Theory of Quantum FEL and the QFEL Project

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- **Fully quantized model**
- **Linear quantum theory and SS instability (wrong and correct)**
- **(Photon statistics and entanglement)**
- **Fundamental limitation for bunching and energy spread**
- **Theory of Quantum SASE**
- **The QFEL Project**

Some references

HIGH-GAIN AND SASE FEL with “UNIVERSAL SCALING”

Classical Theory

(1) R.B, C. Pellegrini and L. Narducci, Opt. Commun. 50, 373 (1984).
(2) R.B, B.W. McNeil, and P. Pierini PRA 40, 4467 (1989)
(3) R.B, L. De Salvo, P.Pierini, N.Piovella, C. Pellegrini, PRL 73, 70 (1994).
(4, 5) R.B. et al,Physics of High Gain FEL and Superradiance, La Rivista del Nuovo Cimento vol. 13. n. 9 (1990) e vol. 15 n.11 (1992)

QUANTUM THEORY

(6) R. B., N. Piovella, G.R.M.Robb, and M.M.Cola, Europhysics Letters, 69, (2005) 55 .
(7) R.B., N. Piovella, G.R.M. Robb, Quantum Theory of SASE-FEL, NIM A 543, 645 (2005)
(8) R. B., N. Piovella, G.R.M.Robb, and M.M.Cola, Optics Commun. 252, 381 (2005)

See also

(9) F.T.Arecchi, R. Bonifacio, “MB equation”, IEEE Quantum Electron., 1 (1965) 169

Fully quantized many particles model for CRL

R. B., N. Piovella, Proc. FEL conf. 2005

$$\left[a \, , \, a^{+} \right] = 1$$

(*) Schroeder, Pellegrini, Chen, (SPC) PRE, 64, 56502 (2001)

Linear Theory

$\rho >> 1$ classical theory

BPN (1984)

If one chose P_1 (SPC),

Incorrect ordering gives incorrect results

[illegible]

$\langle \quad \rangle \langle \;|\; \rangle \quad \langle \quad \rangle \langle \;|\; \rangle \quad \langle \quad \rangle \langle \;|\; \rangle$

$| \rangle \qquad | \rangle$

$\langle \quad \rangle \qquad \langle \quad \rangle \qquad \langle \quad \rangle$

[illegible]

$| \rangle \qquad | \rangle \qquad | \rangle \qquad | \rangle$

$\langle \quad \rangle \qquad \langle \quad \rangle \qquad \langle \quad \rangle$

[illegible]

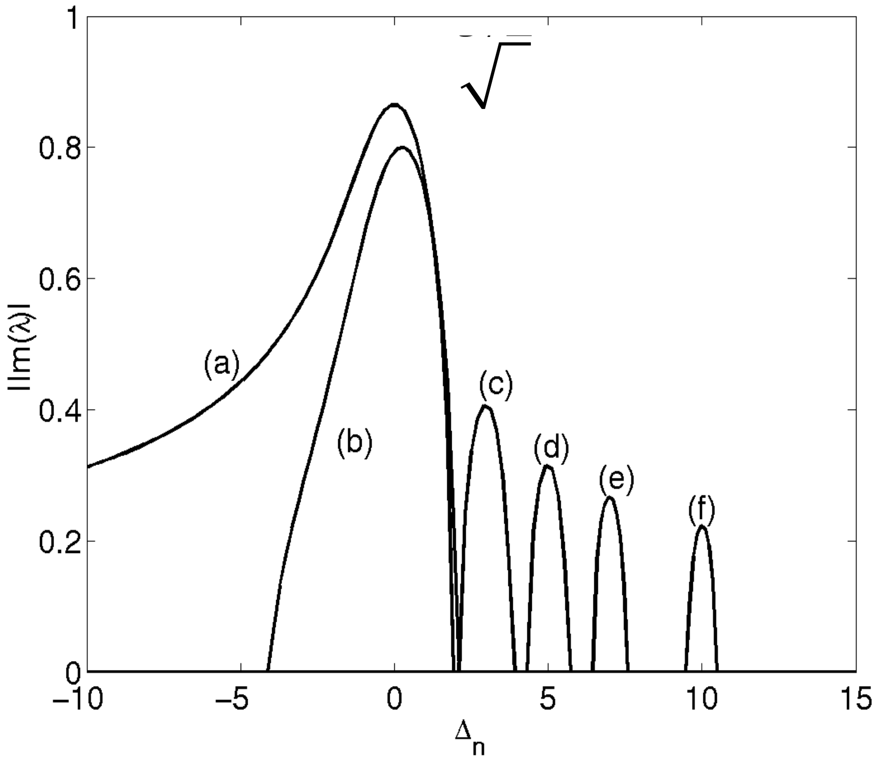
Resonance is not at δ=0 but at

Incorrect ordering → incorrect cubic

(*) Schroeder, Pellegrini, Chen, (SPC) PRE, 64, 56502 (2001)

Linear Theory: QM

As if classical rect. dist.



