One photon per electron: physical limits on narrrow band, inverse Compton scattering X-ray production

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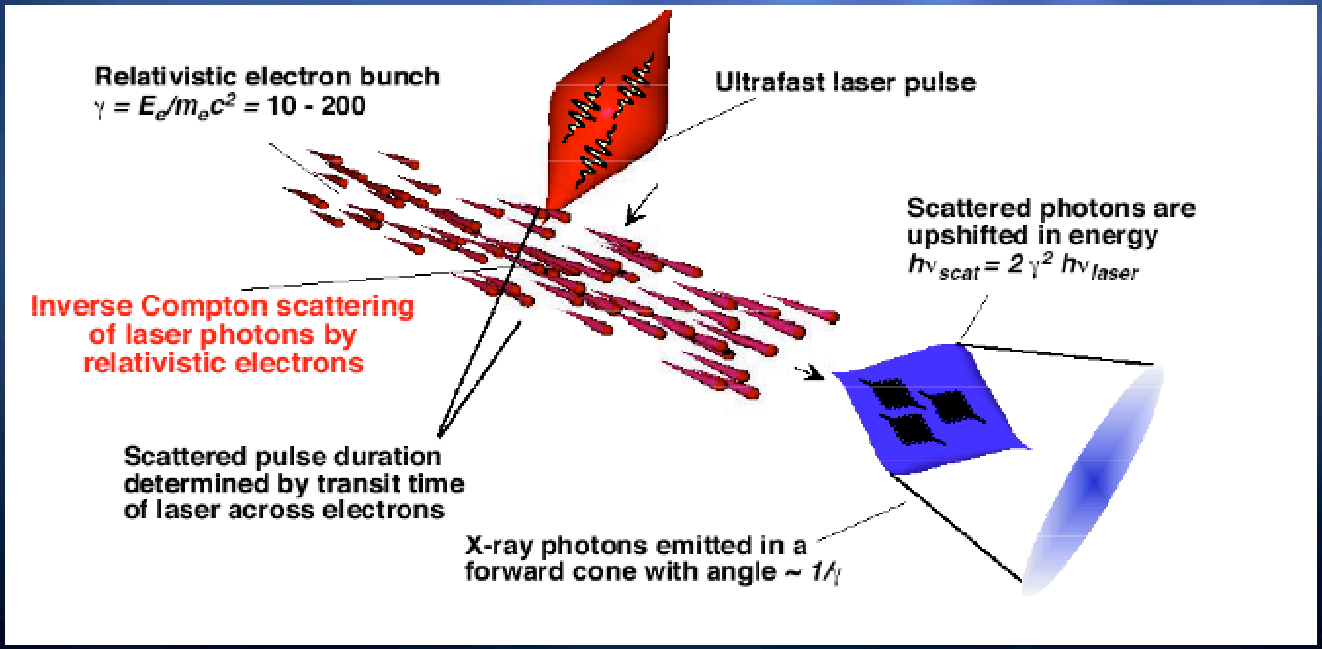
Inverse Compton process

- Collision of *relativistic* electron beam bunch with intense laser pulse
- Scattered light is ~monochromatic, Doppler shifted to v. short wavelength
- Cost is moderate compared to competing techniques
- Difficulty is high, but now w/in state-of-art
- Applications: diverse…

