WAKEFIELD UNDULATOR RADIATION

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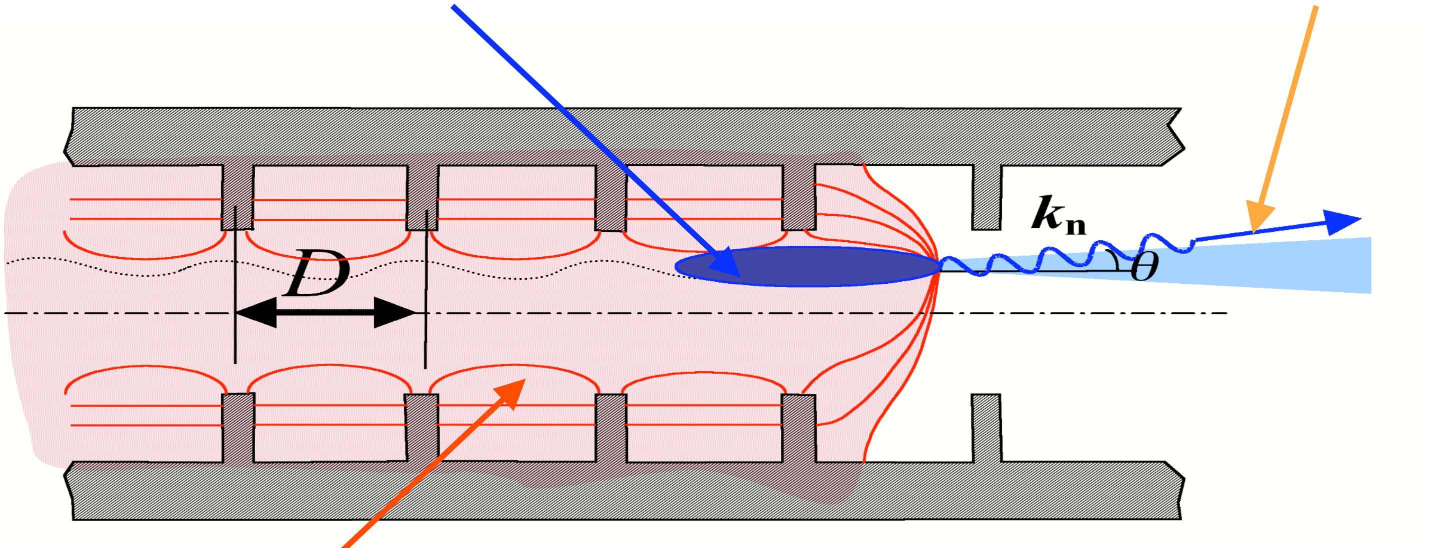
- MECHANISM OF WFU RADIATION
- SPECTRAL -ANGULAR CHARACTERISTICS
- MODEL OF WF UNDULATOR
- WAKEFIELD DISTRIBUTION
- HARD X-RAY GENERATING
- POSSIBILITY of EXPERIMENTAL STUDY

MECHANISM OF WFU RADIATION

A bunch of N electrons

WF Undulator Radiation



$$\lambda_n \approx \frac{D/n}{2\gamma^2}$$

Wakefields (Coherent Parametric-Cherenkov-Radiation)

$$\omega(h) - h v_0 = n \frac{2\pi v_0}{D}$$

MECHANISM OF WFU RADIATION

Wake force in form of Floquet's series

Alternating transverse wake force (**$p\neq0$**) can give rise to undulating the particles with **transverse velocity**

$\gamma >> 1$

Undulator parameter

WF

Coherent

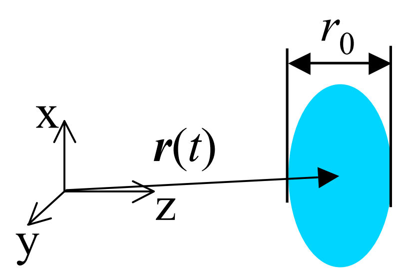
WFU RADIATION

No coherent

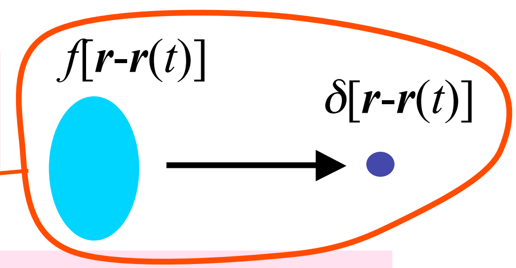
Coherent

Method

Basis *V.L.Ginzburg, V.Ya.Eidman (1959)*
A.I.Akhiezer, Ya.B.Fainberg, G.Ya.Liubarski (1955)



- the charge of particle is distributed, $\rho = f[\mathbf{r}-\mathbf{r}(t)]$
- the equation of motion $$\frac{d}{dt}(m\gamma\mathbf{v}) = e\int\left\{\mathbf{E}(\mathbf{r},t)+\frac{1}{c}\mathbf{v}\times\mathbf{H}(\mathbf{r},t)\right\} f[\mathbf{r}-\mathbf{r}(t)]d^3\mathbf{r}$$
- fields $\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{A}}{\partial t} - \nabla\Phi,\quad \mathbf{H} = rot\mathbf{A}$ are found from $$\Delta\mathbf{A} - \frac{1}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi}{c}\rho\mathbf{v} + \frac{1}{c}\nabla($$
- by Hamilton method (*V.Haitler*) $$\mathbf{A}(\mathbf{r},t) = \mathrm{Re}\sum_\lambda q_\lambda(t)\mathbf{A}_\lambda(\mathbf{r})$$

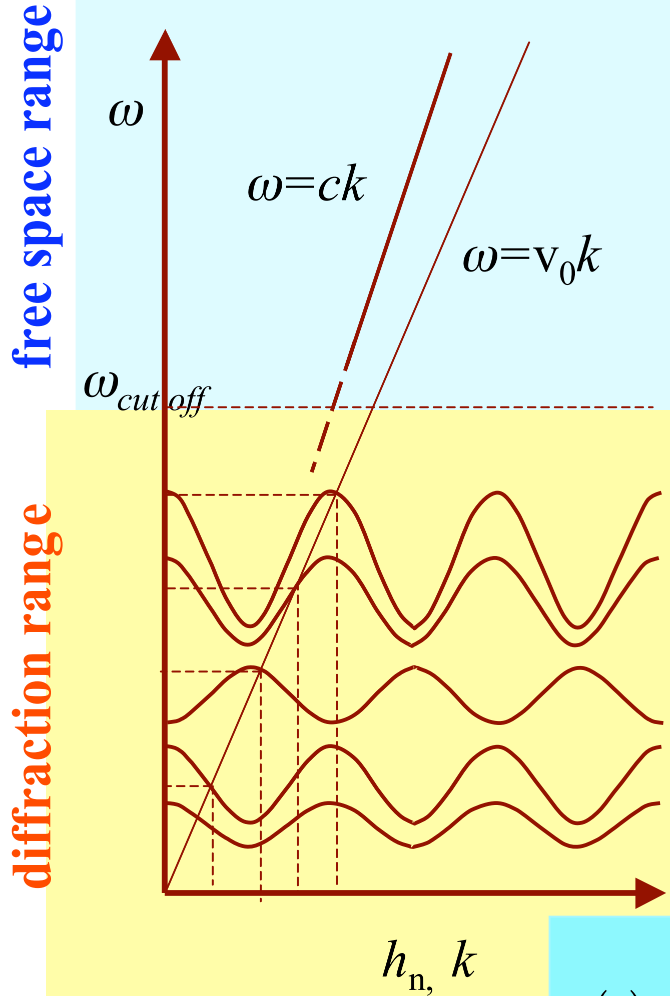


Radiation reaction force

$$\mathbf{F}(\mathbf{v}(t),\mathbf{r}(t),t) = -\frac{e^2}{4c^2V_{tot}}\sum_{\lambda}^{\omega_\lambda<c/r_0}\left\{\left[\mathbf{A}_\lambda(\mathbf{r}(t)) - \frac{\mathbf{v}(t)\times rot\mathbf{A}_\lambda(\mathbf{r}(t))}{i\omega_\lambda}\right]e^{i\omega_\lambda t}\int_0^t \mathbf{v}(t')\mathbf{A}_\lambda^*(\mathbf{r}(t'))e^{-i\omega_\lambda t'}dt' + \right.$$

$$\left. + \left[\mathbf{A}_\lambda(\mathbf{r}(t)) + \frac{\mathbf{v}(t)\times rot\mathbf{A}_\lambda(\mathbf{r}(t))}{i\omega_\lambda}\right]e^{-i\omega_\lambda t}\int_0^t \mathbf{v}(t')\mathbf{A}_\lambda^*(\mathbf{r}(t'))e^{i\omega_\lambda t'}dt'\right\} + c.\tilde{n}.$$