Classical limit (a)

Quantum regime

(*) Carruthers and Nieto, Rev. Mod. Phys. 40, 4411 (1968)

Momentum – bunching uncertainty relation

!!! HUP

Two Level System (QFEL regime)

Let the Hilbert space to be spanned only by 2 eigenstates of the discrete momentum separated by

Let the state of the system to be a superposition of these states with probability P_1 and P_2 such that $P_1 + P_2 = 1$.

It is trivial to show that

Δp has a maximum at = 1/2 for $P_1 = P_2 =$ _

From previous uncertainty relation becomes:

i.e.,

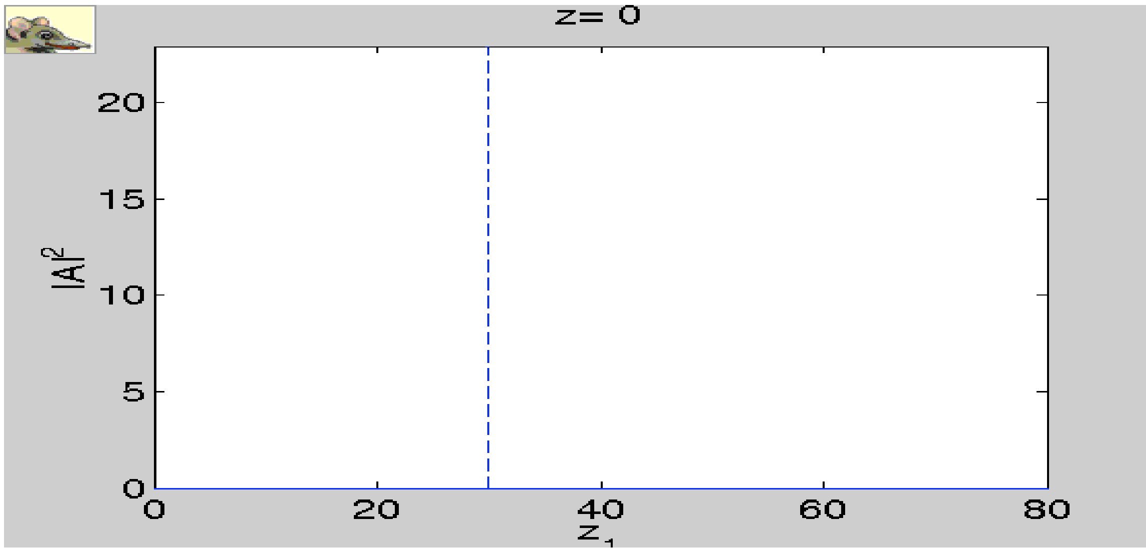
All the numerical simulations are in agreement with the above quantum limitation (max bunching = 0.5 independently on).

Results from quantum linear theory

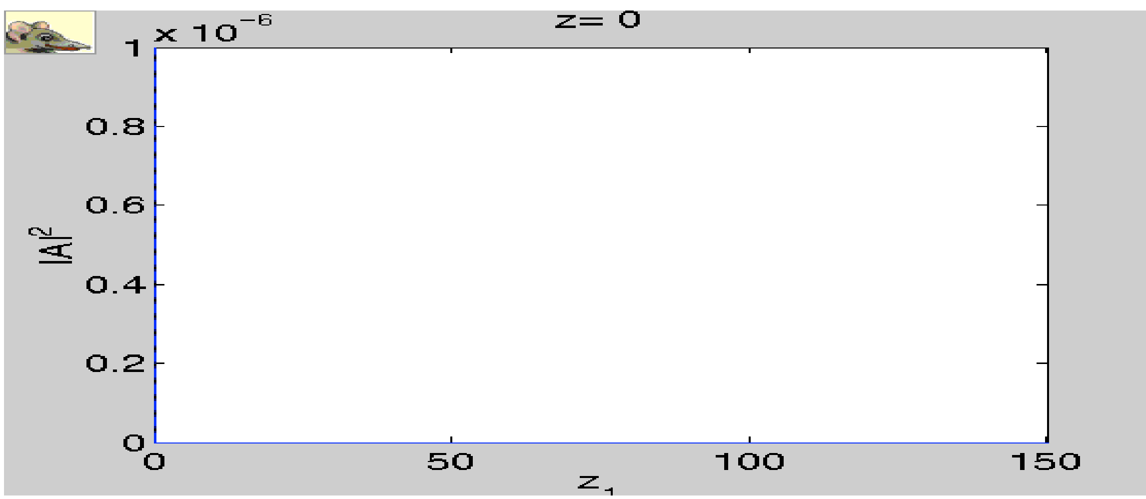
(Quantum fluctuation and entanglement in CARL, PRA, 67, 01387 (2003), N. Piovella, M. Cola and R.B.)