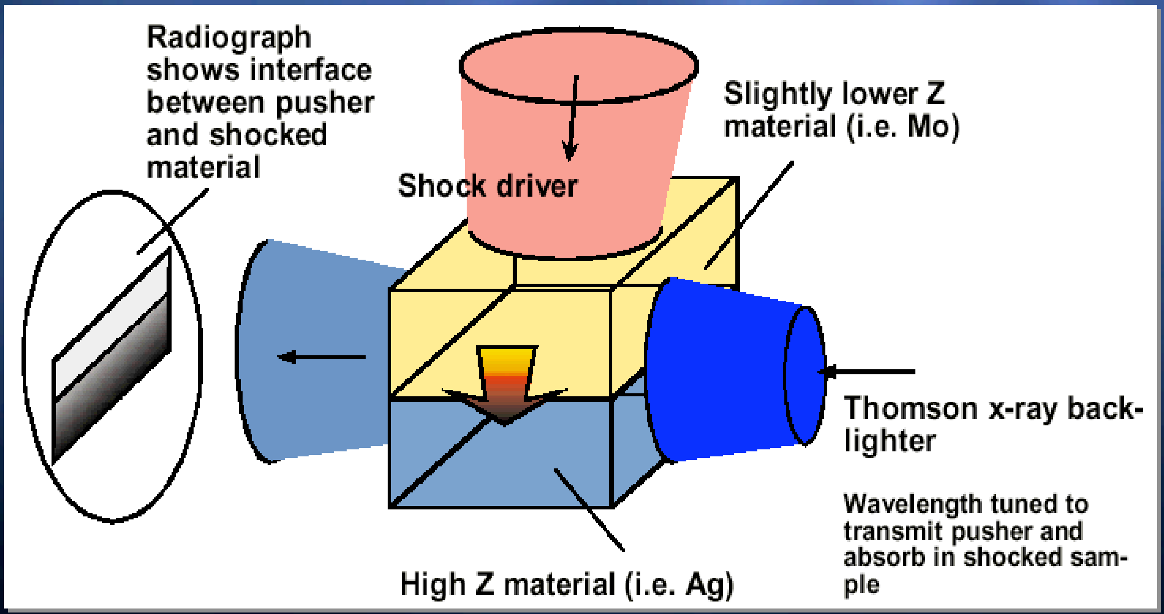


ICS applications

- Ultra-fast materials characterization
  - Original context
  - Laser-intensive community
  - High-end X-ray users
- High energy physics
  - Very high energy photons ($\gamma\gamma$ collider, polarized positrons)
- Biology and medicine
  - Reliable customers for X-rays
  - Potential breakthroughs in diagnosis and therapy

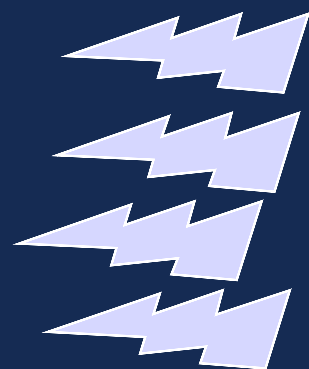
Shock physics

- *Ultra-fast, high energy density* physics
- Fundamental material studies for
  - Inertial confinement fusion
  - Nuclear stockpile stewardship
- Pump-probe systems with high power lasers
- EXAFS, Bragg, radiography in fsec time-scale
- Ultra-fast gives higher laser intensity…

