

Theory of Quantum FEL and the QFEL Project

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Outline

1. Steady State and **Superradiant instability**
2. Classical SASE
3. Quantum FEL Model (QFEL) and Classical Limit
4. Quantum Linear Analysis
5. Quantum SASE: **quantum “purification”** and **discrete lines**
6. LENS experiment: **BEC = QFEL**
7. **QFEL Project**

R. B., N. Piovella, G.R.M. Robb, NIM A 543 (2005) 645 and ref. therein

Some references

HIGH-GAIN AND SASE FEL with “UNIVERSAL SCALING” Classical Theory

- (1) R.B, C. Pellegrini and L. Narducci, *Opt. Commun.* 50, 373 (1984).
- (2) R.B, B.W. McNeil, and P. Pierini *PRA* 40, 4467 (1989)
- (3) R.B, L. De Salvo, P.Pierini, N.Piovella, C. Pellegrini, *PRL* 73, 70 (1994).
- (4, 5) R.B. et al, *Physics of High Gain FEL and Superradiance, La Rivista del Nuovo Cimento* vol. 13. n. 9 (1990) e vol. 15 n.11 (1992)

QUANTUM THEORY

- (6) R. B., N. Piovella, G.R.M.Robb, and M.M.Cola, *Europhysics Letters*, 69, (2005) 55 .
- (7) R.B., N. Piovella, G.R.M. Robb, *Quantum Theory of SASE-FEL, NIM A* 543, 645 (2005), and *proc. FEL Conf. 2005*
- (8) R. B., N. Piovella, G.R.M.Robb, and M.M.Cola, *Optics Commun.* 252, 381 (2005)

See also

- (9) F.T.Arecchi, R. Bonifacio, “MB equation”, *IEEE Quantum Electron.*, 1 (1965) 169

Classical model with "Universal" Scaling

no free parameters

A: scattered field

"Collective FEL parameter"

R. B, C. Pellegrini and L. Narducci,
Opt. Commun. 50, 373 (1984).

Position

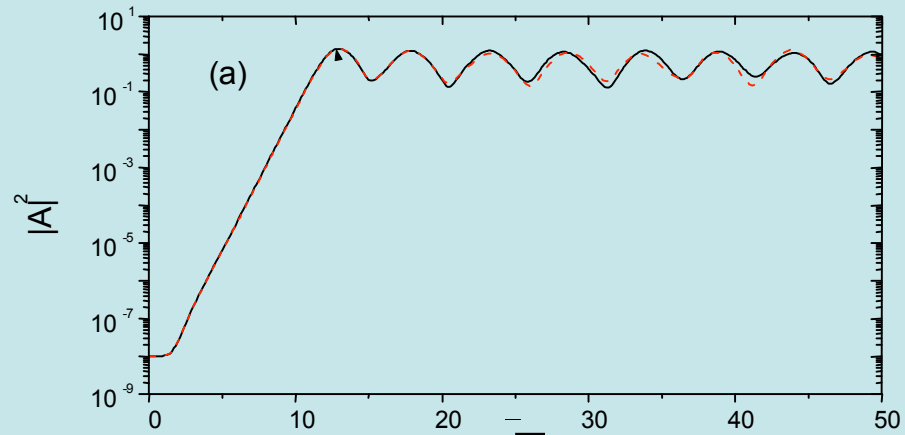
Momentum

Steady State (S.S.) model

Steady State instability (with universal scaling)

R. B. C. Pellegrini and L. Narducci,
Opt. Commun. 50, 373 (1984).

“Collective FEL parameter”

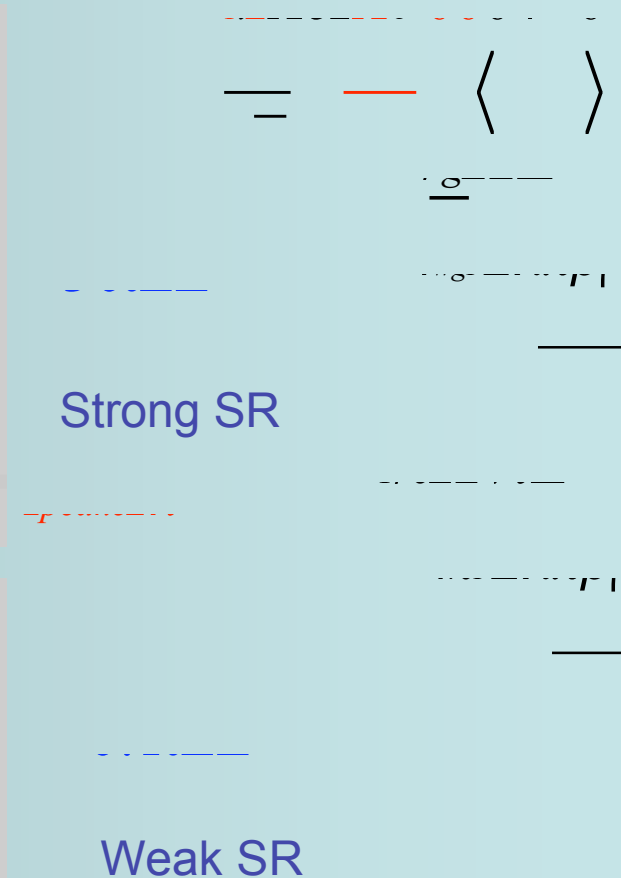
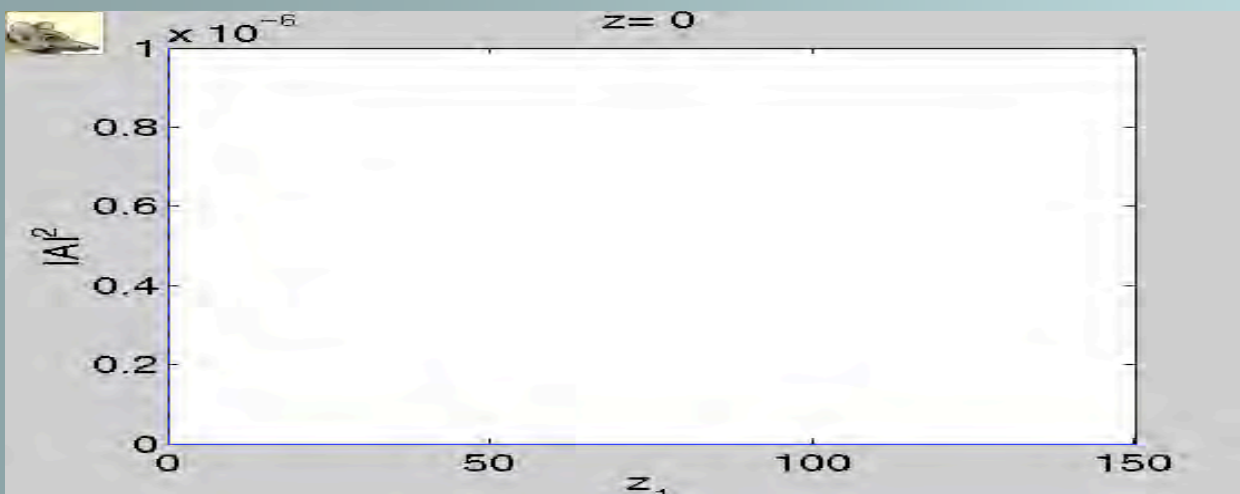
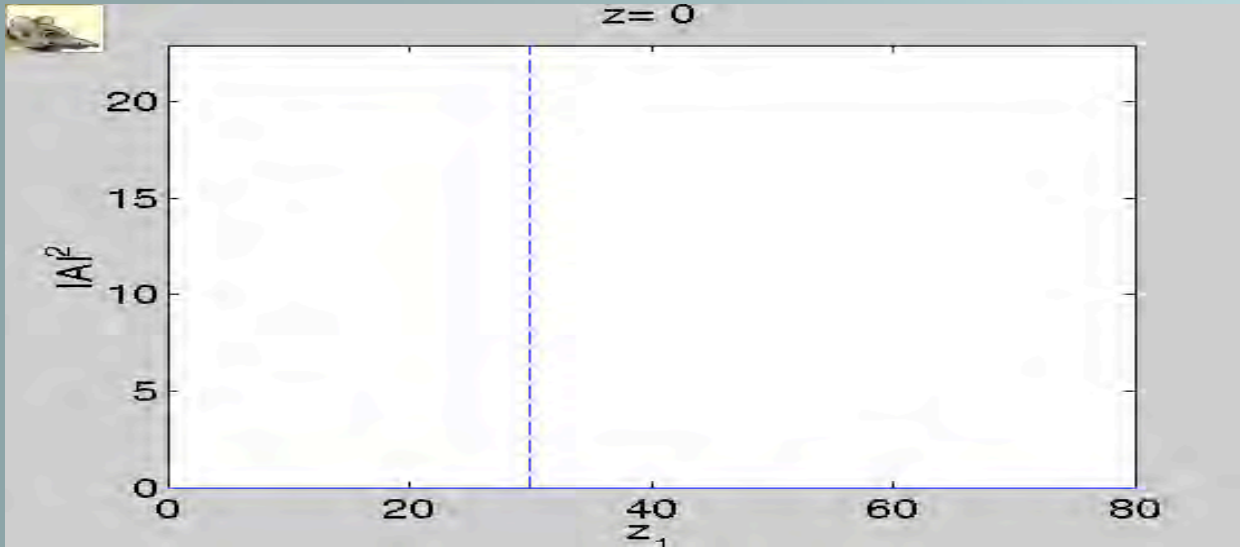


Exponential growth up to $|A| \sim 1$
Independently on the initial value

**Possibility of
start up from noise.**

STEADY STATE AND SUPERRADIANT INSTABILITY, Long and Short Bunch (uniform seed)

Evolution of radiation time structure in the electron rest frame

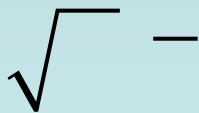
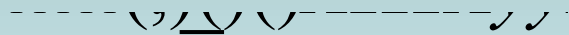


R. Bonifacio, B.W. McNeil, and P. Pierini PRA 40, 4467 (1989)

Soliton-Like solution and Superradiant Regime

(ref. 2,4)