STEADY STATE AND SUPERRADIANT INSTABILITY,
Long and Short Bunch (uniform seed)

Evolution of radiation time structure in the electron rest frame



Strong SR



Weak SR

R. Bonifacio, B.W. McNeil, and P. Pierini PRA 40, 4467

Classical SASE

Ingredients:

i) Start up from noise

ii) Propagation effects (slippage)

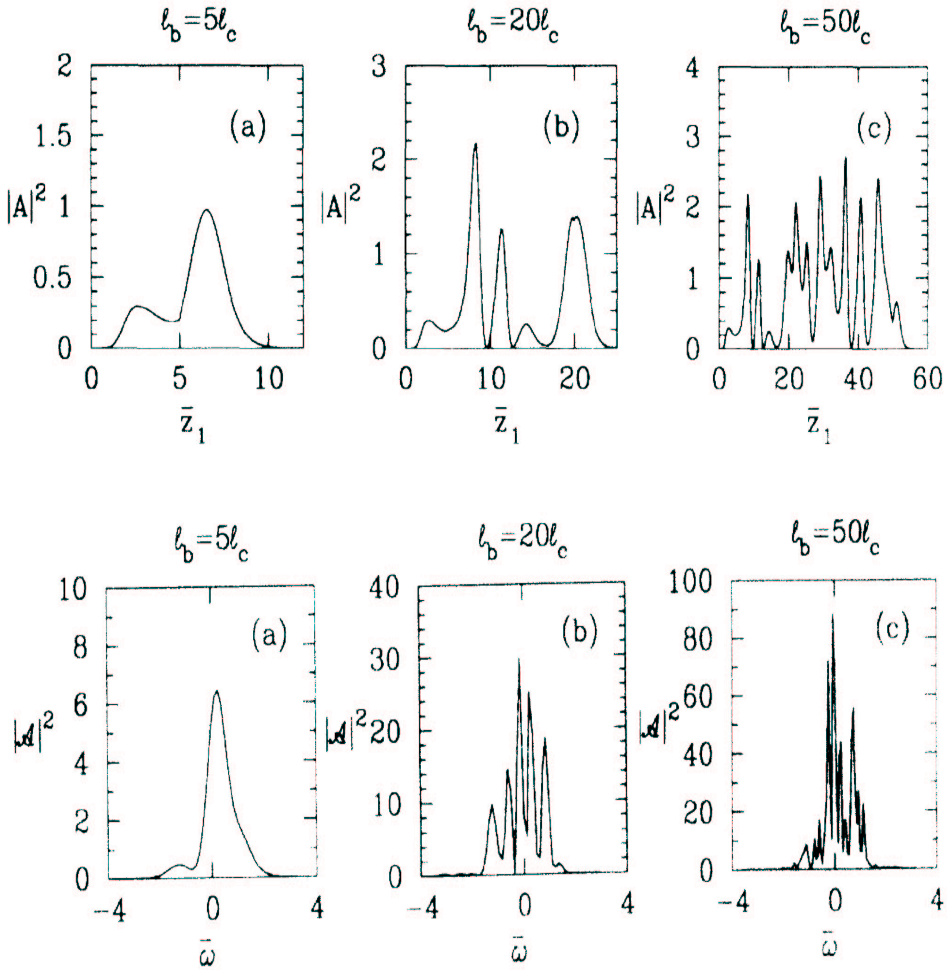
iii) Superradiant instability: (no steady state instability)

