Non-linear Evolution of Short Pulses in FEL Amplifiers and FEL Cascades at Saturation

L. Giannessi, P. Musumeci

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Outline

- Overview of short radiation pulses dynamics in single pass FELs at saturation (Superradiance - Superfluorescence)
- Bunching and harmonic generation
- Cascaded FEL dynamics at saturation
- Harmonic Cascade and SPARC examples

Superradiance

Regime of "field/particles" evolution with

- *Power scaling typical of superradiance a la "Dicke"*
  (Dicke, PR 93, 99 (1954))
- *Solitary wave-like pulse propagation*
- *Peak power exceeding the saturation threshold*
- *Longitudinal self-focusing*

$$P \propto n_e^2$$

First observed in simulations of FEL amplifiers in

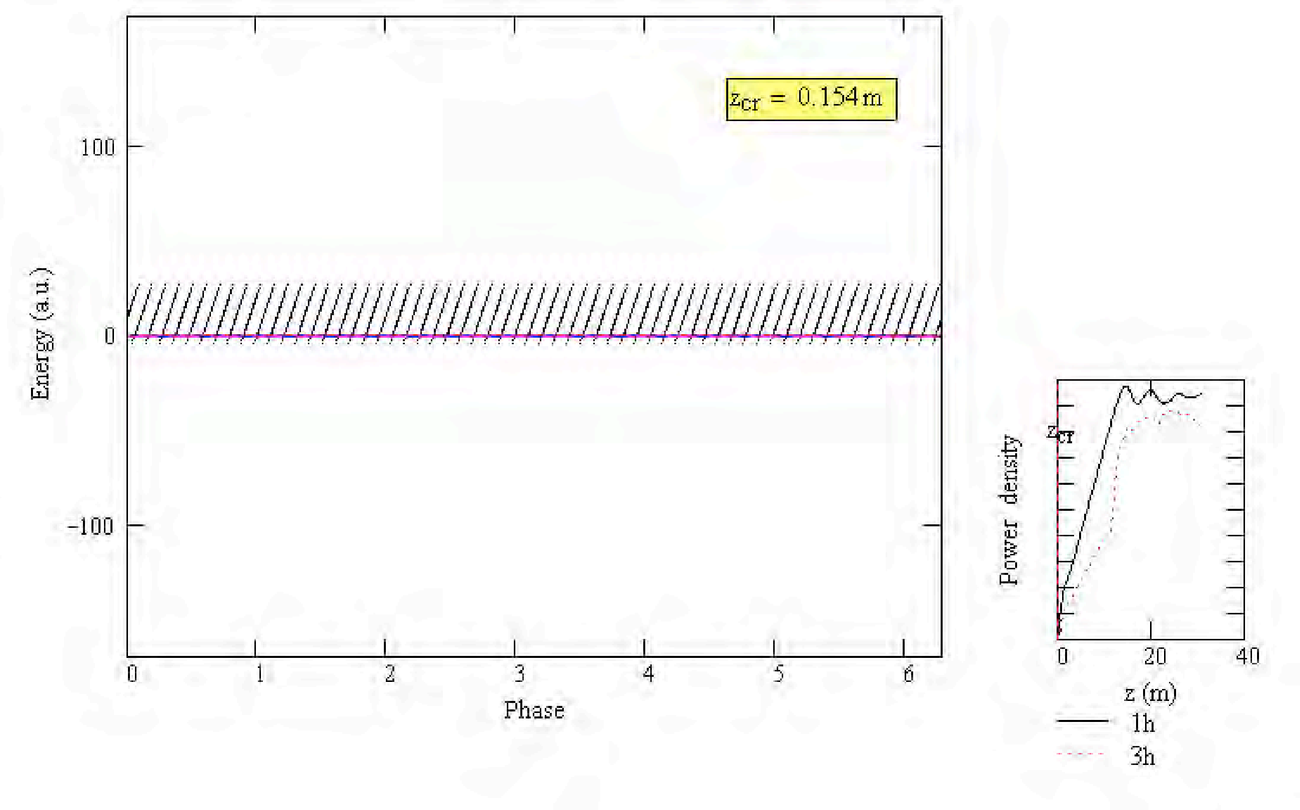
R. Bonifacio, B. W. J. Mc Neil, P. Pierini, PRA 40, 4467 (1989)

R. Bonifacio, L. De Salvo Souza, P. Pierini, N. Piovella, NIM A296, 358 (1990)

Two ingredients

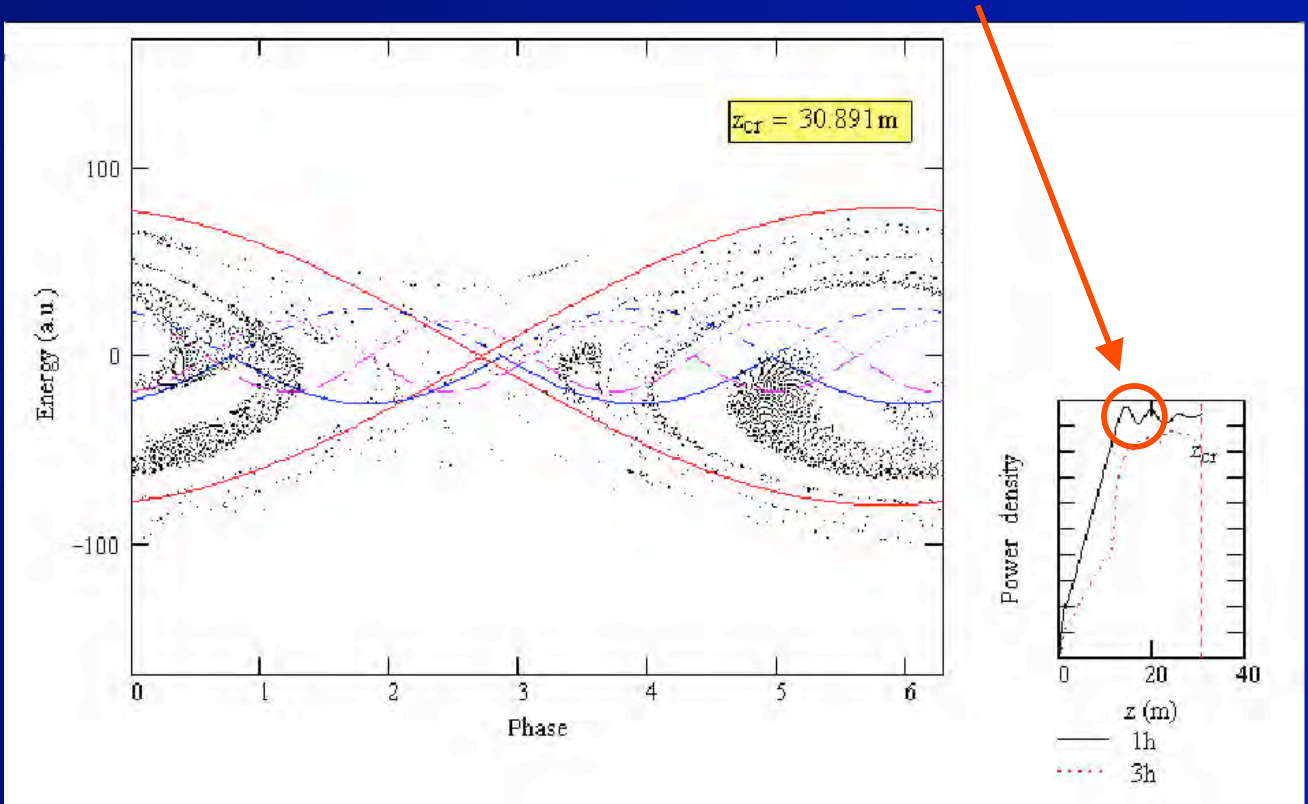
- Slippage
  - The light pulse advances over the electron distribution of a distance N_ in N undulator periods (_ is the resonant wavelength)
- Saturation
  - When the FEL laser power reaches ~_P$_{E,}$ saturation occurs: there is a cyclic energy exchange between electrons and field (in steady state regime – ignoring slippage)

