Wakefields of Subpicosecond Electron Bunches

Karl Bane
Stanford Linear Accelerator Center

October 12, 2005
The Physics and Applications of High-Brightness Beams, Erice, Sicily

Thanks to:
P. Emma for viewgraphs

Introduction

an X-ray FEL project, such as the Linac Coherent Light Source (LCLS), calls for the production of high energy bunches of electrons that are short, intense, and have small emittances

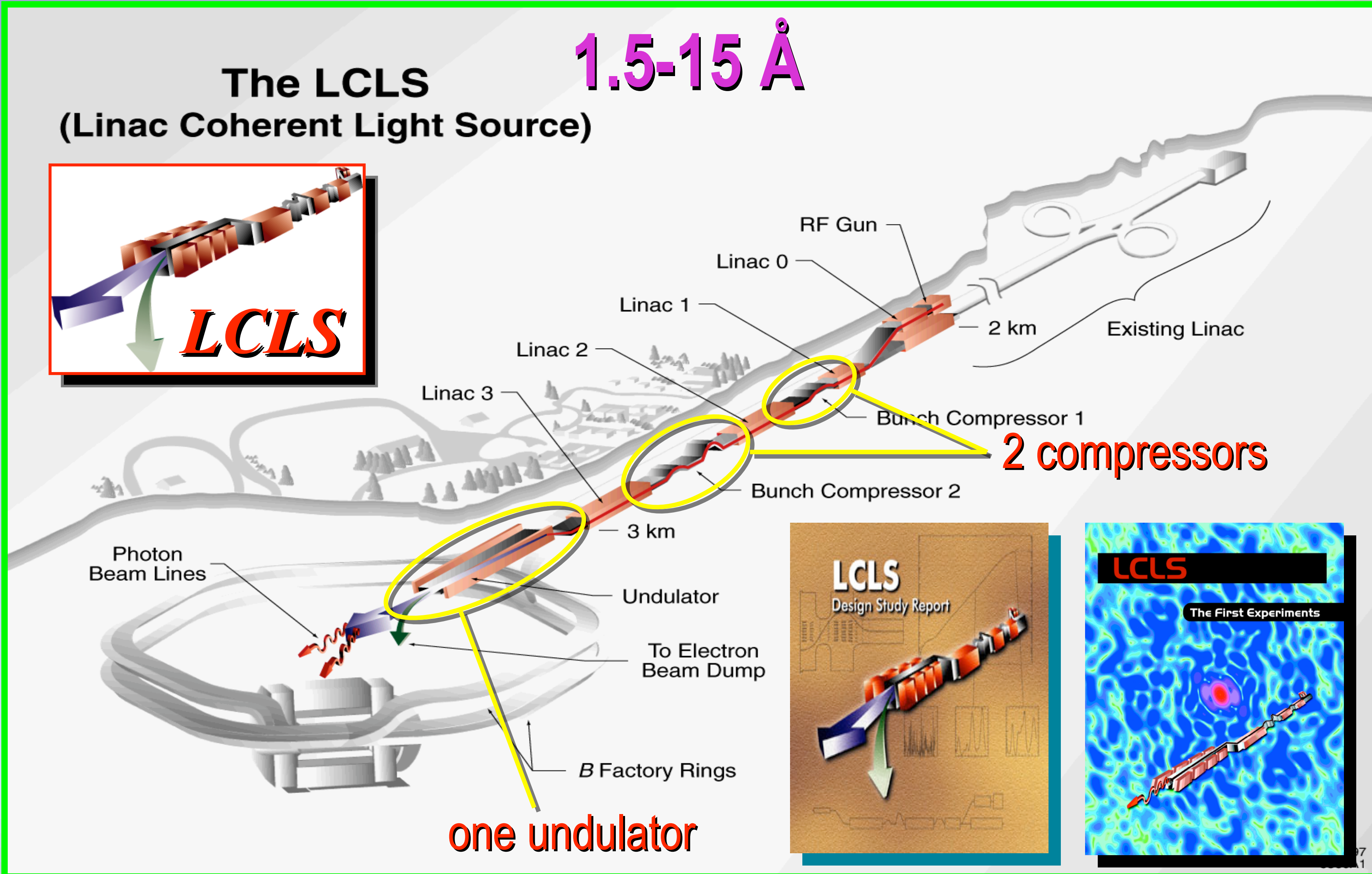
wakefields of the accelerator structures in the linac of the LCLS are an important ingredient of phase space manipulation; other wakes can degrade the beam emittance and, thus, the FEL performance

much understanding in the subject of wakes of extremely short bunches has been gained in the last ~10 years

_ will describe wakes that are important for short bunches; focus on longitudinal plane, analytical expressions

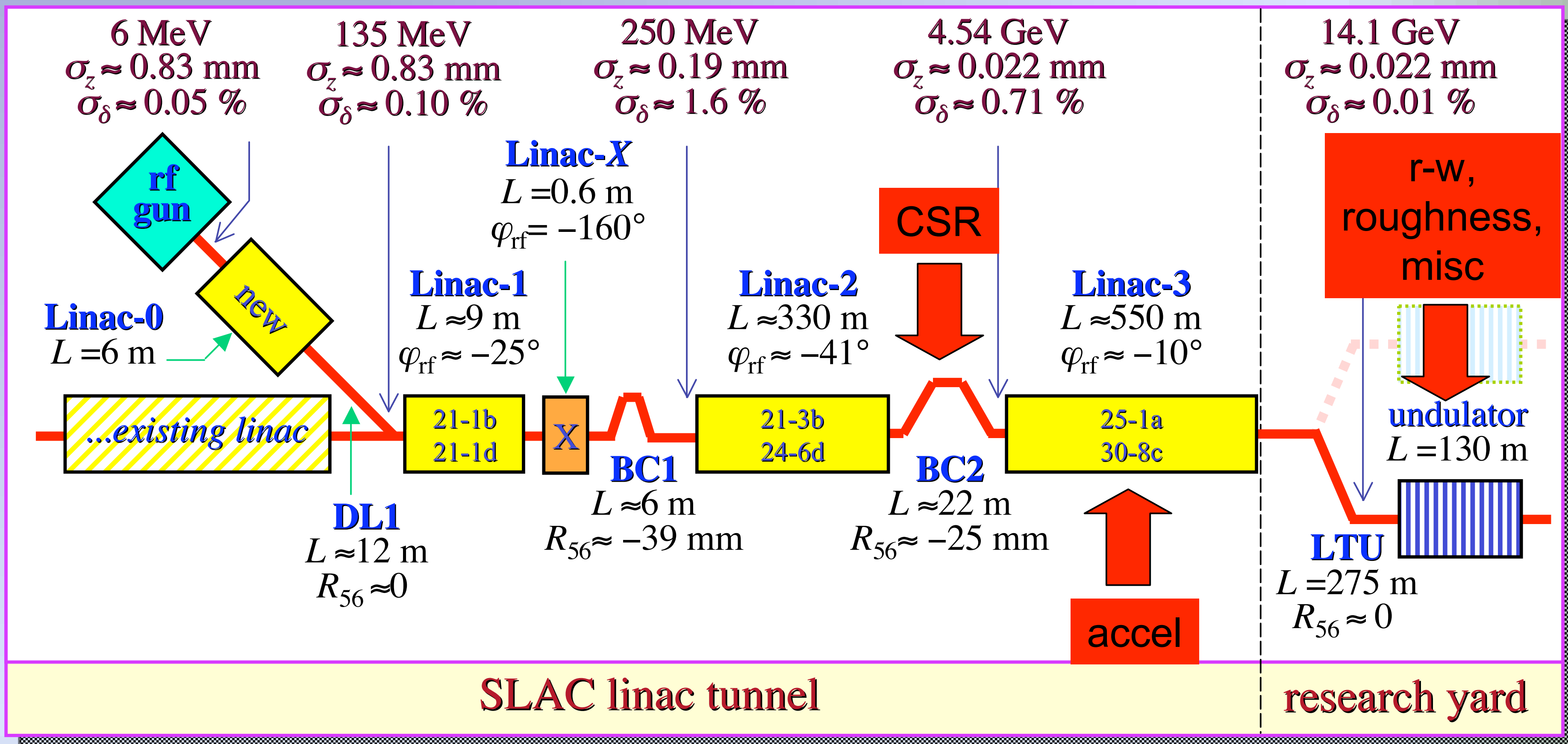
_ will be applied to short-bunch regions of the LCLS, *spec.* for coherent synchrotron radiation (CSR) wake in the BC-2 chicane, accelerator structure wake in Linac-3, and resistive wall and roughness wakes in the undulator

LCLS at *SLAC*



X-FEL based on last 1-km of existing *SLAC* linac

LCLS Accelerator and Compressor Schematic

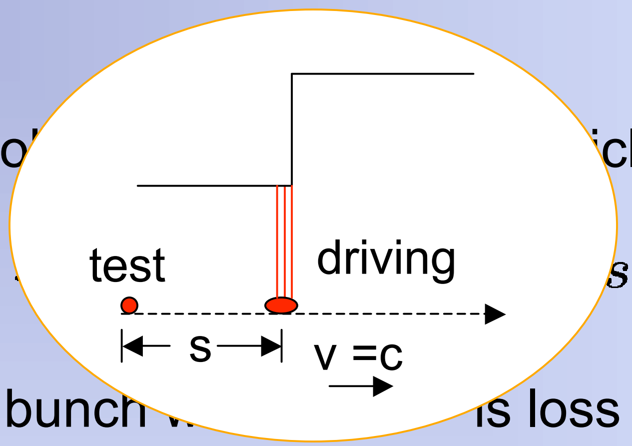


Wakes and Impedances

- consider a particle, moving at speed *c* through a structure, that is followed by a test particle at distance *s*; Wake *W*(*s*) is voltage loss (per structure or per period) experienced by the test particle; *W*(*s*)= 0 for *s*<0.

- bunch wake is vo… …icle in a distribution

$$\mathcal{W}(s) \ldots s')\, ds' \ .$$



average of minus bunch w… …is loss factor; energy spread increase $\delta E_{rms} = eNL\mathcal{W}_{rms}$, with *eN* charge, *L* length of structure (in periodic case).

- impedance

$$Z(k) = \int_0^\infty W(s) e^{iks} ds \ ,$$

- similar for transverse: W_x, Z_x

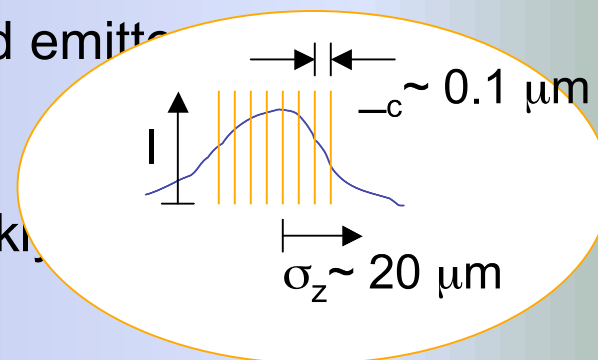
- wakes are used as driving terms in long. or trans. tracking

LCLS Considerations

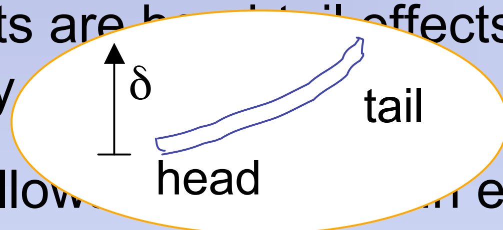
_ in 100 m of undulator, radiation--starting from noise--grows exponentially for ~20 gain lengths, then saturates

_ amplification sensitive to I_{peak} [kA], ε_n [μm], σ_δ [10^{-4}]

- unlike in colliders, not sensitive to projected emitt… amplification sensitive to slice properties



- wakefield effects are h… effects: weak… properties directly



– but limit in allow… energy change in undulator, _δ~ _ (Pierce parameter, ~ 5×10^{-4})

- also specification on bandwidth σ_δ< 0.1% and slice alignment < σ_x

_ longitudinal wakes tend to be most important; linac wakes are important ingredient in final $\lambda(s)$

Considerations for Short Bunches

- numerically obtaining high frequency impedance/short-range wake is typically difficult; often, however, analytic approximations exist

- broad-band impedance suffices (resonances smoothed out)

- can convert between asymptotic behavior of *Z*, *W*, $\mathcal{W}$ (if bunch is smooth) since *W*(0) is finite (at least cut-off by finite γ) and

$$\int_0^\infty W(s)\,ds = 2\pi Z(0) = 0 \;;$$

e.g. long-range resistive-wall wake $W = -As^{-3/2}$; therefore, integrating by parts:

