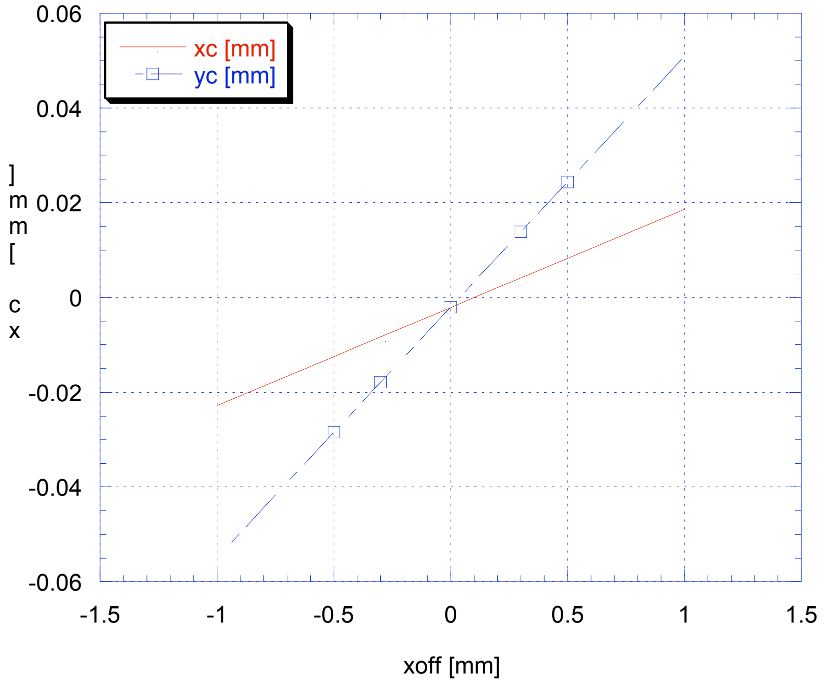

| ex nominal | ex steer off | ex steer on |
|---|---|---|
| 0.79_m | 2.95_m | 0.79_m |

| ey nominal | ey steer off | ey steer on |
|---|---|---|
| 0.79_m | 1.12_m | 0.79_m |