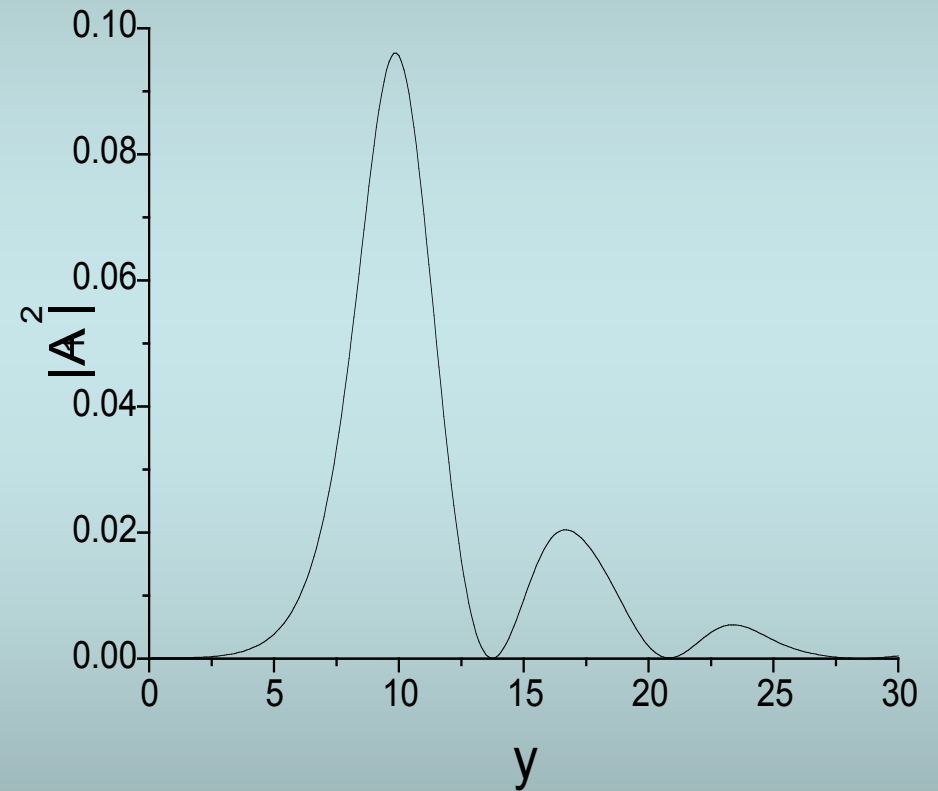
CLASSICAL REGIME:



width



SUPERRADIANCE

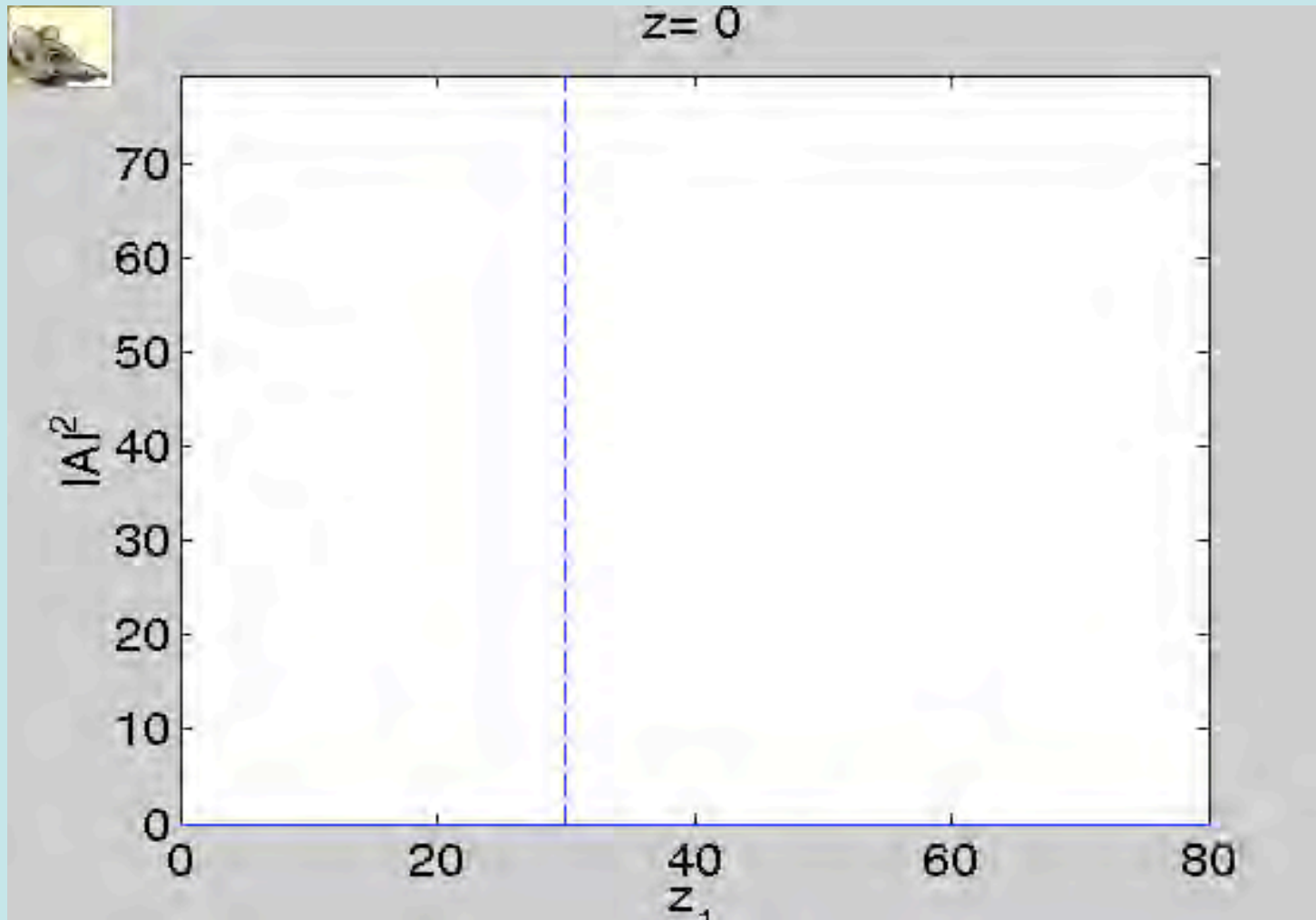


SELF SIMILAR SOLUTION

SUPERRADIANT INSTABILITY, (uniform seed) $L=30L_C$, detuned ($\delta=2$)

R. Bonifacio, B.W. McNeil, and P. Pierini
PRA 40, 4467 (1989)

Evolution of radiation time structure in the electron rest frame



No steady
state
Instability

**SR does not
depend on
detuning and
does not
saturate**

Classical SASE

Ingredients:

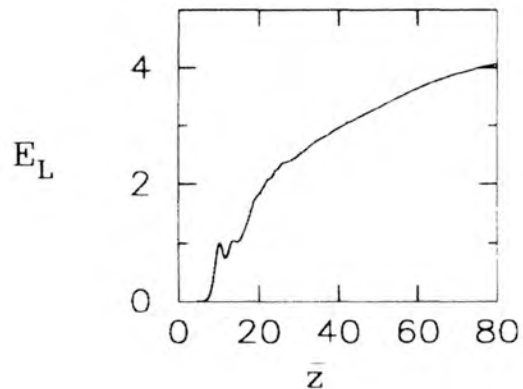
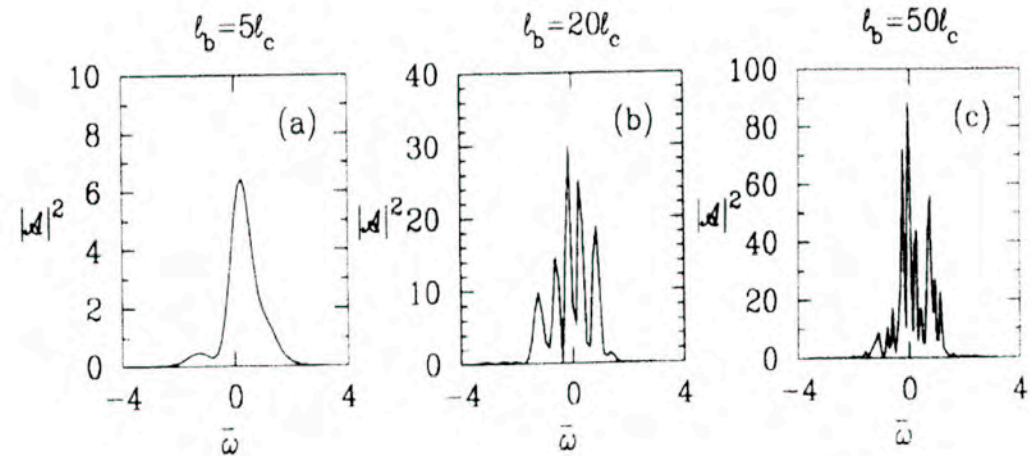
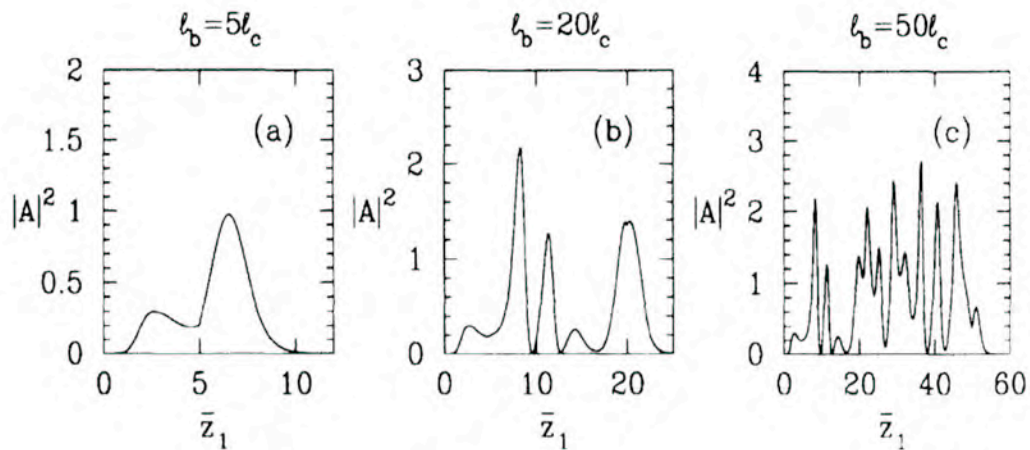
- i) Start up from noise
- ii) Propagation effects (slippage)
- iii) **Superradiant instability**: (no steady state instability)