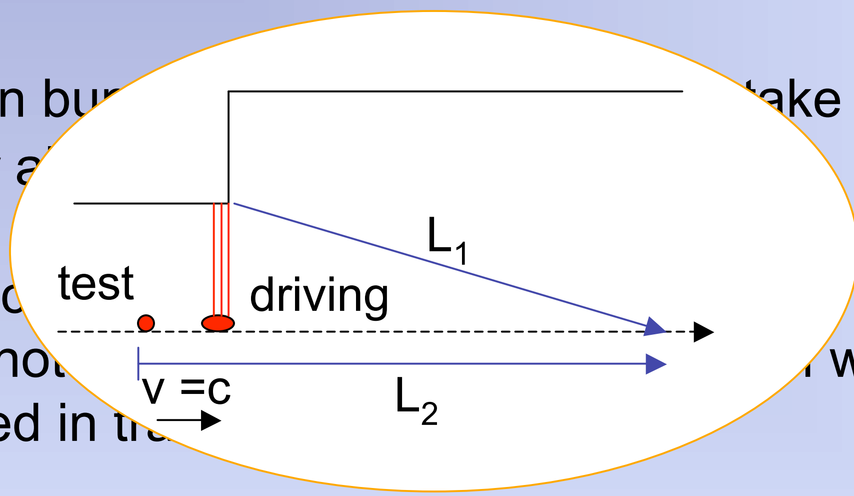
$$\mathcal{W} = -2A\int_0^\infty \frac{d\lambda(s-s')}{ds'}\frac{ds'}{s'^{1/2}}$$

- terminology: from circuit analogy we call wake resistive if $\mathcal{W} \sim \lambda$, inductive if $\mathcal{W} \sim \lambda'$, capacitive if $\mathcal{W} \sim \int \lambda(s)\, ds$

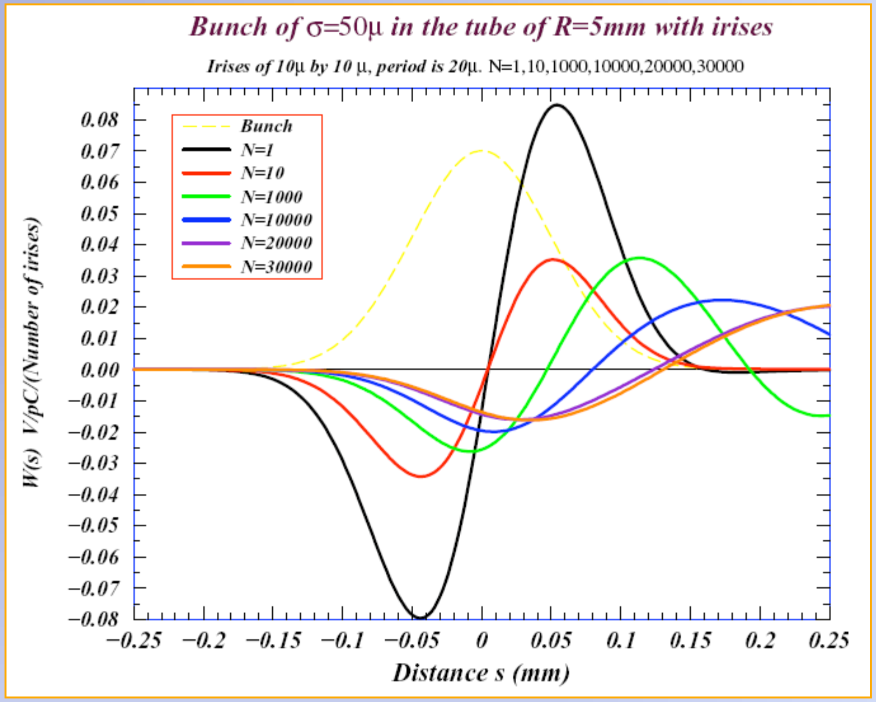
● catch-up distance: if head particle passes e.g. the beginning of a cavity, tail particle doesn't know it until $z= a^2/2s$ (a beam pipe radius, s separation of particles) later. If a= 1cm and s= 20 μm, then z= 2.5 m.

for Gaussian bu… take many times this distance for a…

wake is typic… the catch-up distance is no… wave-length, wake can't be used in tr…



_ transient region: similarly, for periodic structures, there will be a transient regime before steady-state is reached; for Gaussian with length σ_z, transient will last until $z \sim a^2/2\sigma_z$



Simulation of wake per period generated by a bunch in a tube with *N* small corrugations (A. Novokhatski).

limiting value of wake: for periodic, cylindrically symmetric structures whose closest approach to axis is *a*, the steady-state wakes have the property

$$W(0^+) = \frac{Z_0 c}{\pi a^2} \quad \text{and} \quad W_x'(0^+) = \frac{2 Z_0 c}{\pi a^4} \ ,$$

with $W_x(0^+)= 0$, where $Z_0= 377\ \Omega$.

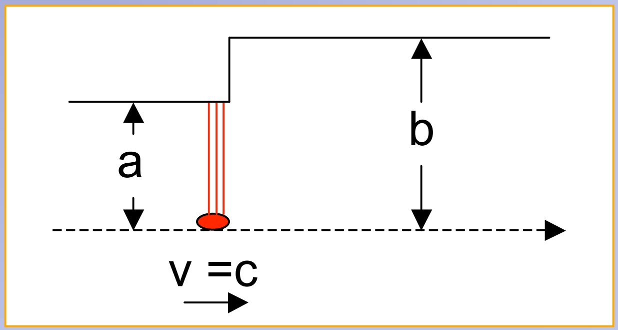
_ this is true for a resistive pipe, a disk-loaded accelerator structure, a pipe with small periodic corrugations, and a dielectric tube within a pipe; it appears to be a general property

_ for very short bunches the longitudinal wake approaches a maximum, the transverse wake zero

finite energy: impedance drops sharply to 0 when $k>\gamma/a$ (γ Lorentz energy factor); for $\sigma_z< a/\gamma$, replace σ_z by a/γ in wake formulas; if a= 1 cm, energy E= 14 GeV, this occurs when $\sigma_z = 0.4$ μm.

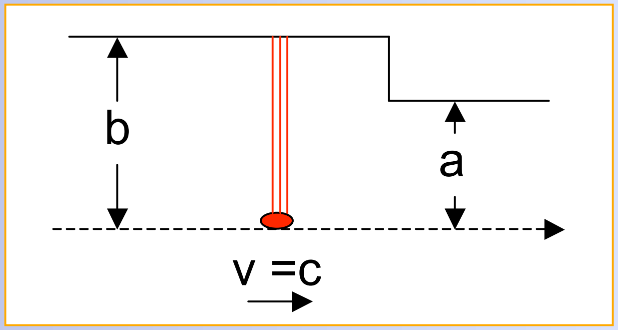
Example Short-Bunch Wakes

a. Pair of Shallow Transitions (Fraunhoffer) diffraction



"out transition"

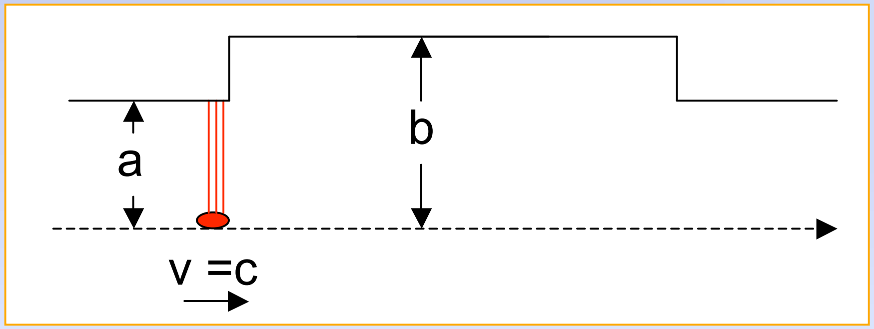
$$Z_{hi} \approx \frac{Z_0}{\pi} \ln(b/a)$$



"in transition"

$$Z_{hi} \approx 0$$

(Heifets & Kheifets)



$$Z_{hi} \approx \frac{Z_0}{\pi} \ln(b/a)$$, a constant

- wake: $$W = \frac{Z_0 c}{\pi} \ln(b/a)\delta(s)$$

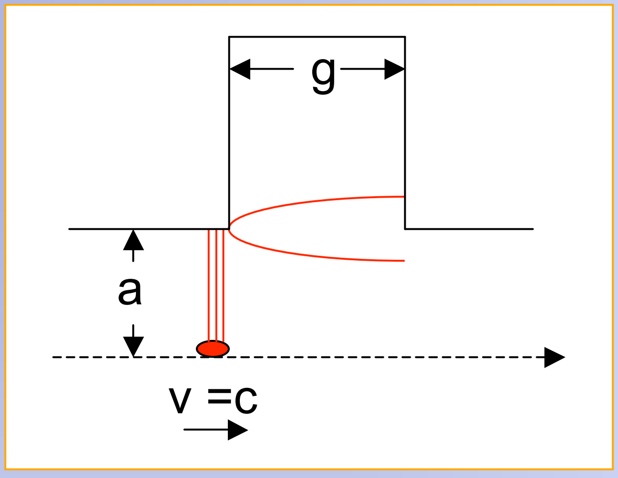
  gaussian wake: $$\mathcal{W}_{gz} = -\frac{Z_0 c \ln(b/a)}{\sqrt{2}\pi^{3/2}\sigma_z} e^{-\frac{1}{2}z^2/\sigma_z^2}$$, resistive

- dipole: $$Z_x = \frac{Z_0}{\pi k}\left(\frac{1}{a^2} - \frac{1}{b^2}\right)$$ (Gianfelice & Palumbo)

  implies $$\mathcal{W}_{gx} \propto \int^s \lambda_z(s')\, ds'$$, capacitive

- tapering doesn't help until tan $\theta \sim \sigma_z/a$; σ_z= 20 μm, *a*= 2.5 mm => $\theta \sim$ 0.5 deg

b. Single Cavity (Fresnel) diffraction



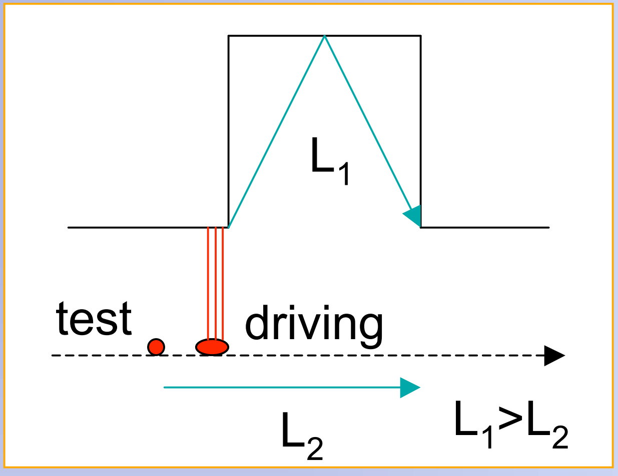
(J.D. Lawson)

- high frequency impedance: $$Re(Z) = \frac{Z_0}{2\pi^{3/2}a}\sqrt{\frac{g}{k}}$$

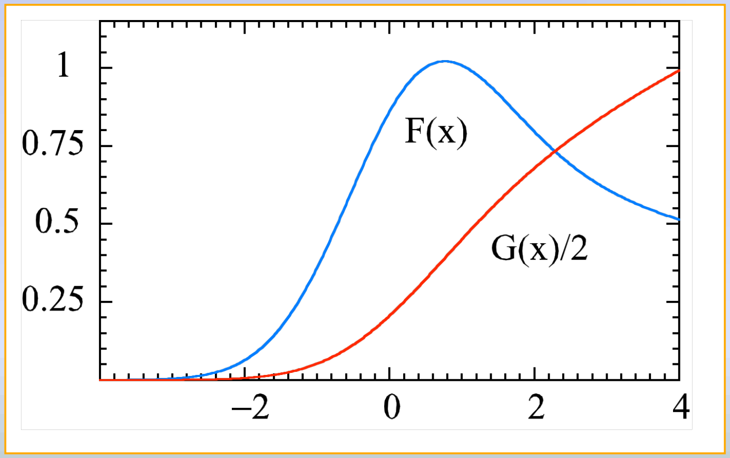
=> short-range wake: $$W = \frac{Z_0 c}{\sqrt{2}\pi^2 a}\sqrt{\frac{g}{s}}$$