Self-wake force



Eigenfunctions

$\omega_\lambda > \omega_{cut\ off}$

$$\mathbf{A}_{\lambda,l}(\mathbf{r}) = c\sqrt{4\pi}\,\boldsymbol{a}_{\lambda,l}e^{i\mathbf{k}_\lambda \mathbf{r}}$$

$\omega_\lambda << \omega_{cut\ off}$

$$\mathbf{A}_\lambda(\mathbf{r}) = \sum_{n=-\infty}^{\infty} \boldsymbol{g}_\lambda^{(n)}(\mathbf{r}_\perp)e^{ih_n z}$$

Zeroth order approximation

$$\mathbf{v} = \mathbf{v}_0 = \mathrm{v}_0\mathbf{e}_z, \qquad \mathbf{r}(t) = \mathbf{r}_{0,\perp} + \mathbf{v}_0 t$$

$$\mathbf{F}(t) = \begin{cases} 0 \\ -e^2 \sum_{p=-\infty}^{\infty} \boldsymbol{w}^{(p)} e^{ip\Omega t} + \ldots \end{cases}$$

the p*th* harmonic of wakefunction

$$\boldsymbol{w}^{(p)} \equiv \frac{D\mathrm{v}_0}{4c^2 V_{cell}} \sum_{n=0}^{\infty} \sum_{\lambda_j'} \frac{g_{z,\lambda_j}^{(n)\,*}}{\left|\mathrm{v}_0 - \frac{d\omega_\lambda}{dh}\right|_{\lambda=\lambda_j}} \left[\boldsymbol{g}_{z,\lambda_j'}^{(n+p)} - i\frac{\mathrm{v}_0}{\omega_\lambda}\nabla_\perp g_{z,\lambda_j}^{(n+p)} - \frac{\Omega p}{\omega_\lambda}\boldsymbol{g}_{\perp,\lambda_j}^{(n+\ldots}\right.$$

The first order approximation

from the equation of motion

$$\frac{d\mathbf{v}_\perp}{dt} \approx \frac{\mathbf{F}_\perp(t)}{m\gamma} = -\frac{e^2}{m\gamma}\sum_{p\neq 0} \boldsymbol{w}_\perp^{(p)} e^{ip\Omega t} + c.c.$$

the corrected law of motion

the small parameter

Radiation power

$$P \equiv -\lim_{t\to\infty}\frac{1}{t}\int_0^t \mathbf{v}(t')\mathbf{F}(\mathbf{v}(t'),\mathbf{r}(t'),t')dt'$$

$$= \lim_{t\to\infty}\frac{1}{t}\int_0^t dt \frac{e^2}{4c^2V_{tot}}\sum_{\lambda}^{\omega_\lambda < c/r_0} \mathbf{v}(t)\mathbf{A}_\lambda(\mathbf{r}(t))\left\{e^{i\omega_\lambda t}\int_0^t \mathbf{v}(t')\mathbf{A}_\lambda^*(\mathbf{r}(t'))e^{-i\omega_\lambda t'}dt' + e^{-i\omega_\lambda t}\int_0^t \mathbf{v}(t')\mathbf{A}_\lambda^*(\mathbf{r}(t'))e^{i\omega_\lambda t'}dt'\right\} + c.\tilde{n}.$$

Radiation power in the first order approximation

free space range

$\omega_\lambda > \omega_{cut\ off}$

$$R(\omega,\theta,p) \equiv \left(1 - \frac{\omega}{\Omega p}\beta_0 \cos\theta\right)^2 - \beta_0^2\left(\frac{\omega}{\Omega p}\right)^2$$

diffraction range

$\omega_\lambda < \omega_{cut\ off}$

Interference of Cherenkov and Undulator-type radiation

Resonant condition

$$\omega_\lambda - h\mathrm{v}_0 = n\Omega$$

WFUR SPECTRAL -ANGULAR CHARACTERISTICS

Single- Particle Radiation

The *spectral-angular power density* of hard UR emitted spontaneously by a single particle of the bunch

Here $d\Omega = sin\theta \, d\theta d\varphi$

WFUR SPECTRAL -ANGULAR CHARACTERISTICS

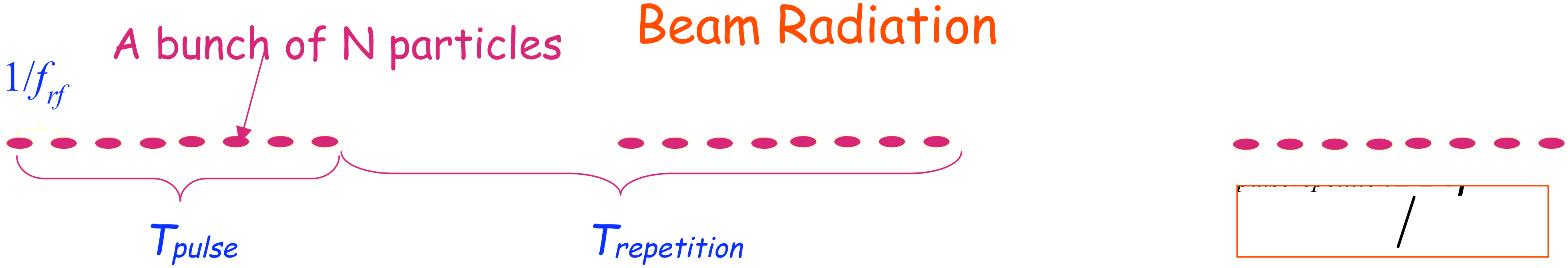
Single-Particle Radiation

Resonant frequencies of the WFUR _th harmonics

The *spectral-angular photon flux density* of the _th harmonics in the forward direction

WFUR SPECTRAL -ANGULAR CHARACTERISTICS

Beam Radiation



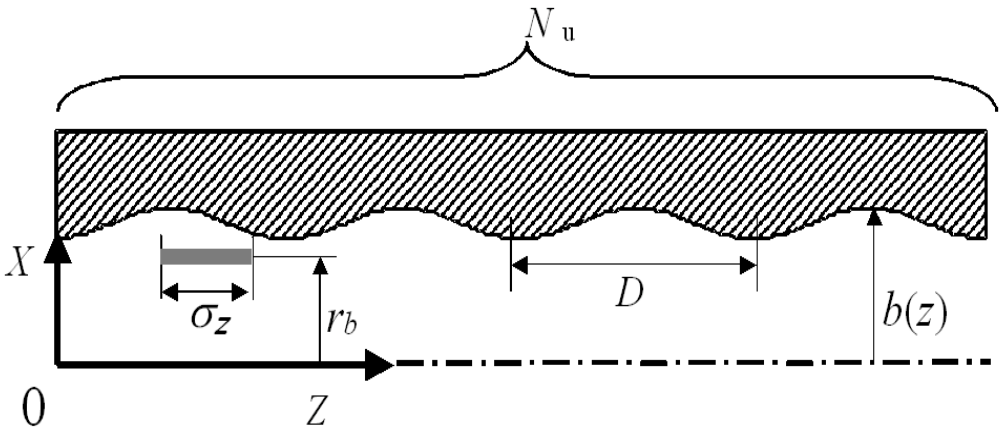
Pulse duty factor

The *spectral-angular distribution of photon flux density* of the th harmonic from a beam in the forward direction