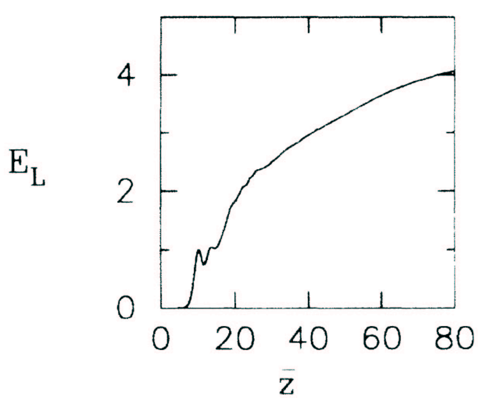
Self Amplified Superradiant Emission

(RB, L. De Salvo, P.Pierini, N.Piovella, C. Pellegrini, PRL 73 (1994) 70)

$\Downarrow$

The electron bunch behaves as if each cooperation length would radiate independently a weak SR spike which gets amplified propagating on the other electrons with no saturation. Spiky time structure and spectrum.





CLASSICAL SASE

reprinted from PRL 73 (1994) 70

Time structure:

Almost chaotic behavior:

number of random spikes goes like .

Spectrum:

is just the envelope of a series of narrow random spikes

If a single SR spike.

At short wavelengths => many random spikes.

Total energy does not saturate (at 1.4).

QFEL propagation model

matter wave

QFEL parameter

R. B., N. Piovella, G.R.M. Robb,
NIM A 543 (2005) 645 and proc. FEL conf. 2005

; Q. F. T. by G. Preparata† (Phys. Rev. A, 38 (1988), 233)

Canonical Quantization

Quantization

The QFEL model for the matter wave

Derived from Q-field theory by G. Preparata (Phys. Rev. A, 38 (1988), 233)

Classical Limit:

One can prove that the Schroedinger equation for the QFEL model reduces to the classical Vlasov Equation for the Quantum Wigner function in the limit:

In the classical limit, with universal scaling, no dependence on

R. B., N. Piovella, G.R.M. Robb,
NIM A 543 (2005) 645

Classical limit when

Wigner function

Classical Vlasov Equation

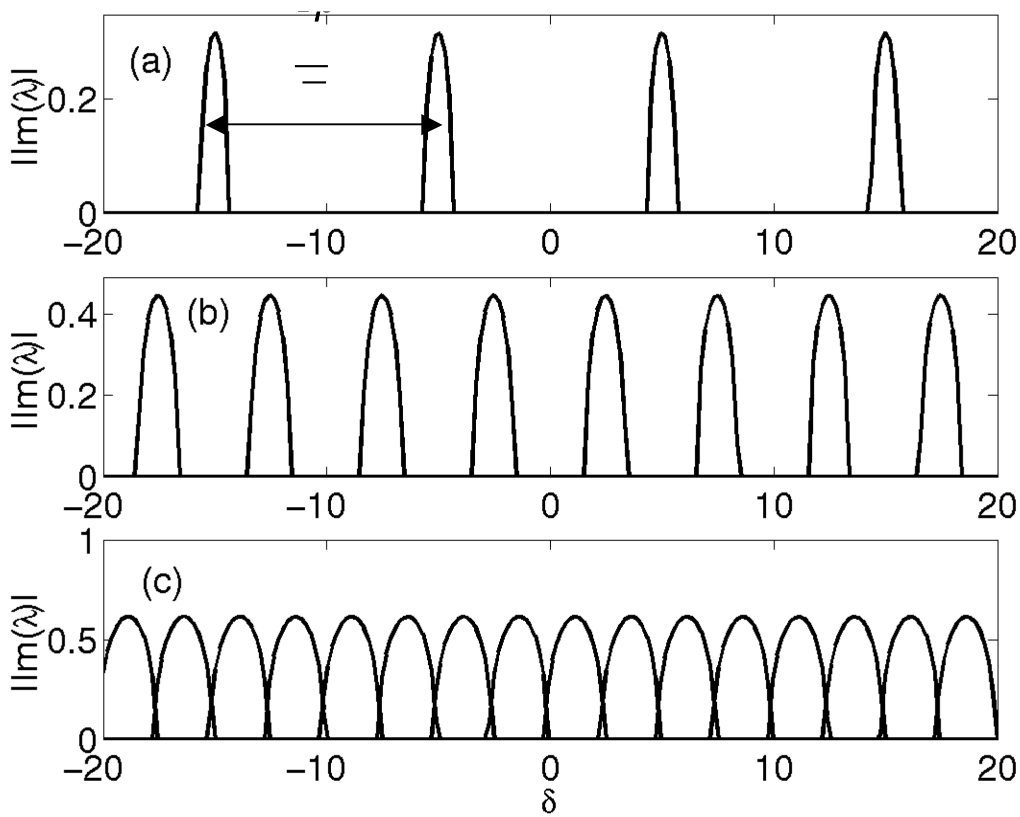
R. B., N. Piovella, G.R.M. Robb, NIM A 543 (2005) 645