Self Amplified Superradiant Emission

(RB, L. De Salvo, P. Pierini, N. Piovella, C. Pellegrini, PRL 73 (1994) 70)



The electron bunch behaves as if each cooperation length would radiate **independently** a weak **SR spike** which gets amplified propagating on the other electrons **with no saturation**. **Spiky time structure and spectrum.**



CLASSICAL SASE

reprinted from PRL 73 (1994) 70

Time structure:

Almost chaotic behavior:

number of random spikes goes like

Spectrum:

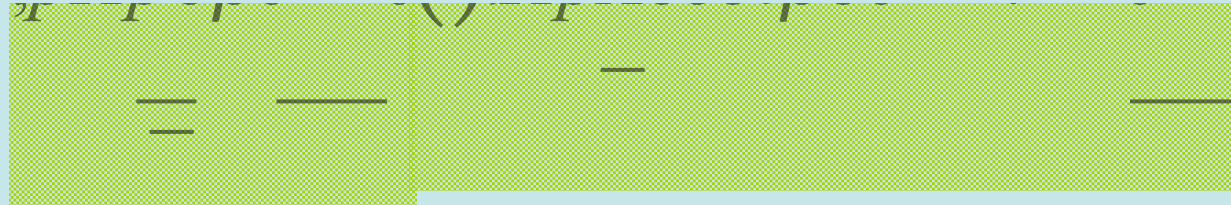
is just the envelope of a series of narrow **random spikes**

If a single SR spike.

At short wavelengths => **many random spikes.**

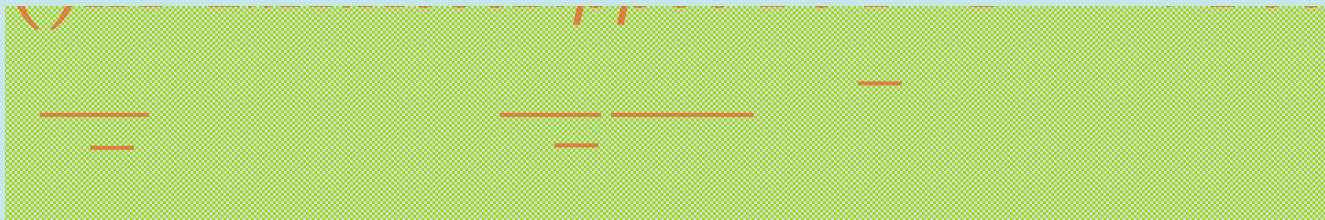
Total energy does not saturate (at 1.4).

Canonical Quantization



Quantization

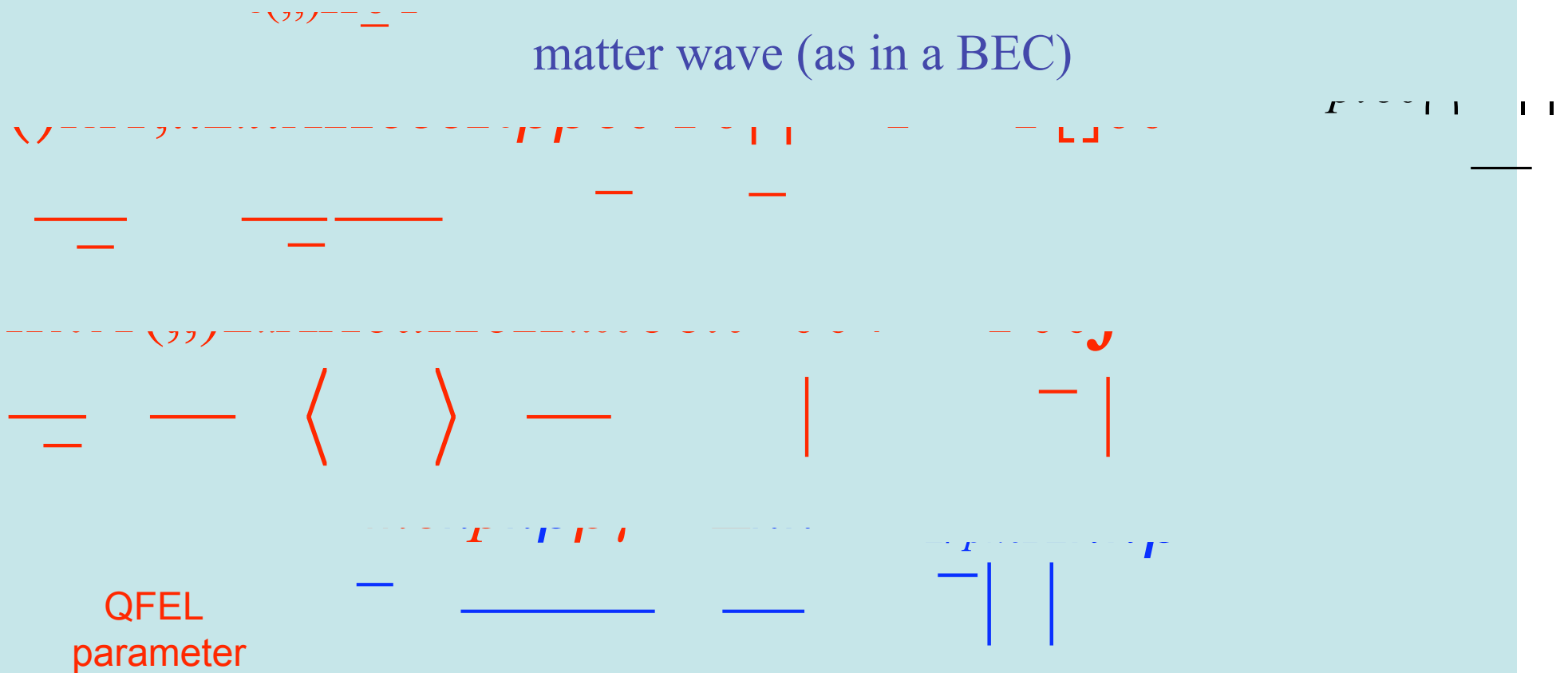
The QFEL model for the matter wave



Derived from Q-field theory by G. Preparata (Phys. Rev. A, 38 (1988), 233)

QFEL (BEC) propagation model (ref.8)

matter wave (as in a BEC)



R. B., N. Piovella, G.R.M. Robb., NIM A 543 (2005) 645
and proc. FEL Conf. 2005 (Stanford)

— ; Q. F. T. by **G. Preparata**[†] (Phys. Rev. A, 38 (1988), 233)

Classical Limit: $\hbar \rightarrow 0$

One can prove that the Schroedinger equation for the QFEL model reduces to the **classical Vlasov Equation** for the **Quantum Wigner function** in the limit:

$\hbar \rightarrow 0$

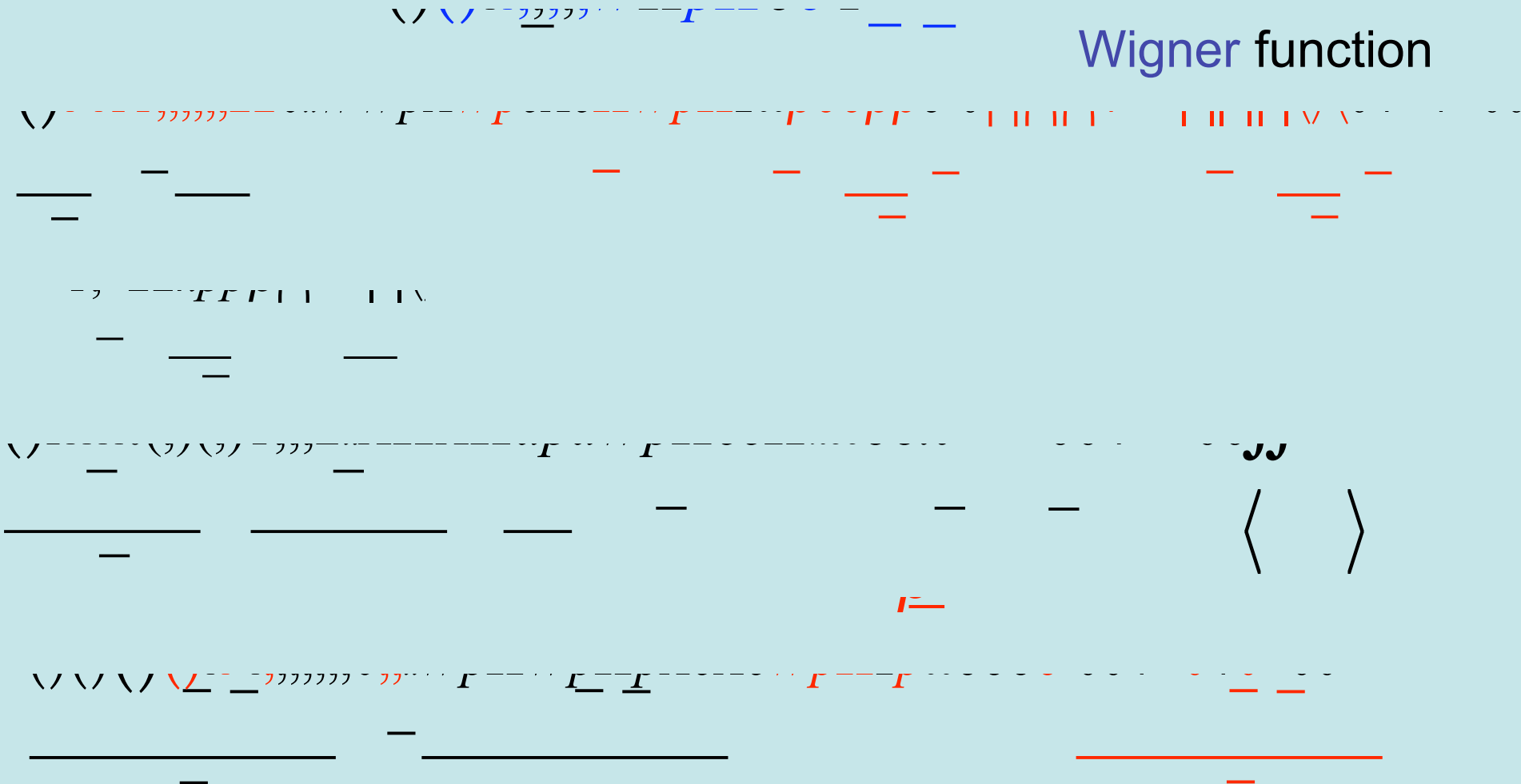
In the classical limit, with **universal scaling**,
no dependence on \hbar

