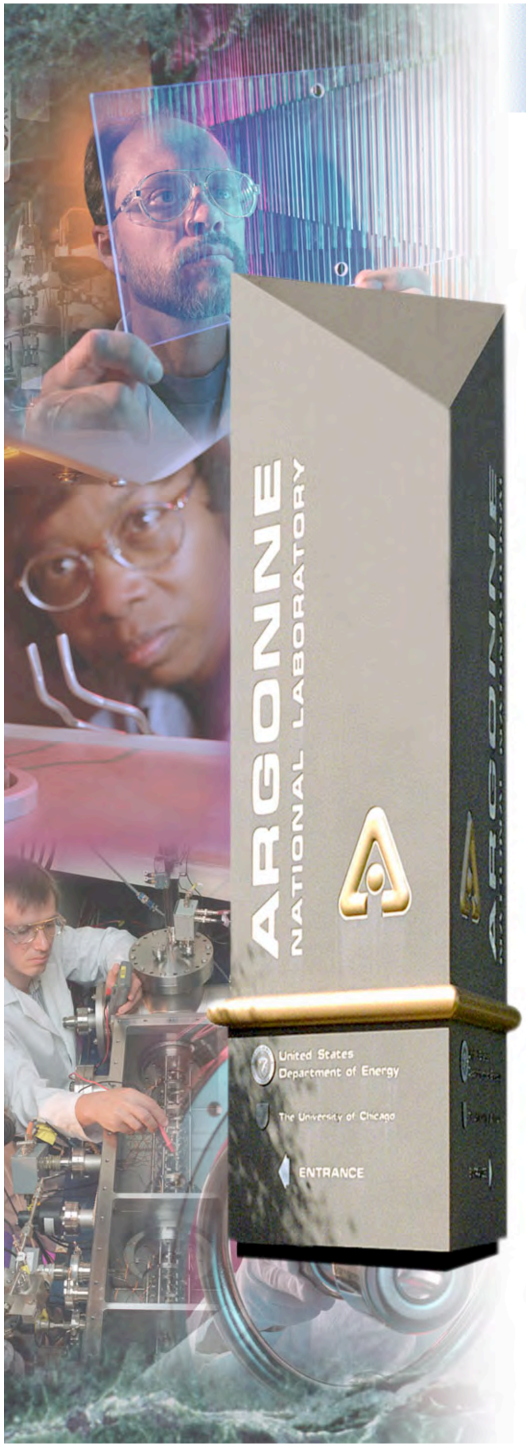
ARGONNE NATIONAL LABORATORY



A Smith-Purcell BWO for Intense Terahertz Radiation

Kwang-Je Kim and Vinit Kumar

ANL and The University of Chicago

The Physics and Applications of High Brightness Electron Beams

Erice, Sicily

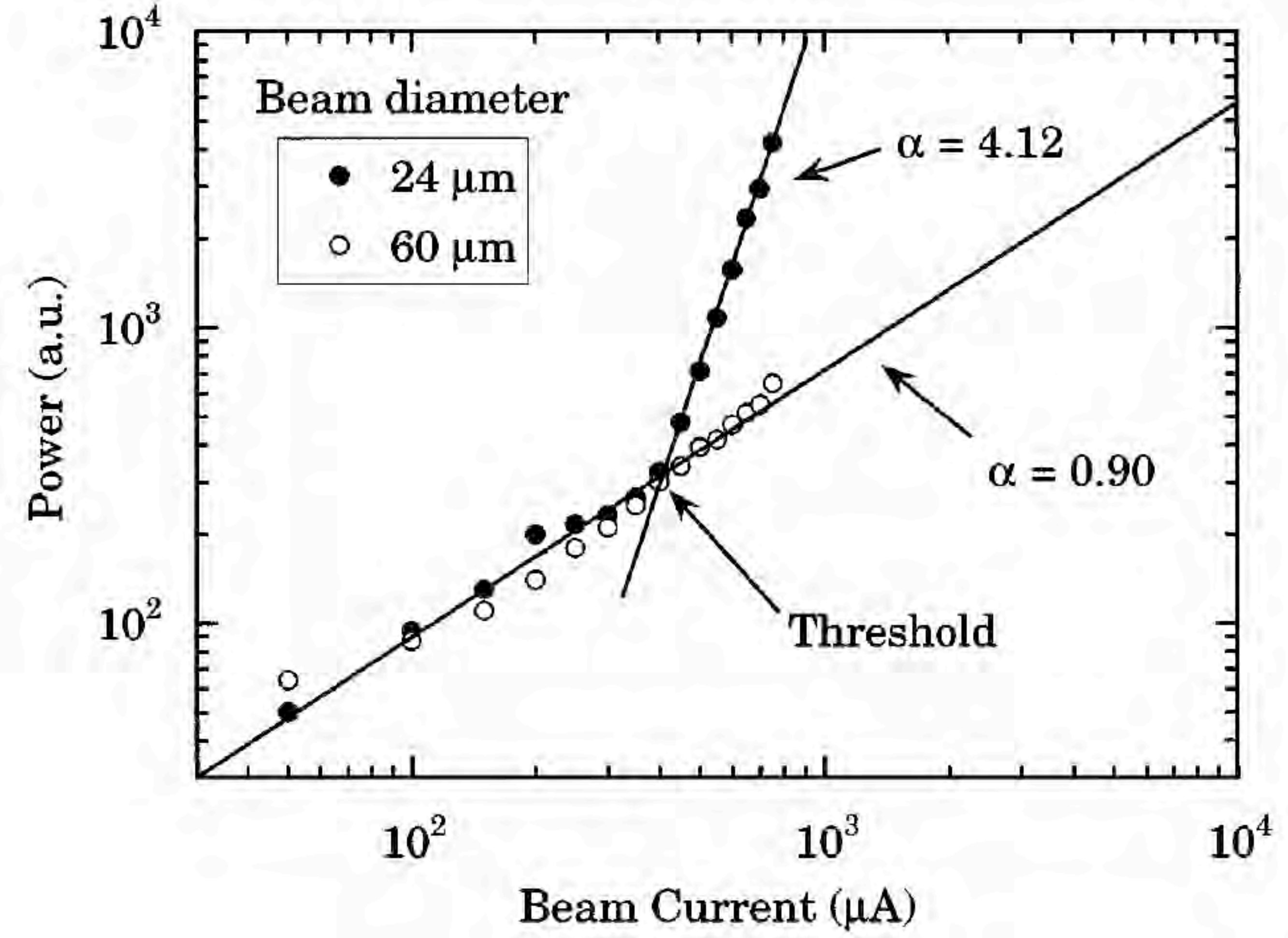
October 9-14, 2005

THE UNIVERSITY OF CHICAGO