The _th harmonic of wake function

FORMULAS FOR QUANTITATIVE ESTIMATIONS

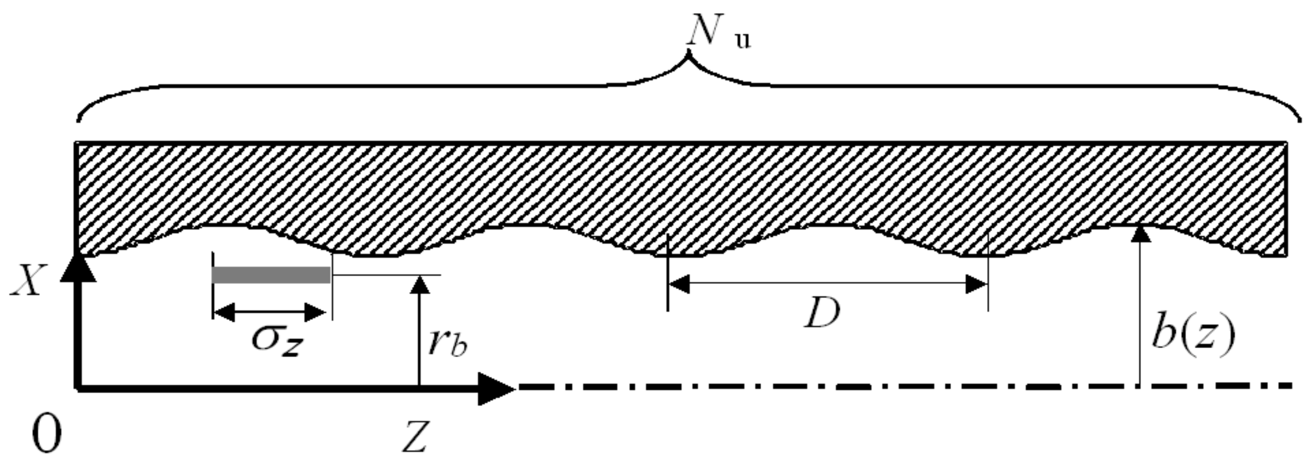
WF characteristics of weakly corrugated waveguide



Radius of waveguide

WF undulator parameter

AXIALLY SYMMETRICAL MODEL OF WAKEFIELD UNDULATOR



The WFU and electron bunch parameters for producing 792 keV photons

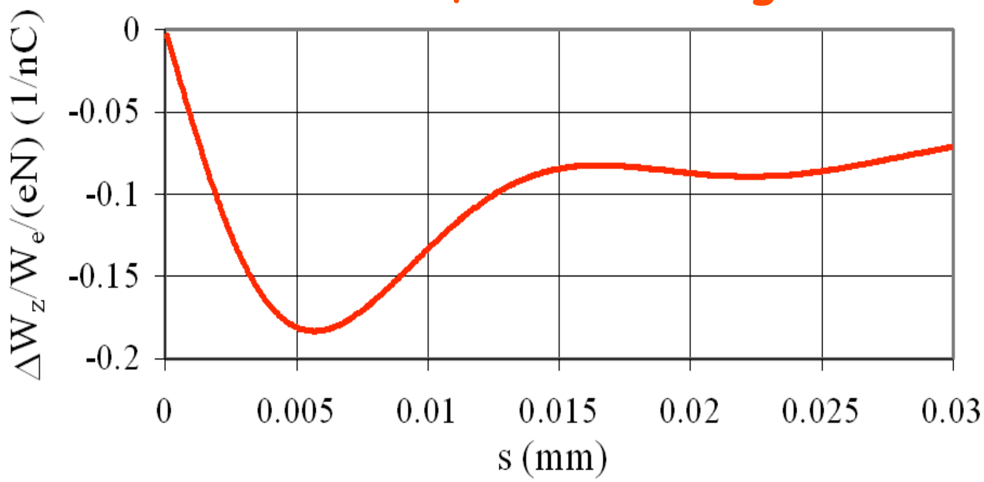
Period	D	300 μm	Energy of electrons	W_e	5 GeV
Average radius	b_0	300 μm	Bunch length	σ_z	30,300 μm
Relative amplitude of corrugations	Δ	30 μm	Bunch distance from axis	r_b	260 μm
Number of periods	N_u	1000	Average beam current	I	100 mA

The beam parameters are typical for conceptual projects of synchrotron X-ray sources based on Energy ERL.