Steady state FEL phase space evolution

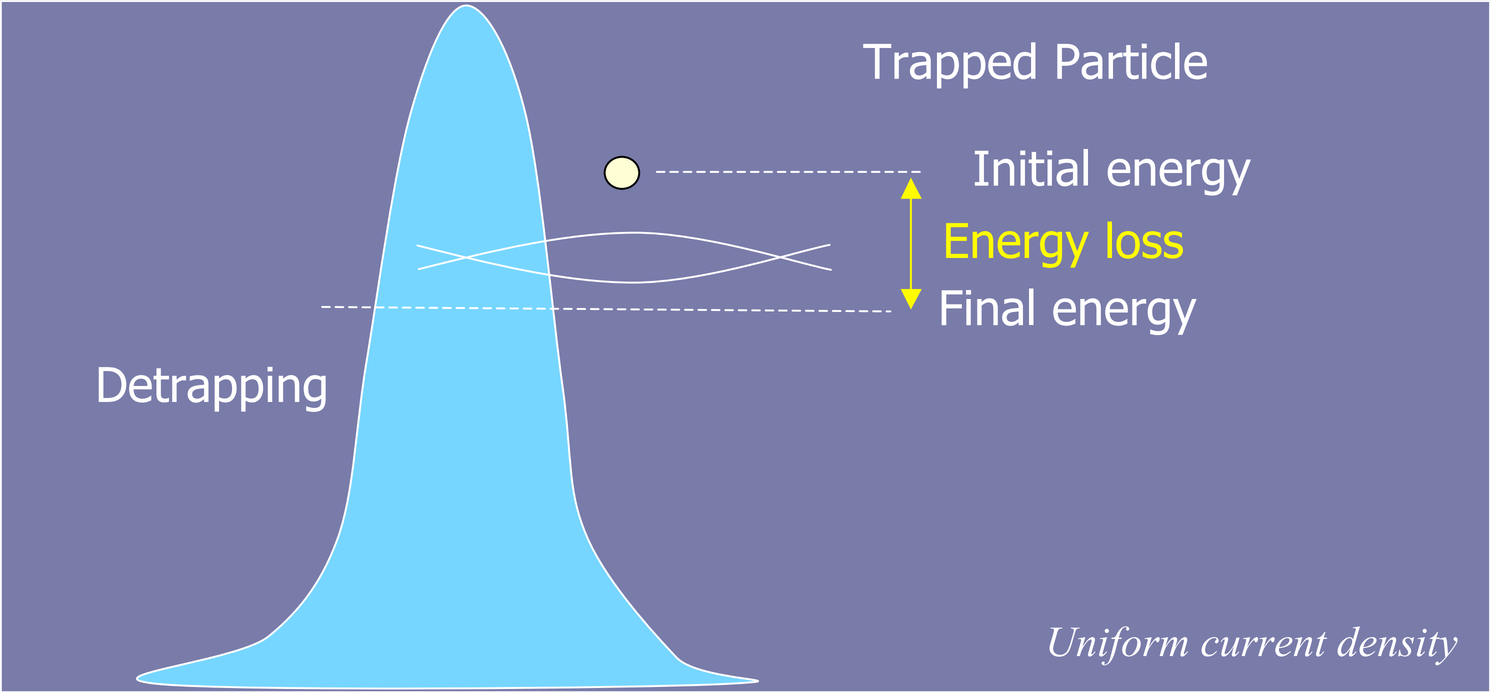


Steady state FEL phase space evolution

What happens if we have a short pulse that slips over the electrons in a time shorter than the synchrotron period ?



Short pulse slipping in a synchrotron period



Condition

- Distance covered by the light in a synchrotron period

$$z_s = \frac{2\pi c}{\omega_s} = \lambda_u \frac{\sqrt{1+K^2/2}}{\sqrt{2KK_R f_B(K)}} \propto \frac{1}{P_L^{1/4}}$$

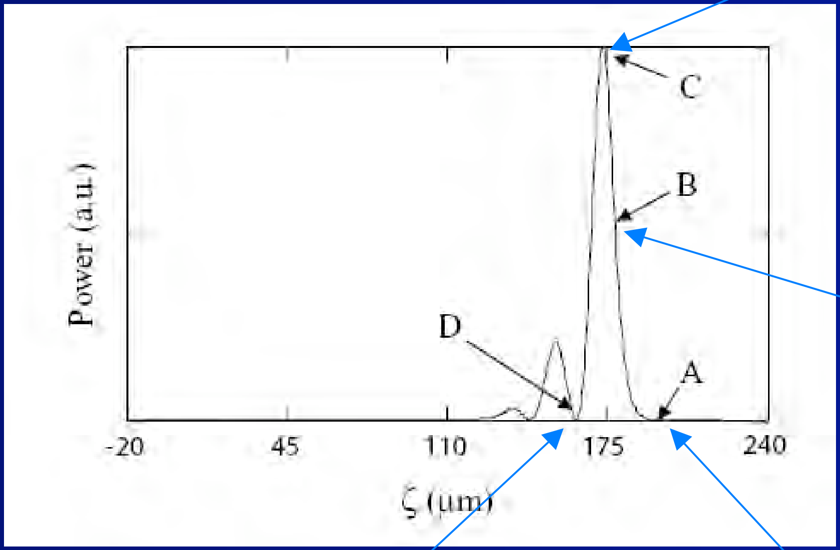
- Slippage length over a synchrotron period

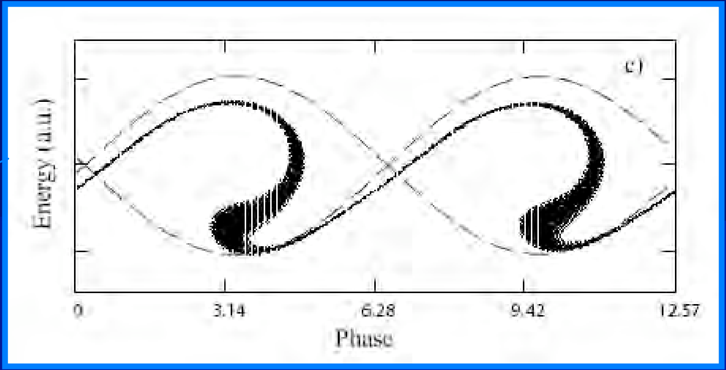
$$z_{sr} \cong z_s \frac{\lambda}{\lambda_u}$$

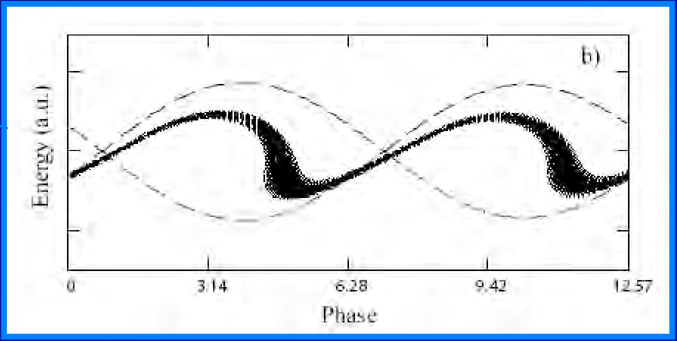
- Relation between pulse length and power $\left(\sigma_{sr} \approx \frac{z_{sr}}{2}\right)$