The Momentum Representation

| | is the probability that an electron has a momentum

QFEL “working equations”

The Discrete frequencies as in a cavity

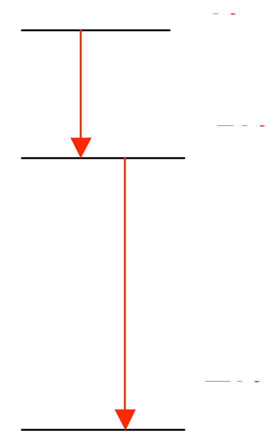


Frequency separation

Full width

Continuous classical limit

Quantum limit : discrete resonance as in a cavity



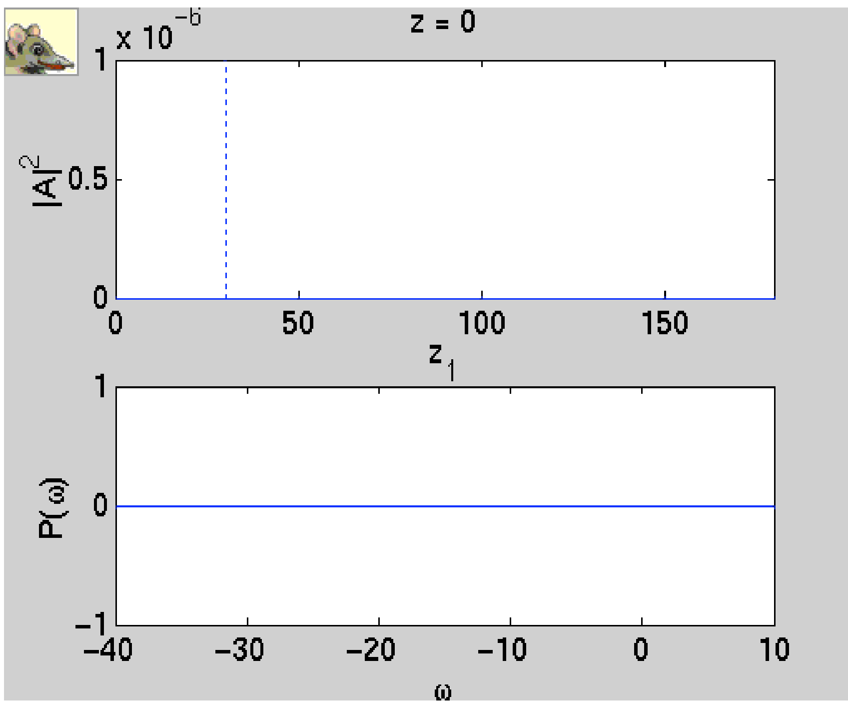
Only recoil : no absorption

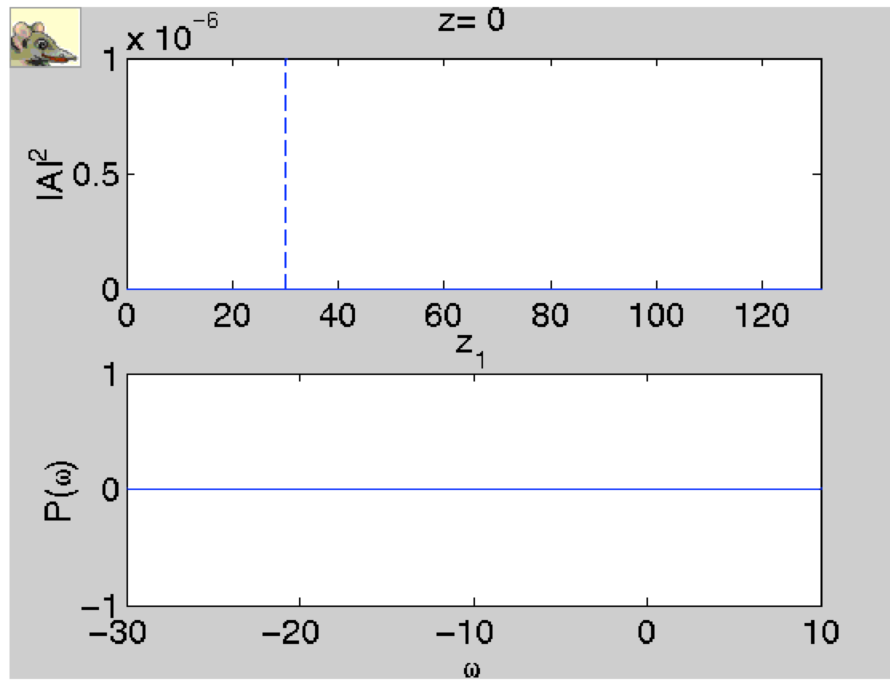
SASE

Quantum — Classical

$= 0.05$ — $= 5$

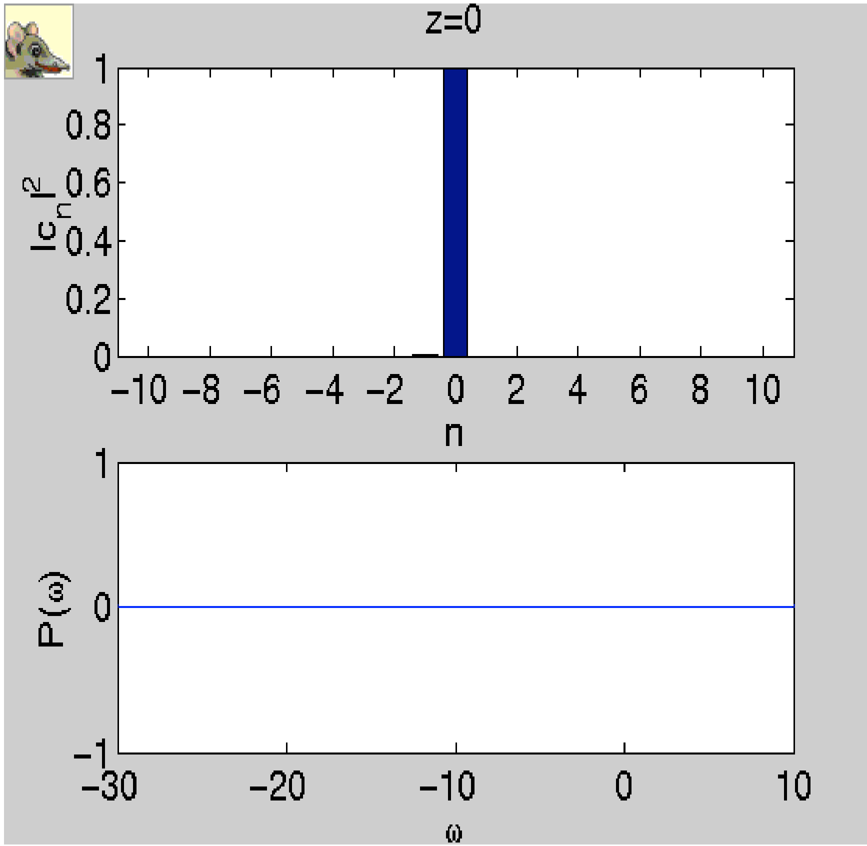
Evolution of radiation time structure in the electron rest frame