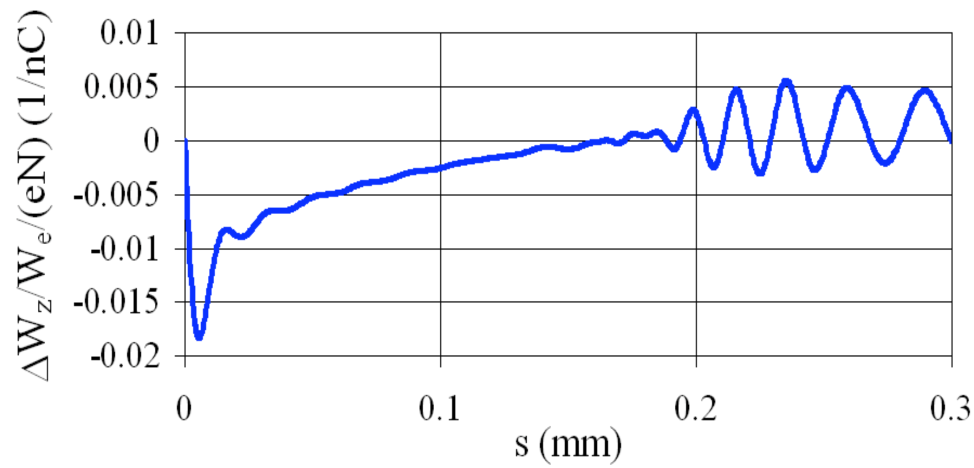
WAKEFIELD DISTRIBUTION INDUCED BY UNIFORM CHARGED BUNCH

The relative energy losses per bunch charge along the bunch

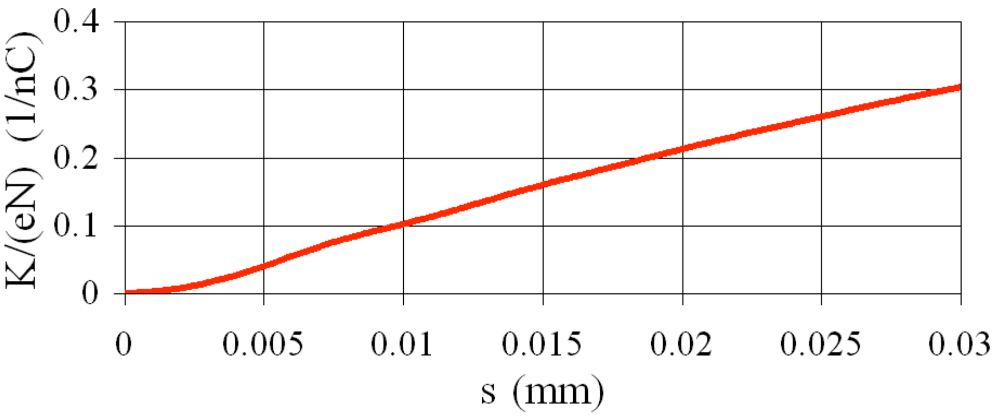
30 μm bunch length

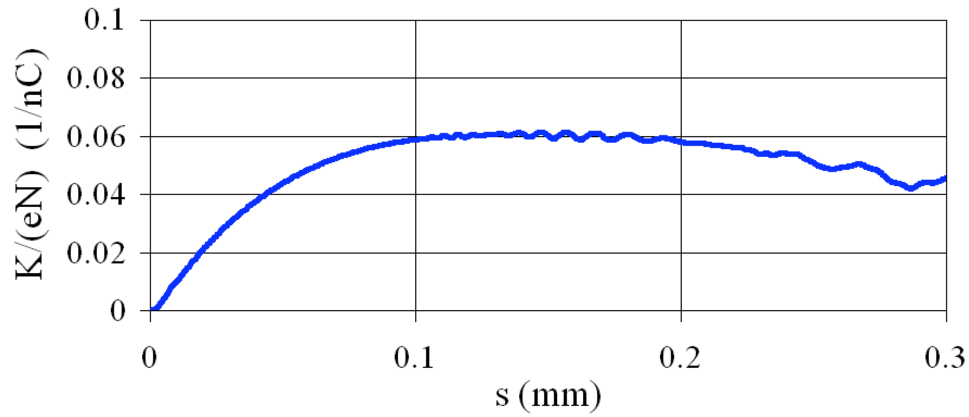


300 μm bunch length

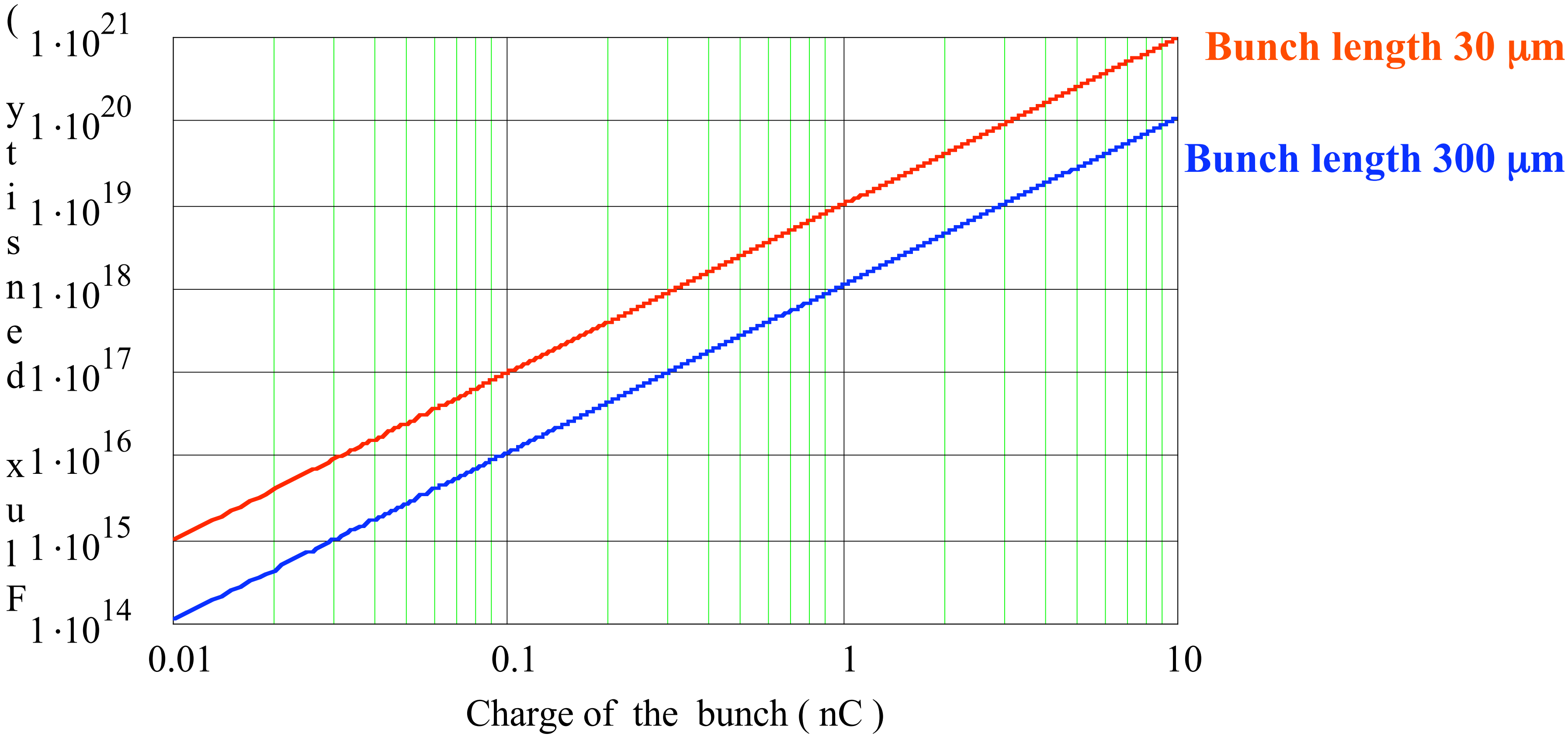


The distributions of the undulator parameter *K* per bunch charge



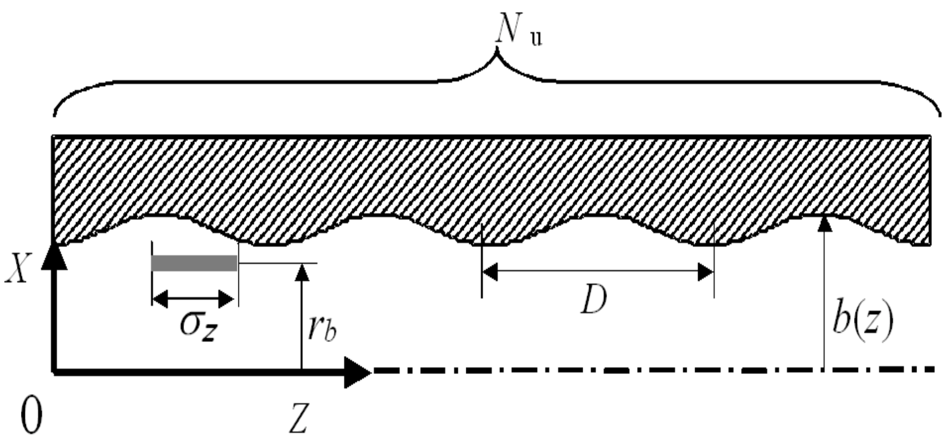


HARD X-RAY GENERATING

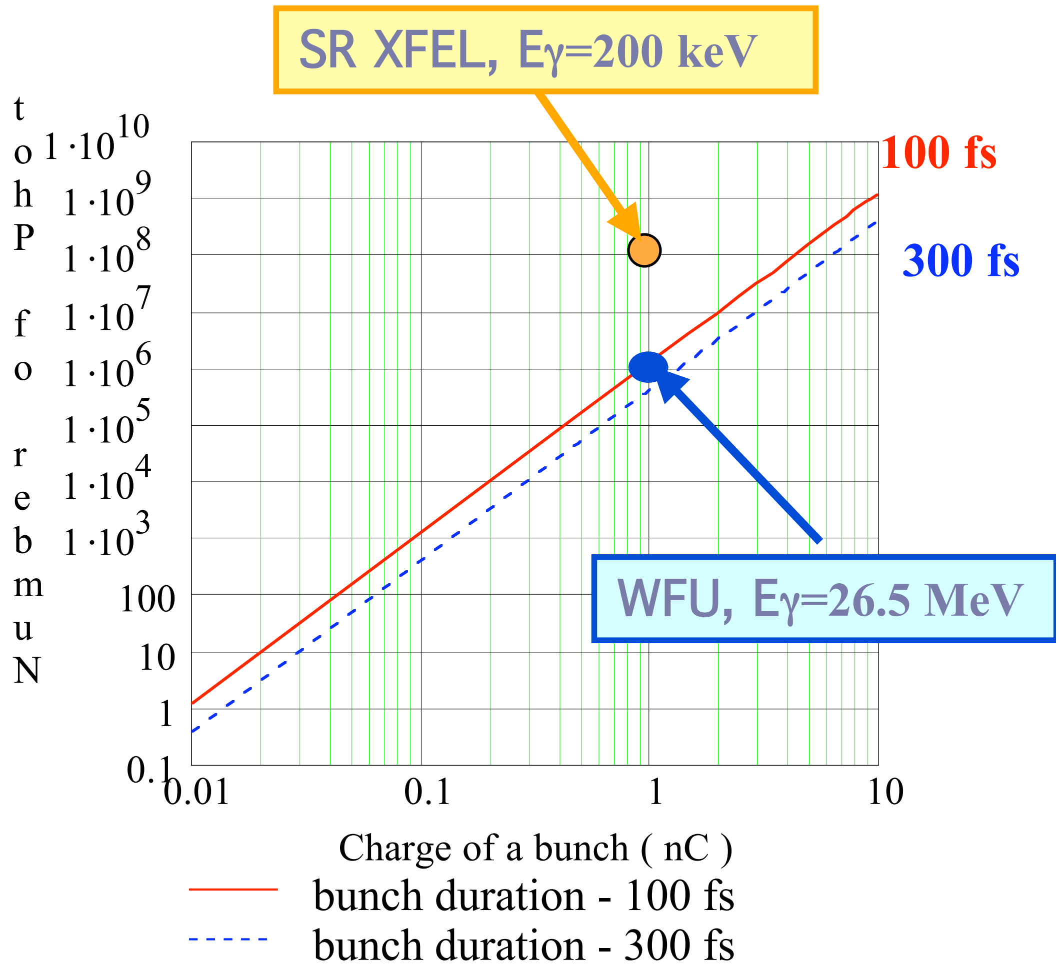


The 0.8 MeV photon flux density v.s. bunch charge.