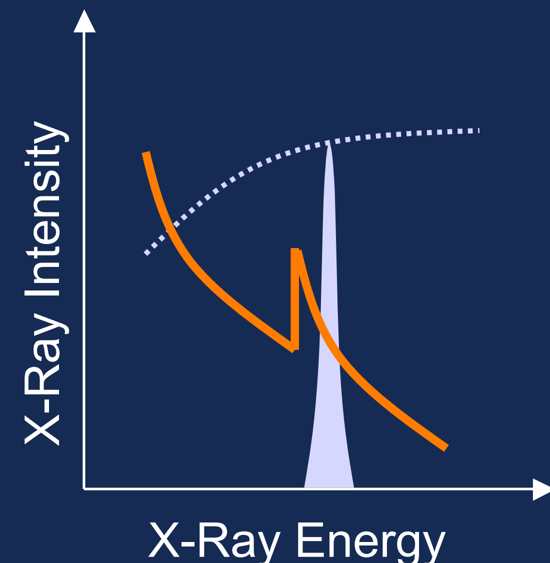


Medical uses: Monochromatic cancer therapy

Tagged Agents Imaged by Noninvasive X-Ray Absorption or Diffraction Spectroscopy



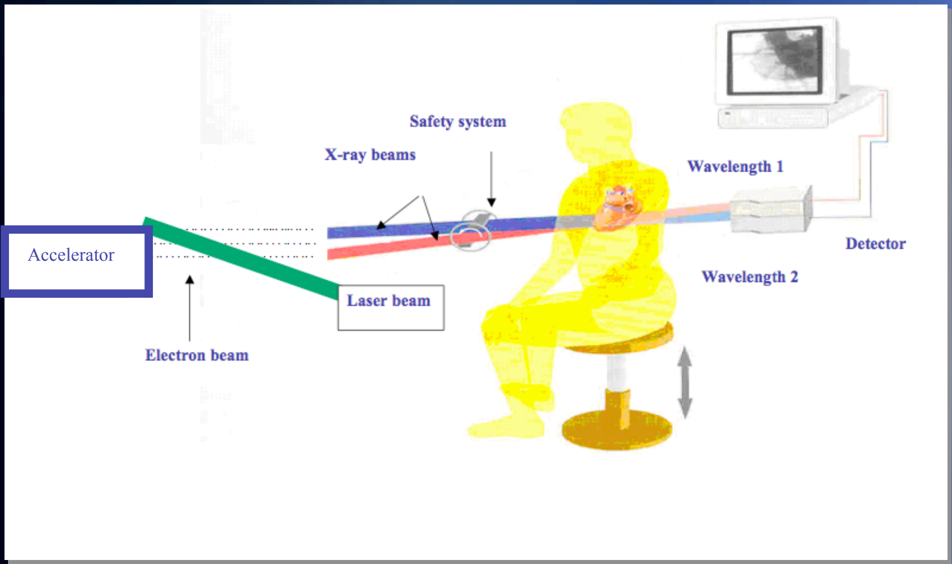
Intensity Increased to Deliver Localized Radiation Dose



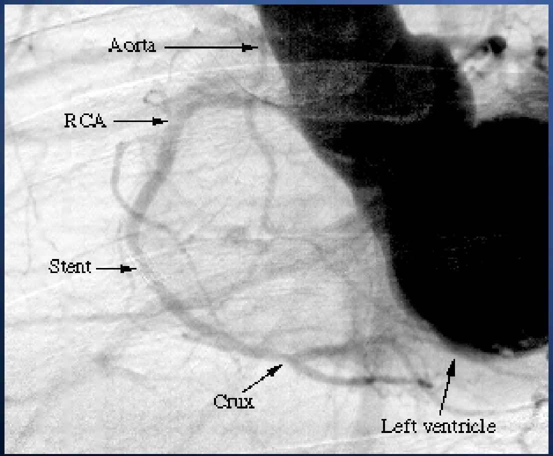
K-edge (~30 keV in iodine)

Simple idea; needs high average flux.

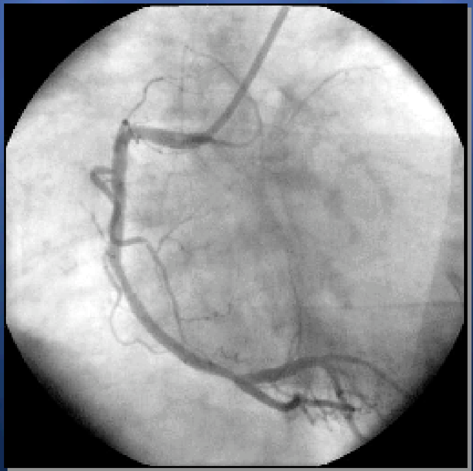
Medical applications: Dichromatic imaging



- Illuminate above and below contrast K-edge
- Digital image subtraction
- Established at synchrotrons
  - Access limited
  - Expensive ($100M's)
- Mitigate risk of angiography