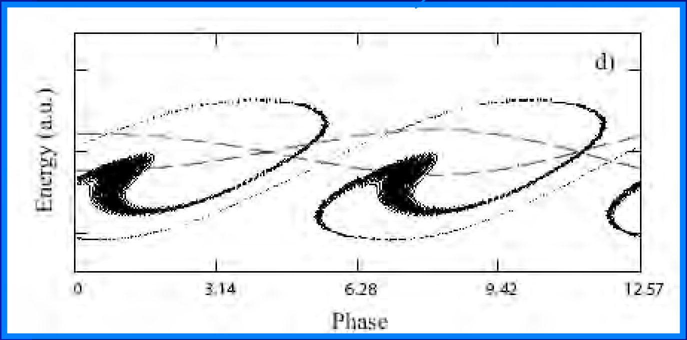
$$\sigma_{sr} P_L^{1/4} = const$$

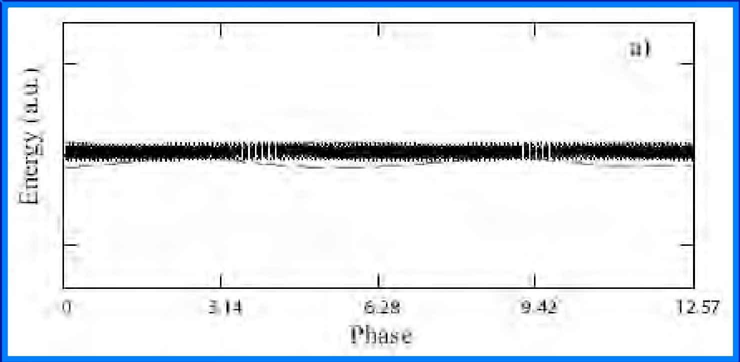
Solitary wave-like superradiant pulse





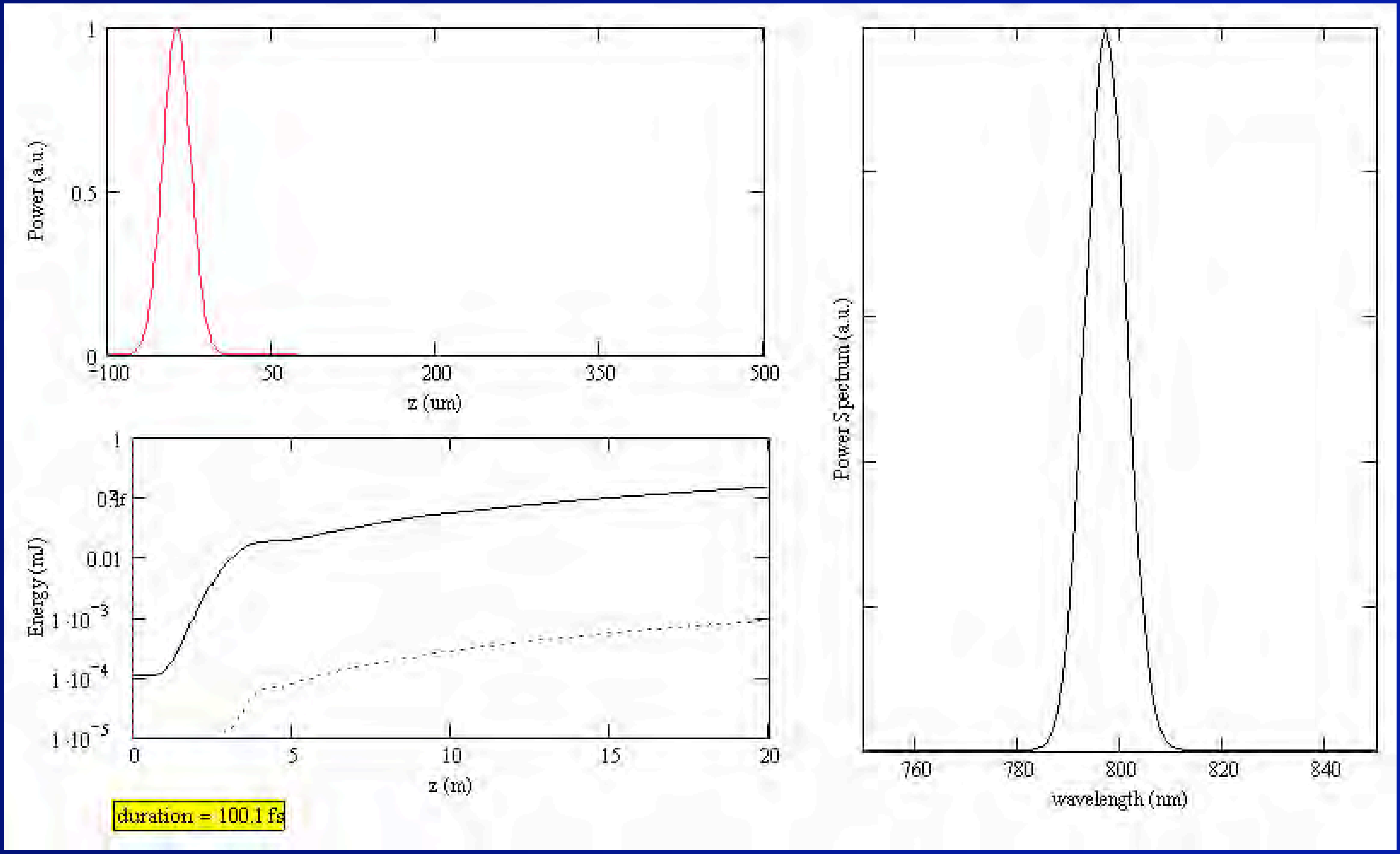






Pulse evolution

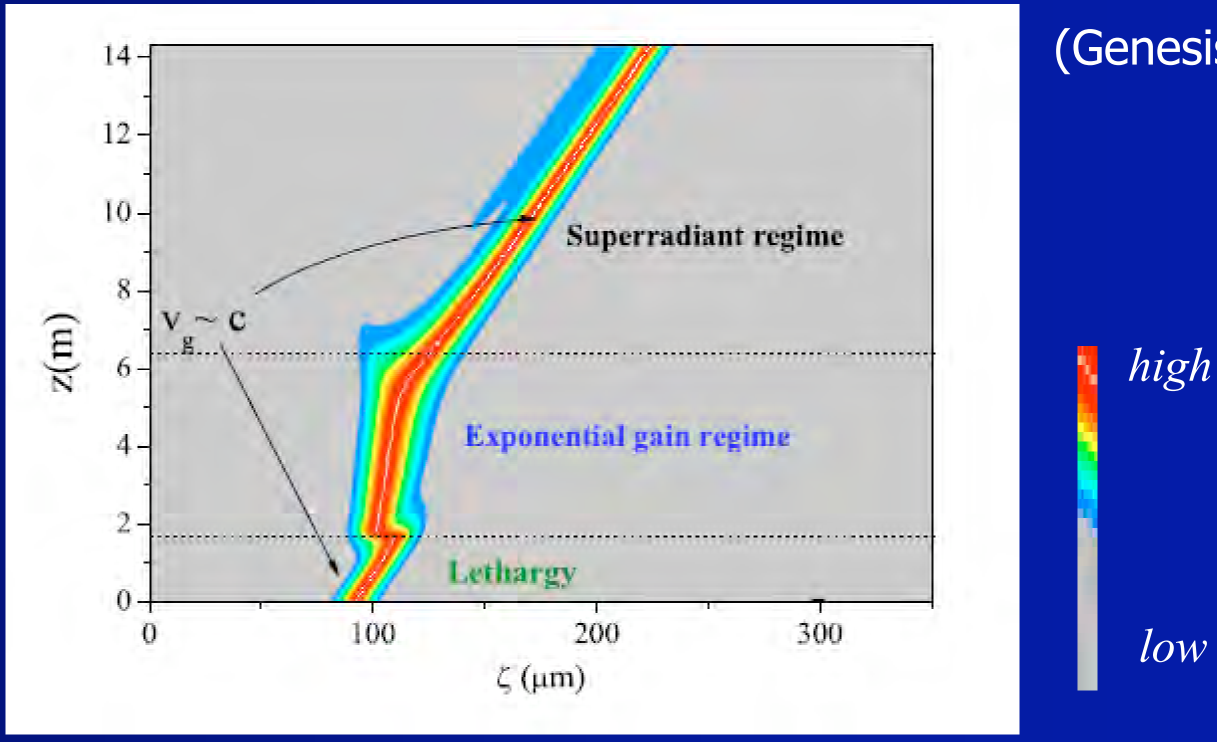
(Perseo http:\\www.perseo.enea.it)



Pulse evolution

Normalized intensity (a.u.)

Distance along the undulator



(Genesis 1.3)

Distance along the e-bunch

Scaling laws

Pulse length $\sigma_{sr} \propto P^{-1/4}$ (1)

Pulse Energy $E = P\sigma_{sr} \propto P^{3/4}$ (2)

… and also: $E \propto z\,\omega_s \propto zP^{1/4}$ (3)

Prop. to number of electrons interacting with the pulse

Prop. to the depth of the bucket in phase space, i.e. Average energy lost by one electron

Power scaling law

Comparing (2) & (3) … $zP^{1/4} \propto P^{3/4} \Rightarrow P \propto z^2$ *… from which follows:*

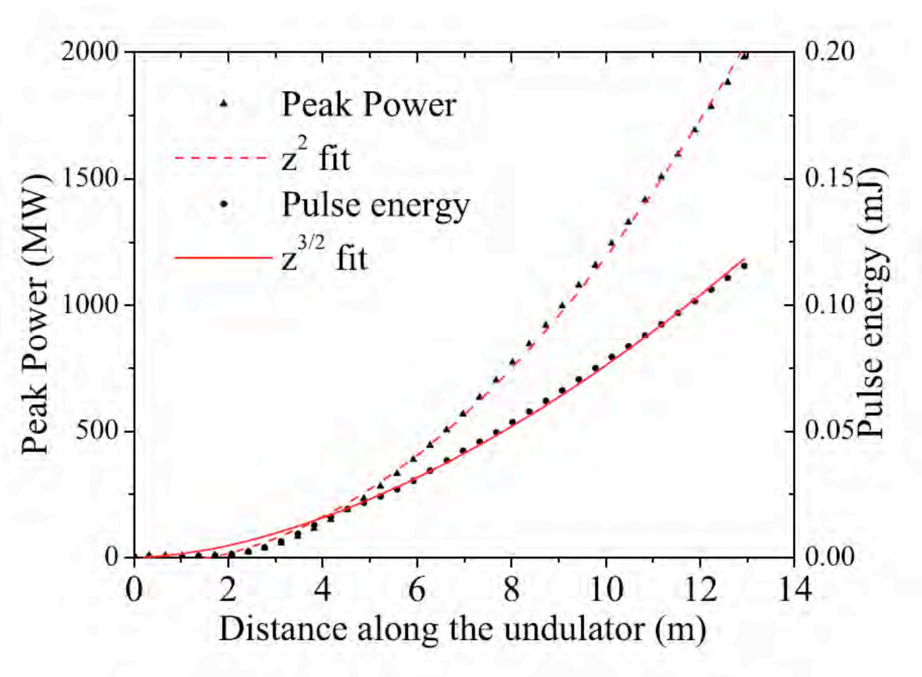
Pulse length

$$\sigma_{sr} \propto z^{-1/2}$$

Pulse energy

$$E \propto z^{3/2}$$

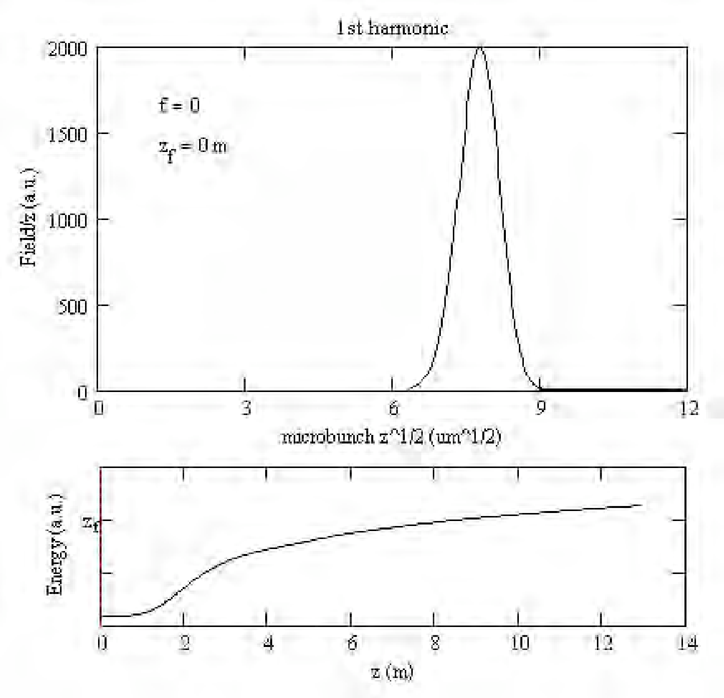
Scaling laws



“Perseo” 1D simulations

Pulse energy and power do not saturate because of the “fresh” electrons injection