WF UNDULATOR AS ULTRA-FAST X-RAY SOURSE



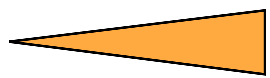
Period	D	**397 μm**
Average radius	b_0	**115 μm**
Relative corrugation	ε_1	**0.065**
Bunch distance from axis	r_b	**54 μm**
Bunch diameter	$2\rho_b$	**≤40μm**



FORMULAS FOR QUANTITATIVE ESTIMATIONS

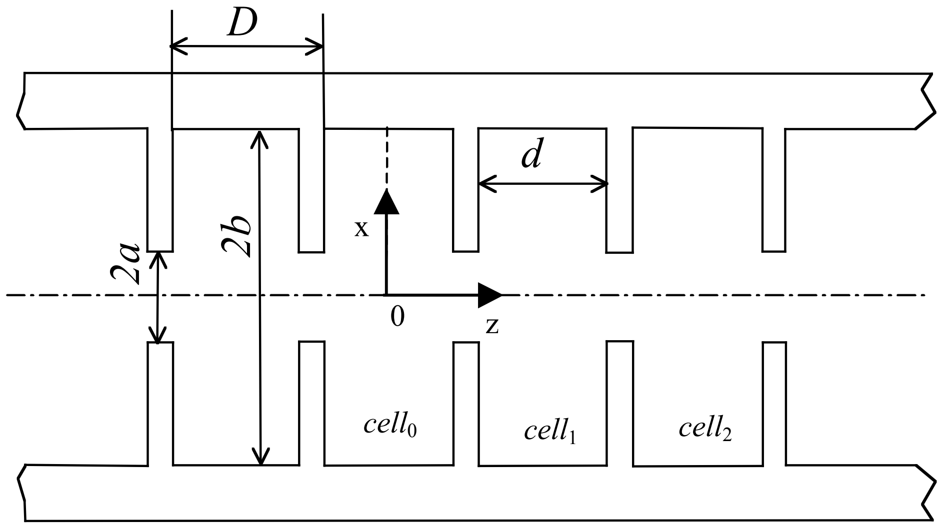
WFU radiation

Spectral flux (photons/s) into a small $\Delta\omega$ of the _th harmonics

Full flux in the central cone  into FWHM bandwidth

Bunch averaging $\langle \ \rangle$

FORMULAS FOR QUANTITATIVE ESTIMATIONS

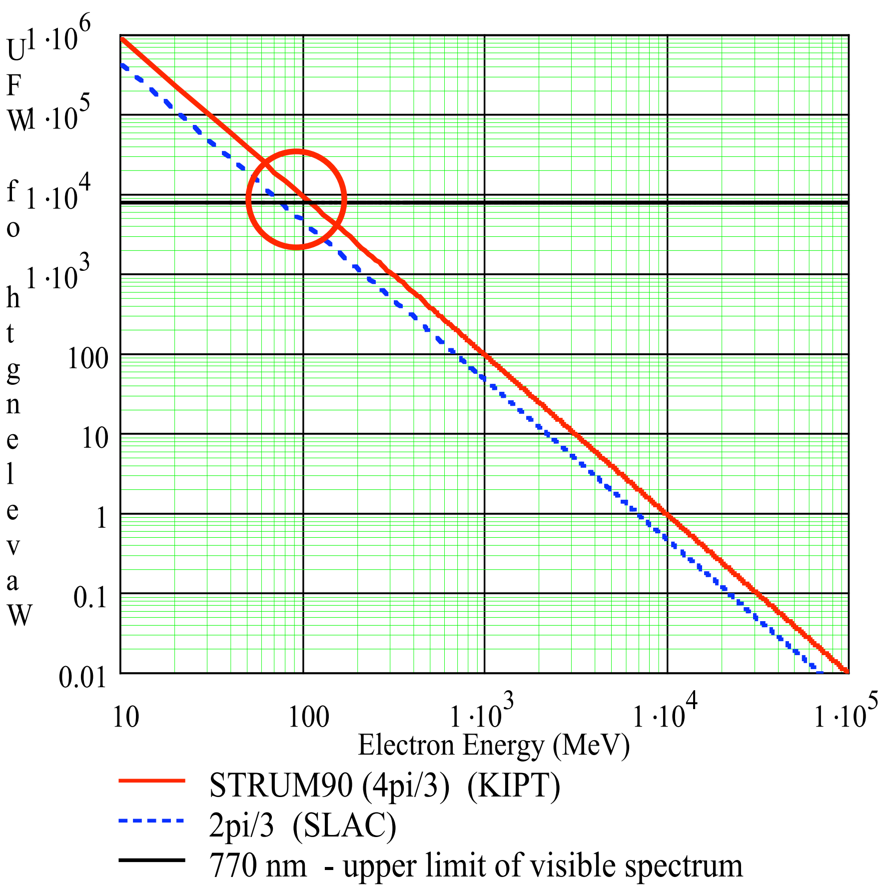


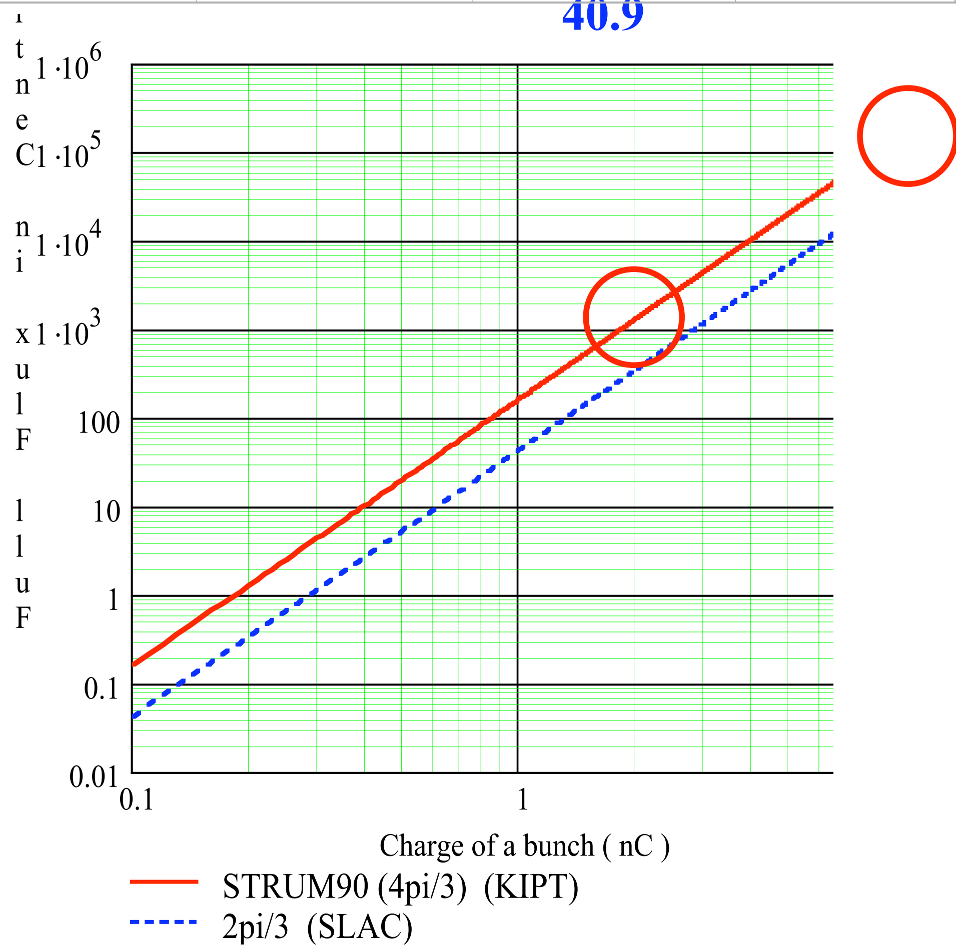
Loss factor