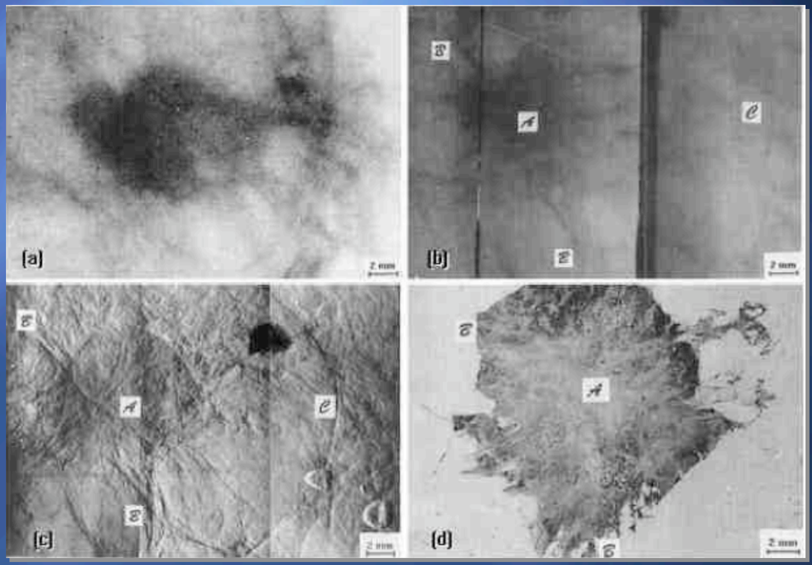
Conventional angiogram



Digital subtraction angiogram (same patient, same day)

Medical applications: mammography

- Conventional X-ray imaging difficult in mammography
  - Soft-tissue contrast poor
- Monochromatic X-rays can enable new techniques
  - Phase contrast imaging



Mammography images of adenocarcinoma. (a) conventional mammogram; (b) monochromatic beam at 22.2 keV; (c) phase contrast image based on monochromatic X-ray beam; (d) histological section.

How many photons?

Table 1. Target specifications for NHLBI Pulsed X-ray Source

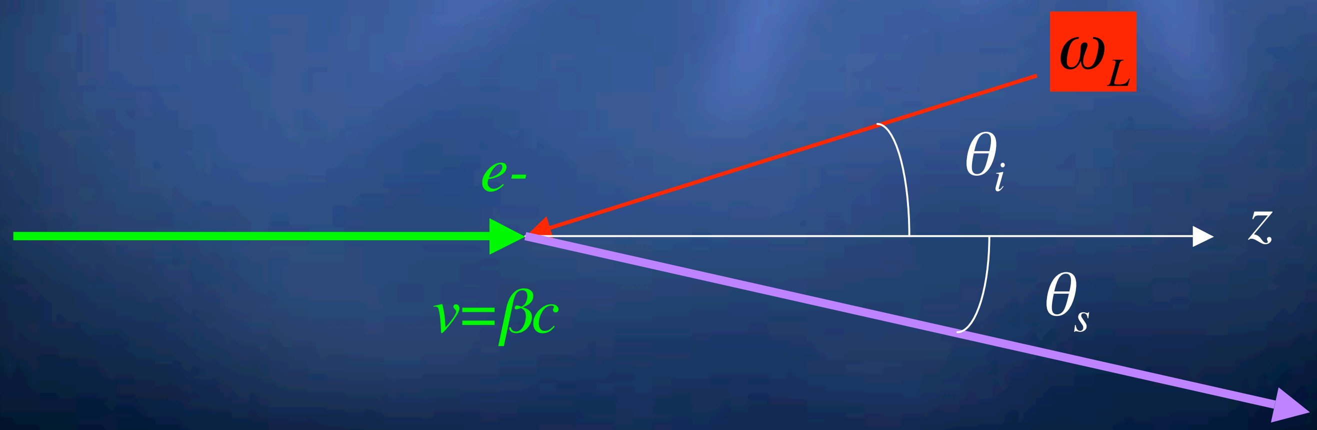
Parameter	NHLBI source (projected)
Peak flux, ph/pulse	**10^10**
Repetition rate, Hz	**1 to 10 (up to 1 kHz in the future)**
Average flux, ph/s	10^11 to 10^13
Peak brilliance, ph/(s mm^2 mrad^2 1%bw)	10^23
Wavelength range, Å	0.4 to 45
Energy range, keV	**0.28 to 30**
Energy bandwidth, %	**0.1 to 10**
Pulse width, ps	**<10**
Source size, um	**<20**
Divergence, mrad	**<2**
Tunability	**repetition rate, bandwidth, wavelength**
Coherence	partial, transverse ?
Suggested dimensions, m	3 x 5 x 1.5

Most critical parameters are given in bold.