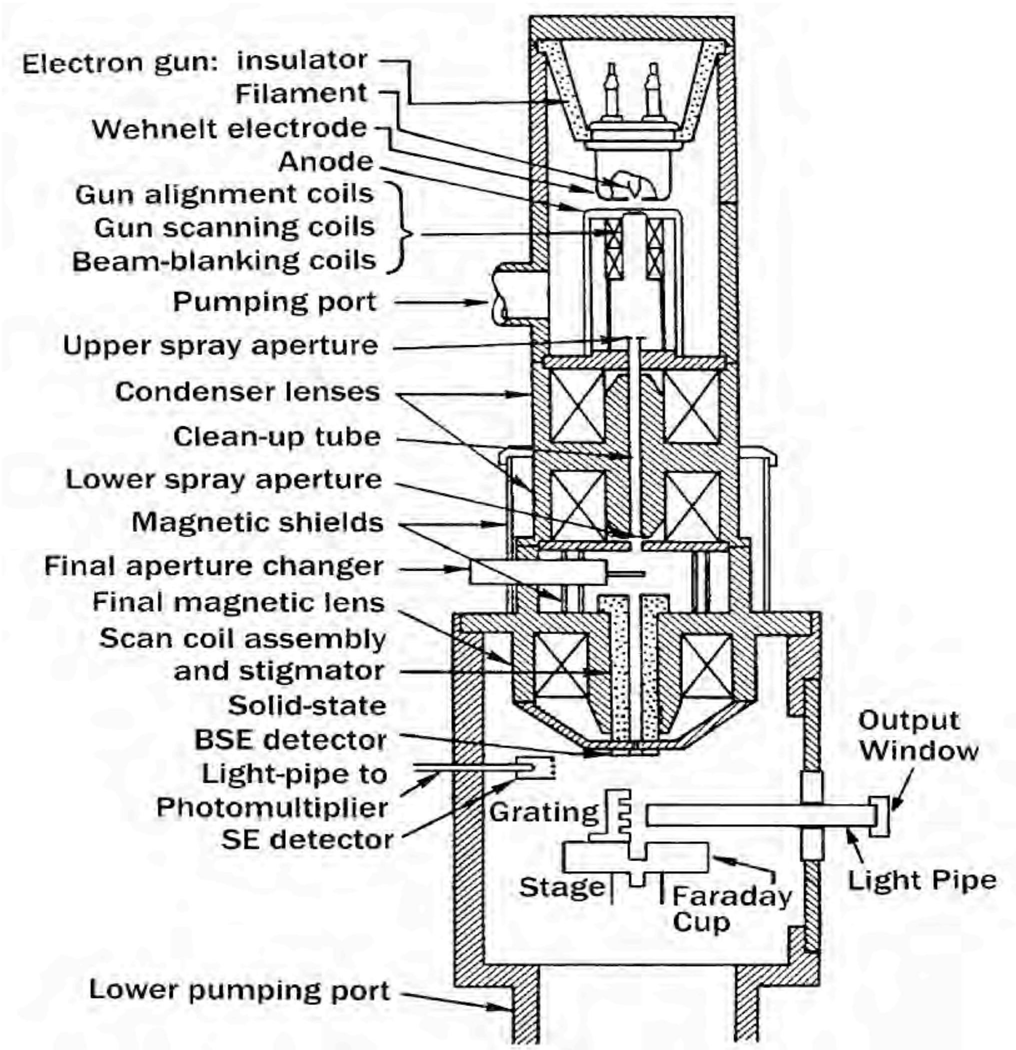
Office of Science U.S. DEPARTMENT OF ENERGY

Argonne National Laboratory is managed by
The University of Chicago for the U.S. Department of Energy

Non-Linear Behavior in Smith-Purcell Radiation ? *(J. Urata et al., PRL 80 (1998) 516-519)*



SEM-Based Smith-Purcell Radiator



β= 0.35 (35 keV)