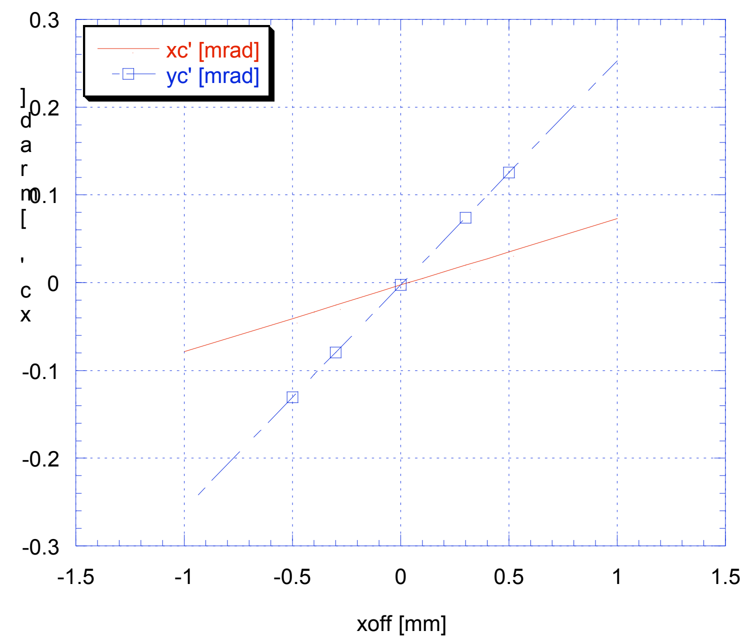
# Initial laser offset effects



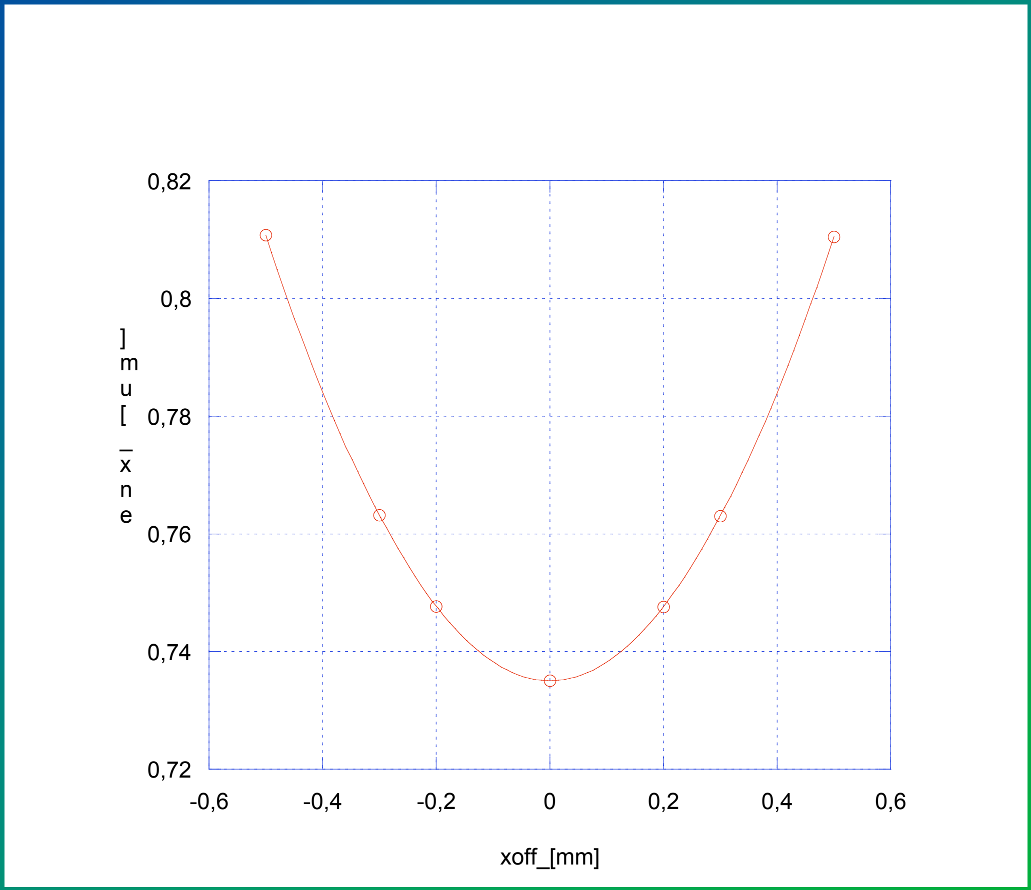
# Centroid position at the undulator entrance