- Medical applications demand
  - large numbers of photons
  - Narrow bandwidth
- Example: NIH pre-solicitation (left)
- Similar numbers for other state-of-art ICS projects
- Are these parameters possible?
- Discuss using simple model of ICS
  - Very similar to sophisticated computations

ICS basics

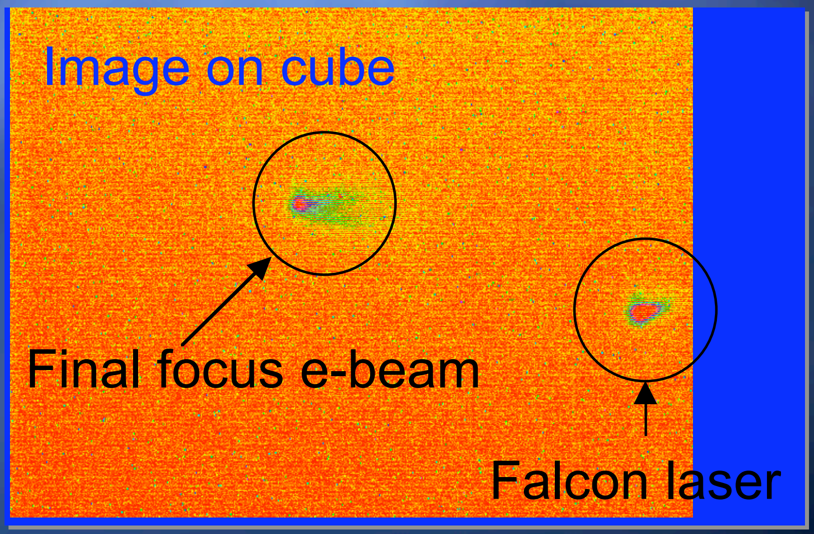
- We consider nearly head-on collision
- Ignore electron beam divergence in this talk
- Work in "Thomson" limit; also quasi-linear
- Look at spectral broadening mechanisms
  - Relate to laser intensity/photon production



Why are we allowed to ignore the electron beam divergence?

- Anecdote: even with very bad final emittance at PLEIADES, the e-beam was smaller at the IP than the laser!
- The laser "emittance" is usually much bigger than the e-beam

$$\varepsilon \cong \frac{\varepsilon_n}{\gamma} << \frac{\lambda_L}{4\pi}$$



- Example: PLEIADES design

$$\varepsilon = 1.6 \times 10^{-8}\,\mathrm{m}, \quad \frac{\lambda_L}{4\pi} = 6.4 \times 10^{-8}\,\mathrm{m}$$

Analysis begins in electron rest frame

- Lorentz transformation of ω-k 4-vector

$$\omega' = \gamma\left(\omega_L + \beta c k_L \cos(\theta_i)\right) = \gamma\omega_L\left(1+\beta\cos(\theta_i)\right)$$

$$k'_\perp = k_L \sin(\theta_i)$$

$$k'_z = \gamma\left(k_L \cos(\theta_i) + \beta\omega_0 / c\right) = \gamma k_L\left(\cos(\theta_i) + \beta\right)$$

- Blue-shifted rest frame photon will scatter with negligible recoil (Thomson limit of Compton scattering), if

$$\hbar\omega' << m_e c^2, \quad \text{or}$$

$$\hbar\omega_L \gamma\left(1+\beta\cos(\theta_i)\right) << m_e c^2$$

Neptune example:
γ=30, $h\omega_L$=0.12 eV

$$\hbar\omega_0\gamma(1+\beta) = 7.2 \text{ eV} << 5.11\times 10^5 \text{eV}$$

Thomson scattering in electron rest frame

- In Thomson limit, ω is independent of emission direction in rest frame θ'

$$\omega_s' = \omega' = \gamma\omega_L\left(1+\beta\cos(\theta_i)\right)$$

- Wave-vector components

$$k_{\perp s}' = \frac{\omega_s'}{c}\sin(\theta') = k_L\gamma\left(1+\beta\cos(\theta_i)\right)\sin(\theta')$$