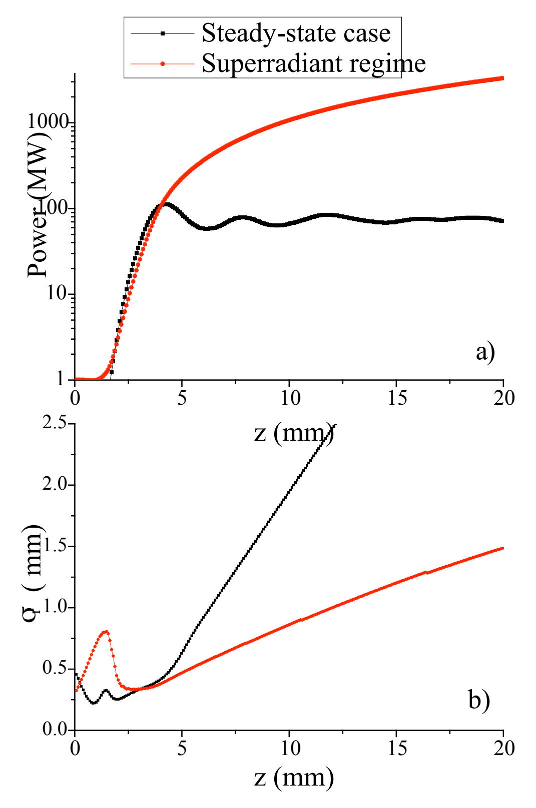
Pulse propagation in “scaled” units: “field/z” vs. “$z^{1/2}$”

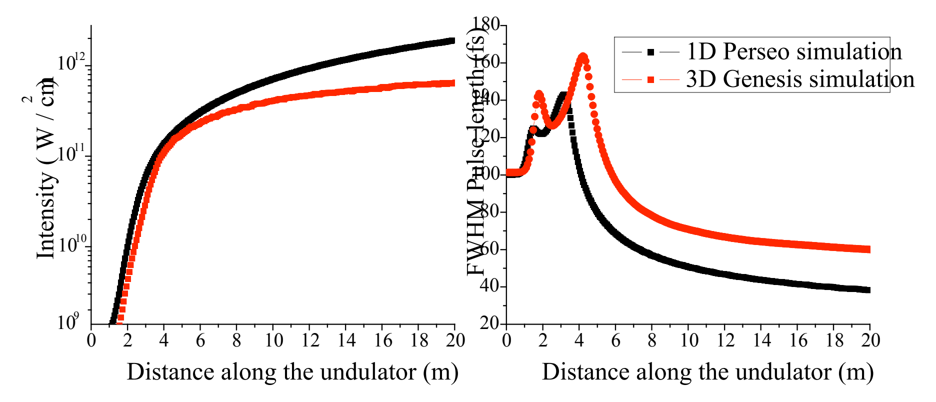


3D with diffraction (Genesis)

Example: $L_{g\_3D} \approx 1.5 L_{g\_(no\ diffraction)}$

Input seed power	1 MW
Wavelength λ	800 nm
Input pulse length (FWHM)	150 fs
Undulator period	3.9 cm
Number of periods	512
Undulator K	1.125
Beam energy	100 MeV
Beam current	350 Amp
Emittance	3mm-mrad

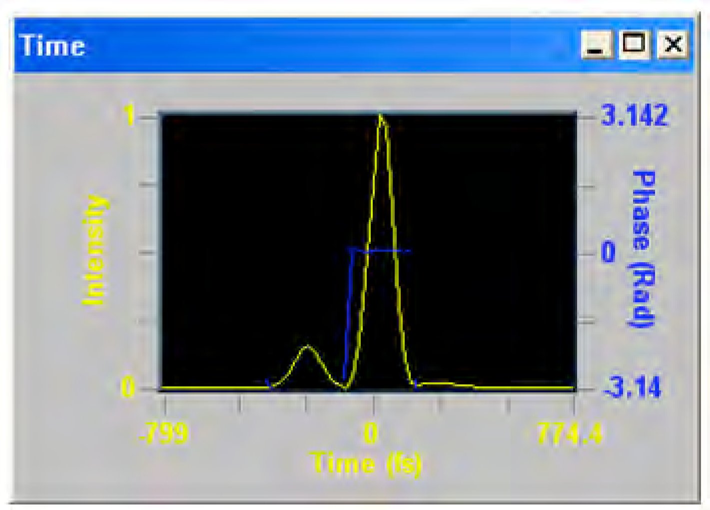




BROOKHAVEN
NATIONAL LABORATORY

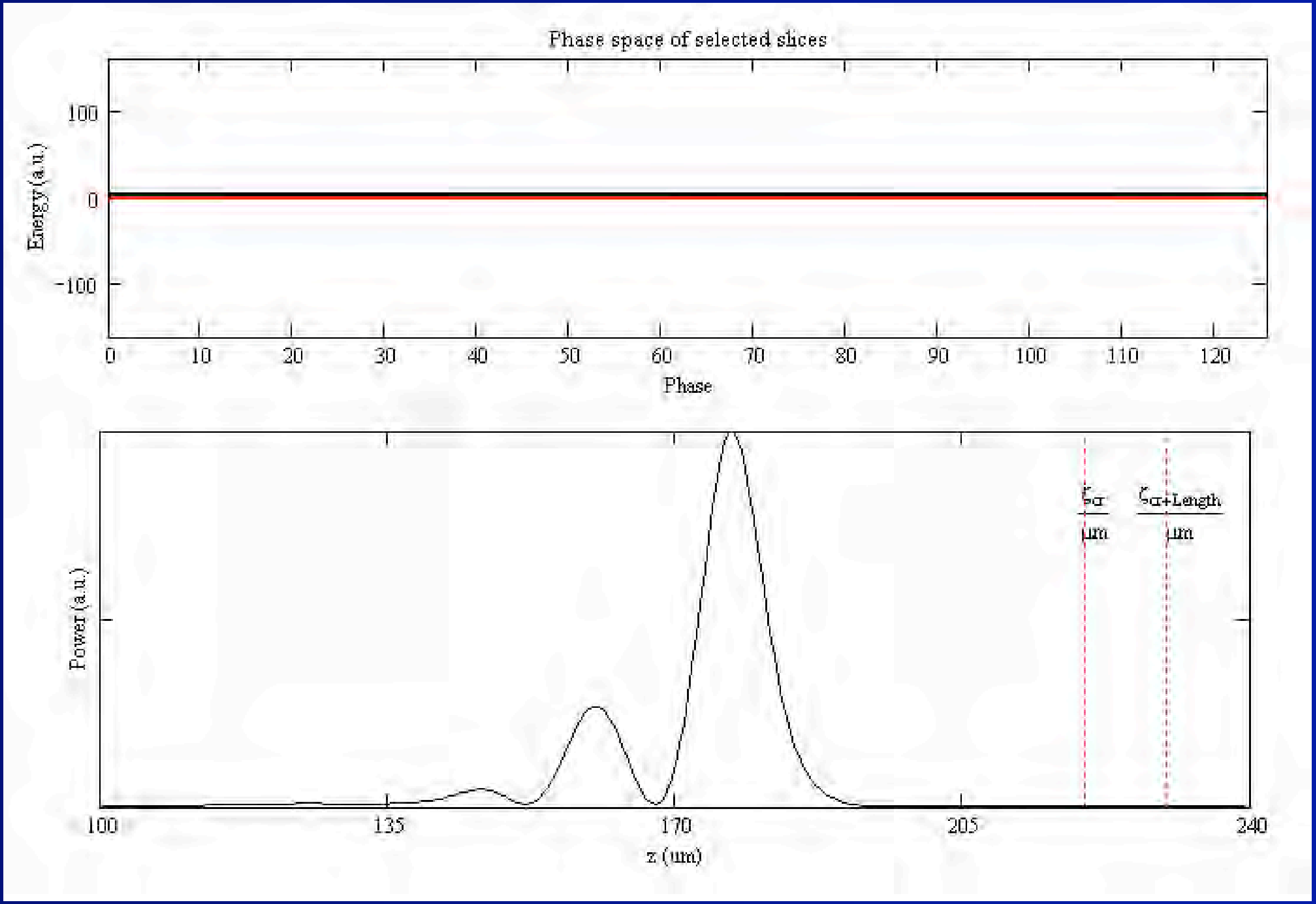
Takahiro Watanabe,
WG3 1° talk this afternoon