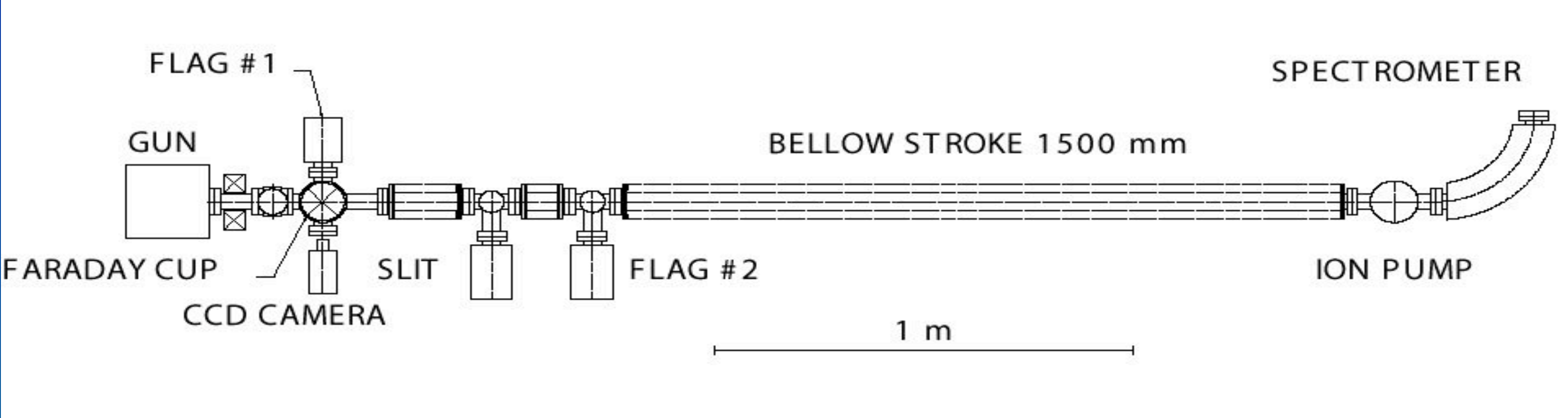

# Angle at the undulator entrance



# Emittance at the entrance of the undulator

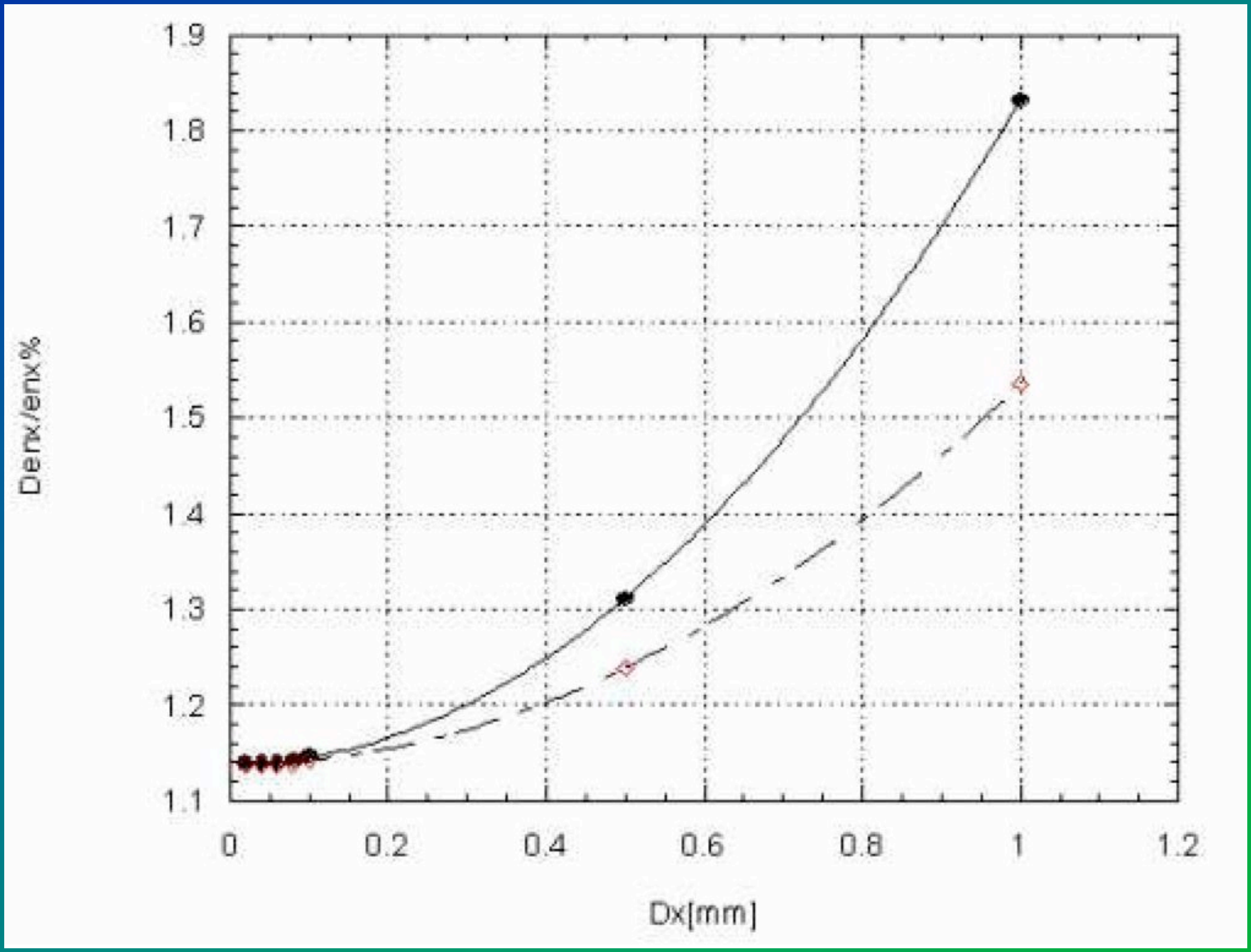


# Energy Spread and Emittance Degradation in the Emittance