$$k_{zs}' = k_L\gamma\left(1+\beta\cos(\theta_i)\right)\cos(\theta')$$

- Note: Power profile in linear limit is also derived from Thomson

$$\frac{dP_s}{d\Omega'} = \frac{e^2\dot{v}^2}{4\pi c^3}\cos^2(\theta')$$

- The total Thomson cross-section is Lorentz invariant

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = \frac{8\pi}{3}r_e^2$$

Back to the lab frame

- Final frequency: $\omega_s = \gamma^2 \omega_L \left(1 + \beta \cos(\theta_i)\right)\left(1 + \beta \cos(\theta')\right)$
- Wave-vector components:

$$k_{\perp s} = k_L \gamma \left(1 + \beta \cos(\theta_i)\right) \sin(\theta')$$
$$k_{zs} = k_L \gamma^2 \left(1 + \beta \cos(\theta_i)\right)\left(\cos(\theta') + \beta\right)$$

- Lab frame angle

$$\tan(\theta_s) = \frac{k_{\perp s}}{k_{zs}} = \frac{1}{\gamma}\left(\frac{\sin(\theta')}{\cos(\theta') + \beta}\right)$$

- Small angle approximation

$$\theta_s \cong \frac{\theta'}{2\gamma}$$

Small angle spectrum

- Approximate small angle spectrum

$$\omega_s = 4\gamma^2\omega_L\left(1-\frac{\theta_i^2}{2}\right)\left(1-\frac{(\gamma\theta_s)^2}{2}\right)$$

Maximum Doppler shift

Incident angle effect

Final angle effect

- Final angle-induced red shift familiar from FEL
  - Resonance: when emitted wave-front overtakes electron by λ_r in λ_U ($\sim\lambda_L/2$. Thomson)
  - Relative red shift always ~ $(\gamma\theta)^2/2$

Angular "efficiency"

- Small bandwidth means small angles accepted into aperture
- In terms of rest frame angle

$$\eta_{acc}(\theta'_{\max}) \equiv \frac{N_{acc}(\theta'_{\max})}{N_{total}} = \tfrac{1}{2}\left(1-\cos^3(\theta'_{\max})\right) \cong \frac{3}{4}\theta'^2_{\max}$$

$$\eta_{acc}(\theta_s) \cong 3(\gamma\theta_s)^2$$

- In terms of *rms* bandwidth

$$\eta_{acc}(\theta_{\max}) \cong 6\sqrt{3}(BW_{rms})_{acc}$$

Red shift from (small) nonlinear effects

- Electron has angle in motion due to laser field $\theta \approx \frac{p_\perp}{p_0} \cong \frac{a_L}{\gamma}, \quad a_L = \frac{eE_L}{m_e c \omega_L}$
- Relative red shift $\frac{(\gamma\theta)^2}{2} = \frac{a_L^2}{2}$
- Result OK for small a_L only
  - Figure-8 motion
  - Harmonics…
- RMS BW (gaussian beams) $\left(BW_{rms}\right)_{NL} \cong \frac{a_L^2}{7.7}$

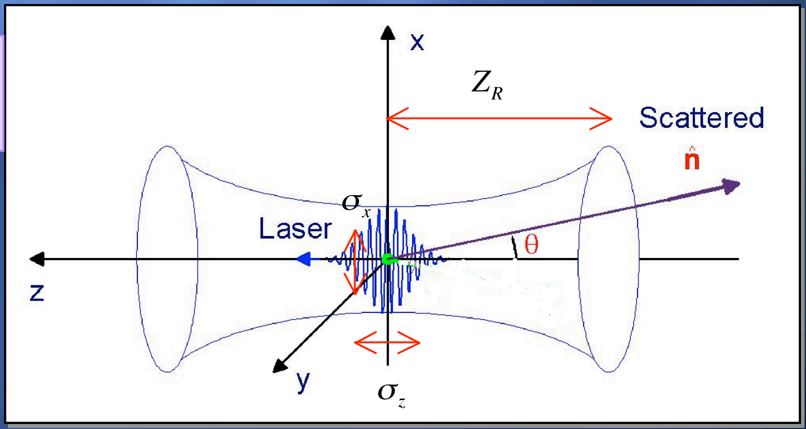
Finite pulse length and focal effects

- Fourier spread in the laser pulse from finite length

$$(BW_{rms})_{FT} \cong \left(\sqrt{2}k_L\sigma_z\right)^{-1} = \frac{\lambda_L}{\sqrt{8}\pi\sigma_z}$$