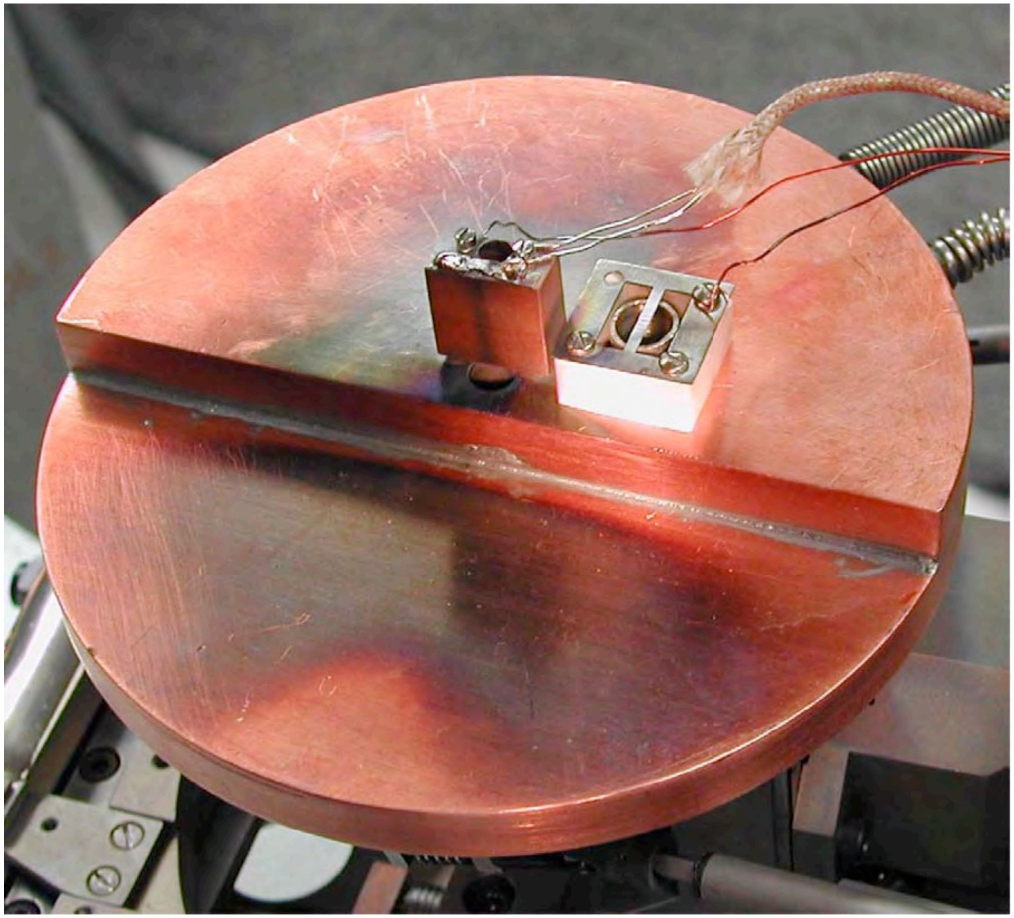
$I \leq 1$ mA

λg = 173 μm, d = 100 mm, w = 62 μm,

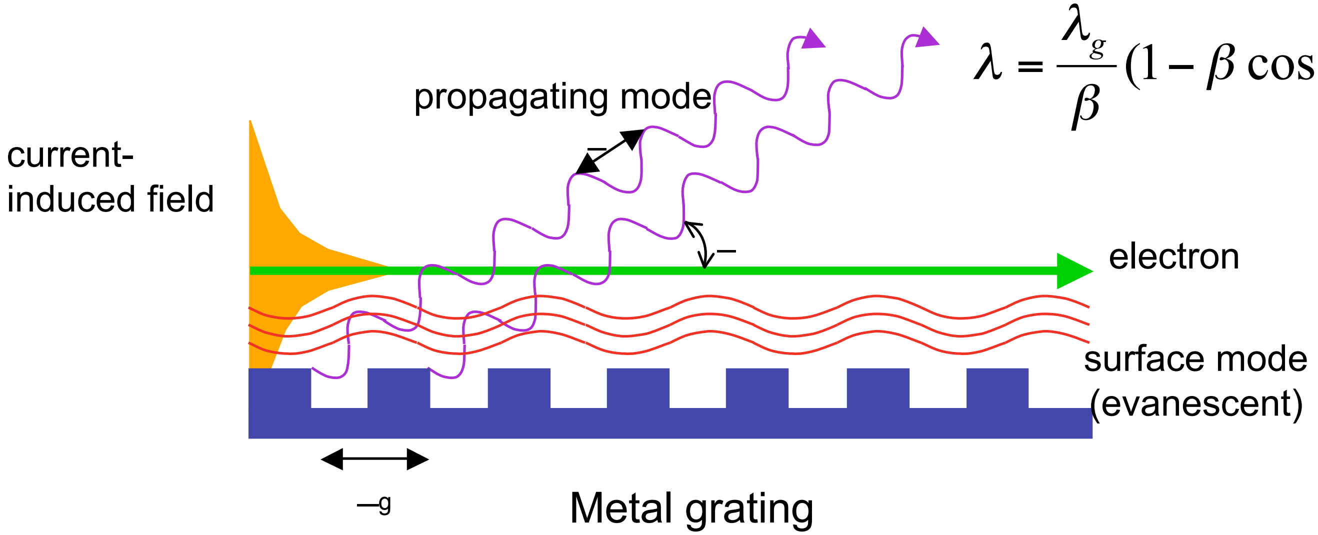
b = 10 μm, L = 12.7 mm

SEM-Based Smith-Purcell Radiator at the U of C, After the Dartmouth Set-Up (O. Kapp, A. Crewe, KJK)

Heated Specimen Stage and Possible Black Body radiation background



Waves on a Grating: Propagating and Evanescent Modes



$$\lambda = \frac{\lambda_g}{\beta}(1 - \beta\cos\theta)$$

*S. J. Smith and E. M. Purcell, Phys. Rev. 92, 1069 (1953)

Sheet Current

Consider a sheet electron beam having current density*

$$J_z(x,z,t)=\frac{q}{\Delta y}\delta(x)\sum_i \delta(z-z_i)v_z=\frac{q}{\Delta y}\delta(x)\sum_i \delta(t-t_i(z))$$

Fourier transform of this current density is given by

$$J_z(x,z,\omega)=\delta(x)\underbrace{\frac{\text{q}}{\text{Äy}}\sum_i \exp(i\omega\xi_i)}_{K(z,\omega)}\exp(ik_0 z)$$

$K(z,\omega)$ ←slowly varying function in *z*

↓

$K_0(\omega)\exp(\mu z)$

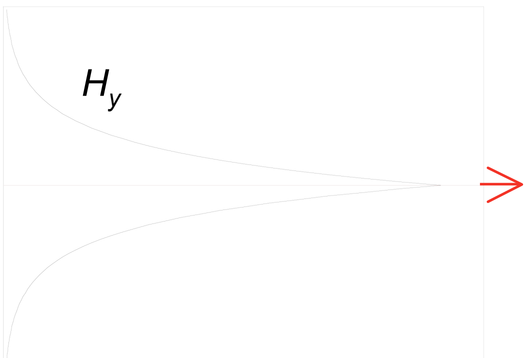
$$k_0=\omega/c\beta,\quad \xi_i=t_i-z/c\beta$$
$$\alpha_0=k_0-i\mu$$

$$J_z(x,z,\omega)=\delta(x)K_0(\omega)\exp(i\alpha_0 z)$$

*K.-J. Kim and S. B. Song, Nucl. Instrum. Methods Phys. Res. A 475, 158 (2001).

EM Fields Induced by a Sheet Current

Solving the Maxwell equations with proper symmetry, we get



$$H_y^I = \frac{1}{2}\varepsilon(x)K_0(\omega)\exp[i\alpha_0 z - \varepsilon(x)\Gamma_0 x]$$

$$E_x^I = \frac{-i}{\omega\varepsilon_0}\frac{\partial H_y^I}{\partial z}$$

$$E_z^I = \frac{i}{\omega\varepsilon_0}\left(\frac{\partial H_y^I}{\partial x} - J_z\right)$$

$$E_y^I = H_x^I = H_z^I = 0$$

$\varepsilon(x)$ = -1 for $x < 0$

+1 for $x > 0$

$$\Gamma_0 = \sqrt{\alpha_0^2 - \omega^2 / c^2} = \omega / c\beta\gamma$$

These are slow plane waves, propagating along z-axis with speed v, but decaying along x-axis with decay constant Γ_0. These are non-radiating, zeroth order evanescent wave.