meter

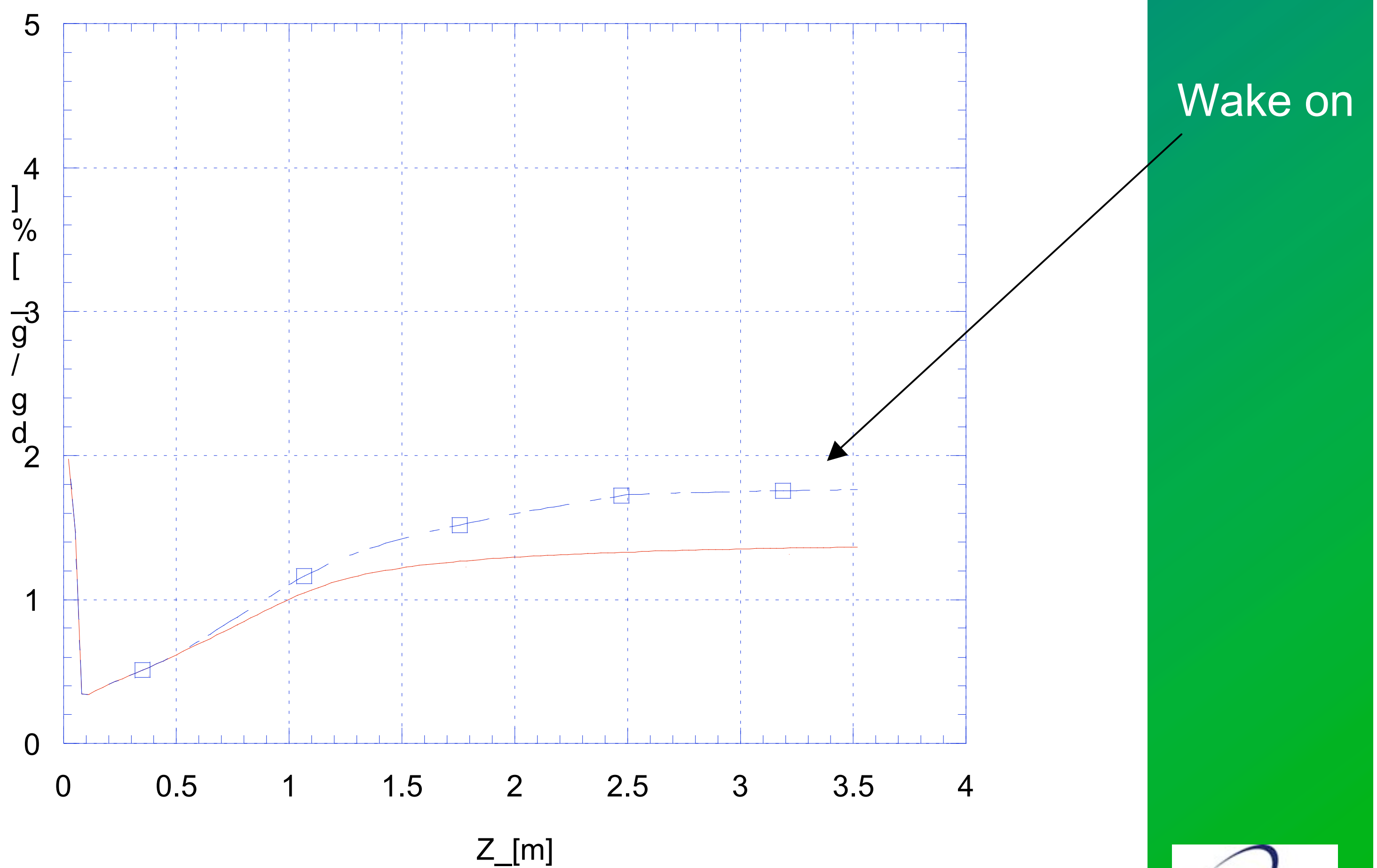



| Bellow | $a$ $mm$ | $b$ $mm$ | $g$ $mm$ |
|---|---|---|---|
| first | 26.0 | 47.5 | 3.40 |
| second | 51.25 | 75.0 | 4.00 |

**Emittance degradation** in percent vs the bellow