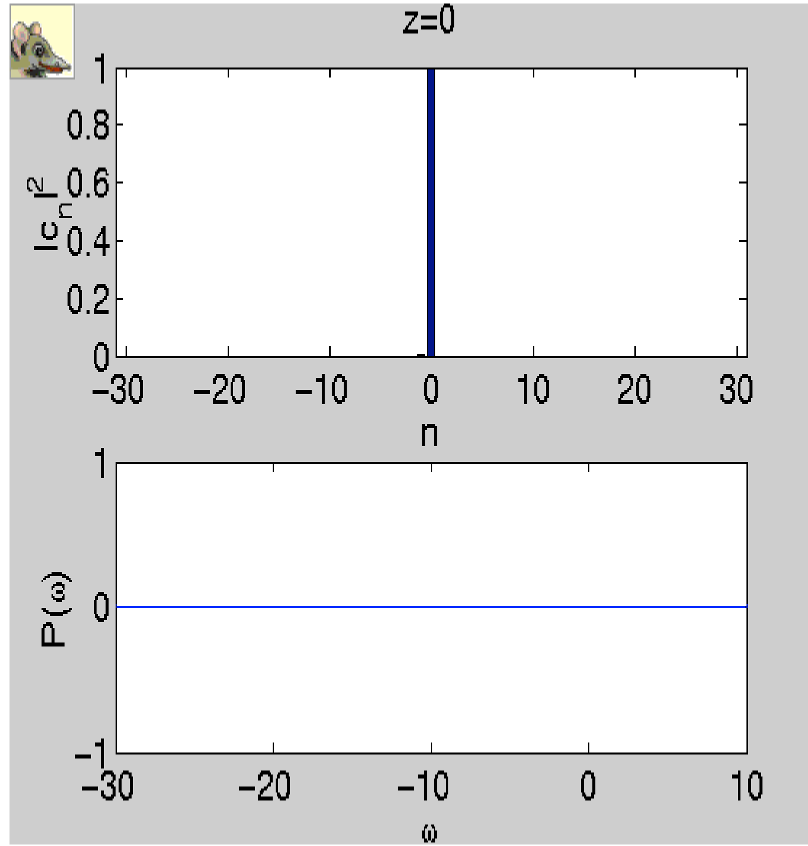


Simulation using QFEL model: Momentum distribution (average)