- dipole wake: $$W_x = \frac{2}{a^2}\int W(s)\, ds$$

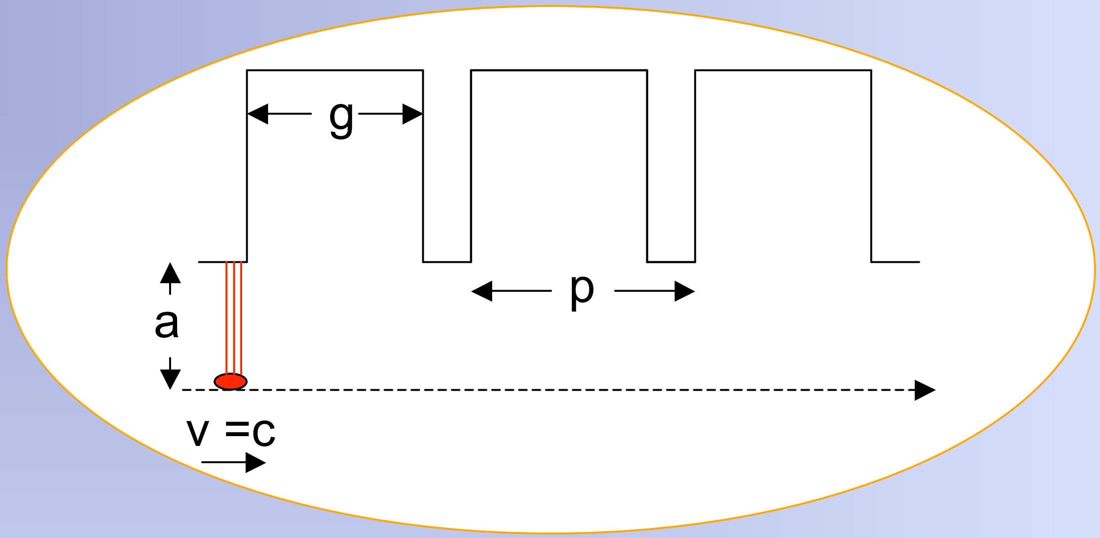
- this model is valid when $g < 2(b-a)^2/l$ where b is outer cavity dimension and l is total bunch length



- gaussian bunch: $\mathcal{W}_{gz} = W(\sigma_z)F(s/\sigma_z)$, $\mathcal{W}_{gx} = W_x(\sigma_z)G(s/\sigma_z)$



c. Accelerator Structure Wake



- high frequency impedance:

$$Z(k) \approx \frac{iZ_0}{\pi k a^2}\left[1+(1+i)\frac{\alpha(g/p)\,p}{a}\left(\frac{\pi}{kg}\right)^{1/2}\right]^{-1},$$ [Gluckstern; Yokoya and Bane]

$$\alpha(x) \approx 1 - 0.465\sqrt{x} - 0.070x$$

- inverse Fourier transform gives very short-range wake:

$$W \approx \frac{Z_0 c}{\pi a^2}\exp\left(\frac{2\pi\alpha^2 p^2 s}{a^2 g}\right)\mathrm{erfc}\left(\frac{\alpha p}{a}\sqrt{\frac{2\pi s}{g}}\right)$$

(e.g. SLAC linac: +4% error at s= 200 μm)

- at high frequency $Z_x = 2Z/(a^2 k)$ [Gluckstern, et al]

_ numerical calculation of wake can be obtained by field matching, or time domain calculation

_ numerical calculation of wake can be fit to (over useful parameter range)

$$W(s) = \frac{Z_0 c}{\pi a^2} \exp\left(-\sqrt{s/s_1}\right) \quad \text{, with} \quad s_1 = 0.41 \frac{a^{1.8} g^{1.6}}{p^{2.4}} .$$

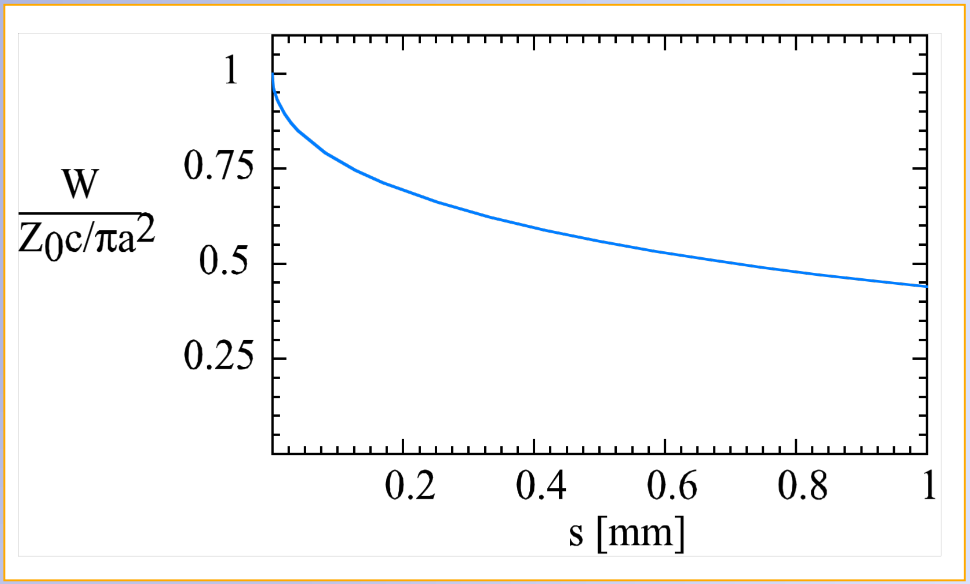
in SLAC linac, s_1=1.5 mm [K. Bane, et al]

_ for LCLS Linac-3, σ_z= 20 µm, W~ constant; note transient regime $z \sim a^2/2\sigma_z \sim$ 3.4 m (small compared to 550 m)

_ same has been done for transverse wake

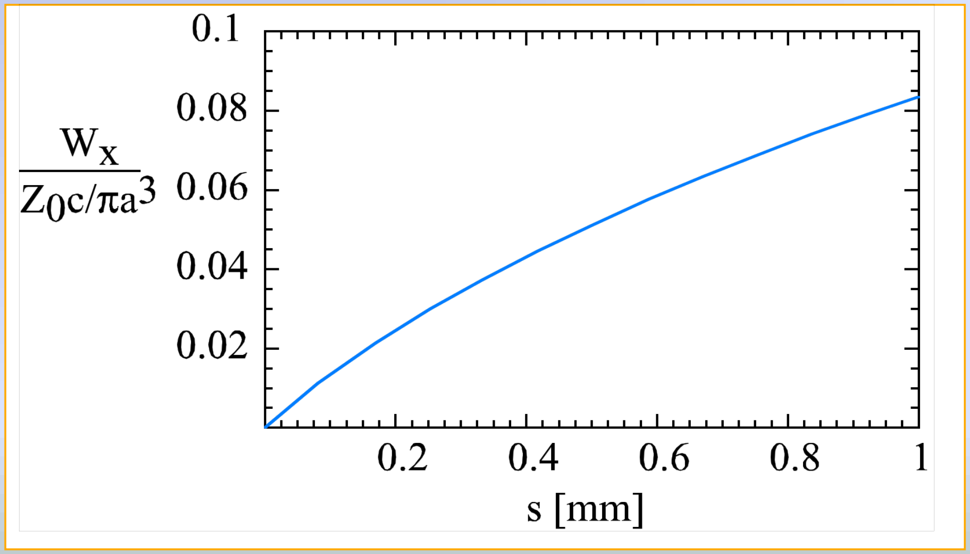
- SLAC linac wakes:

$$\frac{Z_0 c}{\pi a^2} = 0.27\ \frac{\mathrm{GV}}{\mathrm{nC \cdot km}}$$



longitudinal

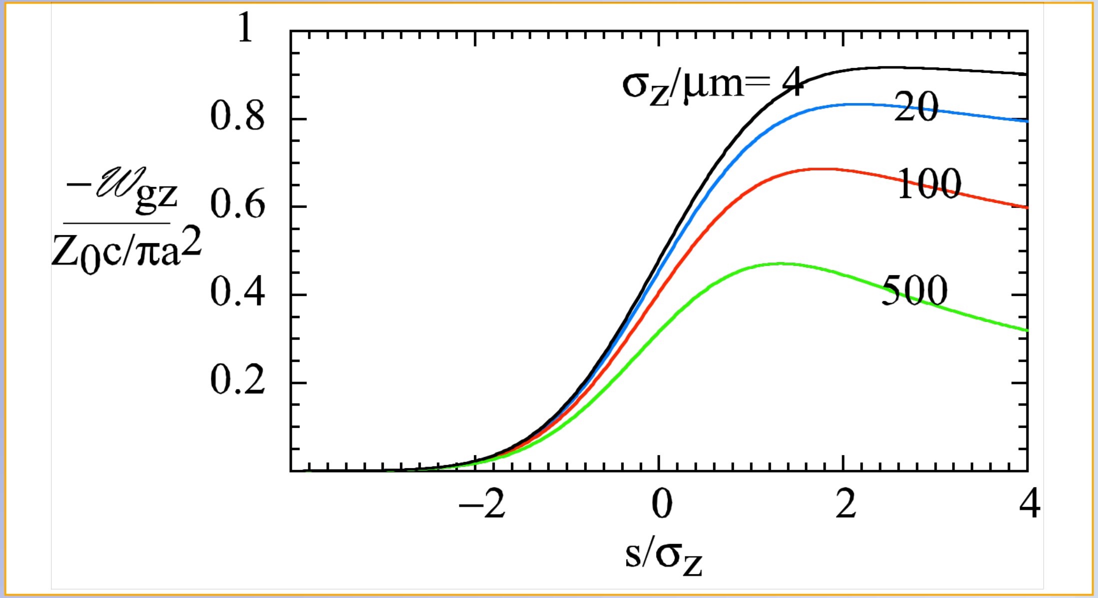
$$\frac{Z_0 c}{\pi a^3} = 23\ \frac{\mathrm{kV}}{\mathrm{nC \cdot m \cdot mm}}$$



transverse

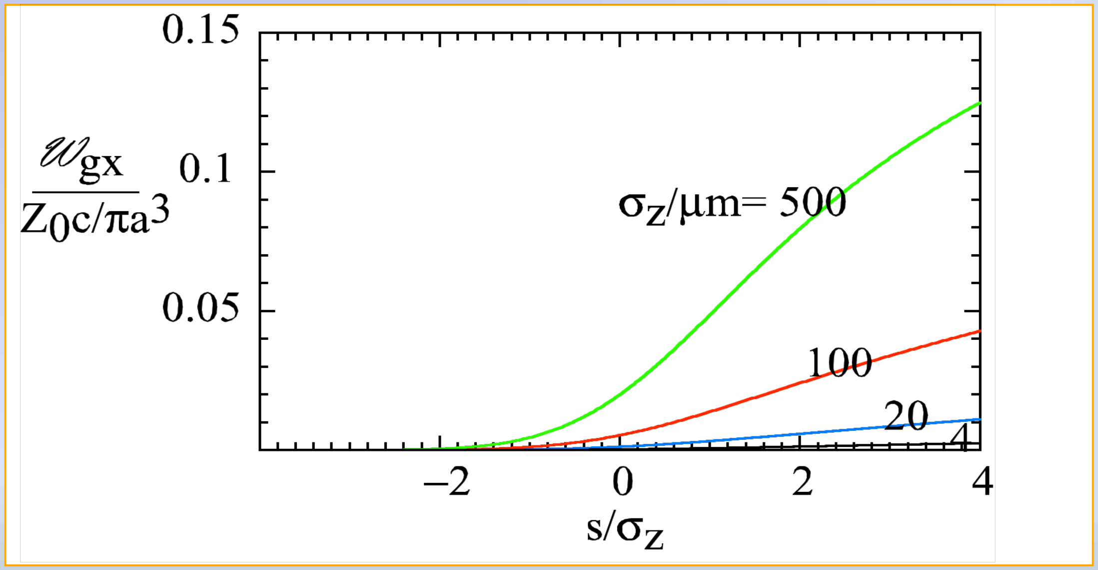
- SLAC linac, gaussian bunch:

$$\frac{Z_0 c}{\pi a^2} = 0.27 \; \frac{\mathrm{GV}}{\mathrm{nC \cdot km}}$$



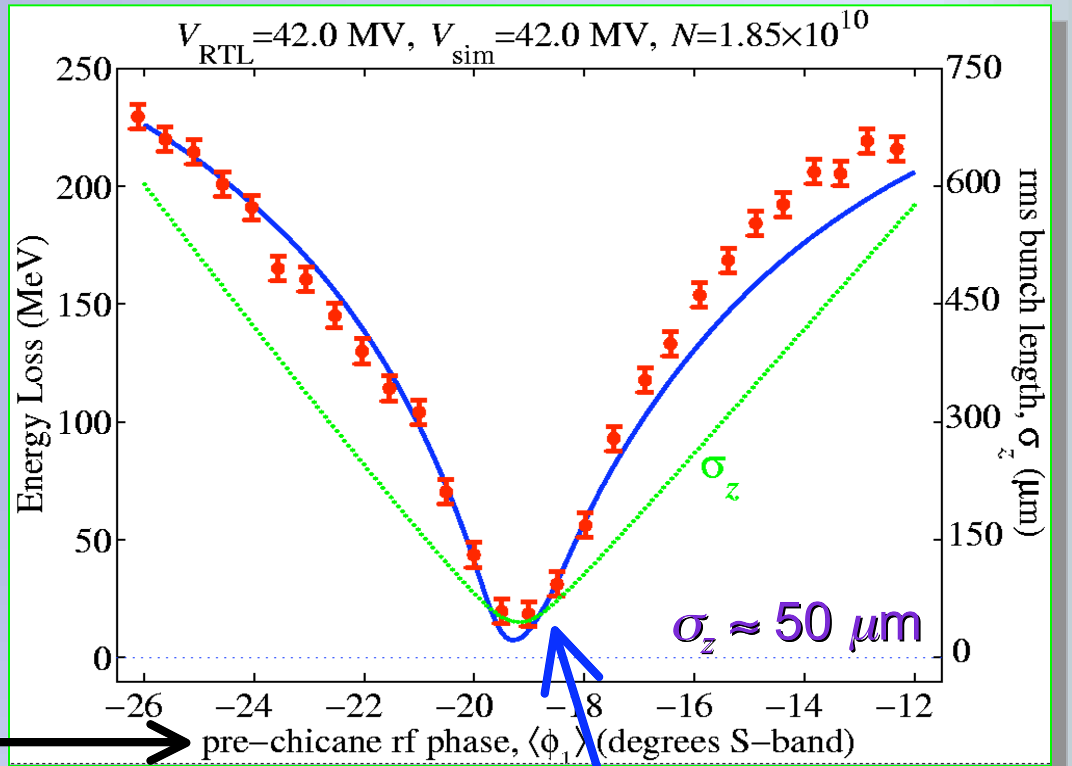
longitudinal

$$\frac{Z_0 c}{\pi a^3} = 23 \; \frac{\mathrm{kV}}{\mathrm{nC \cdot m \cdot mm}}$$

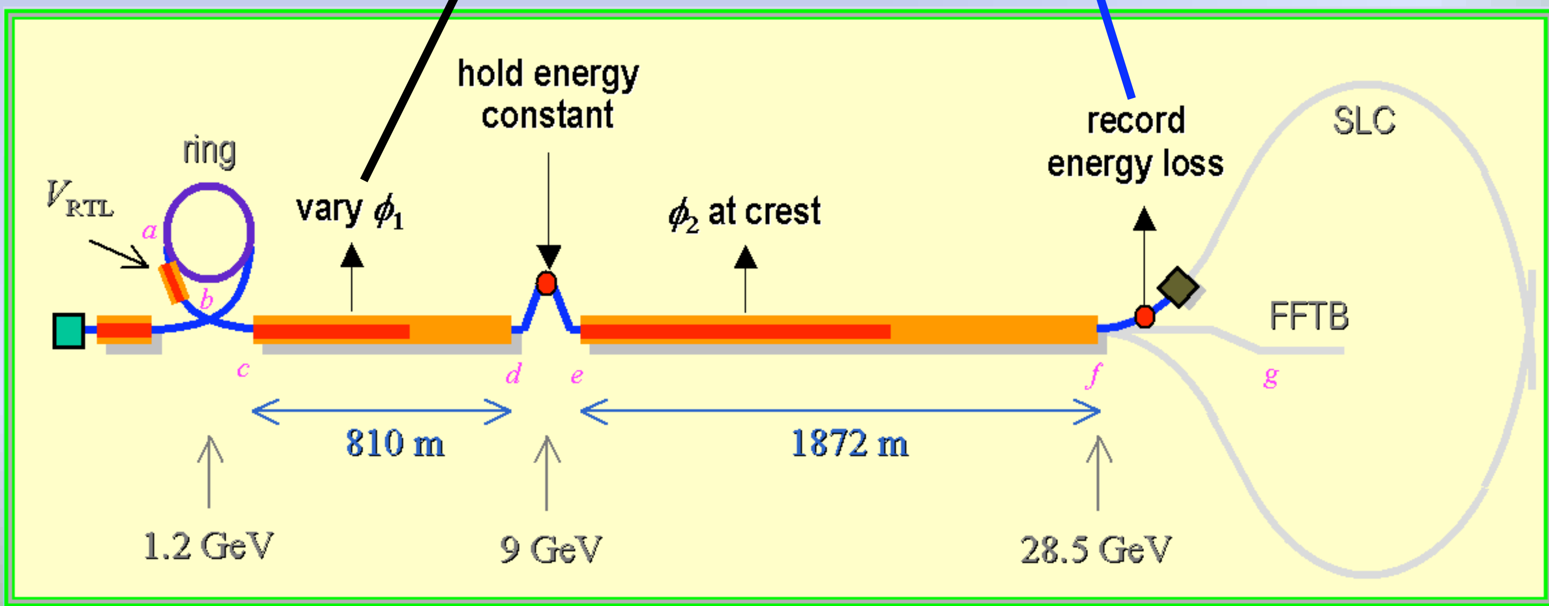


transverse

Wakefield energy-loss used to set and confirm minimum bunch length



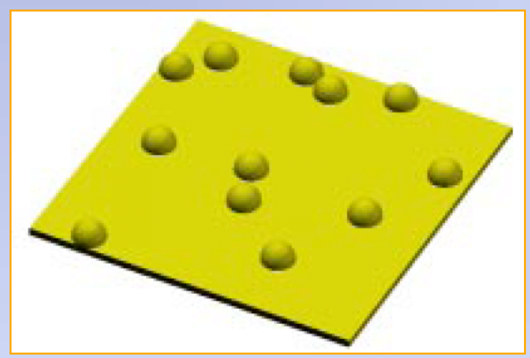
K. Bane *et al.*, PAC'03



d. Roughness Impedance

A metallic beam pipe with a rough surface has an impedance that is enhanced at high frequencies. Two approaches to modeling are (i) random collection of bumps, (ii) small periodic corrugations

(i) Random bumps



Impedance of one hemispherical bump (of radius *h*) for $k \ll 1/h$

$$Z(k) = ikc\mathcal{L}_1 = ik\frac{Z_0 h^3}{4\pi a^2} \ ,$$

[S. Kurennoy]

_ for many bumps (α filling factor, *f* form factor)

$$\mathcal{L}/L = \frac{2\alpha f a \mathcal{L}_1}{h^2} = \frac{\alpha f Z_0 h}{2\pi a c} \;,$$

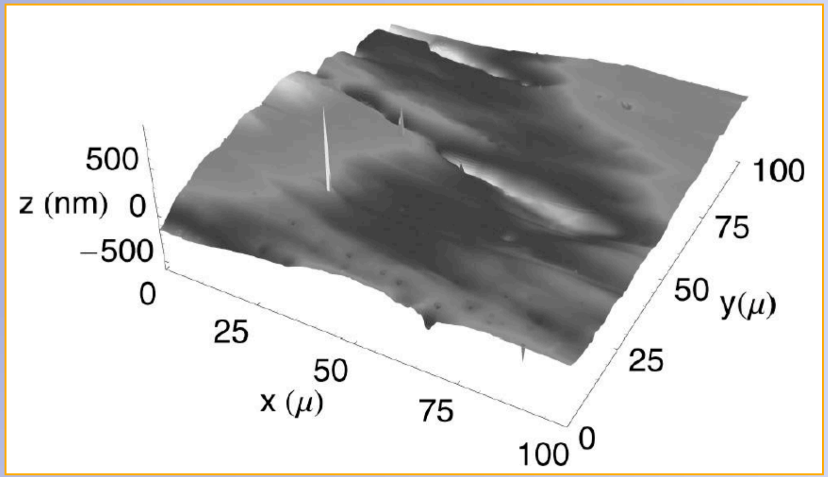
- bunch wake ~λ': can't use model for rectangular or other non-smooth distribution; gaussian $(\mathcal{W}_{gz})_{rms} \approx 0.06 c^2 \mathcal{L}/L \sigma_z^2$

_ idea has been systematized so that, from surface measurement, can find impedance:

$$\mathcal{L}/L = \frac{Z_0}{2\pi c a} \int_{-\infty}^{\infty} \frac{k_z^2}{\sqrt{k_\theta^2 + k_z^2}} S(k_z, k_\theta)\, dk_z dk_\theta \;,$$

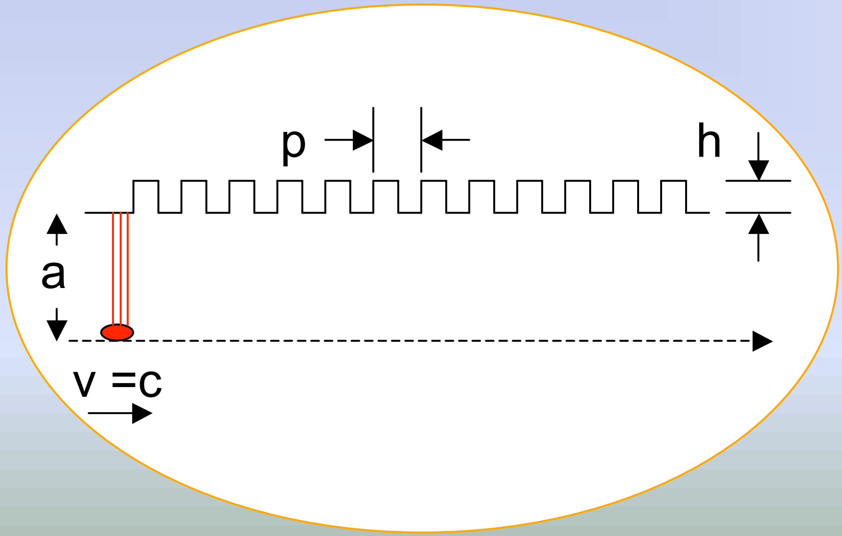
with *S* spectrum of surface, k_z, k_θ, longitudinal, azimuthal wave numbers

[K. Bane, et al; G. Stupakov]



Sample profile measured with atomic force microscope
[from G. Stupakov, et al]

(ii) Small periodic corrugations



motivation: numerical simulations of many randomly placed, small cavities on a pipe found that, in steady state, the short range wake is very similar to truly periodic case

_ consider a beam pipe with small corrugations of height *h*, period *p*, and gap *p*/2. If $h/p \gtrsim 1$, wake

$$W(s) \approx \frac{Z_0 c}{\pi a^2} \cos k_0 s \quad \text{with} \quad k_0 = \frac{2}{\sqrt{ah}} \ .$$

_ for gaussian, with $k_0\sigma_z$□1, becomes inductive with $\mathcal{L}/L = Z_0 h/(4ac)$, similar to earlier model

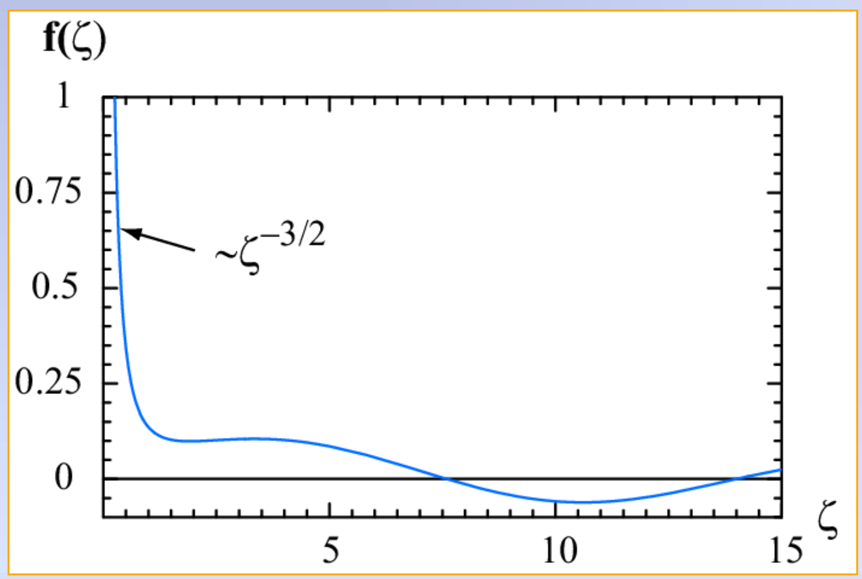
_ can be used with non-smooth bunch distribution