NSLS SDL Facility



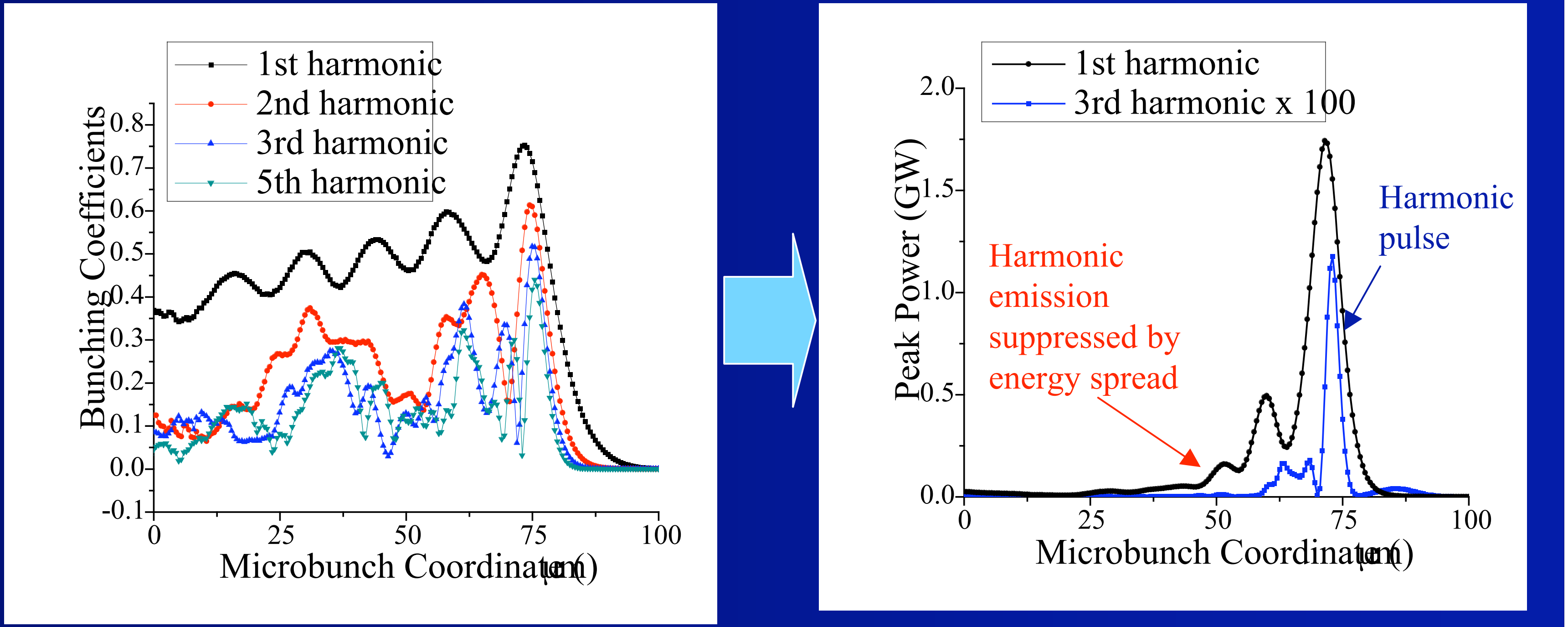
Reconstructed frog trace at the end of the undulator

Phase space in a superradiant pulse (Perseo simulation)



Superradiant harmonics

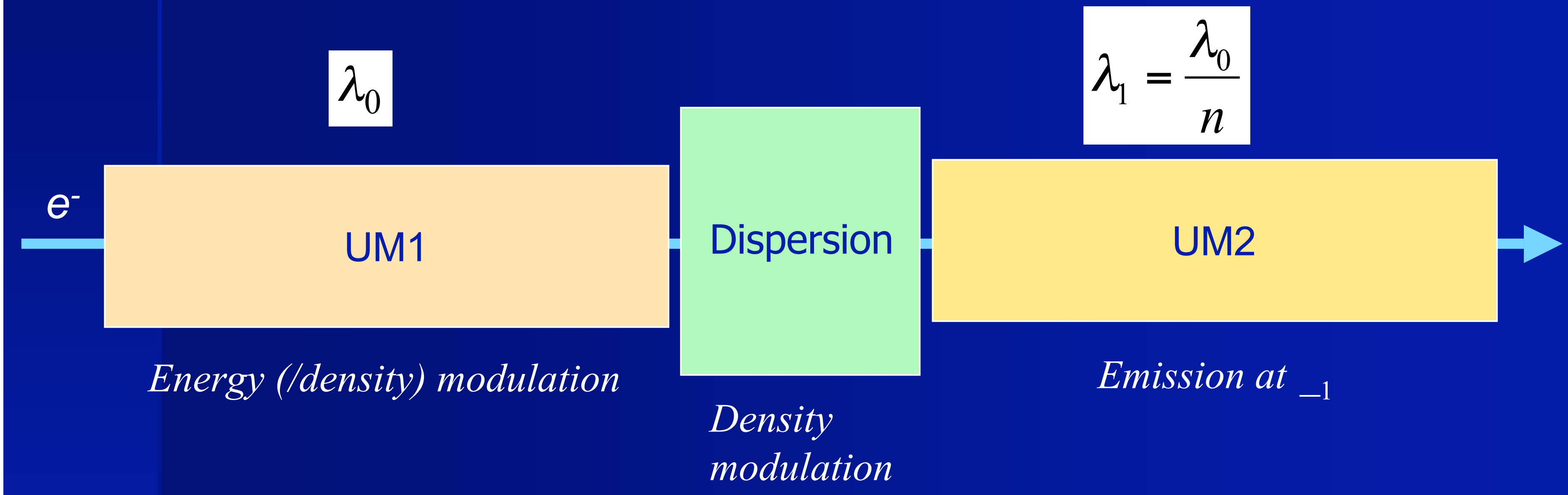
- Short bunching peaks on the pulse front side at the higher order harmonics
- Dynamics for non-linear harmonic evolution “faster” by the harmonic factor n. (i.e. $L_{g,n} \sim L_g / n$)
- Short bursts of harmonic radiation



Harmonic emission suppressed by energy spread

Harmonic pulse

What happens in a “cascade” ?