E- Field, Energy Modulation, and Bunching; Three-Fold Way for FELs

- E_z-Field gives rise to energy modulation

$$\frac{d\eta}{dz} = \frac{q}{\gamma mc^2} E_z(z,t) \qquad \eta = \frac{\gamma - \gamma_0}{\gamma_0}$$

- Energy modulation gives rise to bunching

$$\frac{d\xi}{dz} = -\frac{\eta}{c\beta^3\gamma^2}$$

- Bunching gives rise to surface mode

$$E_z = \frac{i\Gamma_0}{2\varepsilon_0\omega}\left(e_{00}e^{-2\Gamma_0 b} - 1\right)K_0(\omega)e^{i\alpha_0 z}$$

- *Quadratic equation for growth rate if e_{00} is a smooth function**
- *However, e_{00} is singular !*

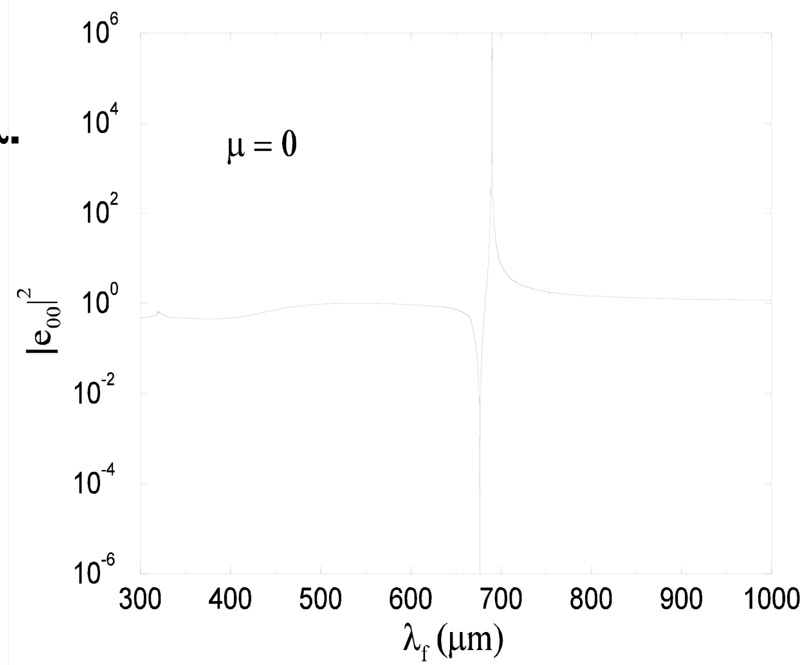
*K.-J. Kim and S. B. Song, Nucl. Instrum. Methods Phys. Res. A 475, 158 (2001).

Singularity in e_{00} and Freely Propagating Surface Mode

- **The reflection coefficient e_{00} diverges at λ=690 m.**
- **Freely propagating surface mode at this λ.**
- **For a non-zero growth rate (μ) it has a simple pole**

$$e_{00}(\mu) = \frac{-i\chi}{\mu} + \chi_1$$

$$\mu E^{sur} = \frac{dE^{sur}}{dz} = \frac{IZ_0\chi}{2\beta\gamma\Delta y} e^{-2\Gamma_0 b}\left\langle e^{-i\psi}\right\rangle$$



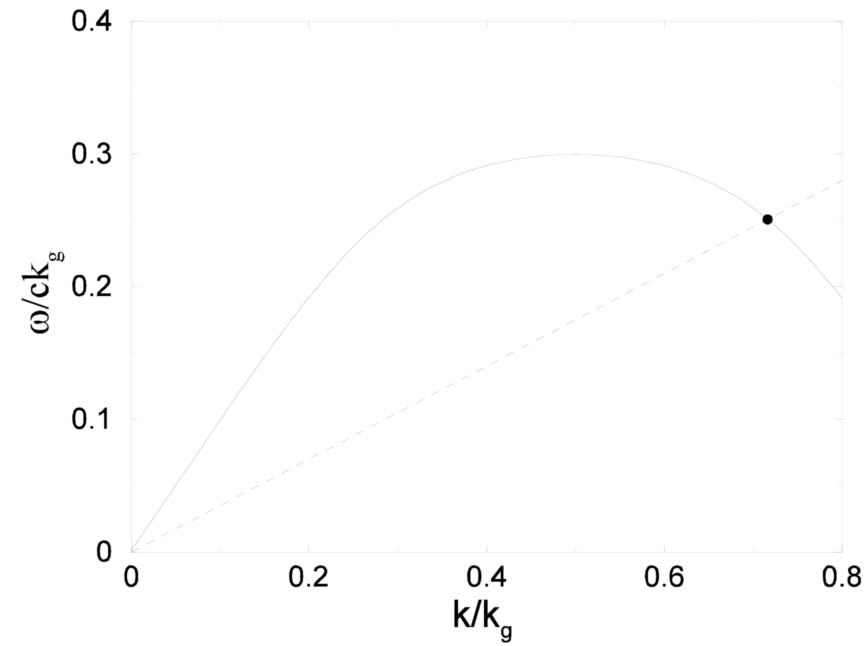
Thus we recover cubic equation !