Quantum regime

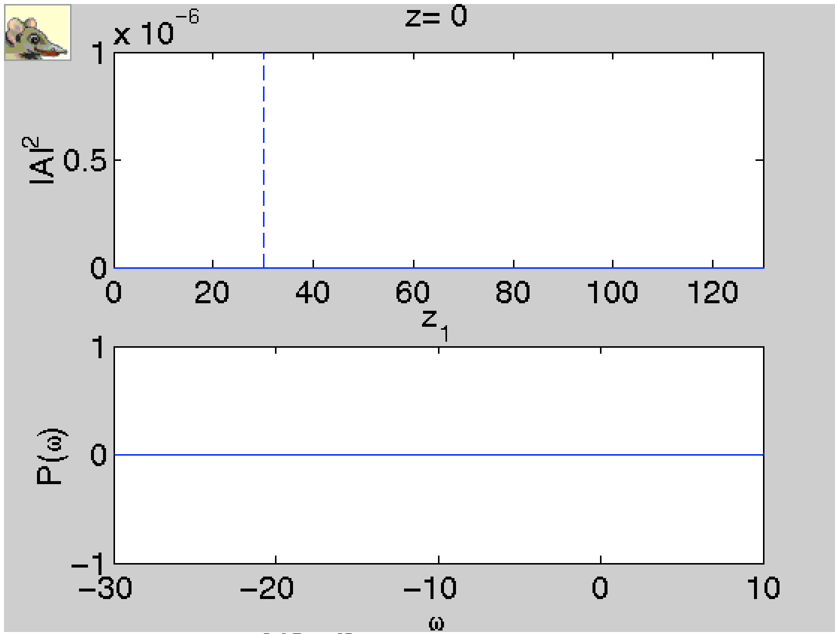


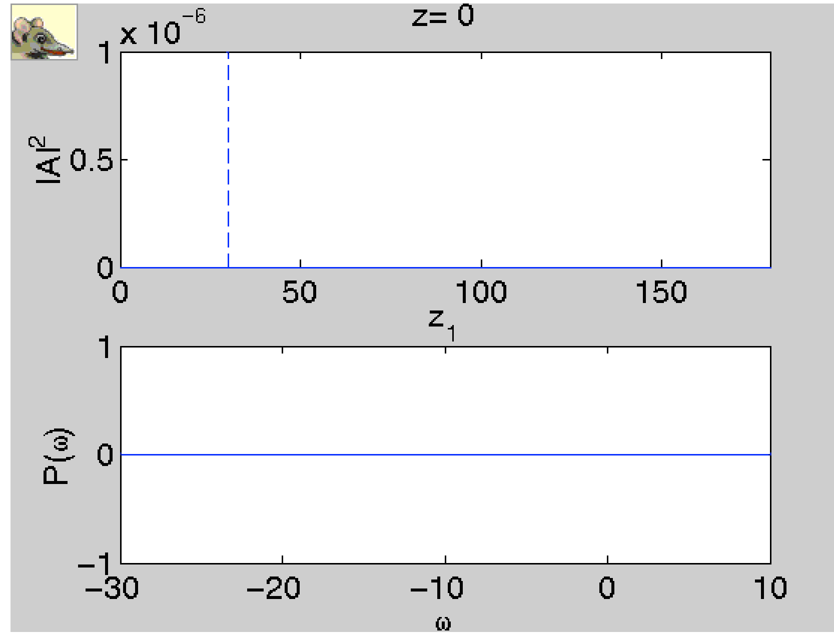
Classical regime



Classical behaviour : both n<0 and n>0 occupied

Quantum behaviour : sequential SR decay, only n<0


z= 0
x 10^-6
|A|^2
z_1