- Bandwidth smaller for longer pulse
- Practical pulse length limited by Rayleigh range

$$\sigma_z \le Z_R = \frac{4\pi\sigma_x^2}{\lambda_L}$$



Inherent angular spread in photons…

- Away from the focus, laser phase fronts have angle, with rms spread

$$\theta_{L,rms} = \frac{\sigma_x}{Z_R} = \frac{\lambda_L}{4\pi\sigma_x}$$

$$(BW_{rms})_\theta = \frac{(\theta_L^2)_{rms}}{2} = \left(\frac{\lambda_L}{4\pi\sigma_x}\right)^2$$

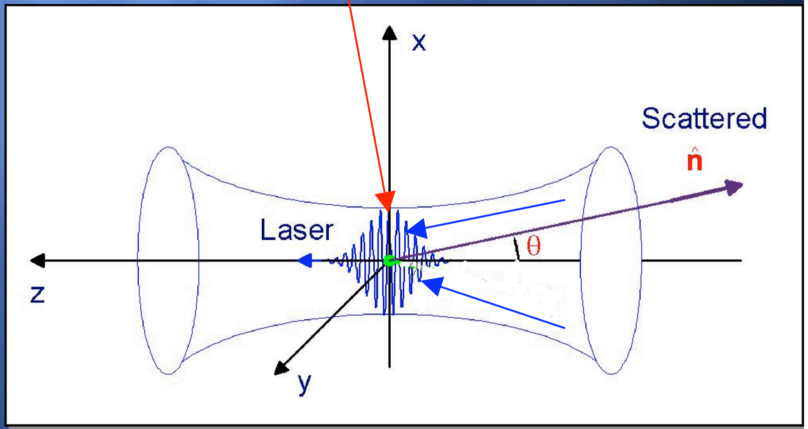
- In focus, phase fronts flatten, but Guoy phase shift changes local k_z

$$\tan(\varphi(z)) = \frac{z}{Z_R} \quad \longrightarrow \quad BW_{\text{shift}} = \frac{\Delta k_z}{k_z} = \frac{1}{k_z}\frac{d\varphi}{dz} \cong \frac{\lambda_L}{2\pi Z_R} = \frac{\lambda_L^2}{8\pi^2\sigma_x^2}$$

- RMS spread same…

From dispersion relation:

$$k_z = \sqrt{(\omega_L/c)^2 - k_\perp^2} \cong \sqrt{k_L^2 - \sigma_x^{-2}} \cong k_L\left(1 - \frac{1}{2}\left(\frac{\lambda_L}{2\pi\sigma_x}\right)^2\right)$$



Photon production and luminosity

- Total scattered photons per pulse: $N_\gamma = \mathcal{L}\sigma_T$
- "Luminosity" per pulse: $\mathcal{L} = \frac{N_L N_{e-}}{4\pi\sigma_x^2}$
- Independent of σ_z; make beam as long as possible to mitigate FT BW: $\sigma_z = Z_R = \frac{4\pi\sigma_x^2}{\lambda_L}$
- *Under this assumption*, angular BW nearly the same as the Fourier transform BW!

$$\theta_{L,rms} = \frac{\lambda_L}{4\pi\sigma_x} = \frac{\lambda_L}{4\pi}\sqrt{\frac{4\pi}{\lambda_L\sigma_z}} = \sqrt{\frac{\lambda_L}{4\pi\sigma_z}} = 2^{-1/4}\sqrt{(BW_{rms})_{FT}}$$