Surface Mode at λ=690 m

- Scattering coefficients from m_{th} to n_{th} spatial modes
- There is a singularity in $e_{00,}$ indicating that a free-propagating surface mode
- Due to linear relation between different e_{mn}, e_{m0} are in general singular
- The m_{th} spatial waves combine to satisfy the grating BC
- ***A surface mode of a perfectly conducting grating does not couple to any propagating modes…If it did, the singularity cannot be infinitely narrow.***

ARGONNE NATIONAL LABORATORY

*Surface Mode Has Negative Group Velocity**



- Phase velocity $=\omega/k_z=\beta c$

,

- $d\omega/dk_z < 0$

- ***Thus SP-FEL is a Backward Wave Oscillator (BWO)***

- **Optical energy accumulates exponentially to saturation *without feedback mirrors***

*H.L. Andrews et al., Phys. Rev. ST Accel. Beams. 8, 050703 (2005)

Including Time Dependence via

Time-dependent Maxwell equation:

$$\frac{\partial E}{\partial t} - v_g \frac{\partial E}{\partial z} = -\frac{IZ_0 \chi v_g}{2\beta\gamma\Delta y} e^{-2\Gamma_0 b} \left\langle e^{-i\psi} \right\rangle$$

$$E^{sc} = \frac{-iIZ_0}{2\beta\gamma\Delta y} \left(1 - \chi_1 e^{-2\Gamma_0 b}\right) \left\langle e^{-i\psi} \right\rangle$$

$\mu \Rightarrow \partial / \partial t \pm v \partial / \partial$

± **According toforward and backward**

Lorentz equation:

$$\frac{\partial \gamma_i}{\partial t} + v \frac{\partial \gamma_i}{\partial z} = \frac{ev}{mc^2} \left(E + E^{sc}\right) e^{i\psi_i} + c.c.$$

$$\frac{\partial \psi_i}{\partial t} + v \frac{\partial \psi_i}{\partial z} = \frac{\omega_s}{\beta^2 \gamma^2} \frac{\left(\gamma_i - \gamma_p\right)}{\gamma_p}$$

*First obtained for microwave circuit by N. S. Ginzburg et al., Sov. Radiophys. Electron., 21, 728 (1979), See also B. Levush et al., IEEE Trans. Plasma Sci., 20, 263 (1992).

Maxwell-Lorentz Equations

Dimensionless variables:

$$\varsigma = z / L$$

$$\tau = \left(\tau - \frac{z}{v_p}\right)\left(\frac{1}{v_p} + \frac{1}{v_g}\right)^{-1}\frac{1}{L}$$

$$\eta_i = \frac{k_s L}{\beta_p^3 \gamma_p^3}(\gamma_i - \gamma_p)$$

$$\varepsilon = \frac{e}{mc^2}\frac{k_s L^2}{c\beta_p^3\gamma_p^3}E$$

$$\varepsilon_s = \frac{e}{mc^2}\frac{k_s L^2}{c\beta_p^3\gamma_p^3}E_s$$

$$J = 2\pi\frac{I}{I_A}\frac{\chi}{\Delta y}\frac{k_s L^3}{\beta_p^4\gamma_p^4}e^{-2\Gamma_0 b}$$

Maxwell-Lorentz equations in dimensionless variables:

$$\frac{\partial\varepsilon}{\partial\tau} - \frac{\partial\varepsilon}{\partial\varsigma} = -J\left\langle e^{-i\psi}\right\rangle$$

$$\frac{\partial\eta_i}{\partial\varsigma} = \left(\varepsilon + \varepsilon^{sc}\right)e^{i\psi_i} + c.c.$$

$$\frac{\partial\psi_i}{\partial\varsigma} = \eta_i$$

$$\varepsilon^{sc} = i\frac{J}{\chi L}\left(\chi_1 - e^{2\Gamma_0 b}\right)\left\langle e^{-i\psi}\right\rangle$$

Boundary conditions:

$\varepsilon(\varsigma = 1,\tau),\ \psi_i(\varsigma = 0,\tau),\ \eta_i(\varsigma = 0,\tau)$ should be known for all τ

Boundary Conditions for a BWO

- No bunching at the entrance of the grating:

$$\psi_i(\varsigma = 0,\tau) = 0$$

- No energy modulation at the entrance

$$\eta_i(\varsigma = 0,\tau) = 0$$

- Oscillation starts when field at the exit vanishes relative to the field at the entrance:

$$\varepsilon(\varsigma = 1,\tau)/\varepsilon(\varsigma = 0,\tau) = 0$$

$$i.e., \varepsilon(\varsigma = 1,\tau) = 0$$

Analytic Solution in the Linear Regime

J.A. Swegle, Phys. Fluids 30, 1201 (1987)

- Collective variables a la Bonifacio

$$\mathrm{E}, \quad B = \left\langle \delta\psi\, e^{-i\psi_0} \right\rangle, P = \left\langle \delta\eta\, e^{-i\psi_0} \right\rangle$$

$$\frac{\partial \mathrm{E}}{\partial \tau} - \frac{\partial \mathrm{E}}{\partial \varsigma} = iJB, \quad \frac{\partial B}{\partial \varsigma} = P, \quad \frac{\partial P}{\partial \varsigma} = \mathrm{E} + QB$$

- Solution of the form exp(ντ)exp(ηζ)

$$\kappa^3 - \nu\kappa^2 - Q\kappa + \nu Q + iJ = 0$$

- General solution:

$$\mathrm{E}(\varsigma,\tau) = e^{\nu\tau}\left[A_1 e^{\kappa_1\varsigma} + A_2 e^{\kappa_2\varsigma} + A_3 e^{\kappa_3\varsigma}\right]$$

- Boundary conditions

B = 0, P = 0 at ζ = 0, E = 0 at ζ = 1

Analytic Solution in the Linear Regime (cont'd)

- Nontrivial solution if

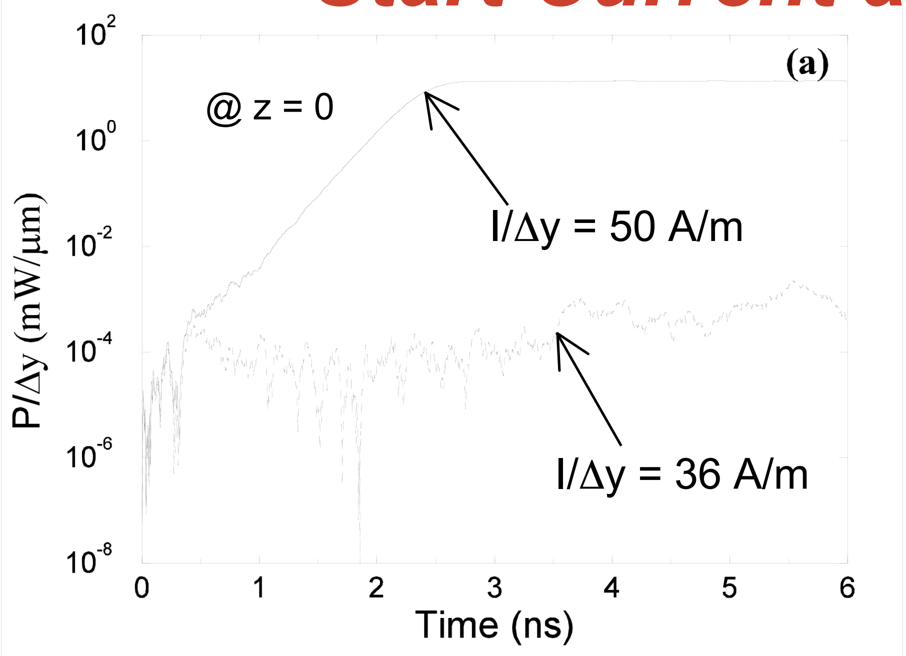
$$(\kappa_1^2 - Q)(\kappa_2 - \kappa_3)e^{\kappa_1} + (\kappa_2^2 - Q)(\kappa_3 - \kappa_1)e^{\kappa_2} + (\kappa_3^2 - Q)(\kappa_1 - \kappa_2)e^{\kappa_3} = 0$$