(Ref. 6, 8)

Classical limit when

(ref.6,8)

Wigner function



Classical Vlasov Equation

The Momentum Representation (ref. 7, 8)

$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \phi(p) e^{ipx/\hbar} dp$$

$|\phi(p)|^2$ is the probability that an electron has a momentum

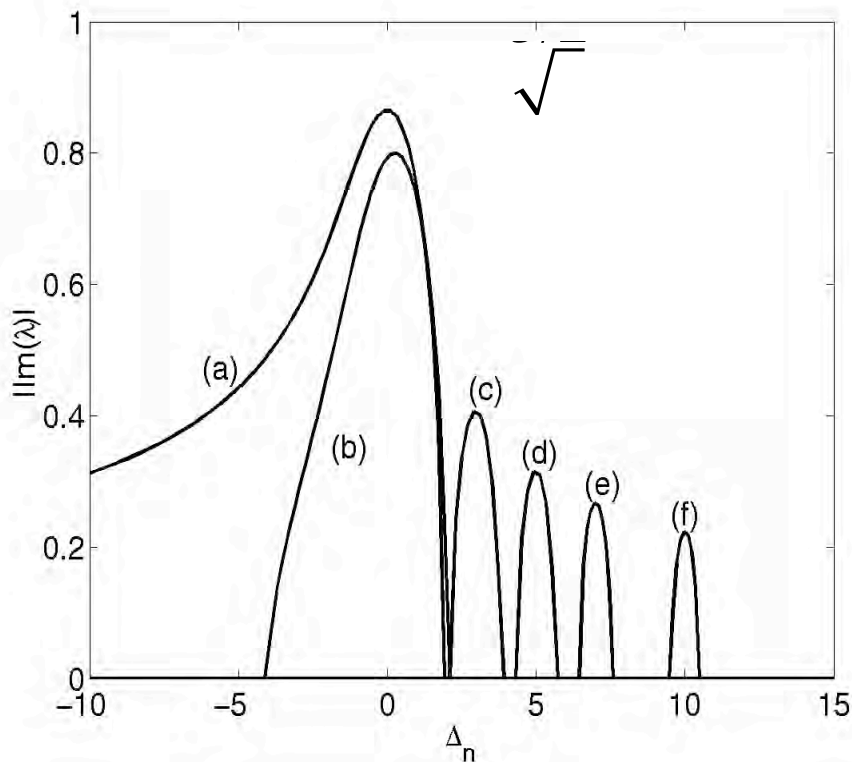
$$\begin{aligned} \langle p \rangle &= \int_{-\infty}^{\infty} p |\phi(p)|^2 dp \\ \langle p^2 \rangle &= \int_{-\infty}^{\infty} p^2 |\phi(p)|^2 dp \end{aligned}$$

QFEL "working equations"

$$\begin{aligned} \langle p \rangle &= \int_{-\infty}^{\infty} p |\phi(p)|^2 dp \\ \langle p^2 \rangle &= \int_{-\infty}^{\infty} p^2 |\phi(p)|^2 dp \end{aligned}$$

Linear Theory: QM

As if classical rect. dist.

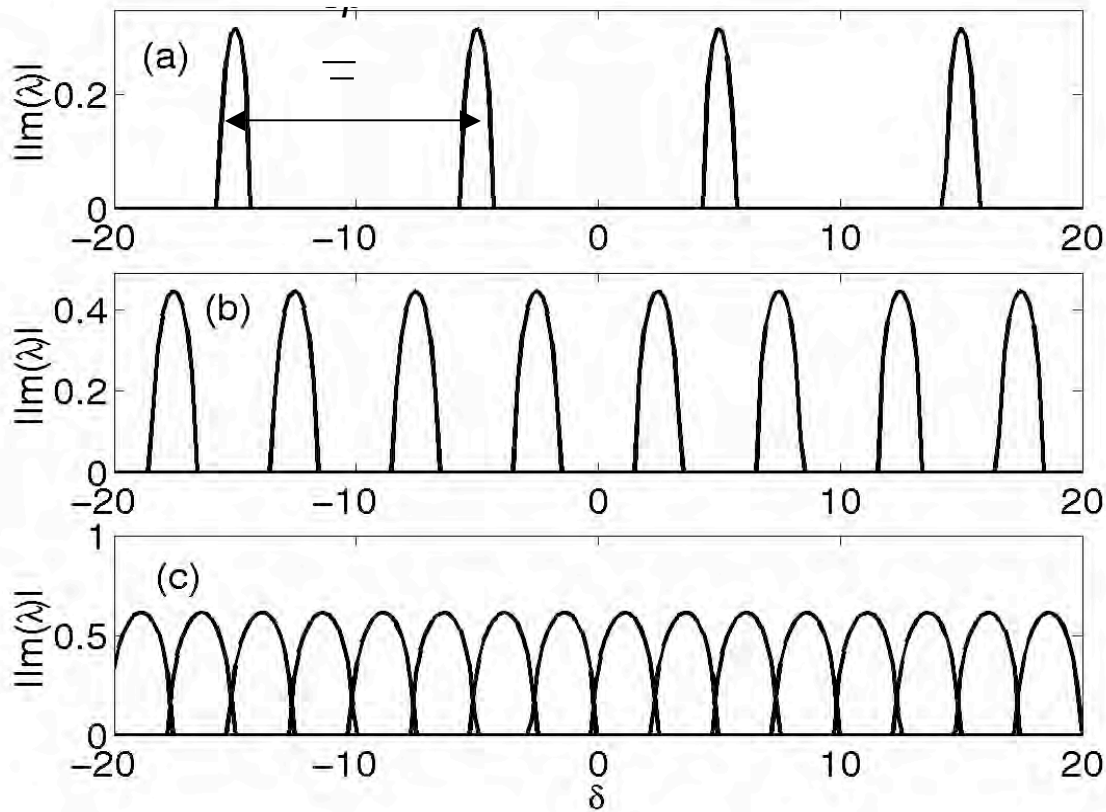


Classical limit (a) (ref. 1)

Quantum regime

(ref. 7, 8)