λ_0

$\lambda_1 = \frac{\lambda_0}{n}$

e^-

UM1

Dispersion

UM2

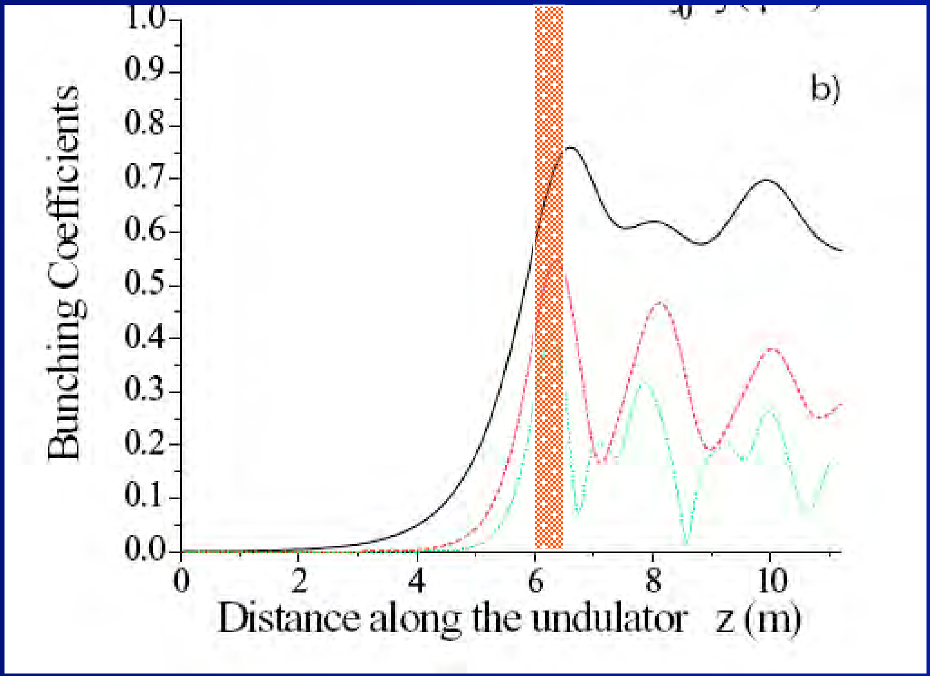
Energy (/density) modulation

Density modulation

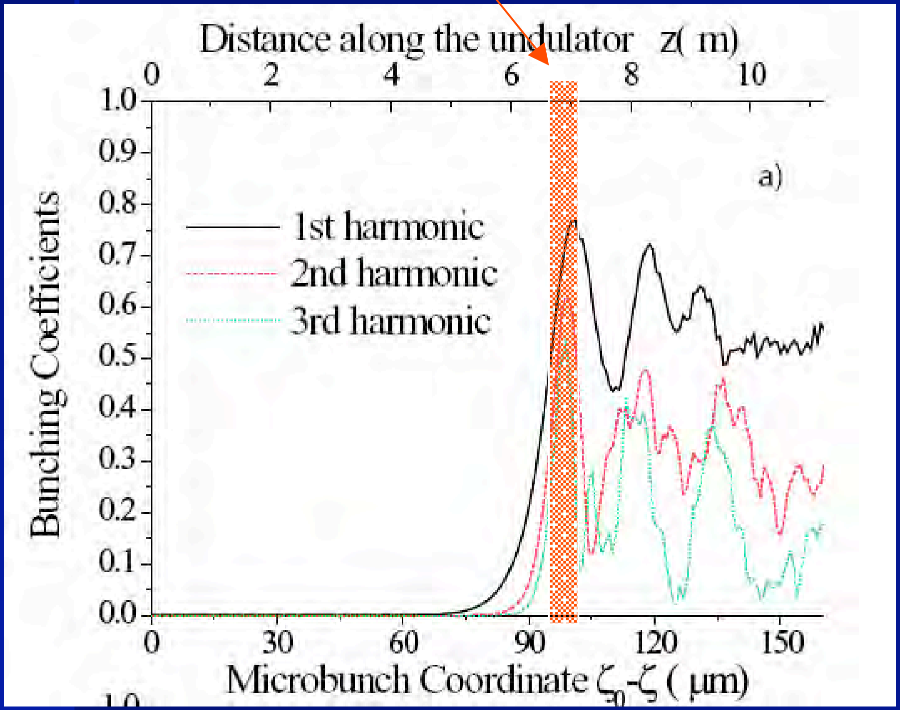
Emission at _1

Bunching coefficients
in the steady state case

Optimized for a cascade



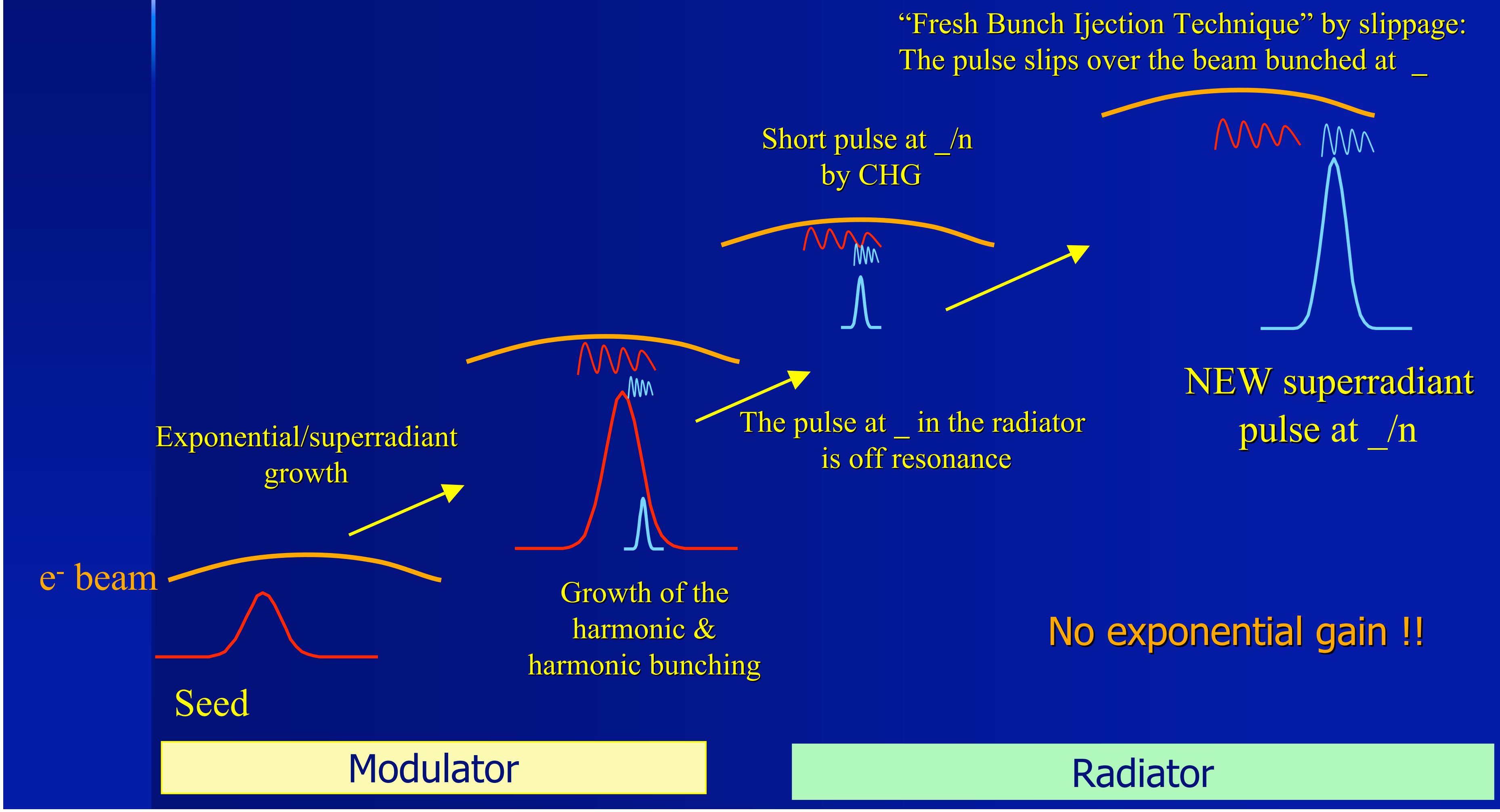
Time dependent - superradiant pulse



Optimized for the cascade

- Higher field induces higher bunching coefficients and a lower sensitivity to “spectral broadening” factors, as energy spread, emittances, magnetic errors, etc.
- A bunching factor optimized for the cascade will always occur somewhere along the pulse
- The coherent spontaneous emission from this portion of the bunch will induce a new pulse at the harmonic wavelength

Evolution in a cascade



e$^-$ beam

Seed

Exponential/superradiant growth

Growth of the harmonic & harmonic bunching

The pulse at _ in the radiator is off resonance

Short pulse at _/n by CHG

“Fresh Bunch Ijection Technique” by slippage:
The pulse slips over the beam bunched at _

NEW superradiant pulse at _/n

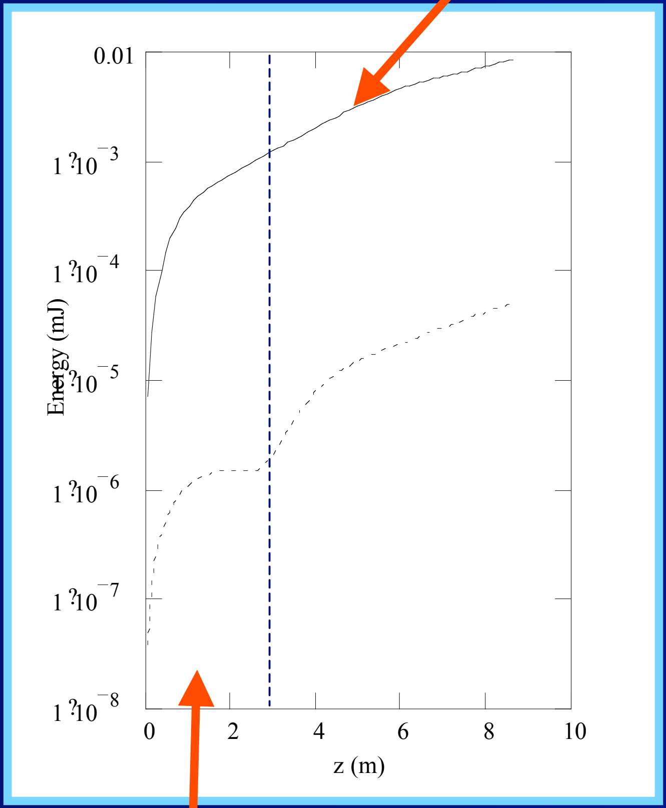
No exponential gain !!