→

$$(BW_{rms})_{foc} = \theta_{L,rms}^2 = \sqrt{2}(BW_{rms})_{FT}$$

Calculate luminosity

- Luminosity per assumption $\mathcal{L} = \frac{N_L N_{e-}}{4\pi\sigma_x^2} = \frac{N_L N_{e-}}{\lambda_L \sigma_z} = \frac{U_L N_{e-}}{hc\sigma_z}$
- Look at laser pulse energy density

$$u_{\mathrm{EM}} = \frac{N_L h\nu_L}{(2\pi)^{3/2}\sigma_z\sigma_x^2} = \frac{U_L}{\sqrt{\pi/2}\lambda_L^2\sigma_z^2}$$

- Pulse intensity and field values…

$$u_{\mathrm{EM}} = \frac{\varepsilon_0 E_L^2}{2} = \frac{U_L}{\sqrt{\pi/2}\lambda_L^2\sigma_z^2}$$

or

$$E_0 = \sqrt{\frac{2u_{EM}}{\varepsilon_0}} = \sqrt{\frac{2U_L}{\sqrt{2/\pi}\varepsilon_0\lambda_L\sigma_z^2}}$$

- Put in terms of maximum vector potential to relate to NL effects

Maximum laser energy

- Laser energy

$$U_L = \frac{\varepsilon_0 E_L^2 \sigma_z^2 \lambda_L}{(2\pi)^{1/2}}$$

- Maximum in terms of $a_{L,\max}$ (BW...)

$$U_{L,\max} = \frac{k_L \sigma_z^2 a_{L,\max}^2}{(8\pi)^{1/2} r_e} m_e c^2$$

- Relate to luminosity...

Maximum photon production

- Luminosity

$$\mathcal{L}_{\max} = \frac{U_{L,\max} N_{e-}}{hc\sigma_z} = \frac{k_L a_{L,\max}^2 \sigma_z m_e c^2}{\sqrt{8\pi} r_e hc} N_{e-} = \frac{\alpha a_{L,\max}^2 k_L \sigma_z}{\sqrt{8\pi} r_e^2} N_{e-}$$

- Photons per pulse

$$N_\gamma = \mathcal{L}_{\max} \sigma_T = \frac{\sqrt{8\pi}\alpha (k_L \sigma_z) a_{L,\max}^2}{6} N_{e-}$$

- Number within design angular acceptance (BW)

$$N_\gamma = \frac{(2\pi)^{3/2} \alpha (k_L \sigma_z) a_{L,\max}^2}{6} \eta(\theta_{\max}) N_{e-}$$

Relation of photon production to bandwidth(s)

- We can cast the photon number in terms of all of the relevant bandwidths

$$N_\gamma \cong 0.76 \frac{(BW_{rms})_{acc}(BW_{rms})_{NL}}{(BW_{rms})_{foc}} N_{e-}$$

- Note: laser energy scales as $(BW_{rms})_{foc}^{-4}$

Hard to exploit larger size beam

- If NL and angular BW are chosen ~1%, and focus/FT BW is ~1E-4

$$N_\gamma \approx N_{e-}$$

How do you design laser?

- Choose a laser wavelength
  - Because of limits on NL motion

$$U_L \propto \lambda_L$$

- Specify the "focus" bandwidth
  - Now you know the beam dimensions
- Specify nonlinear BW
  - Now you know the laser pulse energy
- Specify angular acceptance for desired BW
- In terms of BWs

$$U_L = 0.1 \frac{(BW_{rms})_{NL}}{(BW_{rms})^2_{foc}} \frac{\lambda_L}{r_e} m_e c^2$$

A 1% RMS BW Example

- Laser wavelength: 800 nm
- Focus BW: 1.5E-4, means

$\sigma_z = 450\ \mu\text{m (1.5 ps)}$ and $\sigma_x = 5.3\ \mu\text{m}$

  - Strain electron focus at moderate energy…
- NL (+ angular) bandwidth: 1%
- Looks familiar! $U_L = 1\text{ J}$
- For these choices $N_\gamma \approx 0.5 N_{e-}$
- Then… N_γ=3x10^9/nC, with good emittance

$$\varepsilon_n << \gamma\lambda/4\pi = 5\text{ mm-mrad (38 MeV e-/35 keV }\gamma)$$