- This is a transcendental equation on ν. Find that there is a threshold value of J above which ν has a positive real part.

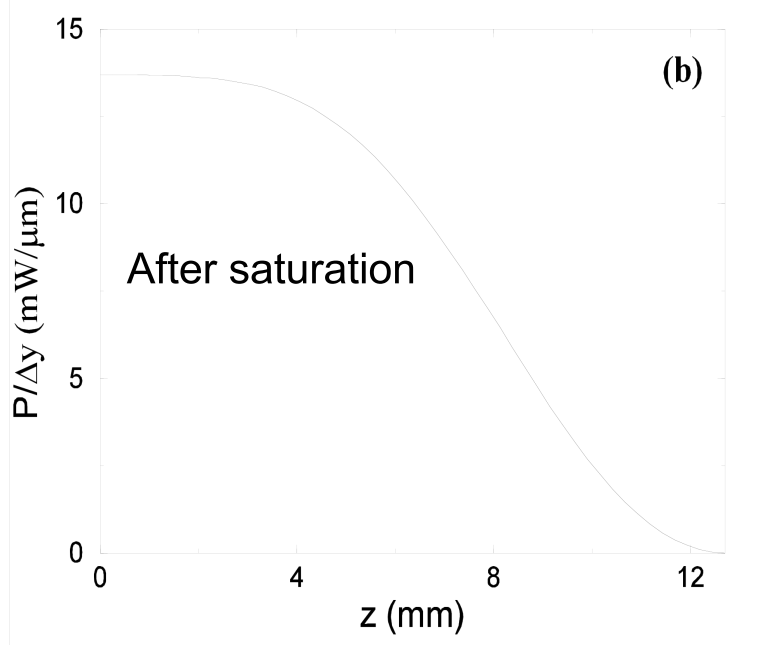
$\Rightarrow$ Start current condition

$$\frac{I_s}{\Delta y} = 7.685\, I_A \frac{\beta^4 \gamma^4 \lambda}{2\pi^2 \chi L^3} e^{2\Gamma_0 b}$$

Simulation Results: Start Current and Saturation



Energy conversion efficiency = 0.8%



For I/Δy = 50 A/m, at saturation, P/Δy = 13.7 mW/μm

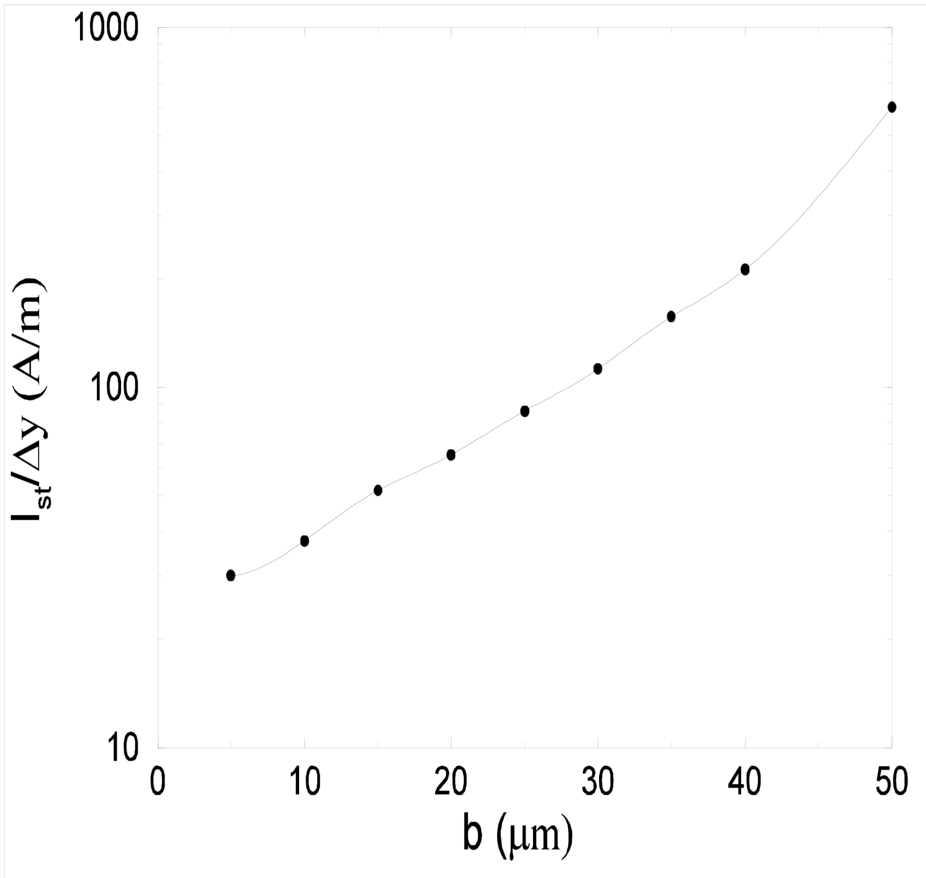
Power e-folding time = 0.2 ns (simulation)