[A. Novokhatski, et al; K. Bane and A. Novokhatski]

- however, when *h*/*p*□ 1

$$W(s) = \frac{Z_0 c h^2 k_1^3}{4\pi a} f(k_1 s)\ , \quad f(\zeta) = -\frac{1}{2\sqrt{\pi}}\frac{\partial}{\partial\zeta}\frac{\cos(\zeta/2)+\sin(\zeta/2)}{\sqrt{\zeta}}$$

with $k_1 = 2\pi/p$



[G. Stupakov]

_ for $k_1 s \lesssim 1$ (but not too small):

_ $W \sim s^{-3/2}$; for bunch $\mathcal{W}_{zg} \sim \sigma_z^{-3/2}$ (vs. σ_z^{-2} for other model)

_ bunch wake weaker by ~*h*/*p* than single mode model

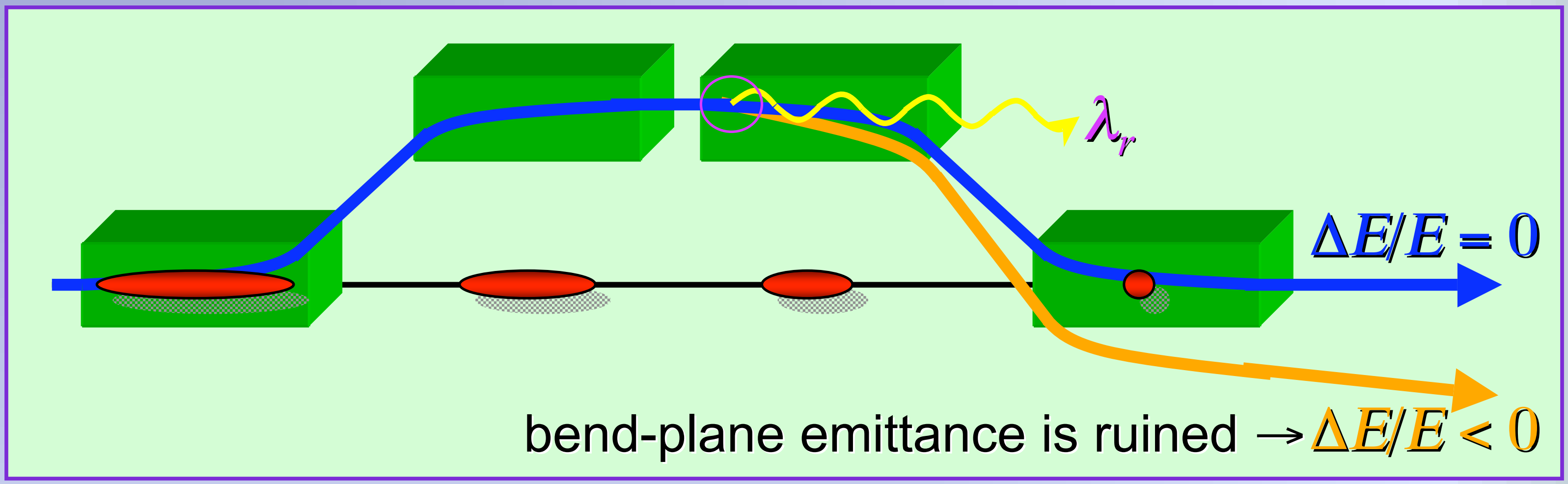
_ for LCLS, if we assume (earlier displayed) measured surface profile is representative of undulator beam pipe ($h \sim 0.5$ µm, $p \sim 100$ µm) and $\sigma_z = 20$ µm, then this model applies, and

$\Rightarrow$roughness wake 0.15 as strong as resistive wall wake (with Cu)

_ some measurements have been done (DESY, Brookhaven) but more needed

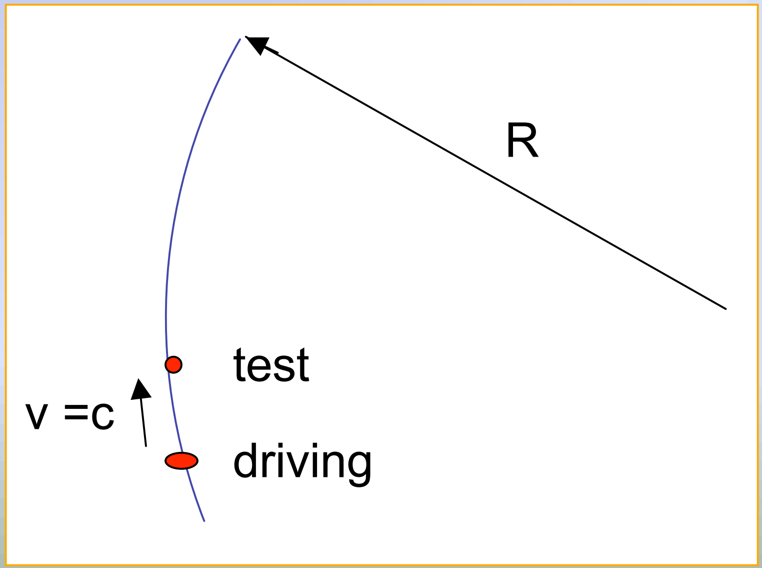
e. CSR Wake

_ CSR effect on bunch can be described as a wakefield effect



_ Consider ultra-relativistic particle (and test particle) moving on circle of radius *R* in free space

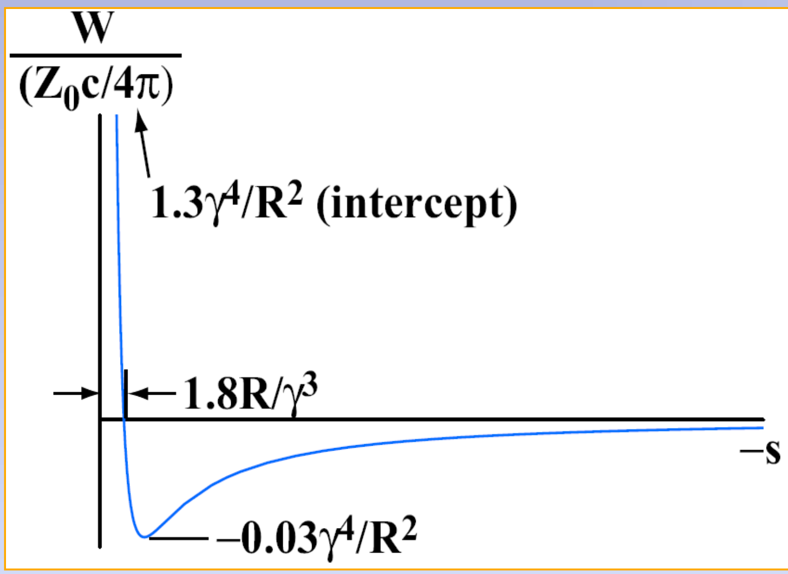
_ for effect test charge needs to be *ahead* of driving charge



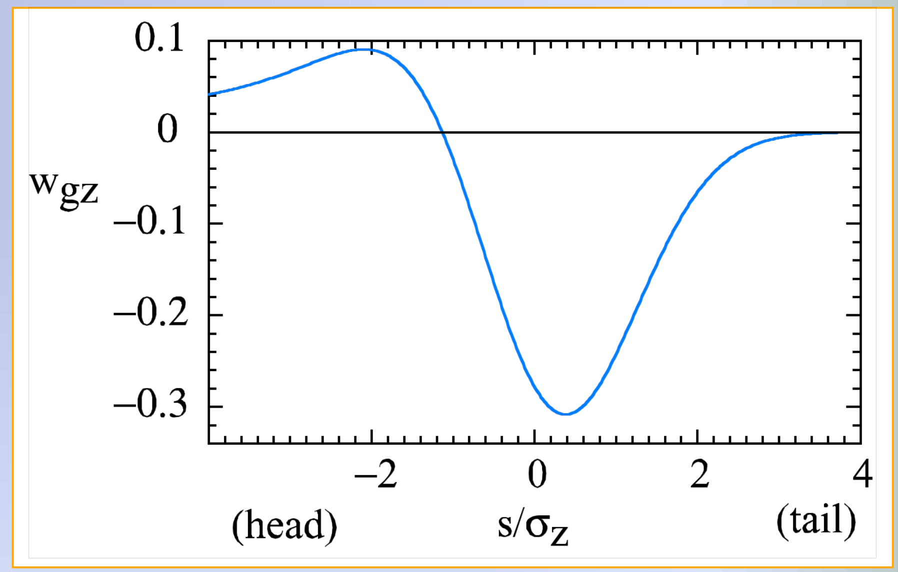
- (Steady-state) wake, for $(-s) \gg R/\gamma^3$

$$W(s) = -\frac{Z_0 c}{2 \cdot 3^{4/3} \pi R^{2/3} (-s)^{4/3}} \qquad s < 0 \ ,$$

while $W(0^-) = Z_0 c \gamma^4 / (3\pi R^2)$



point charge wake



gaussian wake:

$$\mathcal{W}_{gz} = \frac{Z_0 c}{2\pi} w_{gz} R^{-2/3} \sigma_z^{-4/3}$$

- $Z \sim k^{1/3}$

[J. Murphy, et al; Y. Derbenev, et al]

_ shielded by beam pipe if $\sigma_z/a < (a/R)^{1/2}$; for BC2 of LCLS σ_z= 20 μm, a= 1cm, R= 15 m ⇒ bunch is 13 times too short for shielding

_ on entering a bend, the distance of transient wakes is $z \approx (24R^2\sigma_z)^{1/3}$; for above example transient z= 0.5 m

_ to simulate CSR force in a chicane including transients, 1D, 2D, 3D computer programs are available (e.g. TRAFIC4)

_ If compression factor is large, one can estimate CSR energy loss from "compression work". For a gaussian

$$\langle \delta E \rangle = -\frac{e^2 N Z_0 c}{4\pi^{3/2}\sigma_z} \ln\left(\frac{\gamma\sigma_z}{\sigma_x + \sigma_y}\right)$$

where beam sizes are final quantities, and the rms spread $\delta E_{rms} \approx -0.4 \langle \delta E \rangle$

[M. Dohlus; K. Bane and A. Chao]

f. Resistive Wall Wake

- impedance (see A. Chao): $$Z = \left(\frac{Z_0}{2\pi a}\right)\frac{1}{\frac{\lambda}{k} - \frac{ika}{2}}$$

with $\lambda = \sqrt{\frac{2\pi\sigma|k|}{c}}\,[i + sgn(k)]$

- low frequency $Z \sim k^{1/2}$ => familiar long-range wake:

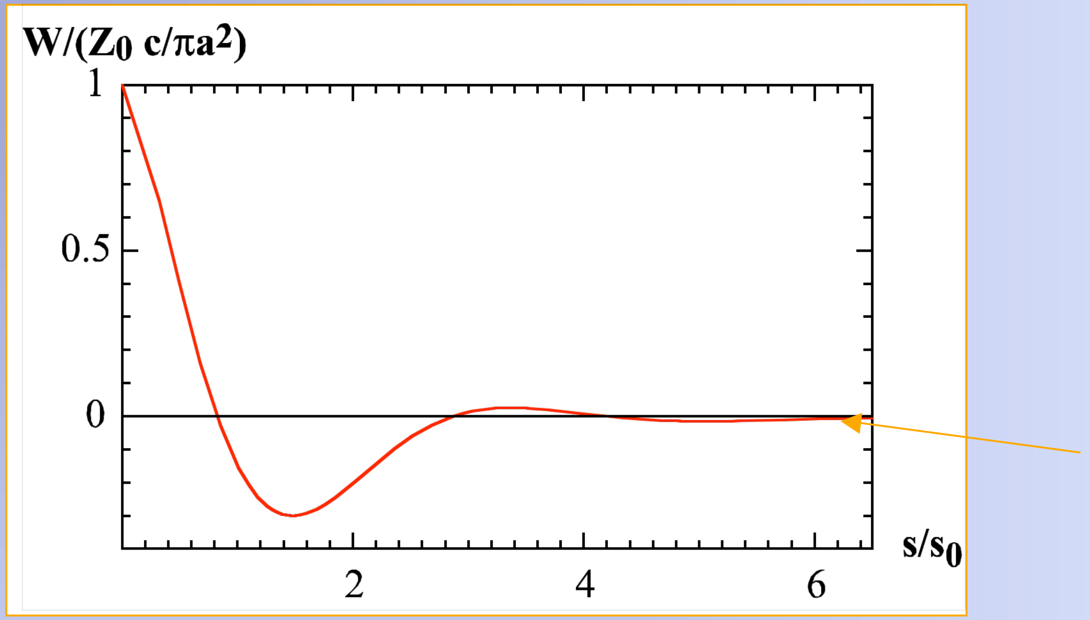
$$W(s) = -\frac{c}{4\pi^{3/2}a}\sqrt{\frac{Z_0}{\sigma}}\frac{1}{s^{3/2}} \; ,$$