The Discrete frequencies as in a cavity

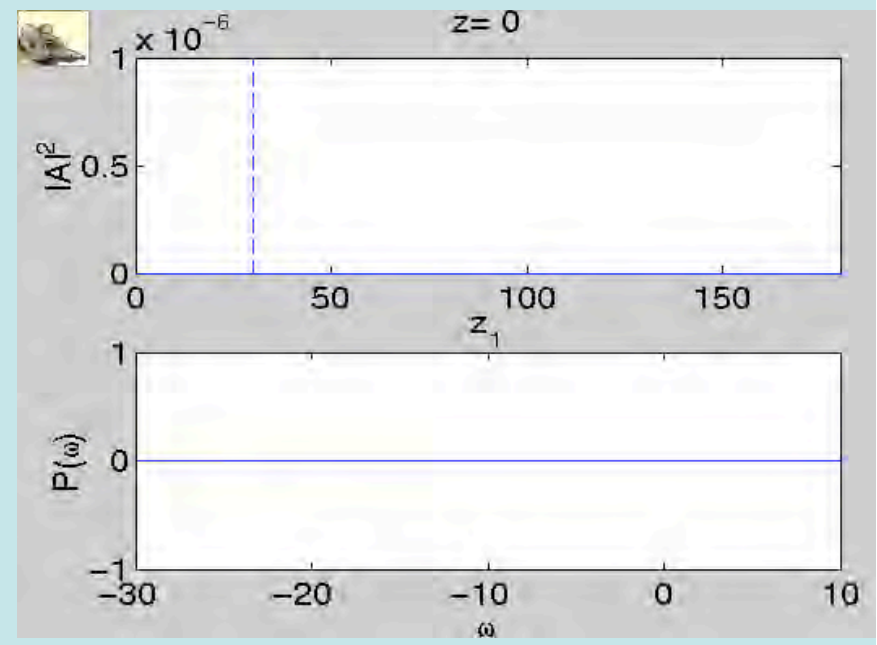
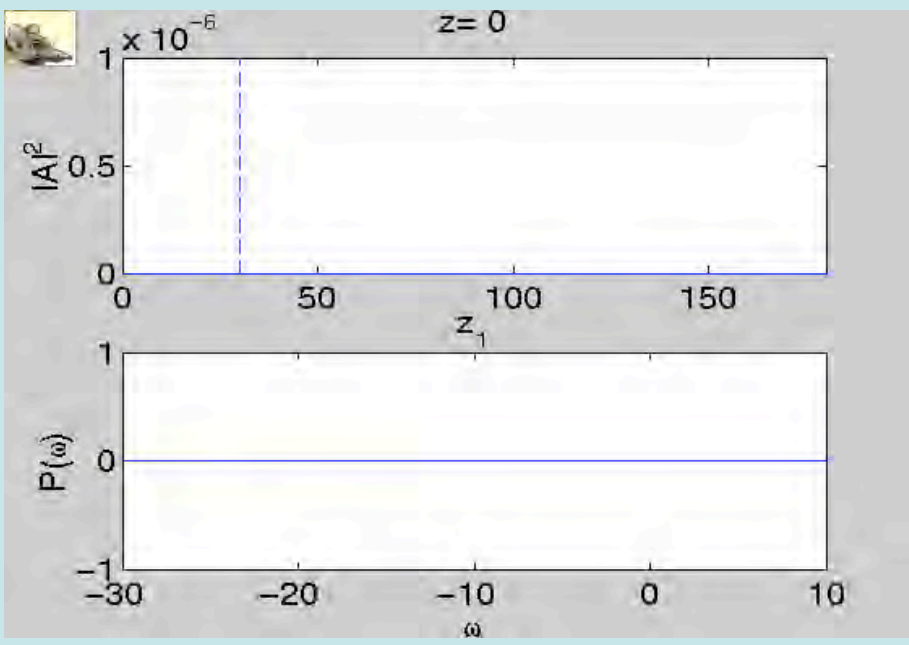
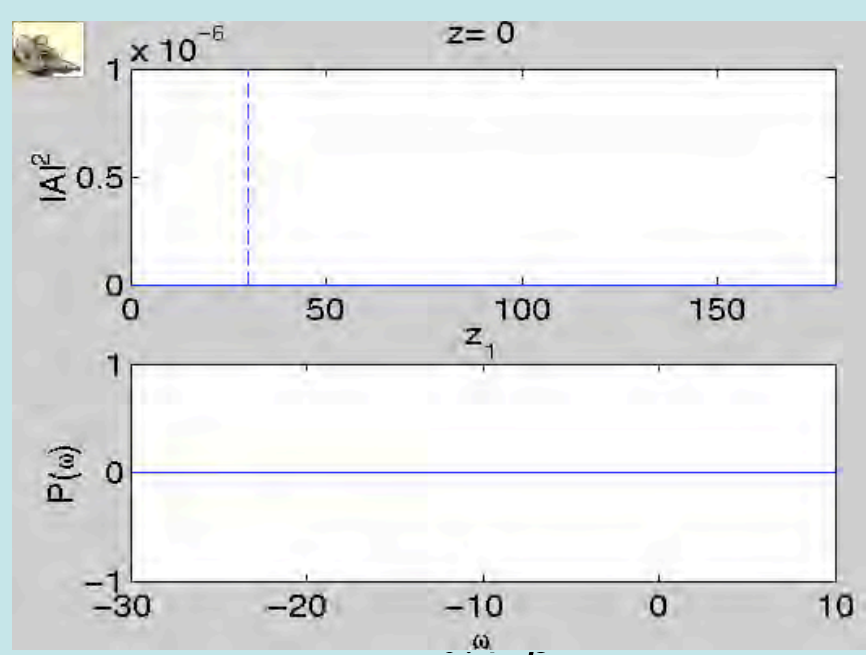
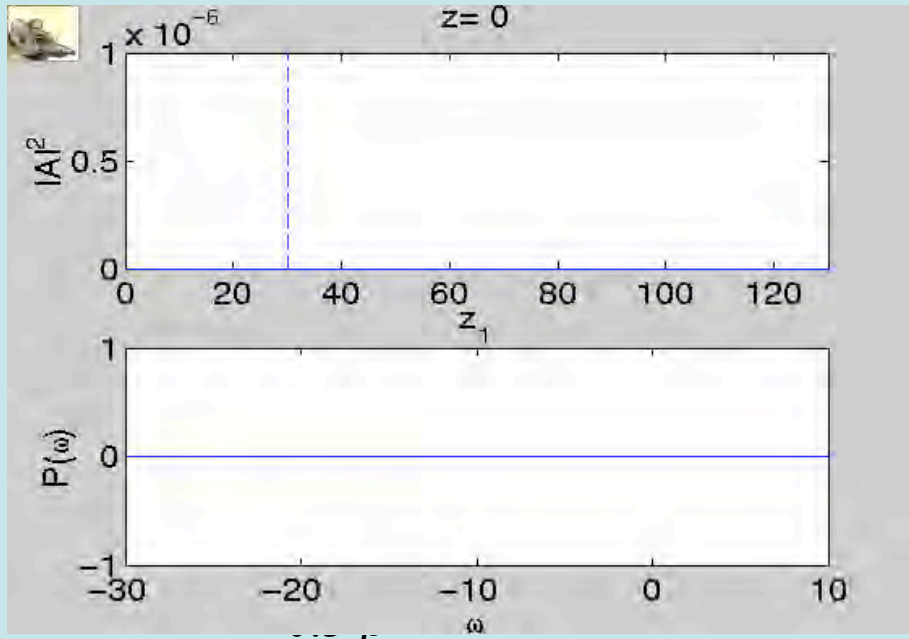


Frequency separation

Full width

Continuous classical limit

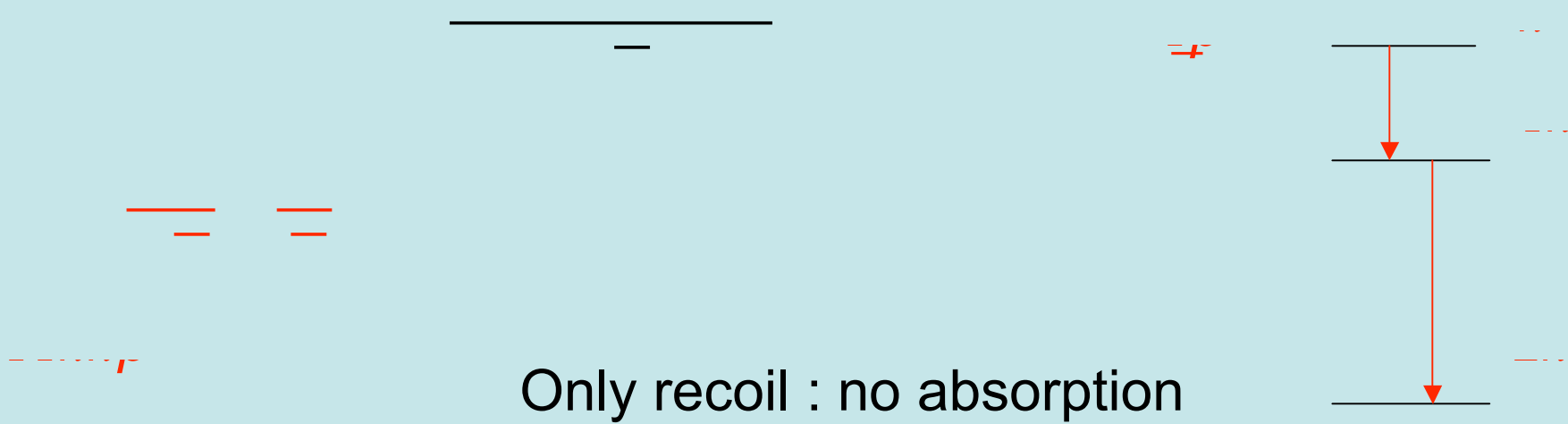
(ref. 8)



Quantum limit : discrete resonance as in a cavity

“Intuitive description”

$$= \quad | \rangle \quad | \rangle \quad =$$



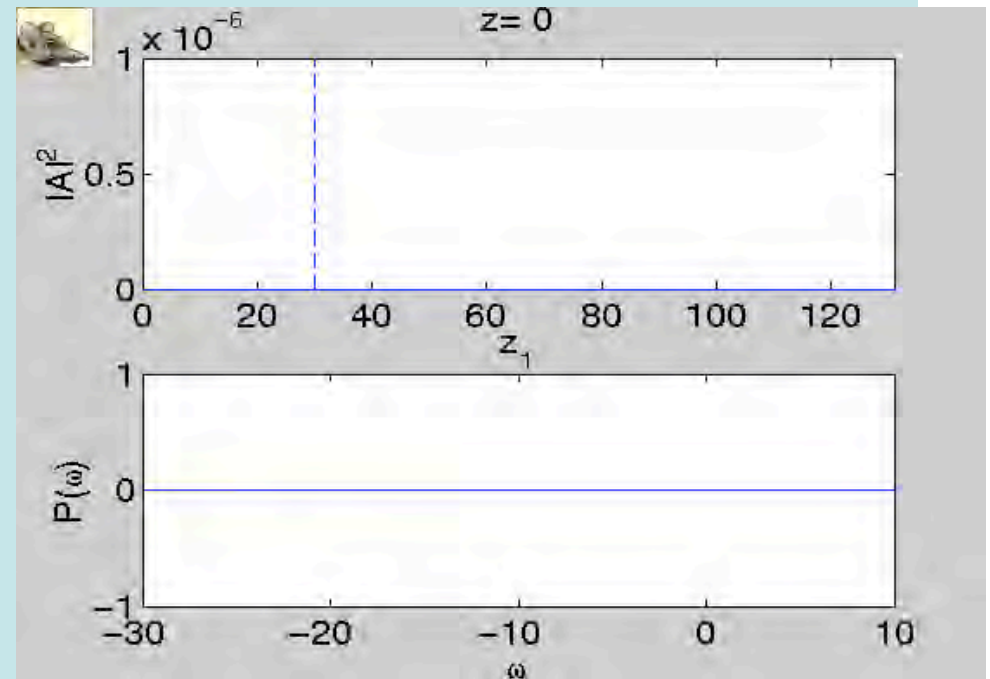
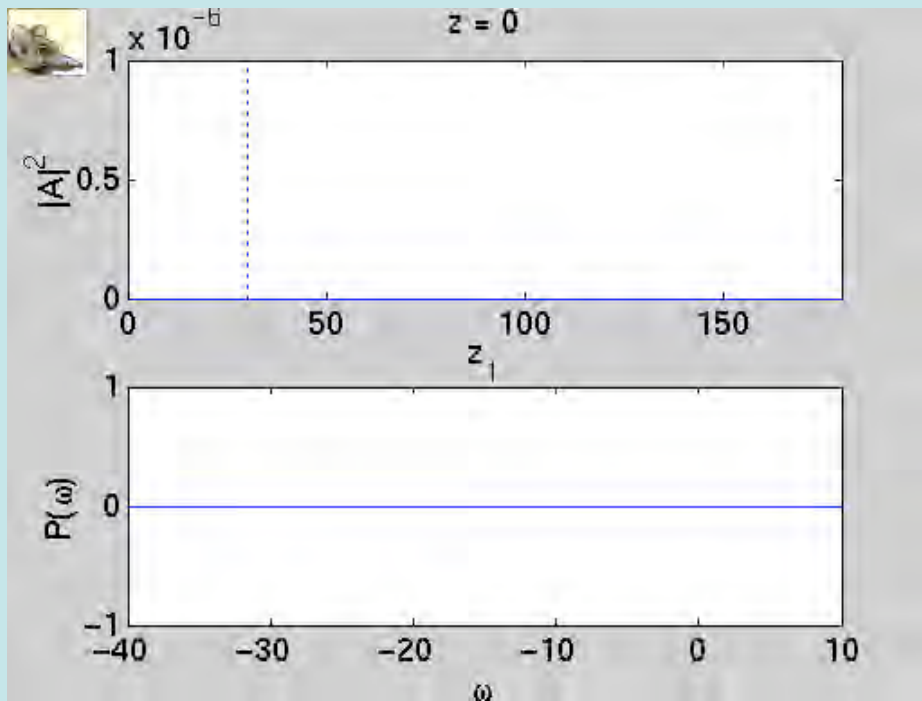
$$= \quad \sqrt{=} \quad \sqrt{=} \quad = \quad =$$