WF undulator parameter

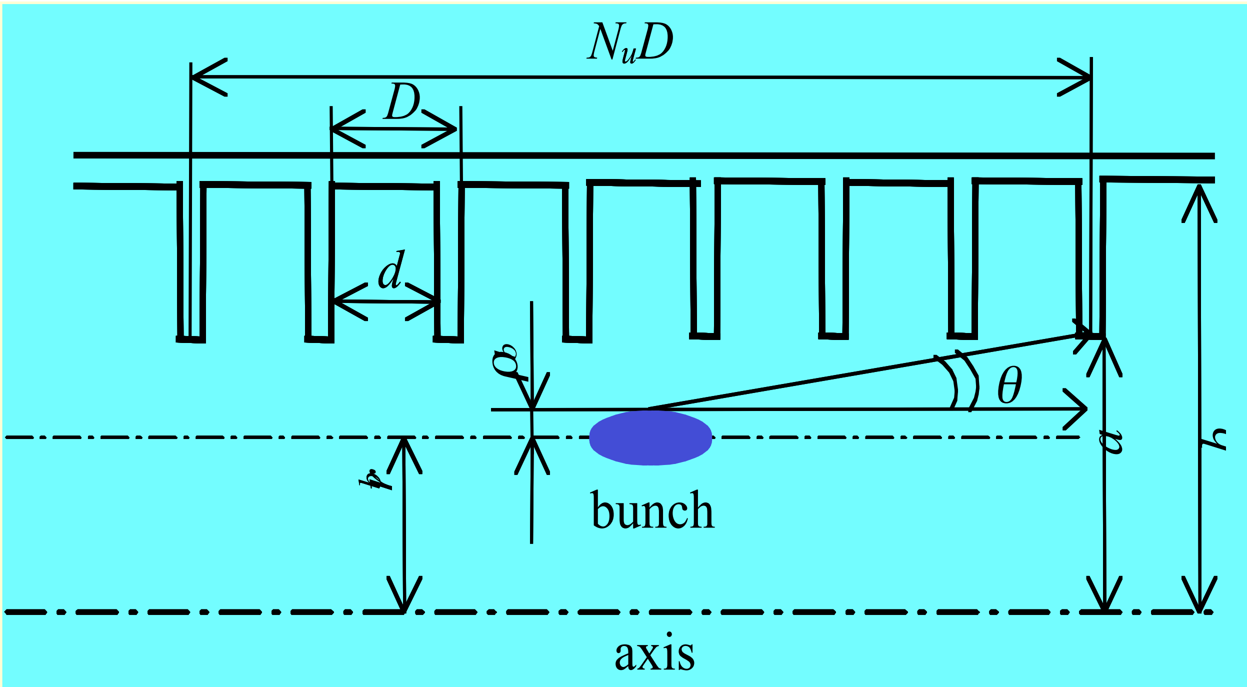
RADIATION BY A HIGH_ENERGY ELECTRON BUNCH
Under-estimation of photon flux

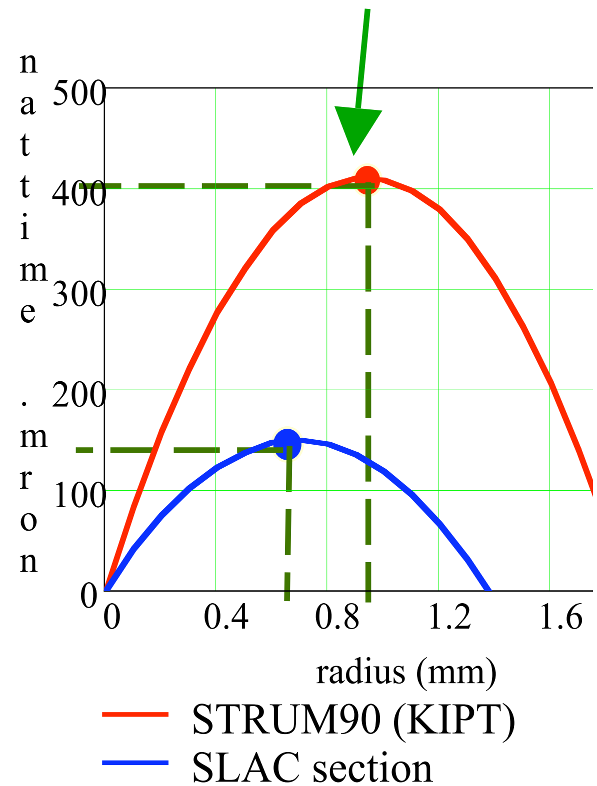
DLW	mode	*D* (mm)	*d*(mm)	*a*(mm)	*b*(mm)	*L*(m)
STRUM-90	4π/3	71.45	67.45	15	41.2	1.7
SLAC-type	2π/3	35.99	30.15	13.1-9.6	41.7-40.9	3.05



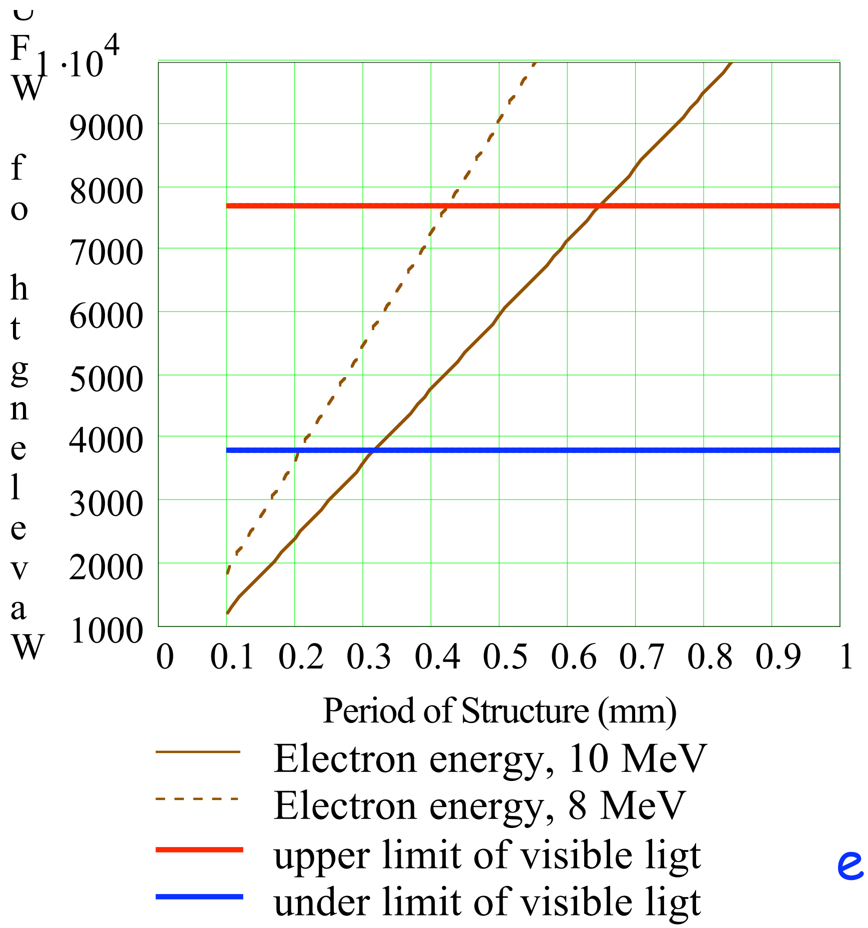


Upper-estimation of transverse emittance





RADIATION BY A LOW ENERGY ELECTRON BUNCH
Under-estimation of photon flux



TU/e Photo-injector

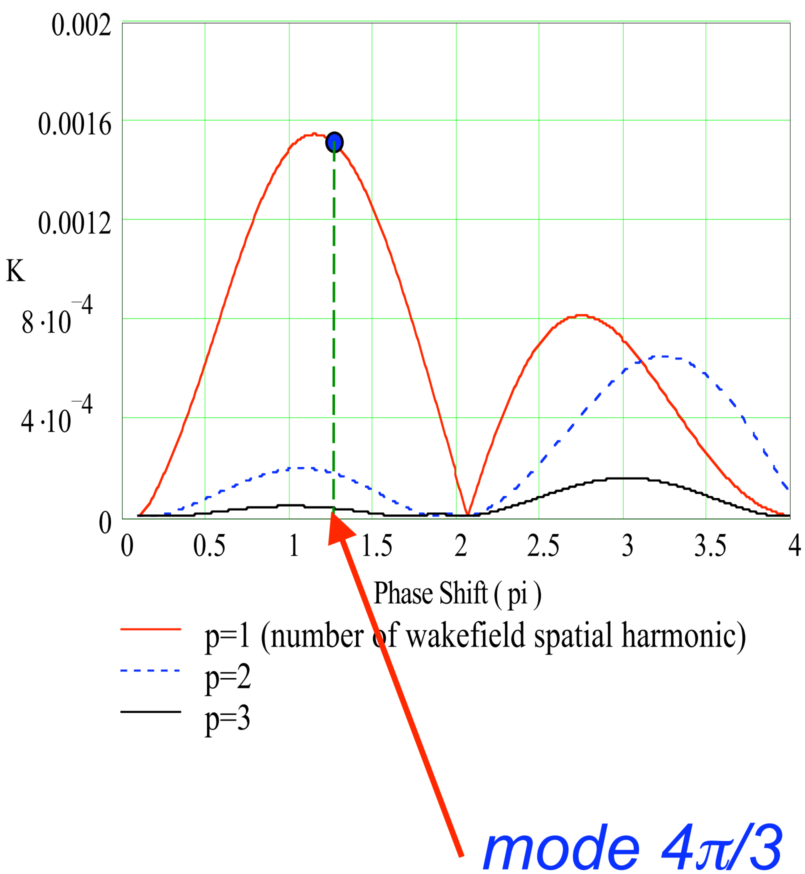
Energy:	10 MeV
Peak Current:	1 kA
Emittance:	1 mm mrad
Length:	100 fs
Charge:	100 pC