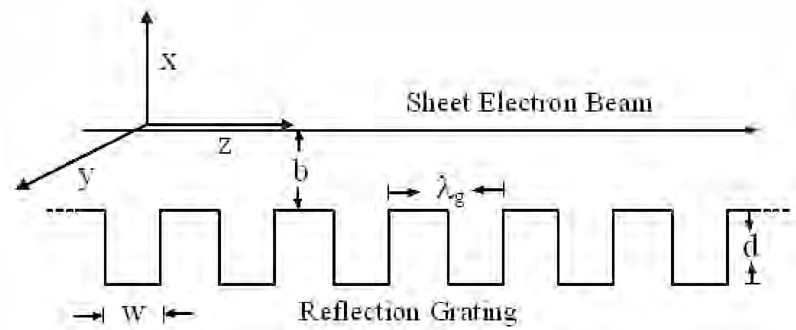
0.17 ns (analytic formula)

Lasing wavelength = 694.5 μm (simulation)

694 μm (analytic formula)

Simulation Results: Start Current as a Function of Gap Distance





For b = 10 μm,

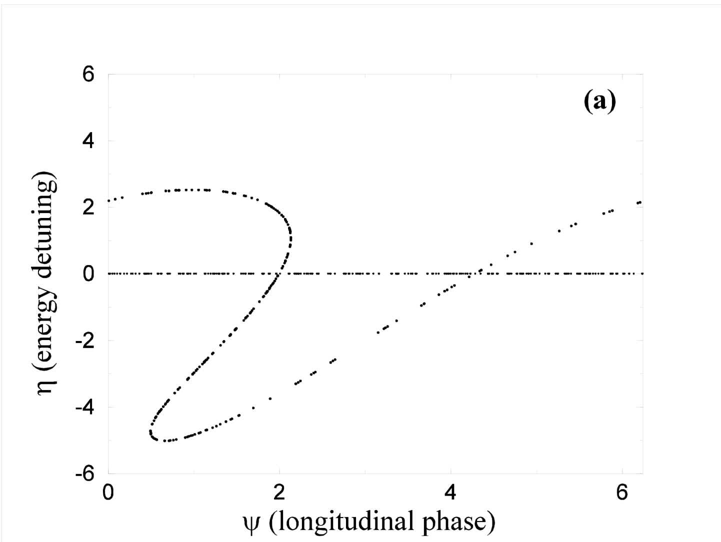
$I_{st}/\Delta y$ = 37.5 A/m (simulation)

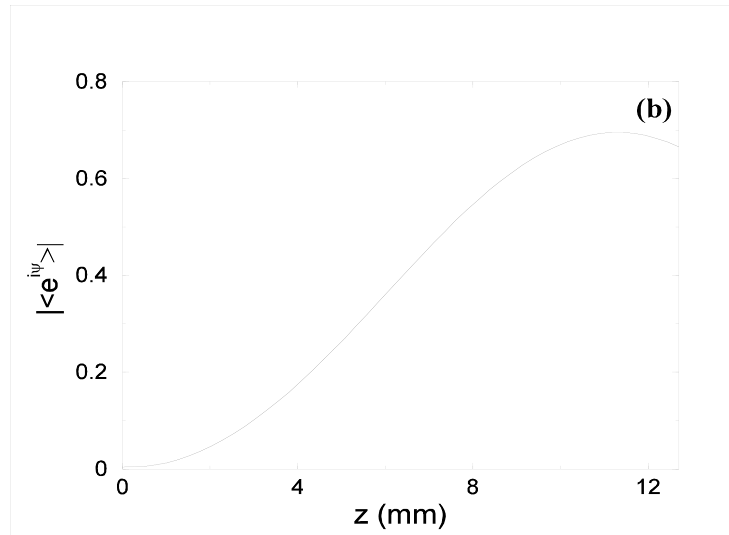
= 36 A/m (analytic formula)

- If we maintain an rms average beam radius of 10 μm over the entire interaction regime, the start surface current density is 37.5 A/m

Simulation results

Evolution of longitudinal phase space





Electron beam becomes bunched due to SP-FEL interaction

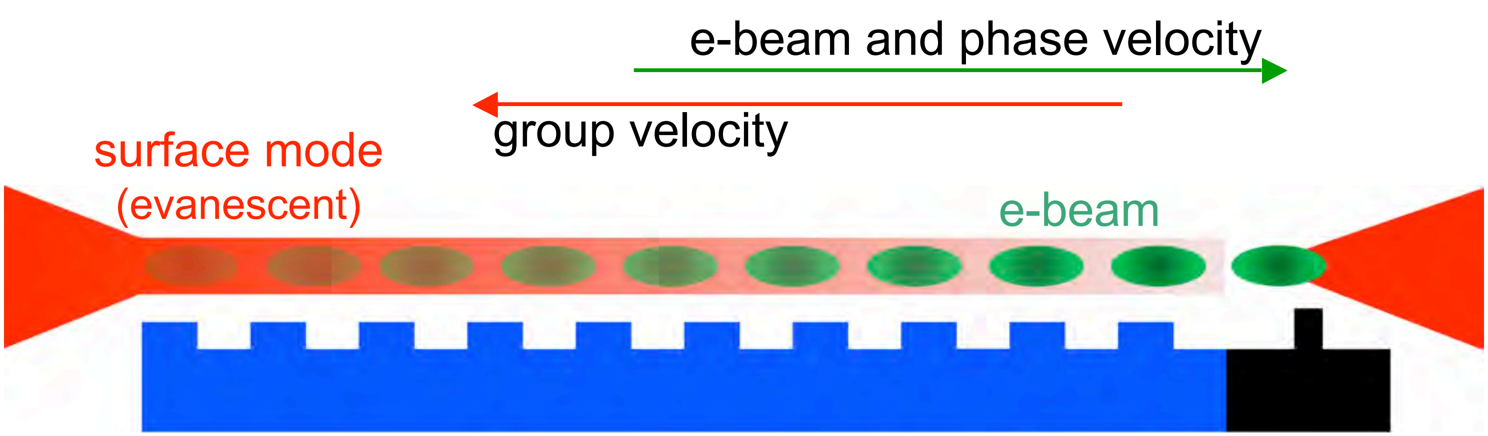
Outcoupling

- Maximum efficiency

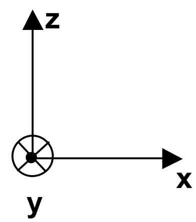
$$\eta_{eff} \approx \frac{1}{2}\frac{\lambda}{L}\frac{(\beta\gamma)^3}{\gamma - 1} \leq 1\%$$