SASE

Quantum
 $r = 0.05$

Classical
 $r = 5$

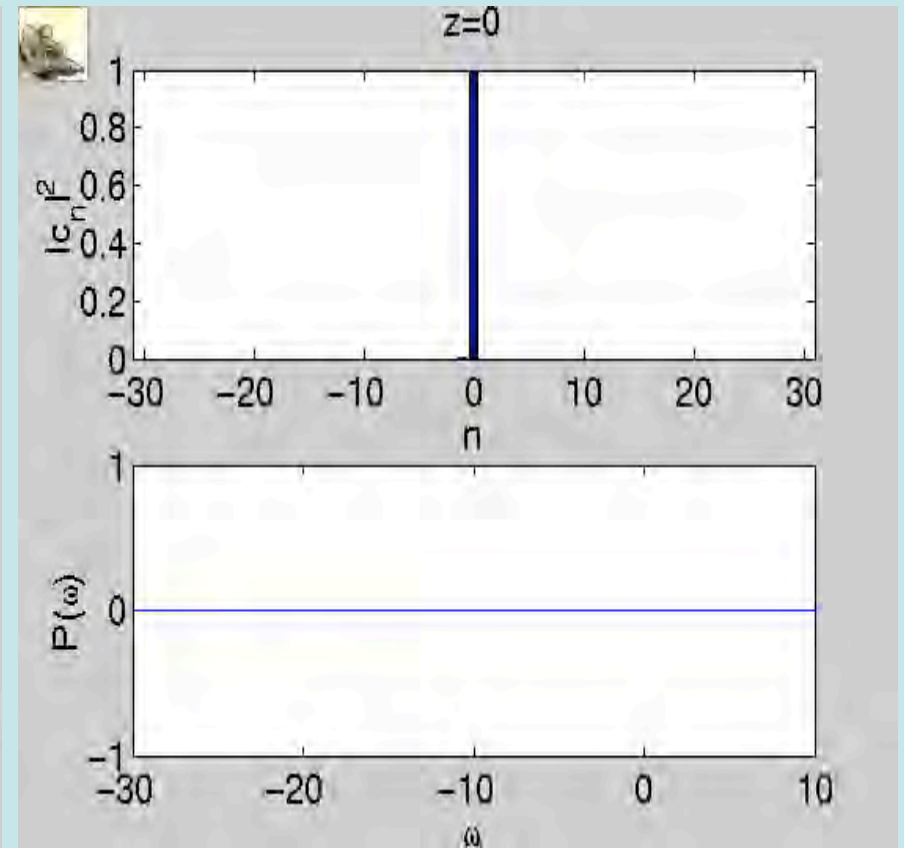
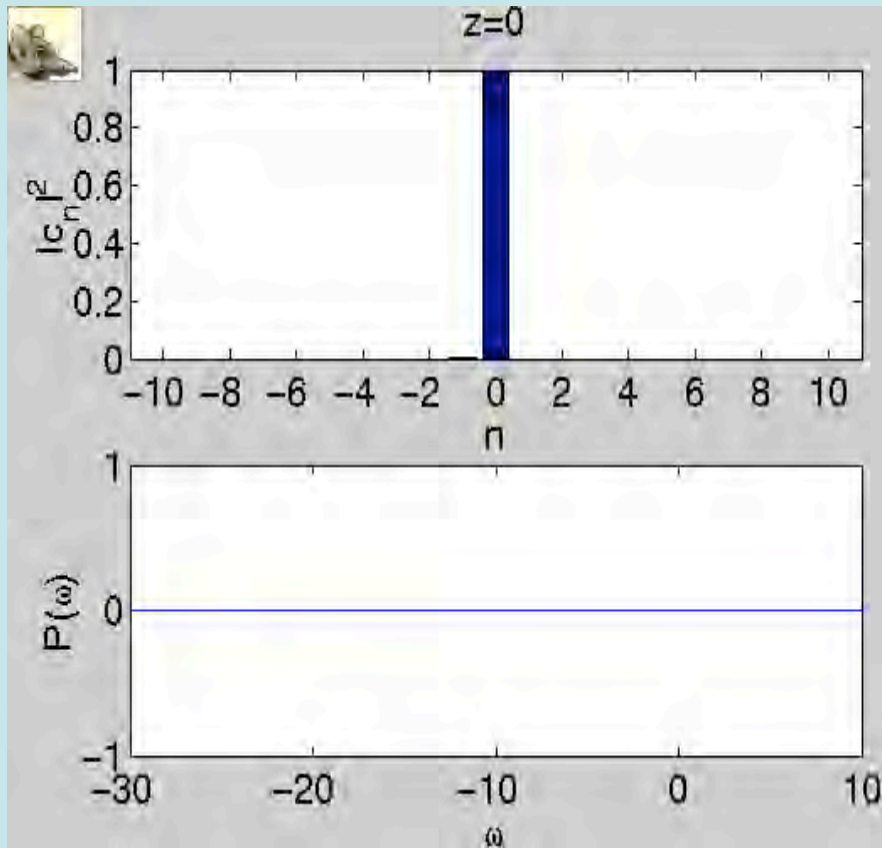
Evolution of radiation time structure in the electron rest frame



Simulation using QFEL model: Momentum distribution (average)

Quantum regime $\frac{\hbar}{\lambda} \ll \gamma$

Classical regime $\frac{\hbar}{\lambda} \gg \gamma$



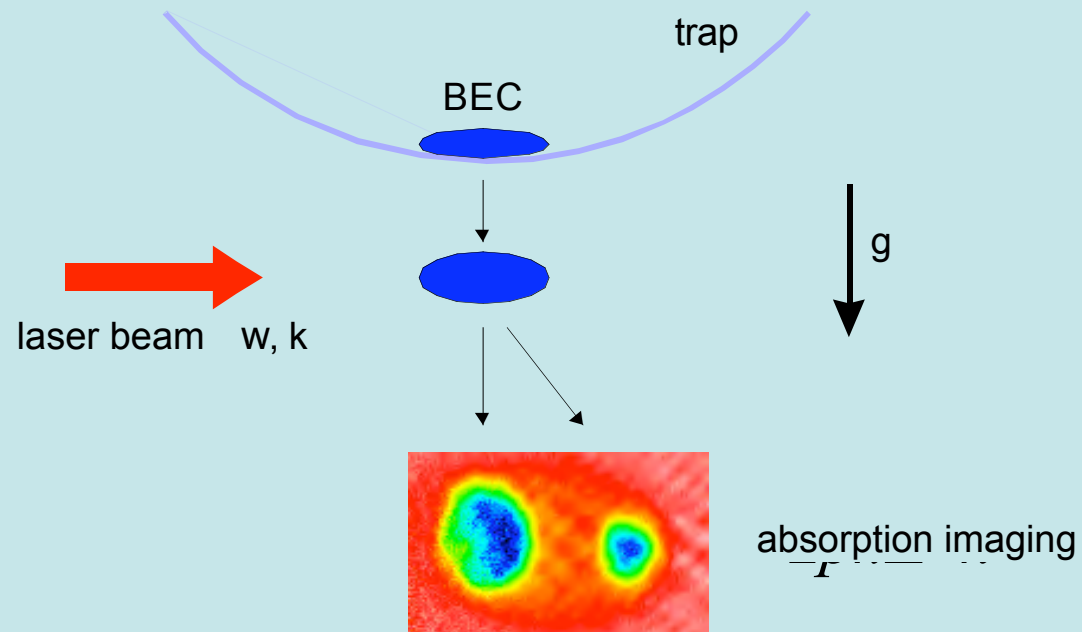
Classical behaviour : both $n < 0$ and $n > 0$ occupied

Quantum behaviour : sequential SR decay, only $n < 0$

Experimental Evidence of Quantum Dynamics

The **LENS** (Firenze) Experiment

- Production of an elongated ^{87}Rb BEC in a magnetic trap
- Laser pulse during first expansion of the condensate
- Absorption imaging of the momentum components of the cloud



Experimental values:

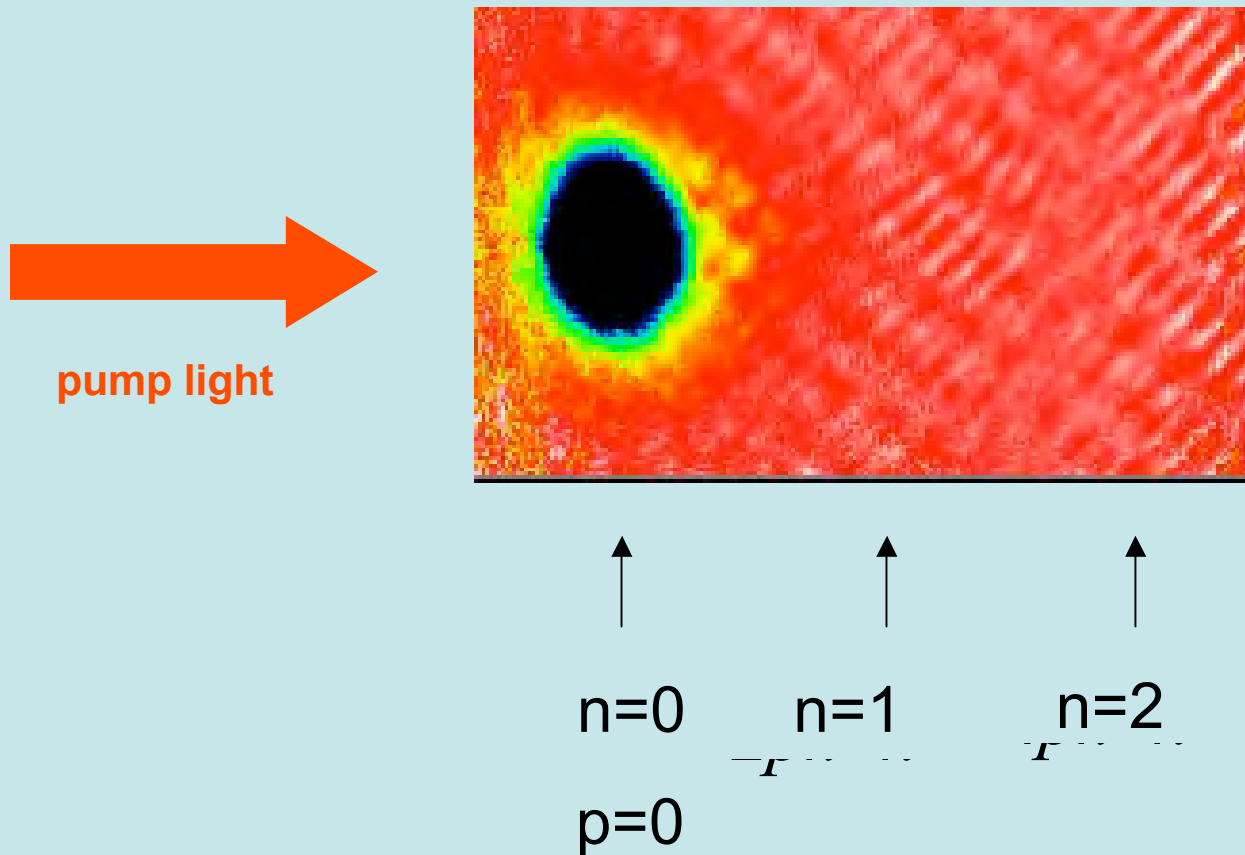
$\Delta = 13 \text{ GHz}$
 $w = 750 \text{ }\mu\text{m}$
 $P = 13 \text{ mW}$

R. B., **F.S. Cataliotti**, M.M. Cola, L. Fallani, C. Fort, N. Piovella, **M. Inguscio**

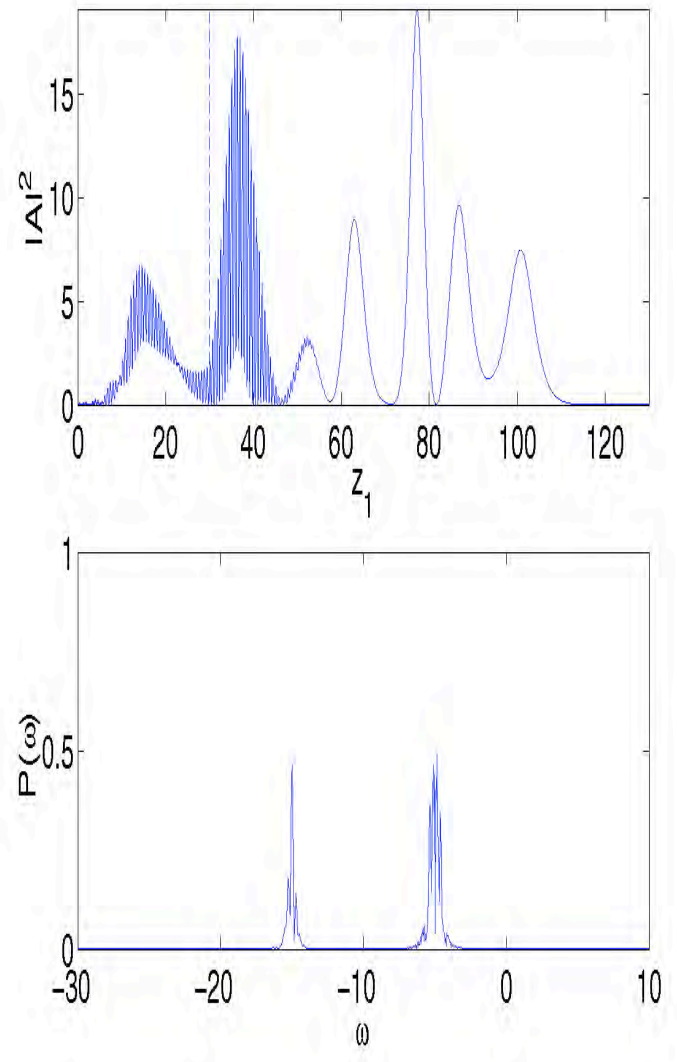
J. Mod. Opt. **51**, 785 (2004), Optics Comm. **233**, 155(2004) and Phys. Rev. A **71**, 033612 (2005)

The Experiment

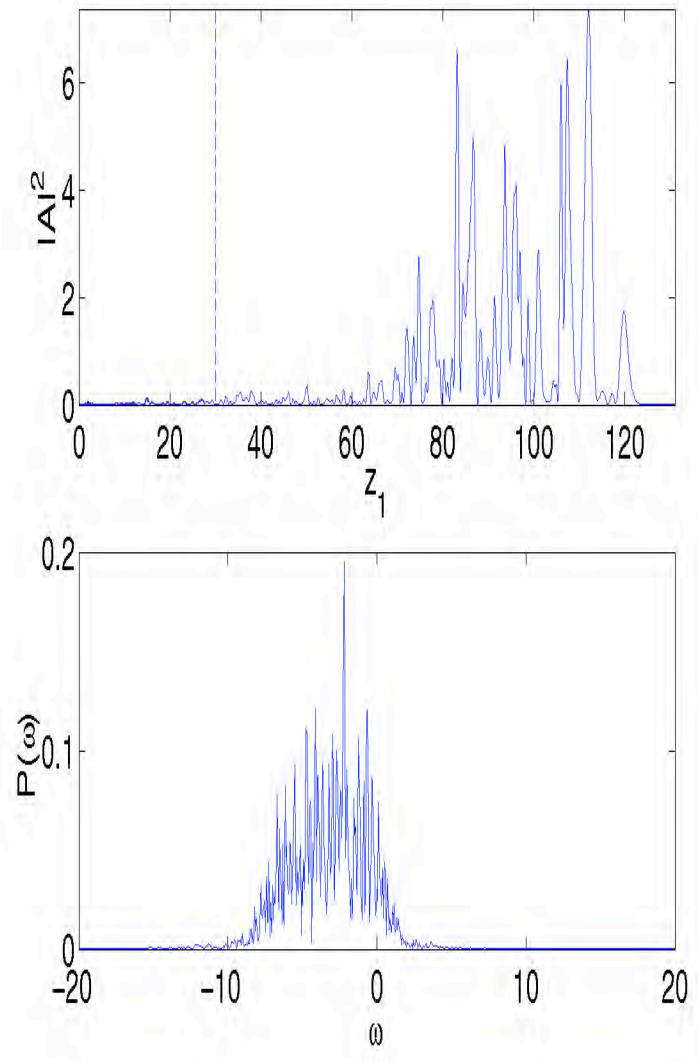
Temporal evolution of the population in the first three atomic momentum states during the application of the light pulse.



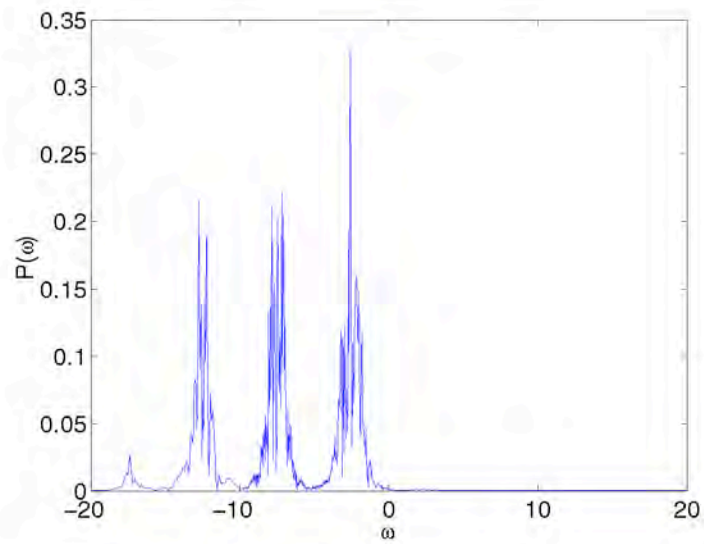
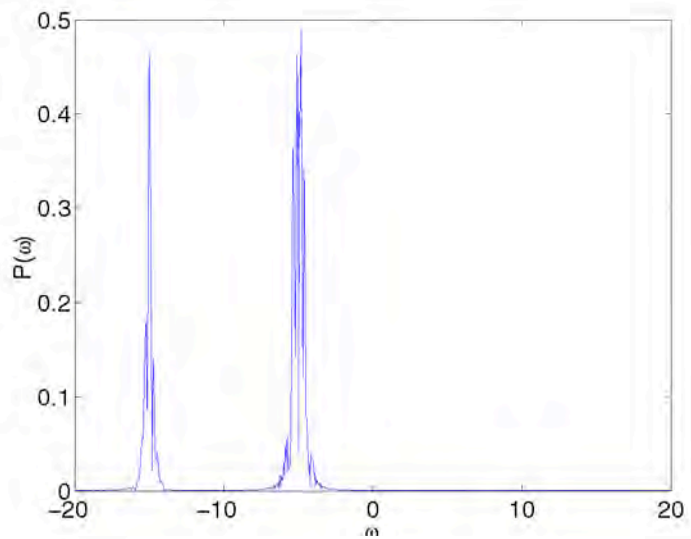
Quantum $\omega = -1$