misalignemnt



## **Energy spread** degradation along the bellow structure



## Conclusions

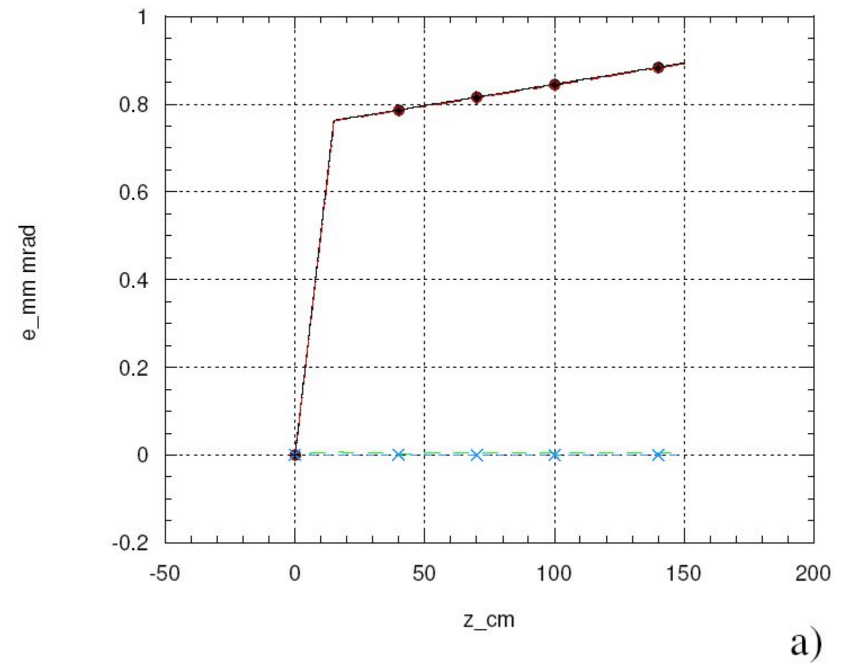
- **The Homdyn code has been improved, including off axis beam dynamics and wake fields**