- general solution:

$$W = \frac{4Z_0 c}{\pi a^2}\left(\frac{e^{-s/s_0}}{3}\cos\frac{\sqrt{3}s}{s_0} - \frac{\sqrt{2}}{\pi}\int_0^\infty \frac{dx\, x^2 e^{-x^2 s/s_0}}{x^6 + 8}\right)$$

- characteristic distance: $s_0 = \left(\frac{2a^2}{Z_0\sigma}\right)^{\frac{1}{3}}$

(for Cu with a= 2.5mm, s_0= 8.1μm)



wake for dc conductivity

- note: for $\sigma_z \sim s_0$, $\mathcal{W}_{gz}$ does *not* scale as $\sigma^{-1/2}$

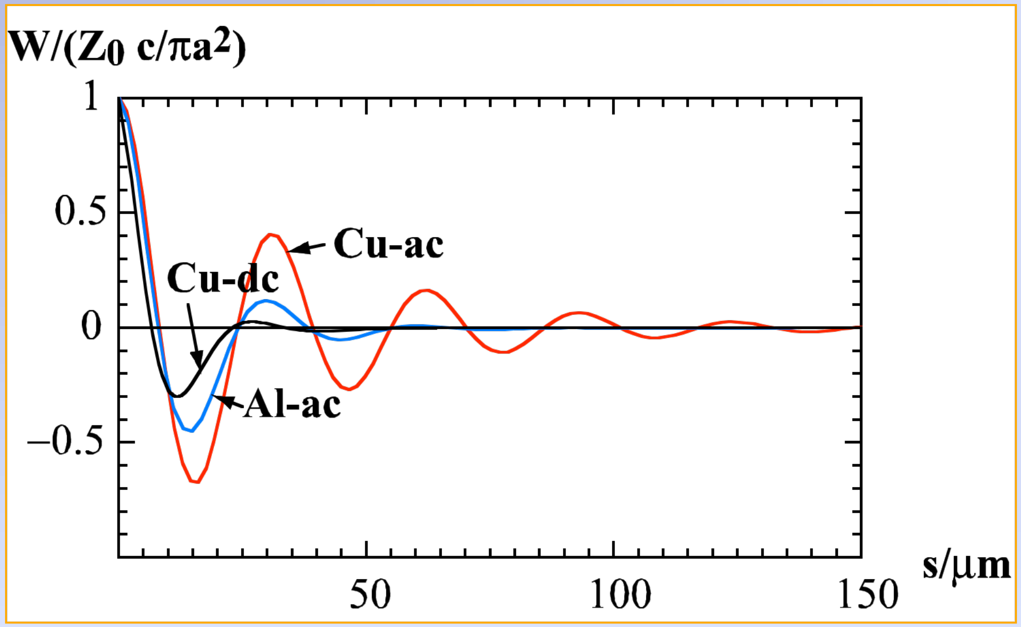
- but at high frequencies dc impedance model not consistent with Drude-Sommerfeld free-electron model of conductivity, since

dc conductivity: $\sigma = ne^2\tau/m$; ac conductivity: $\tilde{\sigma} = \dfrac{\sigma}{1 - ikc\tau}$

- new parameter $\Gamma = c\tau/s_0$.

- for Cu with beam pipe radius a= 2.5 mm, s_0= 8 μm, $c\tau$= 8 μm, Γ= 1.0; for Al, s_0= 9.3 μm, $c\tau$= 2.4 μm, Γ= 0.26.

- for ac conductivity replace σ with $\tilde{\sigma}$ in parameter λ; then again take inverse Fourier transform of *Z* for wake



point charge wake when a= 2.5 mm

[K. Bane and M. Sands]

- for $\Gamma \gtrsim 1$, can approximate

$$W_z(s) = \frac{Z_0 c}{\pi a^2} e^{-s/4c\tau} \cos\left[\sqrt{\frac{2k_p}{a}} s\right]$$

with the plasma frequency $k_p = \sqrt{4\pi\sigma/\tau}/c$

- for LCLS with Cu, plasma wave number k_p= 1/0.02μm; mode wave number k_r= 1/5μm, damping time $c\tau$= 32μm

- for reduced response, Al is preferable to Cu

•*Anomalous skin effect* (Reuter and Sondheimer)

when mean free path l> $\delta = c/\sqrt{2\pi\omega\sigma}$ the skin depth, ASE occurs, the fields don't drop exponentially with distance into metal

for Cu at room temperature, l= 0.04μm and for *k*= 1/20μm, δ= 0.04μm

ASE parameter $\alpha=1.5l^2/\delta^2$; normalized parameter $\Lambda= \alpha/kc\tau$; for Cu at room temperature Λ= 3.4

have obtained the wakefield with ASE, and find the effect is small (~10% for LCLS)

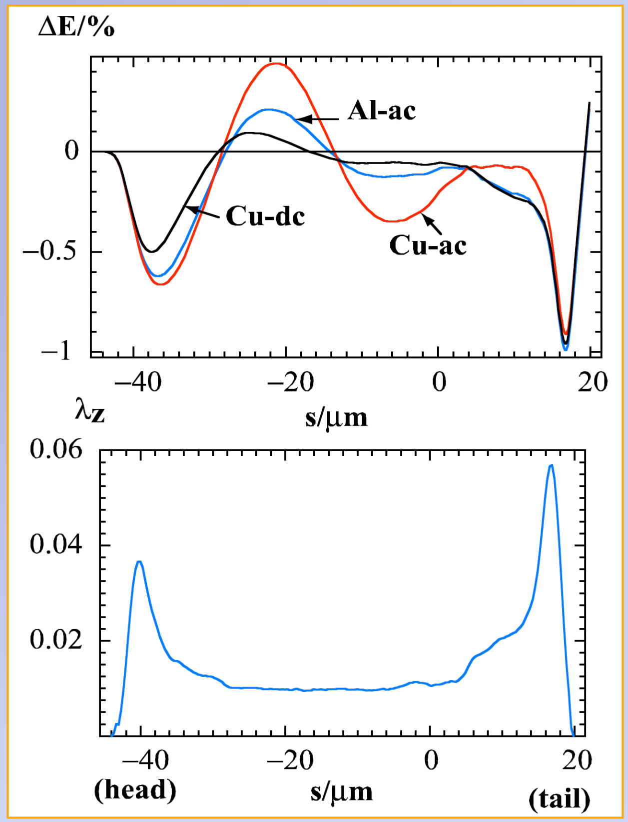
•*Flat chamber*

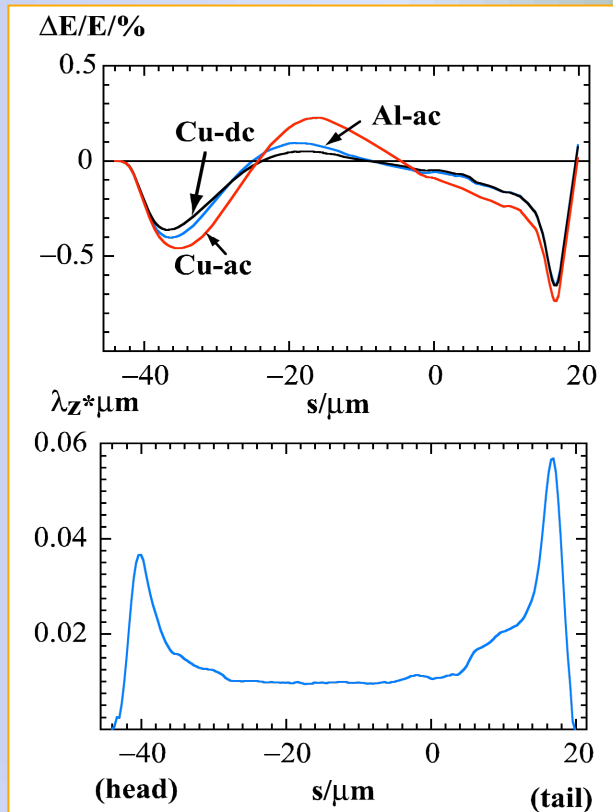
keeping the vertical aperture fixed, expect a flat chamber to have reduced wake

have done this calculation; find LCLS wake effect reduced by ~30%

LCLS parameters

charge—1 nC,
energy—14 GeV,
tube radius—2.5 mm,
tube length—130 m





induced energy deviation for round chamber

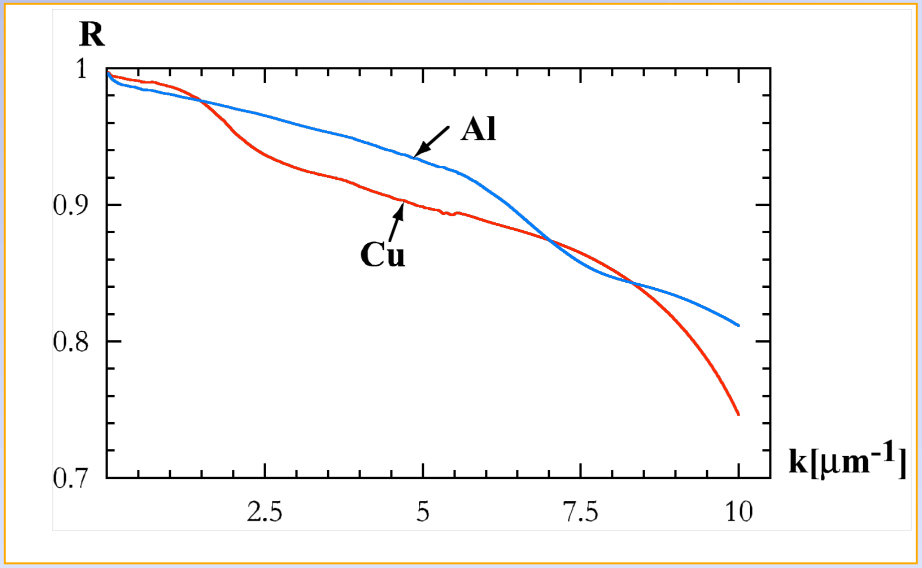
induced energy deviation for flat chamber

[K. Bane and G. Stupakov]

Brookhaven Reflectivity Measurements

- is the free-electron model accurate over our frequency range of interest? In literature not much data.