Modulator

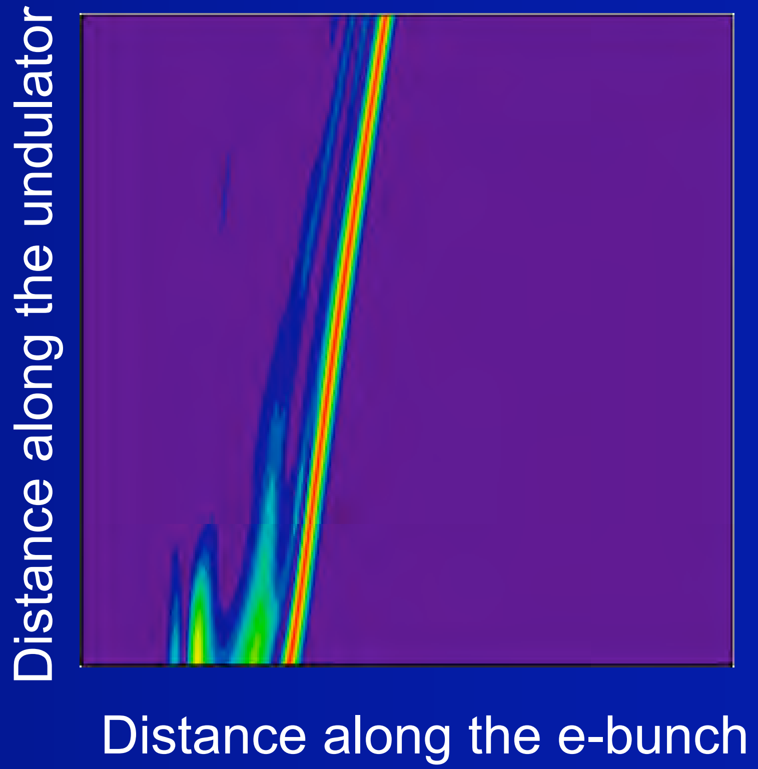
Radiator

Transition in a cascade

Superradiant regime



Coherent spontaneous emission



Example of a cascade

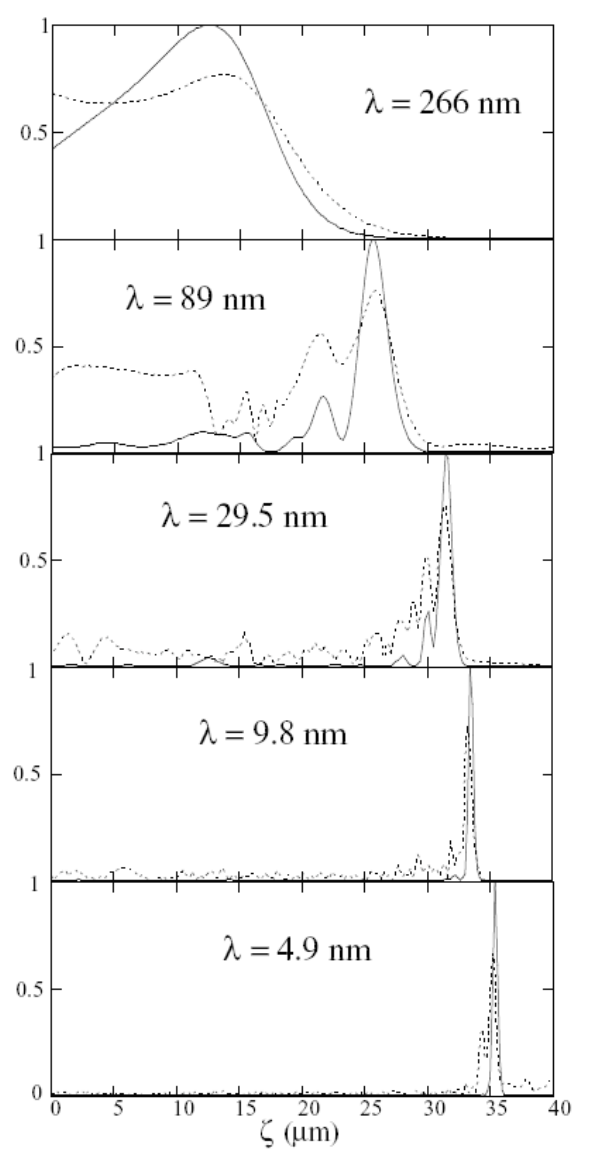
TABLE II: Radiation seed and electron beam parameters

Electron beam energy	800 MeV
Current	1 KA
Emittance	1 mm-mrad
Average β	4 m
$\delta\gamma/\gamma$	$5 \cdot 10^{-4}$
Seed wavelength λ	266 nm
Seed power	41 MW
Seed pulse width (rms)	15 fs

TABLE III: Parameters of the FEL Cascade.

Cascade stage	1	2	3	4	5
$\rho\ (\cdot 10^{-3})$	11	5.8	3.3	1.9	1.3
K	4.9	3.92	2.88	1.8	1.2
Period $\lambda_w (cm)$	10	5	2.8	1.8	1.4
Resonant Wavelength λ (nm)	266	89	29.5	9.8	4.9
Peak power (GW)	1.2	4.8	7	2.5	3.1
Pulse energy (μJ)	63	72	35	4.6	5.3
Pulse width (fwhm - fs)	53	8.4	3.4	1.5	1.4
Undulator periods	60	100	180	200	480

Power & 1° bunching coefficient along the cascade



Harmonic Cascaded FEL

Resonant at λ_1

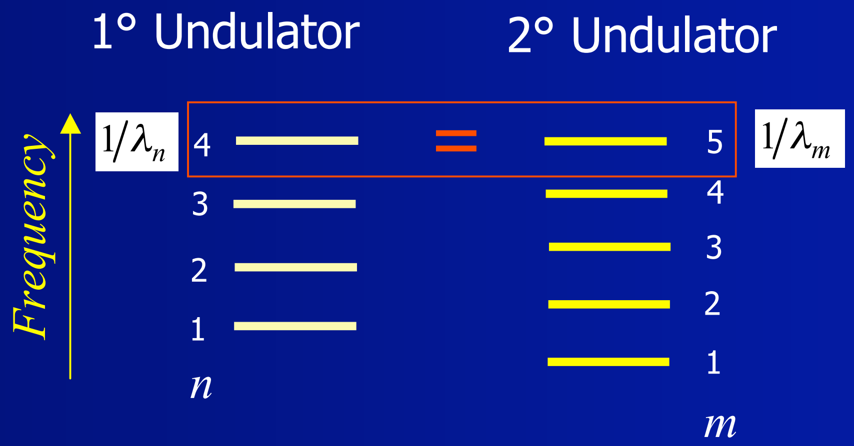
Resonant at $\lambda_2 = \frac{m}{n}\lambda_1, \quad m \neq n$

UM1 → UM2

Emitted wavelength: $\lambda_m = \frac{\lambda_2}{m} = \lambda_n$

Bunching at $\lambda_n = \frac{\lambda_1}{n}$

Harmonics Spectrum of the two undulators

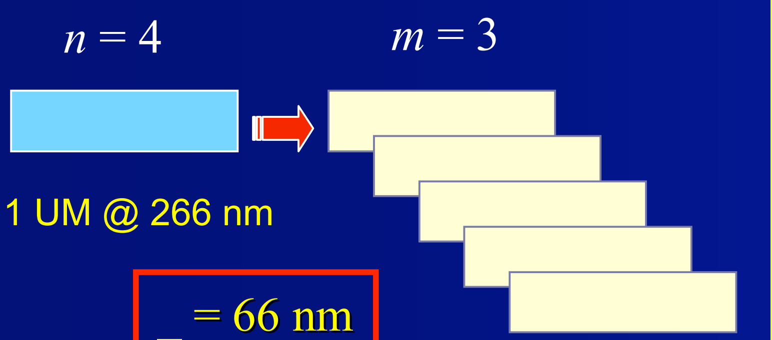


Harmonic Cascade

- The gain (beam current) is not playing an essential role. The important parameters are:
  - Slippage length (N_)
  - Depth of the potential bucket (Laser Intensity + coupling parameters)

- In the second undulator:
  - The first harmonic field (the seed) is off resonance
  - The harmonic pulse at the wavelength $\lambda_m = \frac{\lambda_2}{m} = \lambda_n$ still slips over the electron with velocity _$_2$ per period !!

Example with *Sparc* undulator/beam parameters
1D *Perseo* simulation *(http://www.perseo.enea.it)*



$n = 4$

$m = 3$

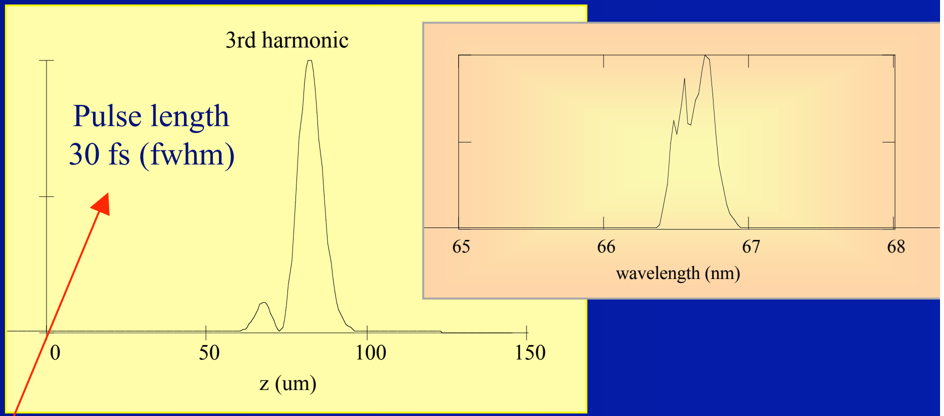
1 UM @ 266 nm

_ = 66 nm

5 UM @ 200 nm

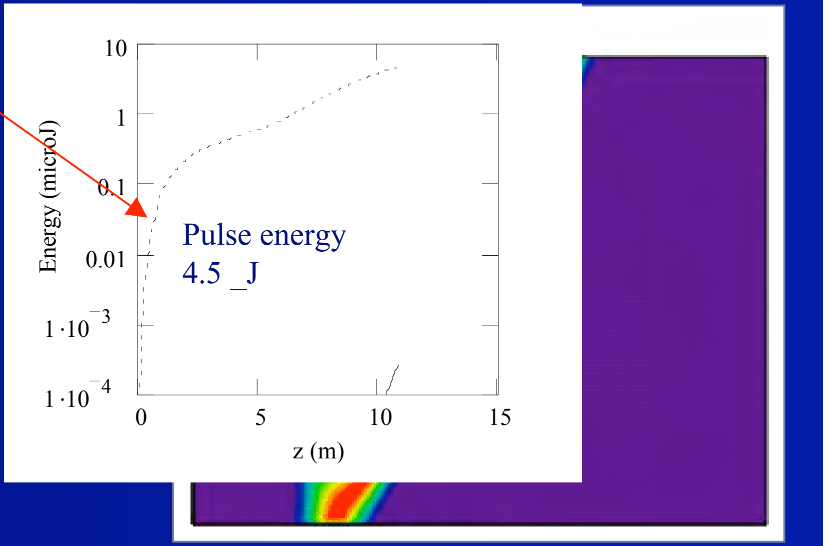
Main parameters

Undulator period	2.8 cm
Undulator K (UM1/UM2)	1.95 / 1.53
Number of periods	77 / 77*5
Beam energy	200 MeV
Res. wavelength (nm)	266 / 200
E-beam current	110 Amp
Energy spread	10^{-4}
Emittance	1 mm-mrad
Input Laser power	2 MW
Input pulse length (fwhm)	100 fs