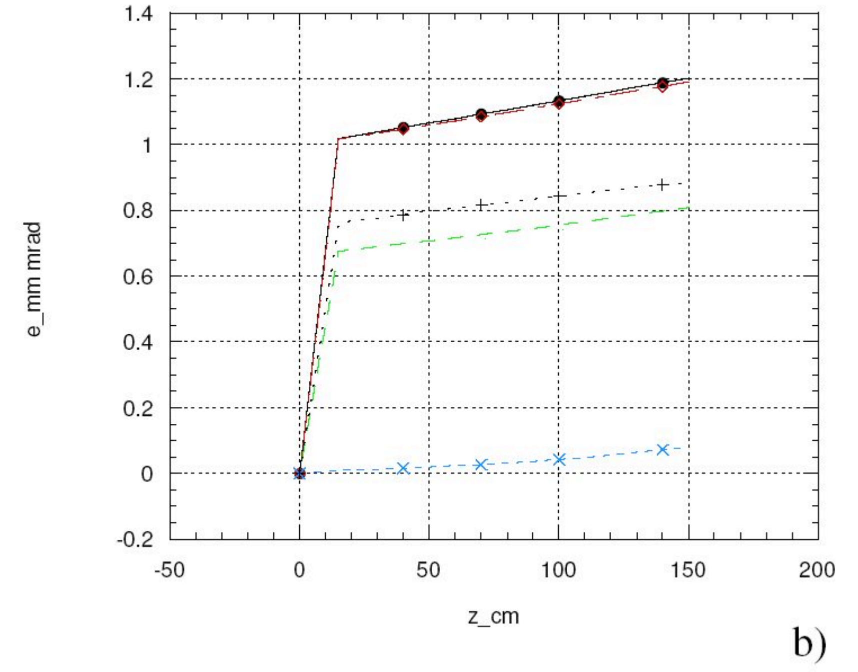
- **The code has been used for the study of a misaligned correction scheme in the SPARC project :**
  - **-The study shows the scheme can correct the centroid position until the entrance of the undulator**
  - **-The preliminary study of the laser pointing instability shows that a stability of 100 _m can satisfy the undulator matching condition**

- **The code allowed the study of the wake fields in the emittance meter thus a geomtery for the emittance meter could be chosen**

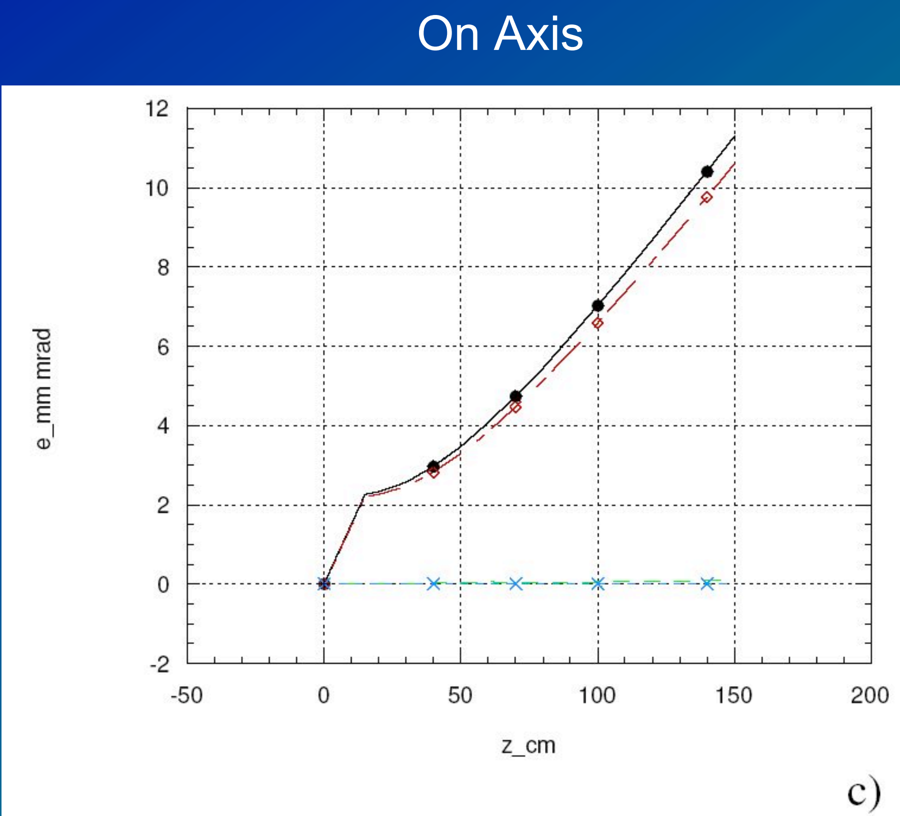
# Validation with Parmela of the Emittance Computation