electron energies 8 - 10 MeV → Periods 0.25-0.5 mm

RADIATION BY A LOW ENERGY ELECTRON BUNCH

Under-estimation of photon flux

Optimization



The optimal dimensions of a S-band waveguide

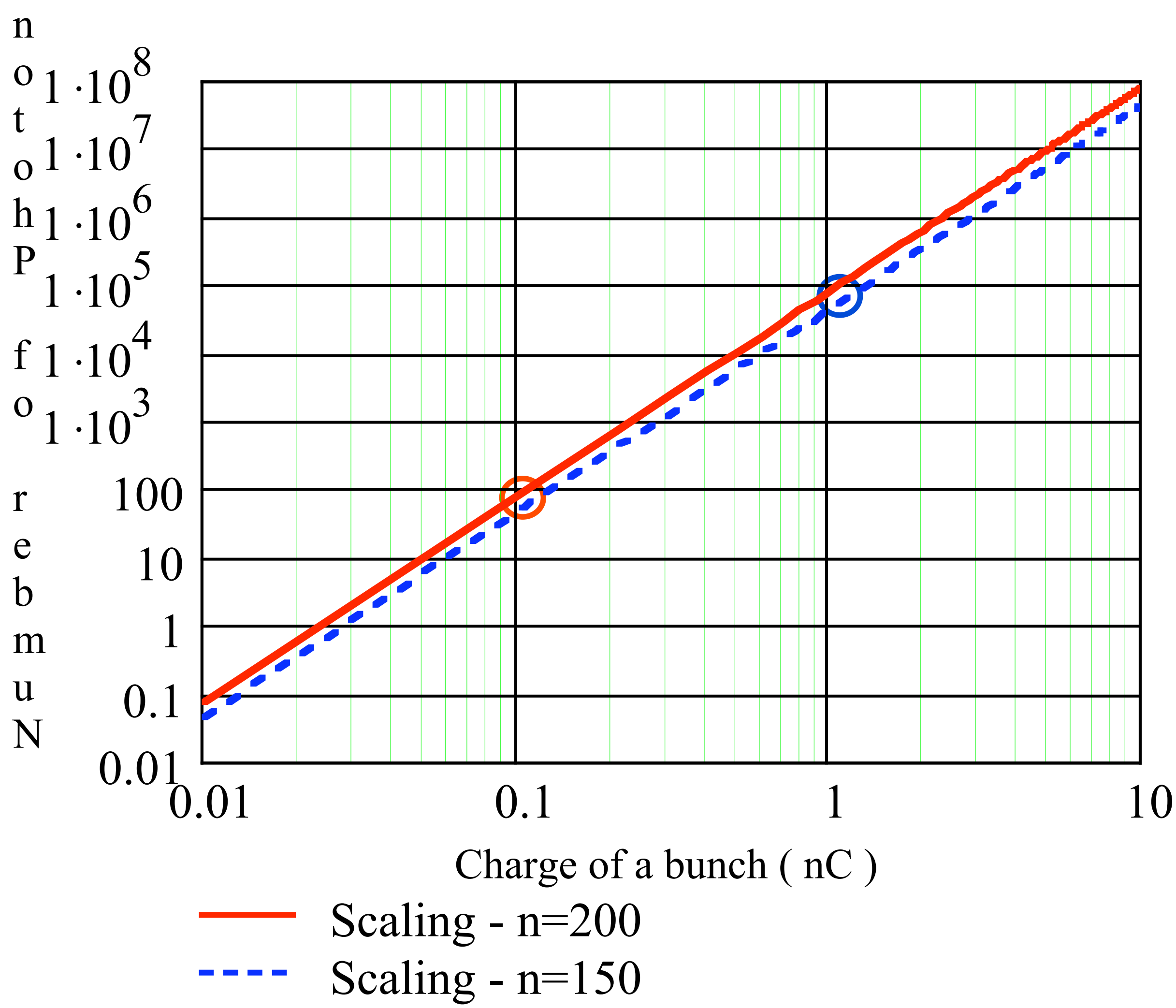
mode	*D* mm	*d* mm	*a* mm	*b* mm
4π/3	71.45	39.5	20	41.2

The optimal dimensions of a S-band waveguide

Reduction ratio, *n*	*D′* mm	*d′* mm	*a′* mm	*b′* mm
150	0.48	0.26	0.13	0.28
180	0.4	0.22	0.11	0.23
200	0.37	0.2	0.1	0.2

RADIATION BY A LOW ENERGY ELECTRON BUNCH

Under-estimation of photon flux



Upper-estimation of transverse emittance

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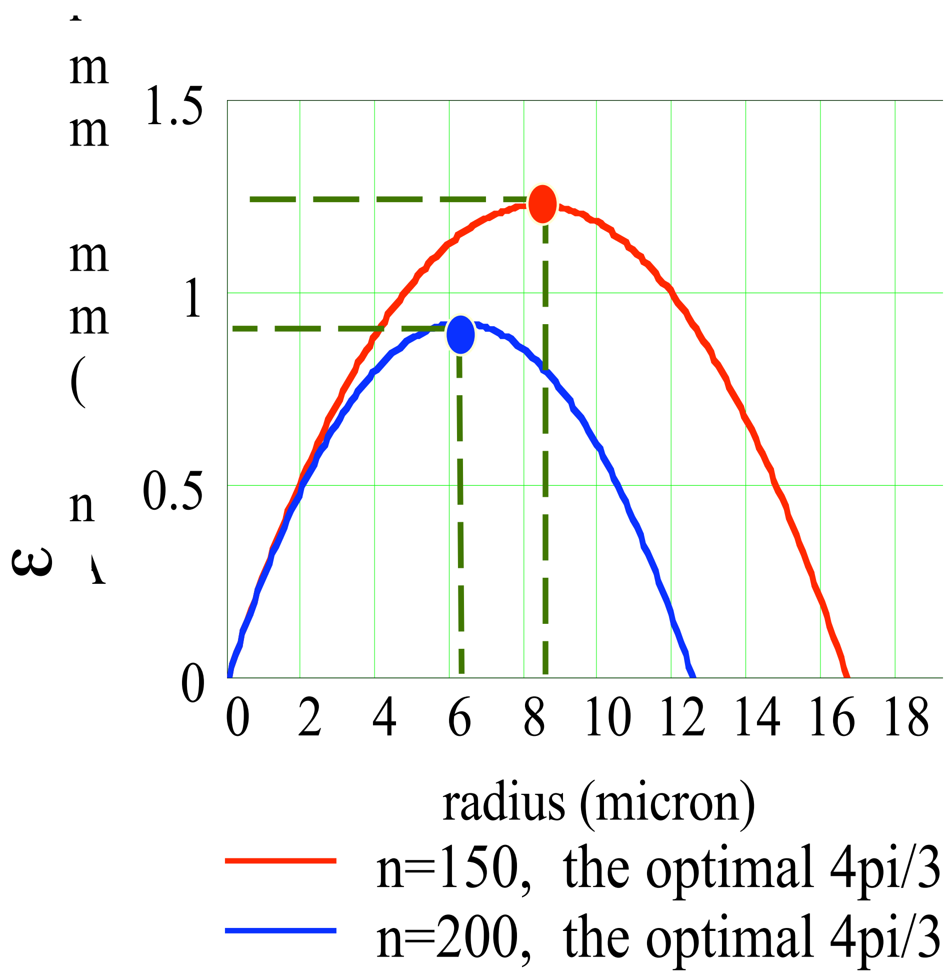
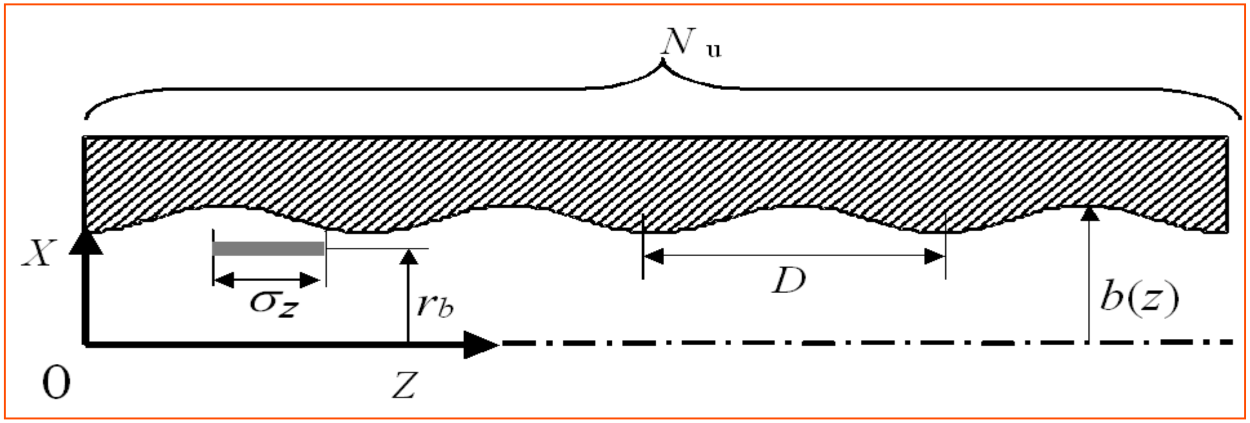