Pulse length
30 fs (fwhm)

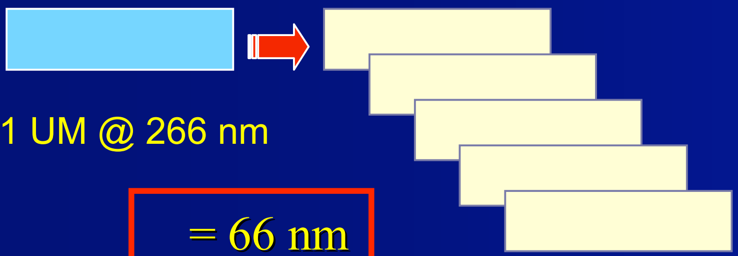
Peak power
≈ 130 MW



Pulse energy
4.5 _J

As before ... but with *n* = 4 and *m* = 5

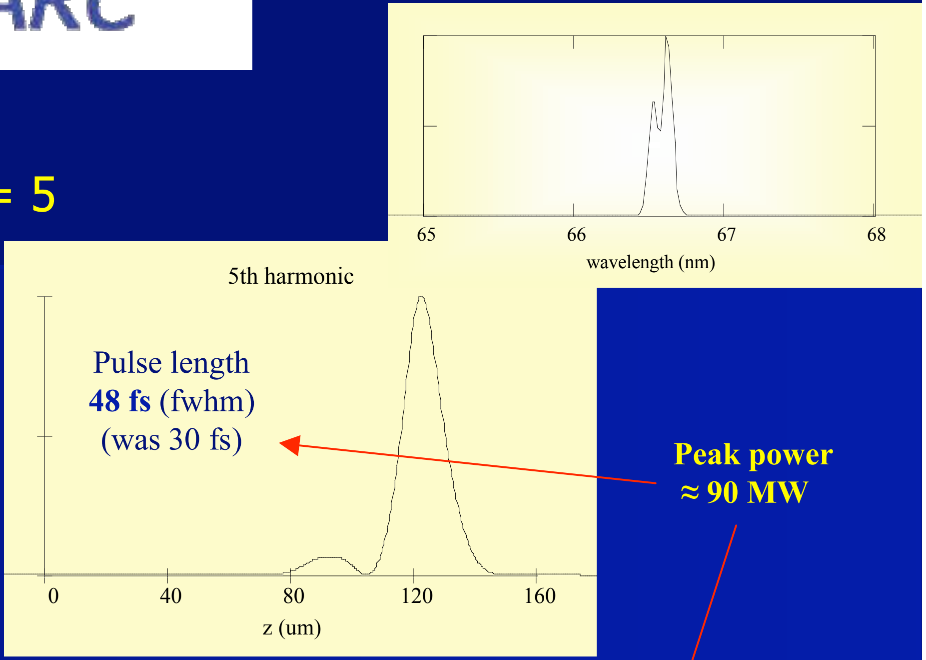
n = 4 *m* = 5



1 UM @ 266 nm

_ = 66 nm

5 UM @ 332 nm

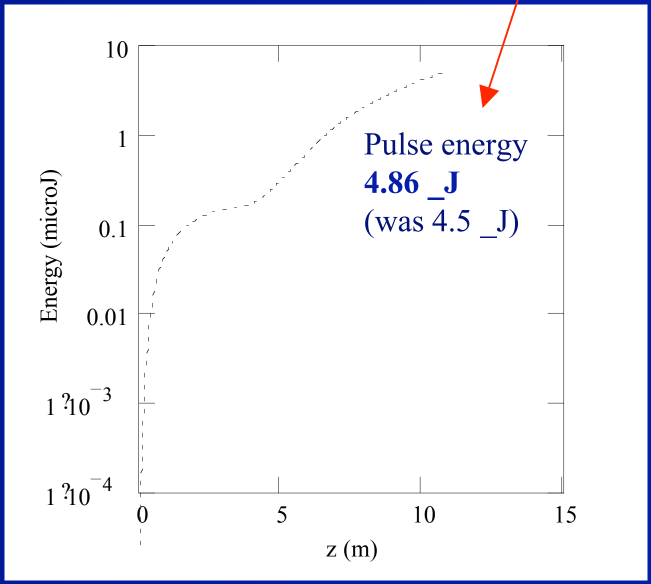


Pulse length
48 fs (fwhm)
(was 30 fs)

Peak power
≈ 90 MW

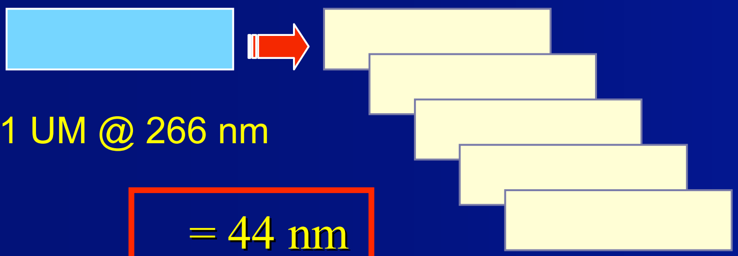
Main parameters

Undulator period	2.8 cm
Undulator K (UM1/UM2)	1.95 / 2.4
Number of periods	77 / 77*5
Beam energy	200 MeV
Res. wavelength (nm)	266 / 332
E-beam current	110 Amp
Energy spread	10^{-4}
Emittance	1 mm-mrad
Input pulse length (fwhm)	100 fs



Pulse energy
4.86 _J
(was 4.5 _J)

As before … but with *n* = 6 and *m* = 5



n = 6

m = 5

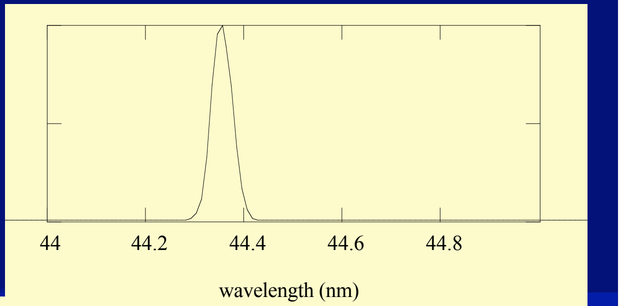
1 UM @ 266 nm

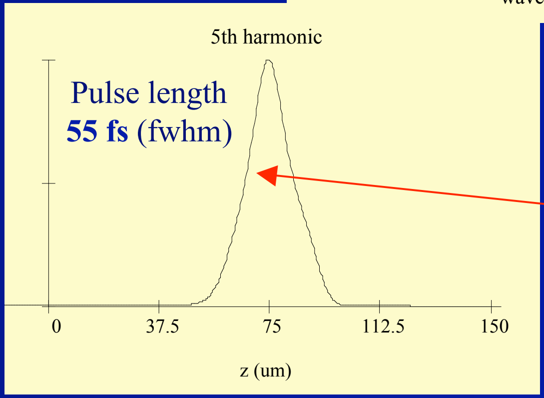
_ = 44 nm

5 UM @ 222 nm

Main parameters

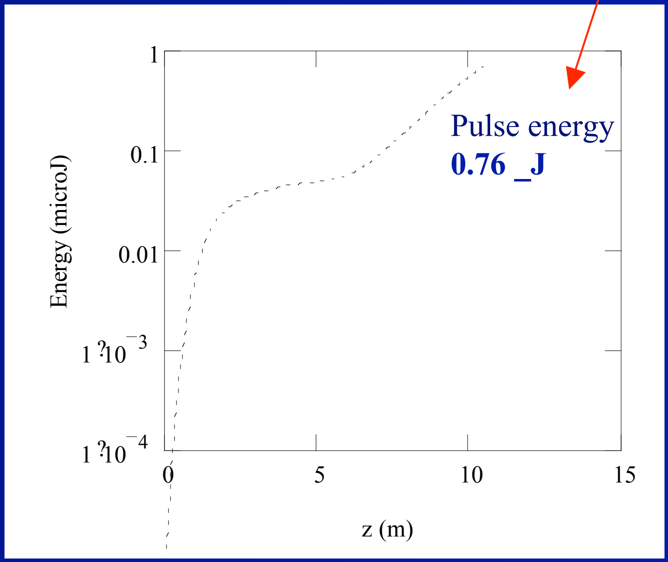
Undulator period	2.8 cm
Undulator K (UM1/UM2)	1.95 / 1.69
Number of periods	77 / 77*5
Beam energy	200 MeV
Res. wavelength (nm)	266 / 222
E-beam current	110 Amp
Energy spread	10^{-4}
Emittance	1 mm-mrad
Input pulse length (fwhm)	100 fs





Pulse length
55 fs (fwhm)

Peak power
≈ 12 MW



Pulse energy
0.76 _J

Conclusions

- Simpler cascade scheme with multiple stages
- Significative extension of the FEL cascade operating range with the harmonic cascade
- Reduced sensitivity to tolerances and fluctuations:
  - No exponential gain
  - The bunching is transmitted through the cascade (with a characteristic length scaling with $z^{1/2}$)
- Pulse shape determined by FEL dynamics
- Sub-fs pulses at short wavelength (Soft X-rays)