- Jiufeng Tu measured normal incidence reflectivity on Cu and Al samples at VUV ring at Brookhaven



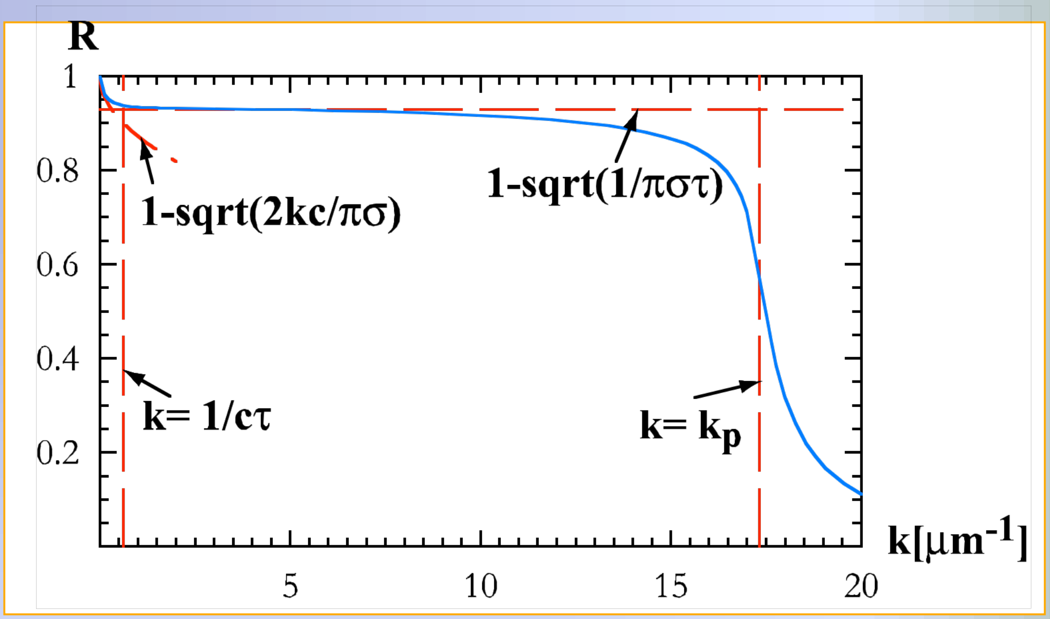
reflectance data

Behavior of R for free-electron model

$$\tilde{\sigma} = \frac{\sigma}{1 - ik c\tau} \ ,$$

$$\tilde{n} = \sqrt{\tilde{\epsilon}} = \sqrt{1 + \frac{4\pi i \tilde{\sigma}}{kc}} \ ,$$

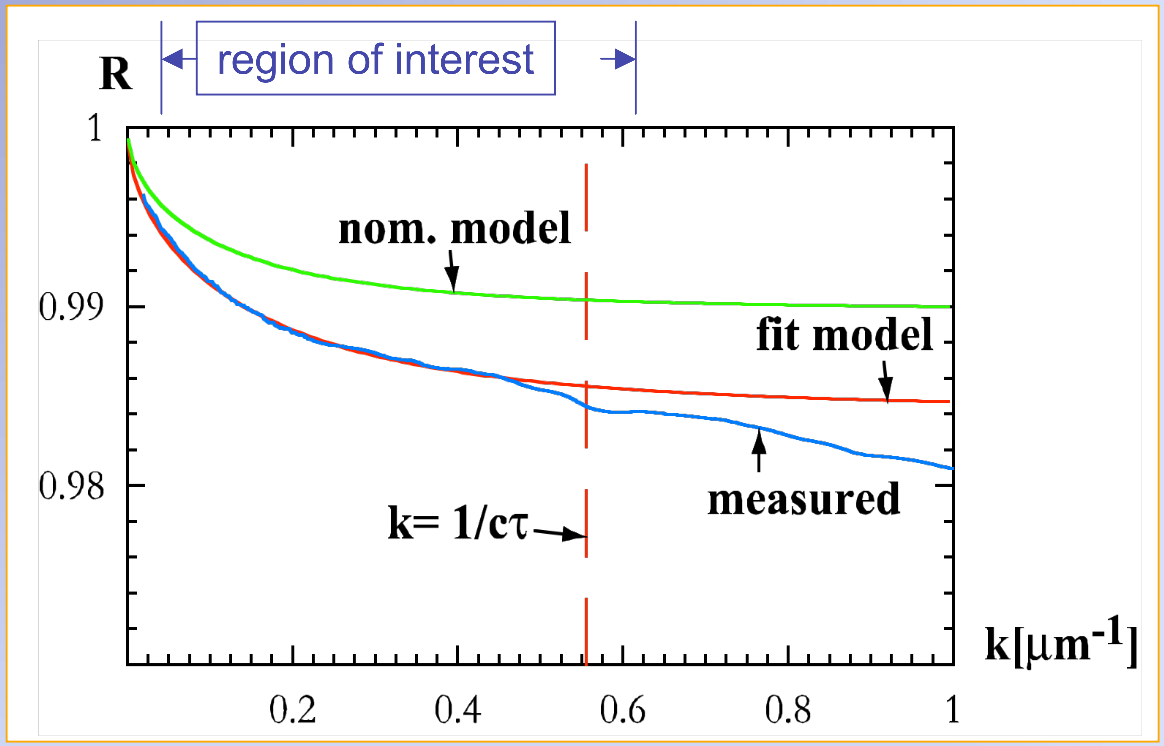
$$R = \left| \frac{\tilde{n} - 1}{\tilde{n} + 1} \right|^2 \ .$$



Frequency *k* vs. reflectivity *R* for a metallic conductor, assuming the free electron model (solid line). For this example $\sigma = 1.2\times10^{16}$ /s and $\tau = 5.4\times10^{-15}$ s. Analytic guideposts are also given (dashes).

--for LCLS bunch interested in λ: [10,100] μm, or *k*: [0.06,0.6]μm^{-1}

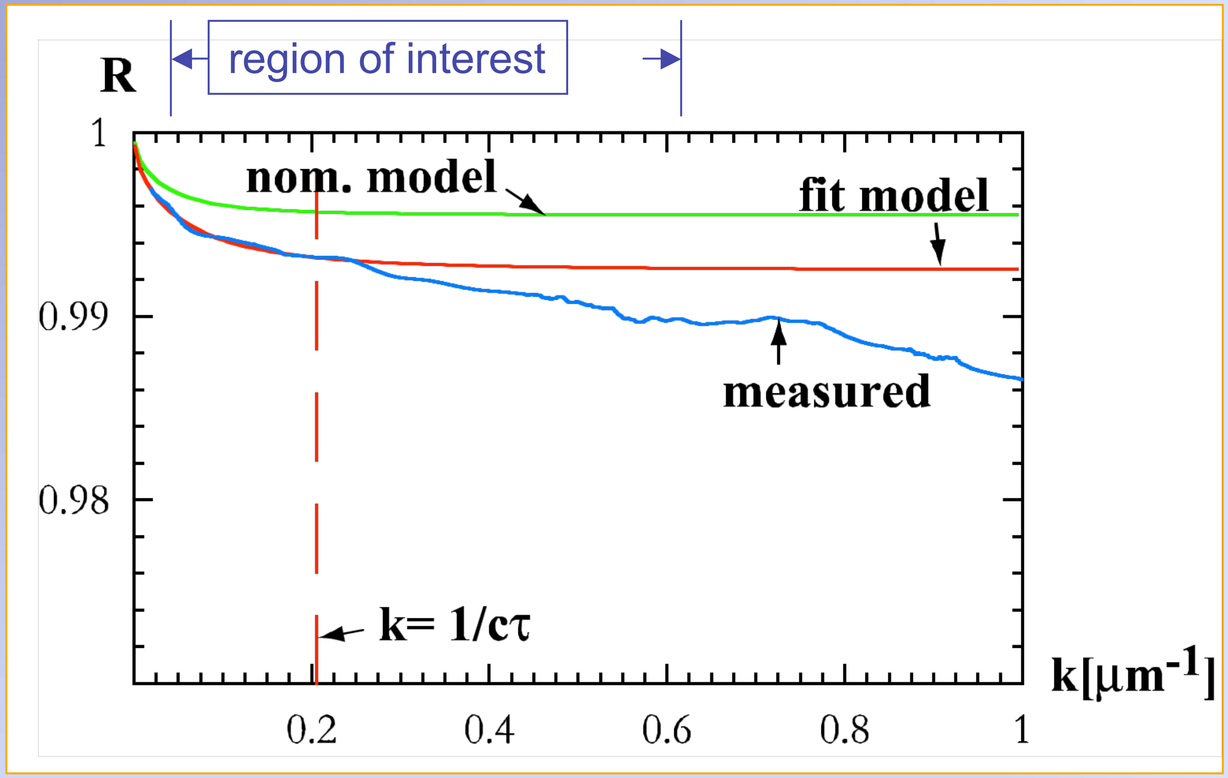
Al fit



Aluminum reflectivity: comparison of measurements (blue) with calculations using nominal σ, τ (green); and fitted values: 0.63 nominal σ, 0.78 nominal τ (red).

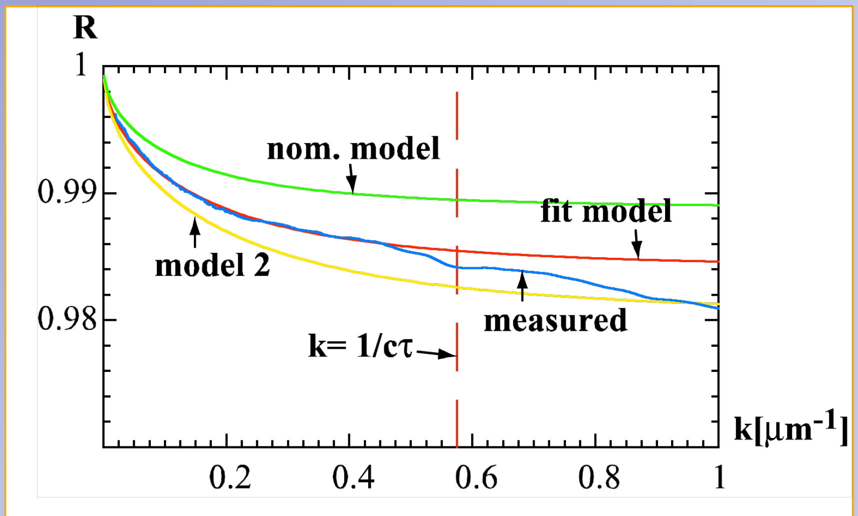
--in literature no data below 0.2 μm^{-1}

Cu fit



Copper reflectivity: comparison of measurements (blue) with calculations using nominal σ, τ (green); and fitted values: 0.66 nominal σ, 0.67 nominal τ (red).

--in literature no data below 0.3 μm^{-1}



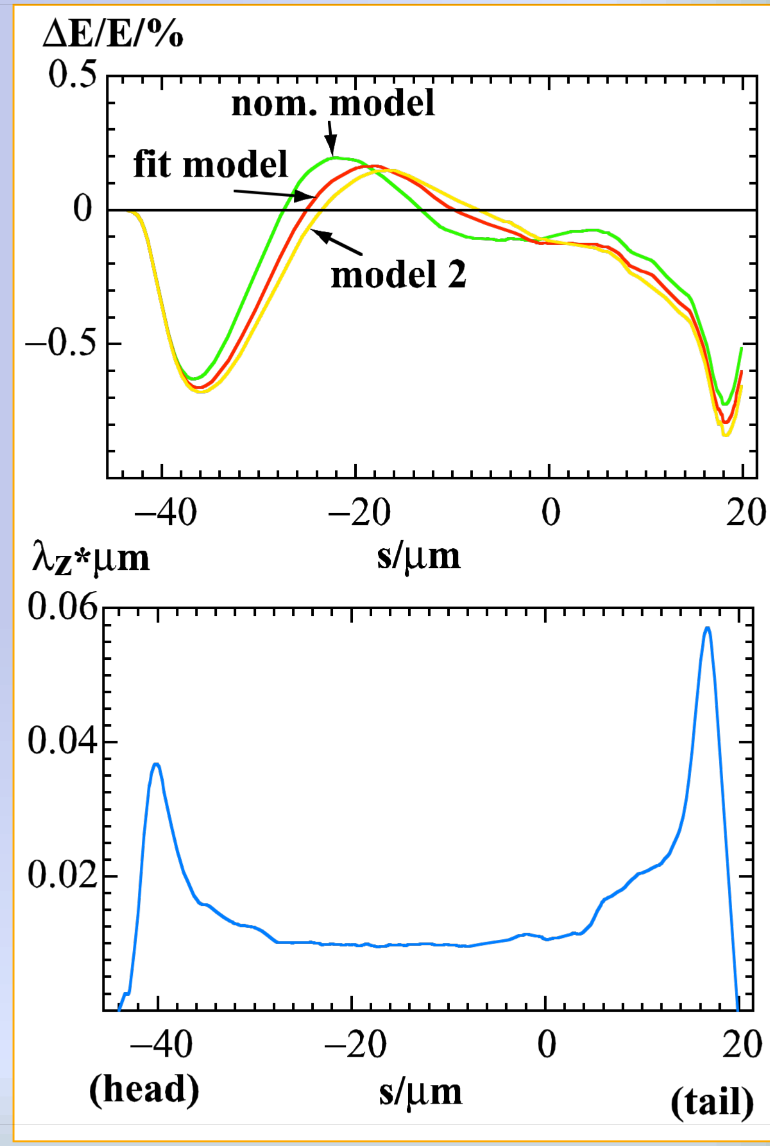
For Al, how does the fit taken affect ΔE for the LCLS?

Consider (σ,τ):

nom.– (3.35×10^{17}/s, 0.75×10^{-14} s)

fit– (2.12×10^{17}/s, 0.58×10^{-14} s)

model2– (1.68×10^{17}/s, 0.48×10^{-14} s)



LCLS Impedance Budget

consider wake effects in LCLS BC-2, Linac-3, undulator: eN= 1 nC, bunch shape uniform with σ_z= 20 µm; before Linac-3, E= 4.5 GeV, after E= 14 GeV; length of Linac-3, L= 550 m, of undulator, L= 130 m.

Linac-3

_ longitudinal wake is good: it is used to take out residual chirp after BC-2. $W \approx Z_0 c/(\pi a^2)$; induced chirp is almost linear with $\delta E_{rms}/E = e^2 N \mathcal{W}_{rms} L/E = 0.3\%$.