RADIATION BY A LOW ENERGY ELECTRON BUNCH

WFU radiation from weakly corrugated waveguide



Waveguide

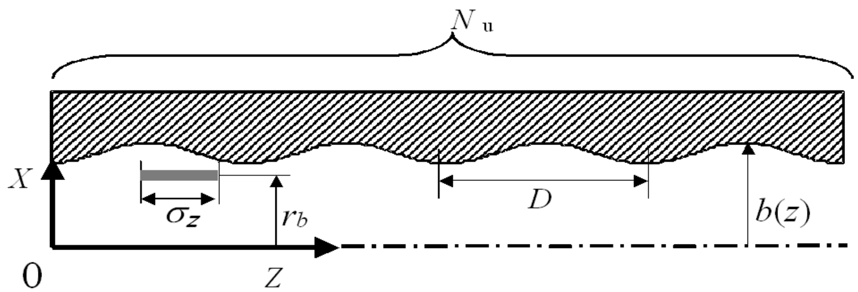
Period	D	0.4 mm
Average radius	b_0	0.1 mm
Amplitude of corrugations	ε_1	0.065
Number of periods	N_u	30

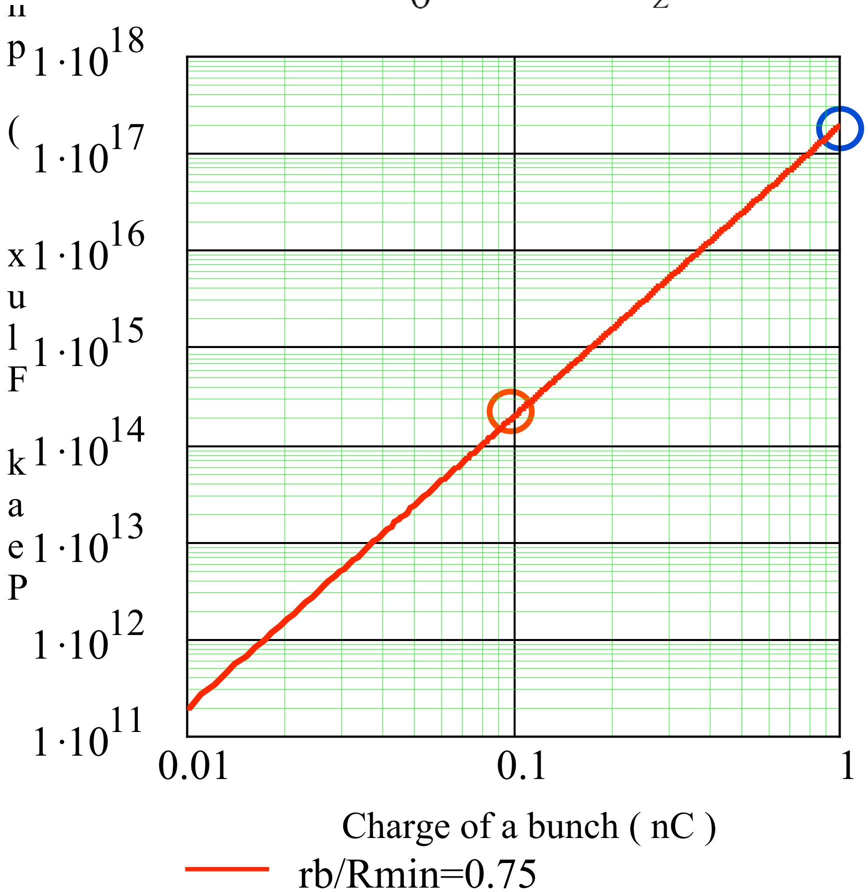
Beam

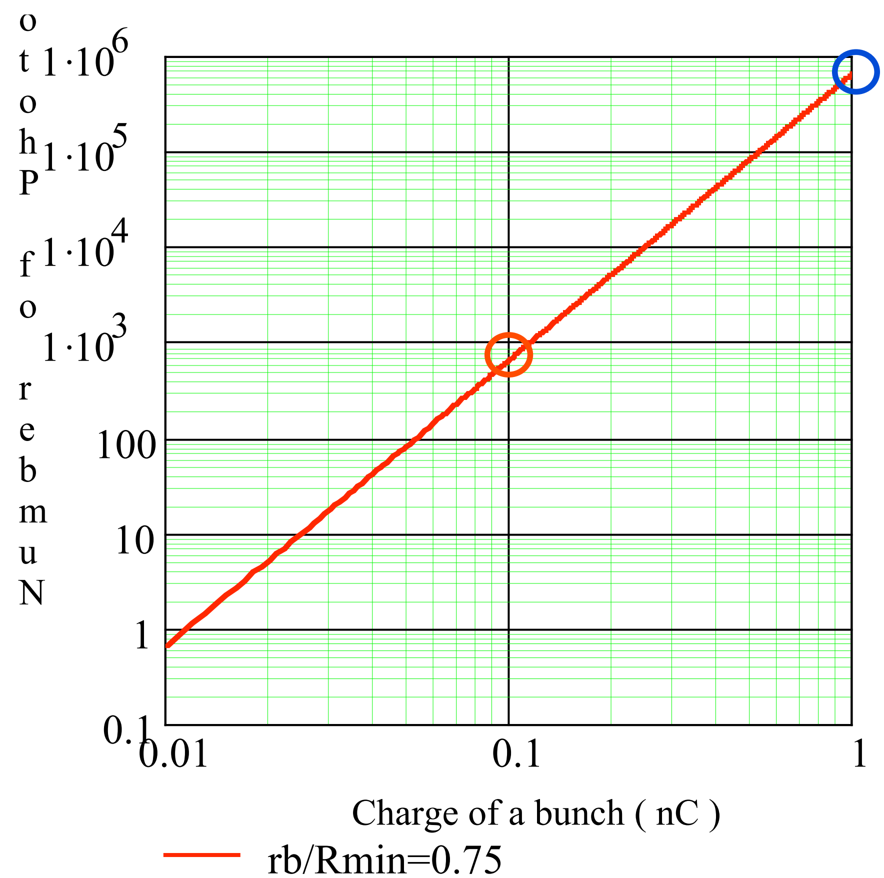
Energy	W_e	10 MeV
Duration	$\Delta\tau$	100 fs
Distance from axis	r_b	75 μm

RADIATION BY A LOW ENERGY ELECTRON BUNCH

WFU radiation from weakly corrugated waveguide







CONCLUSION

1. **For experimental study WFU radiation** ***~ 10⁴ – 10⁵ ph/sec*** **in S-band structures,**
   the electron beams are required with the parameters :

bunch charge ~ 10 nC,

electron energy ≥ 100 _eV

beam radius ≈ 1 mm

normalized emittance (for 3 m sections) ≤ 150 μm

SLAC
-12 nC

2. For observation WFU radiation *10^2 ph/bunch* with TU/e Photo-injector,

It is required the parameters

sub-mm corrugated waveguide with

:

period ~ 0.3 –0.6 mm

transverse sizes ~ 0.1-0.2 mm

beam radius ≈ 1 mm

normalized emittance ~ 1 μm

radius beam ≈ 8 μm

3. For experimental study WFU radiation ***~ 10^4 – 10^5 ph/sec*** **by short pulsed beams in S-band structures (2π/3, 4π/3)**

the electron beams are required with

duration ≈ 1 - 80 ns

charge of pulse ≥ 10 nC,

electron energy ≥ 100 _eV

beam radius ≈ 1 mm

normalized emittance (for 3 m sections) ≤ 150 μm