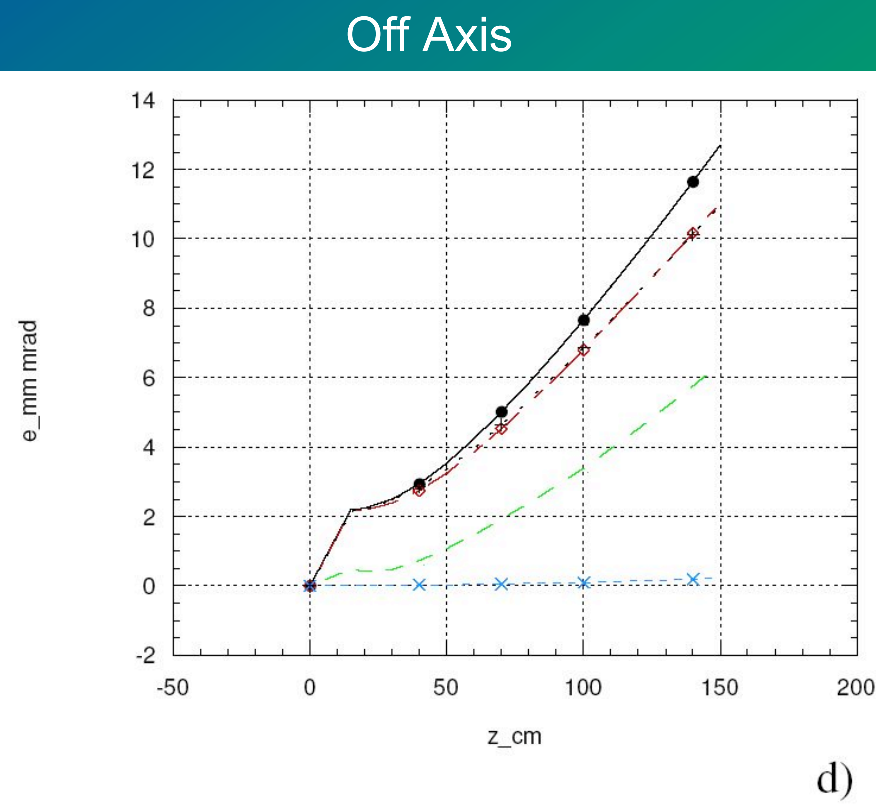
## Without space charge



## With space charge



| UNDULATOR & FEL | A | B |
|---|---|---|
| | | |
| Undulator period (cm) | 3.0 | 3.0 |
| # Undulator sections | 6 | 6 |
| Undulator parameter | 1.4 | 1.4 |
| Undulator field on axis (T) | | |
| Undulator gap (mm) | 11 | 11 |
| Undulator section length (m) | 2.13 | 2.13 |
| Drifts between undulator sections (m) | 0.36 | 0.36 |
| FEL wavelength (nm) | 500 | 290 |
| Saturation length (m, geometrical) | < 14 | NA |
| FEL pulse length (ps) | 8 | NA |
| FEL power @ saturation (MW) | > 80 | NA |
| Brilliance (st. units) | | NA |
| # Photons/pulse | $10^{15}$ | NA |
| FEL power @ sat. (MW) $3^{rd}$ harm. | > 10 | NA |
| FEL power @ sat. (MW) $5^{th}$ harm. | > 0.7 | NA |

| | |
|---|---|
| 0.04 mm | Xc |
| 0.02 mmrad | Xc' |