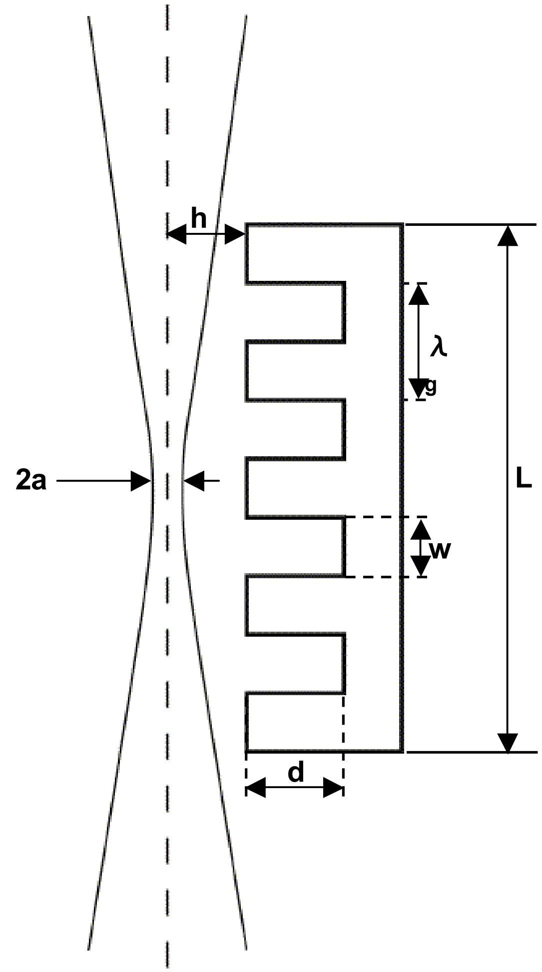
- Outcoupling via
  - Mode conversion at entrance
  - Bunched beam radiation at exit

Smith-Purcell FEL is a Backward Wave Oscillator



E-Beam and Grating Parameters





Reference case:

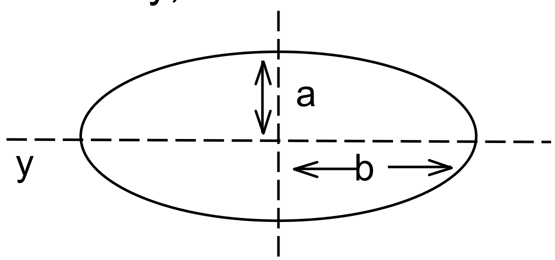
λ_g = 173 μm, β = 0.35 (35 keV)

d = 100 μm, w = 62 μm,

b = 10 μm, L = 12.7 mm

Beam Design for SP-FELs

- For clarity, assume KV distribution



$\sigma_x = a/2$

$\sigma_y = b/2$

- Choose $\beta^* = L$ at the grating center (beam size variation is small)

$$\sigma_x = \sqrt{\varepsilon_x L}, \quad \sigma_y = \sqrt{\varepsilon_y L}$$

- For a good overlap of evanescent wave with e-beam

$$a \le h \le \frac{\lambda\beta\gamma}{4\pi}$$

- *Diffraction condition in y-direction*

$$\varepsilon_y \lesssim \frac{\lambda\beta}{4\pi}$$

- ***These conditions are satisfied by sheet beam (a << b). Thus the theory for sheet beam developed in the above can be used for practical SP-FEL design***

Beam Design Examples

- Start current condition

$$\frac{dI}{dy} > \frac{dI_s}{dy} = 7.7\ I_A \frac{(\beta\gamma)^4 \lambda}{2\pi^2 \chi L^3} e^{2\Gamma_o h}$$

- For Dartmouth parameter the coupling parameter _ = 10/cm
- (Case A) A set of beam parameters satisfying these conditions
  a = 20 μ, b = 500 μ, $\varepsilon_x = 0.8 \times 10^{-8}$ m-r, $\varepsilon_y = 5 \times 10^{-6}$ m-r, I_s = 65 mA
- Condition that space change force is less than the emittance force in the beam envelope equation:

$$\frac{\varepsilon_x^{\ 3}}{\sigma_x^{\ 3}} \geq \frac{I}{(\beta\gamma)^3 I_A (\sigma_x + \sigma_y)}$$

- Case A violates the space change condition by a factor of 5.

Phase Velocity, Group Velocity, and Diffraction

- A wave evanescent in the x-direction and diffracting in y, with waist σ_y at z=0:

$$\int d\phi \exp[-ikct + ik_z(1-\frac{1}{2}\phi^2)z - \Gamma x + ik_z\phi y - \frac{1}{4}k_z^2\sigma_y^2\phi^2]$$

- This satisfies free space wave equation if $k^2=k_z^2-\Gamma^2$

- The phase velocity and diffraction property are determined by the operating value of k and k_z. For example, the diffraction angle $\sigma_{y'}=1/2k_z\sigma_y$, the phase front curvature $R=(z^2+Z_R^2)/z$, etc.
- The group velocity, including its sign, is determined by how k_z changes as a function of k near the operating point.
- For example let $\Gamma=gk(1-\alpha k)$, thus $k_z = k\sqrt{1+g^2(1-\alpha k)^2}$. The group velocity is negative if $\alpha k=3/4$.

Beam Designs Satisfying Also the Condition That Space Charge Emittance Growth Is Small

- (Case B) **Increase the depth of groove d:100_ 150 μ.**

⇒ _ ***increases from*** **10 to 100 /cm** ⇒ I_s reduced by a factor of 10.
The wavelength increases also, but only by about 10%.

- (Case C) **Increase L:1.25 _ 5 cm.**
  a = 20 μ, b = 200, $\varepsilon_x = 2.0 \times 10^{-9}$ m-r
  $\varepsilon_y = 1.25 \times 10^{-6}$ m-r, I_s = 0.36 mA

Conclusions

- We have developed a theory of SP-FELs driven by sheet beams operating as a BWO, using Maxwell-Lorentz equations.
- Simple formula for start current is derived from linear analysis .
- Results from a simulation code based on Maxwell-Lorentz equations agree with linear theory where applicable and give saturation behavior.
- The sheet beam theory can be used for designing a **portable** SP FEL for THz radiation.