Classical $\omega = -1$

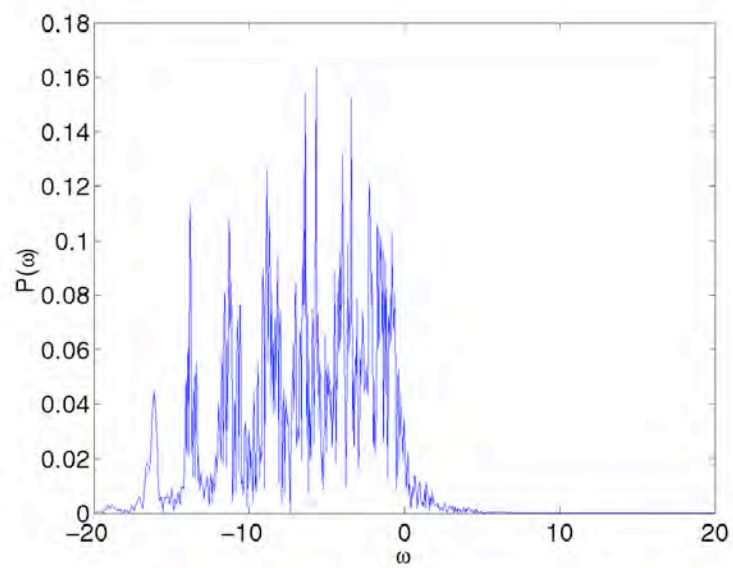
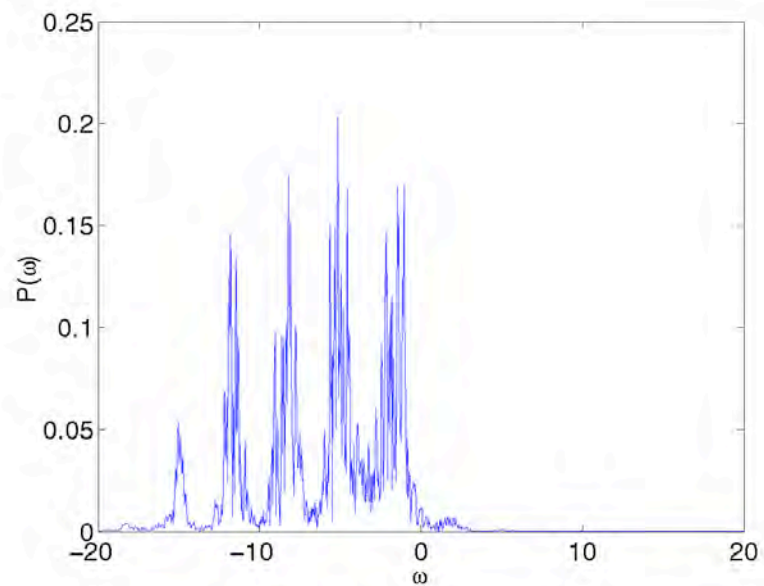


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Conclusions

- Classical description of SASE valid IF $\hbar \omega \ll \hbar \omega_c$
- IF $\hbar \omega \gtrsim \hbar \omega_c$ one has **quantum SASE**: the gain bandwidth **decreases** as $\propto \sqrt{\omega_c - \omega}$ and **line narrowing, temporal coherence**. \Rightarrow
- **Multiple lines Spectrum**:
 - separation $\propto \sqrt{\omega_c - \omega}$, linewidth $\propto \sqrt{\omega_c - \omega}$
- Classical limit: increasing ω_c separation $\propto \sqrt{\omega_c - \omega}$ linewidth $\propto \sqrt{\omega_c - \omega}$ \rightarrow continuous spiky classical spectrum.
- **Experimental evidence of Quantum FEL in a BEC**
- **QFEL Project**

For experimental setup see R.B., NIM A 546 (2005) 634, proc. FEL conf. 2005

Quantum Free Electron Laser

QFEL

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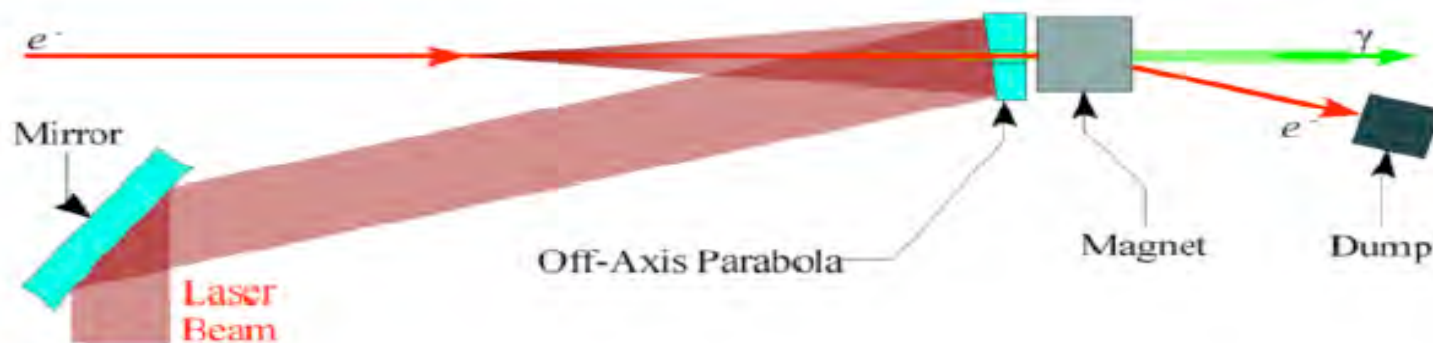
[^] Dipartimento di Energetica, Universita' di Roma "La Sapienza"



Ingredients of Quantum FEL SASE:

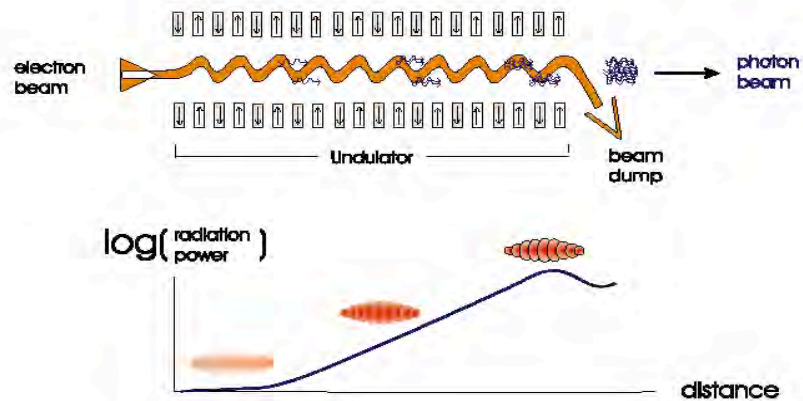
- electron beam 15-100 MeV, 100 A , $\epsilon_n < 2$ mm mrad
- Laser wiggler at 0.8 micron , 10-100 TW (Ti:Sa)

Both under development for SPARC/PLASMON_X



Radiation in QFEL:

- $\sim 10^{10}$ photons at $\lambda \sim 1 \text{ \AA}$ for some ps
- monochromaticity ($\Delta\lambda/\lambda < 10^{-4}$)



Classical **FEL SASE** experiments (**DESY, SLAC**):

- GeV linac (Km) and long undulators (100 m)
- Radiation spectrum broad and chaotic (spikes)
- High cost (10^9 U\$) and large dimensions

Quantum **FEL SASE**:

- **quantum purification** (monochromatic spectrum)
- must use a **laser undulator**
- reduced cost (10^6 U\$) and compact device (m)

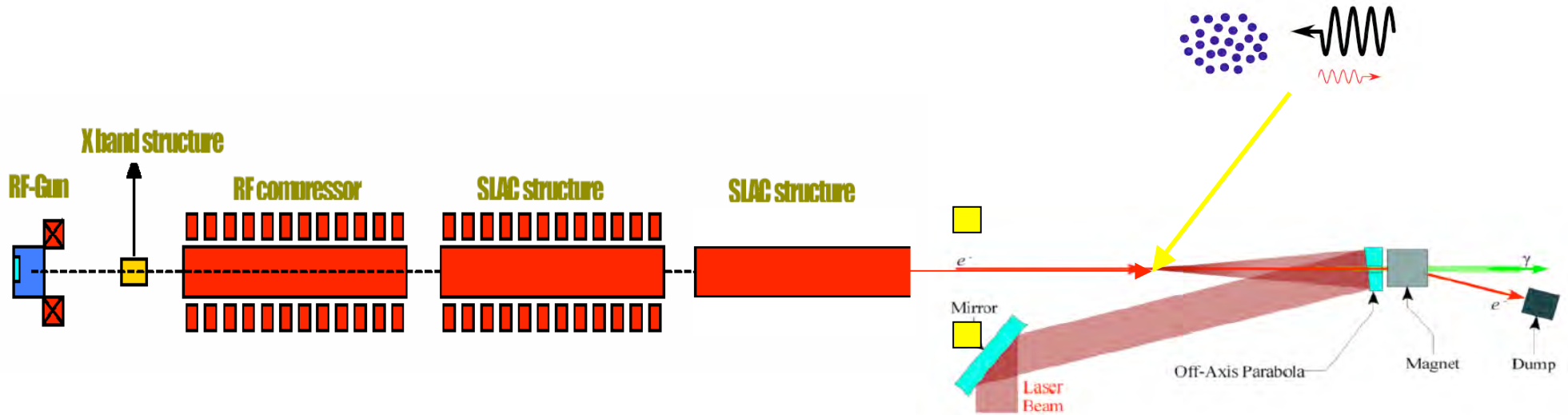




Previous studies are based on a 1D quantum model.

Future work: analytical/numerical study of a **3D Quantum model** in order to demonstrate the feasibility of a **Quantum SASE** experiment at INFN-LNF

Preliminary parameters list for QFEL



Electron beam

| | |
|--------------------------------|-------|
| E [MeV] | 20 |
| I [A] | 40 |
| ϵ_n [μm] | 1 |
| $\delta\gamma/\gamma$ [%] | 0.03 |
| β^* [mm] | 0.5-1 |

Laser beam

| | |
|-----------------------------|--------|
| λ [μm] | 0.8 |
| P [TW] | 1 |
| E [J] | 4 |
| w_0 [μm] | 5-10 |
| Z_r [μm] | 80-300 |

QFEL beam

| | |
|-----------------|-----|
| λ_r [Å] | 1.7 |
| P_r [MW] | 0.3 |





Thank you and see you in my office in Brazil