_ longitudinal wake is bad: 3rd order variation results in horns in the distribution

_ effect of transverse wake: $W_x \approx 2Z_0 cs/(\pi a^4)$; due to a betatron oscillation $\delta\varepsilon/\varepsilon \approx \upsilon^2/2$ with $\upsilon = e^2 NL \langle \mathcal{W}_x \rangle \beta/(2E) = 0.06$ ⇒ $\delta\varepsilon/\varepsilon$ is insignificant

compressor BC-2, CSR

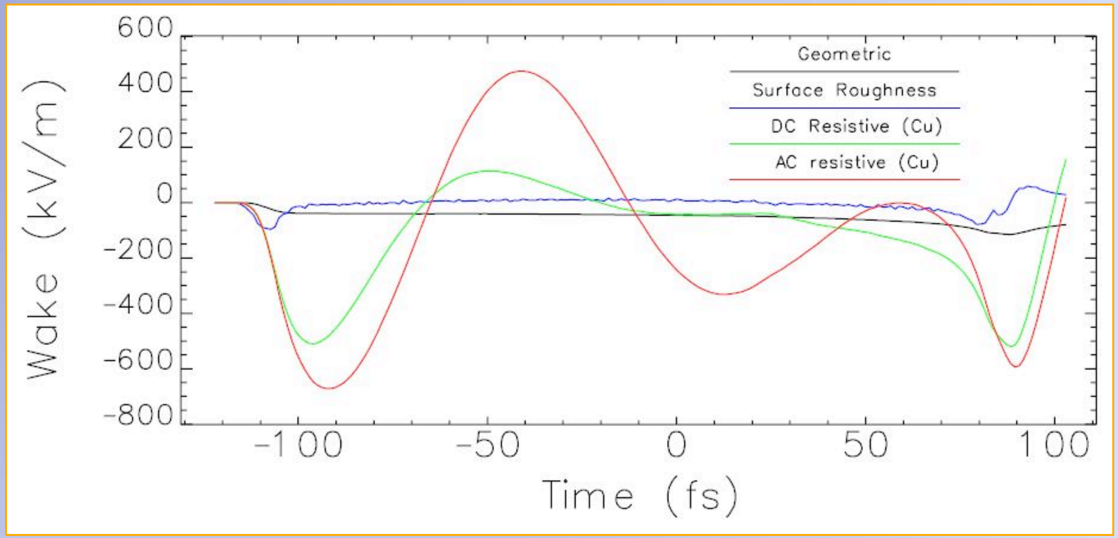
- model: CSR is generated in 3rd bend, becomes emittance growth in 4th bend.

steady-state CSR formula: $\delta E_{rms}/E = 0.02\, Z_0 c e^2 L/(E R^{2/3} \sigma_z^{4/3}) = 0.007\%$ (L= 0.5 m, R= 15 m).
compression work formula: $\delta E_{rms}/E = 0.016\%$.
1D simulation (with gaussian): $\delta E_{\mathrm{rms}}/E = 0.018\%$, leading to $\delta\varepsilon/\varepsilon = 38\%$.

undulator

- the idea that Al pipe is better for the lasing process than Cu is supported by perturbation analysis (Z. Huang and G. Stupakov)

- resistive wall wake dominates over other sources; to ameliorate the effect the vacuum chamber surface will be Al, and the shape will be rectangular



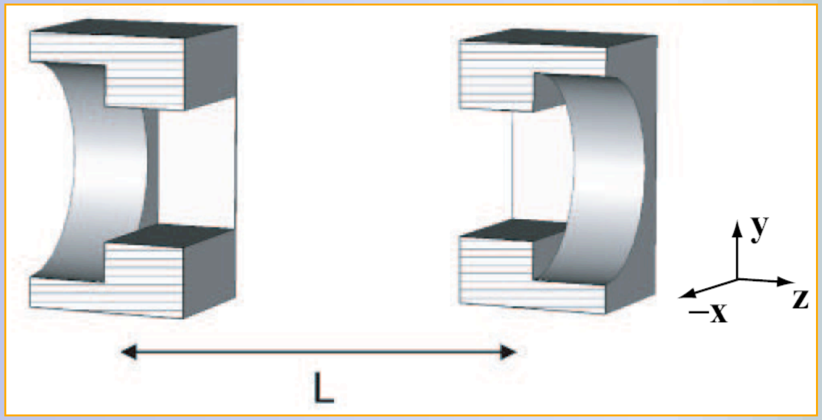
W. Fawley, et al

wake contributors in the undulator region of the LCLS

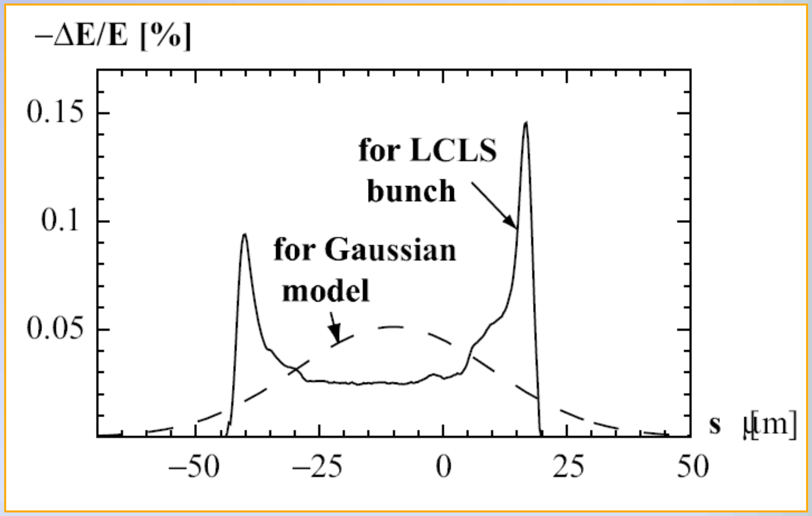
Examples: Rectangular-to-Round Transitions

K. Bane and
I. Zagorodnov

- 3d problems are difficult
- ECHO is a 3d, finite-difference (time-domain) wakefield calculating program
- as with shallow, round transitions, the short-bunch wake is resistive
- effect is 30% less than the inscribed round-to-round transitions

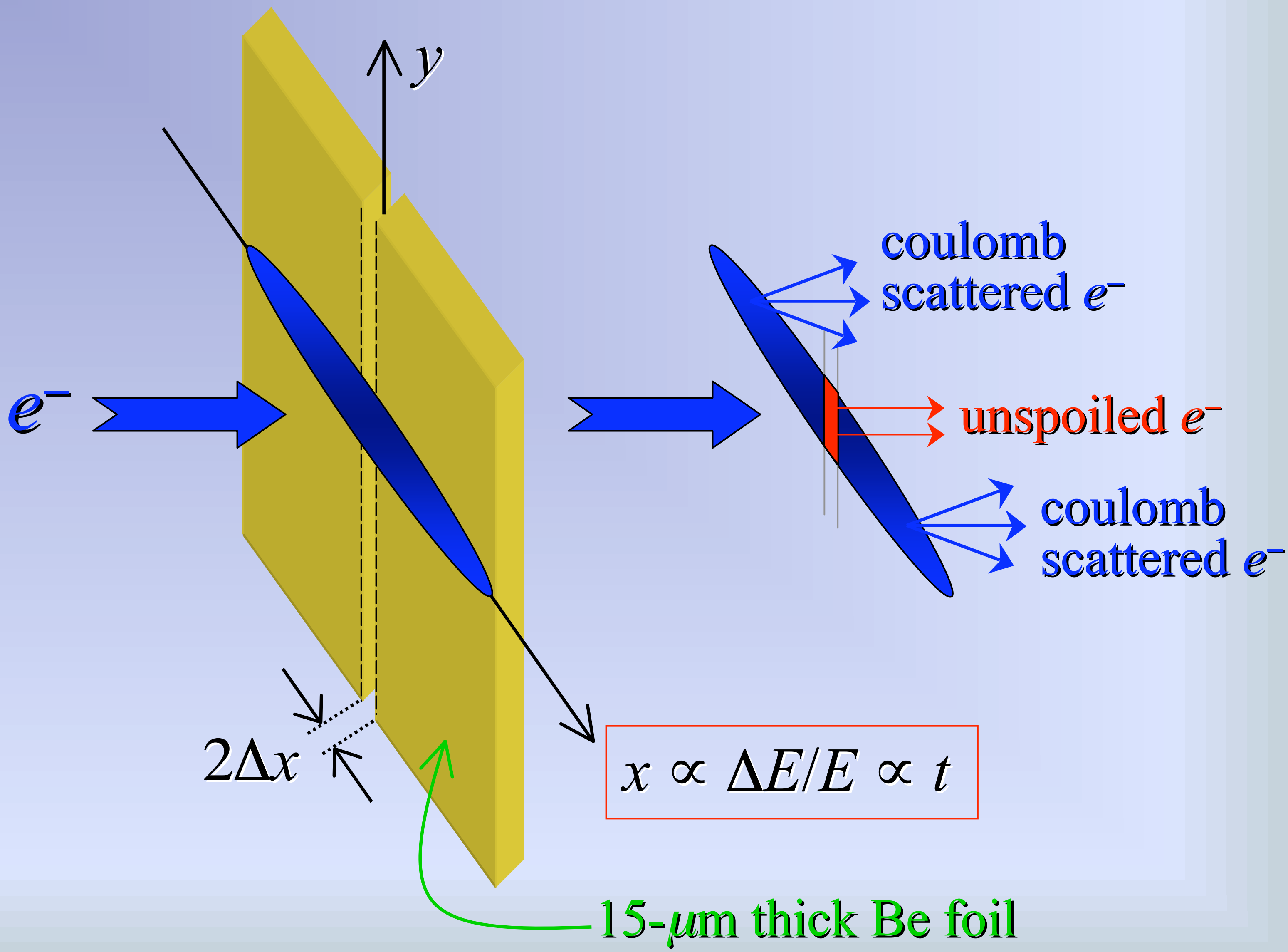


a transition pair



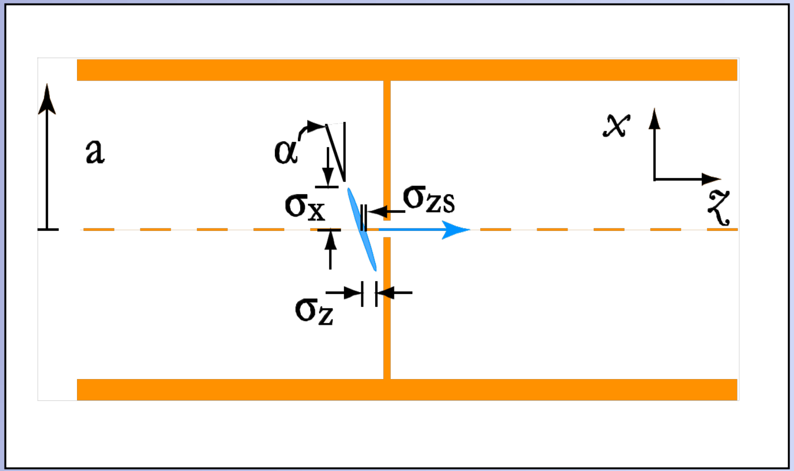
induced energy change of 33 transition pairs

Spoiler: add thin slotted foil in center of chicane



PRL **92**, 074801 (2004).

P. Emma, M. Cornacchia, K. Bane, Z. Huang, H. Schlarb, G. Stupakov, D. Walz (SLAC)



Sketch of problem

(K. Bane and G. Stupakov)

wake effect

Direction	Average		Rms	Units
	Analytical	Numerical	Numerical	
Horizontal (x')	4.8	3.9	0.3	μrad
Vertical (y')	0	0	1.1	μrad
Longitudinal (δ)	−18.	−16.	0.9	10^{-5}

$\sigma_{y'}$= 2 μrad

- 3d problem with a>> σ_x>> σ_y, σ_z>> σ_{zs} (a= 20 mm, σ_x= 2.6 mm, σ_y= 0.1 mm, σ_{zs}= 2μm); aperture= 0.2 mm
- can perform high-frequency analytical calculation, assuming no slot
- have not found numerical program than can solve this problem

Conclusion

- recent accelerator projects, such as x-ray FEL’s (also e.g. the ILC), involve transporting intense, short, low-emittance bunches, and understanding and controlling the very short-range wakes is essential

- have discussed recently gained understanding of the short-range wakes of: accelerator structures, resistive walls, surface roughness, coherent synchrotron radiation, etc.

- have applied the calculations to short-bunch regions of the LCLS