P(ω)
ω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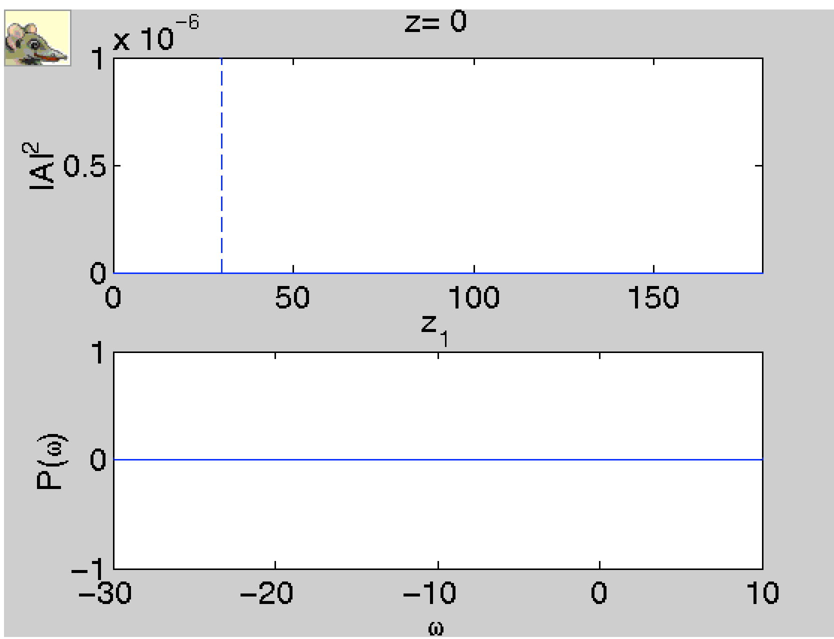
x 10^-6
|A|^2
z_1
P(ω)
ω


z= 0
x 10^-6
|A|^2
z_1
P(ω)
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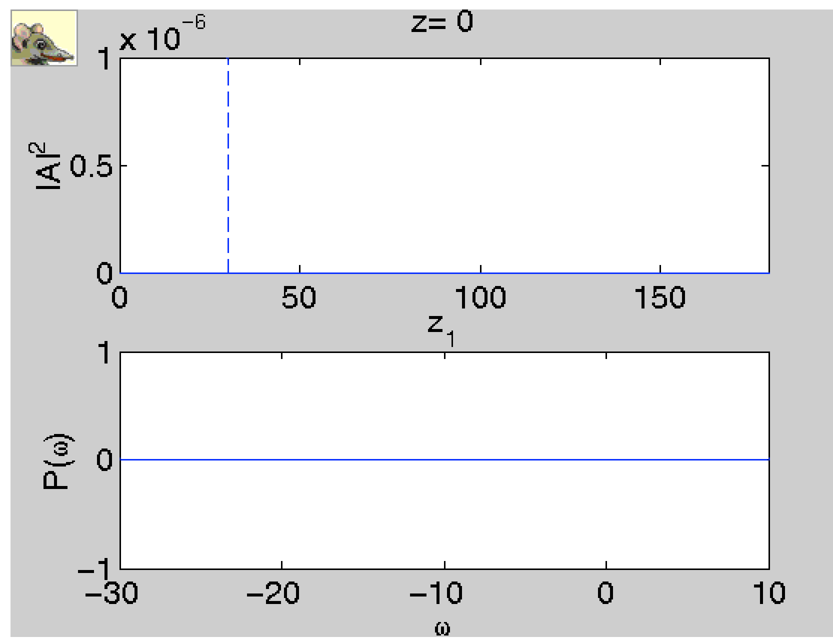
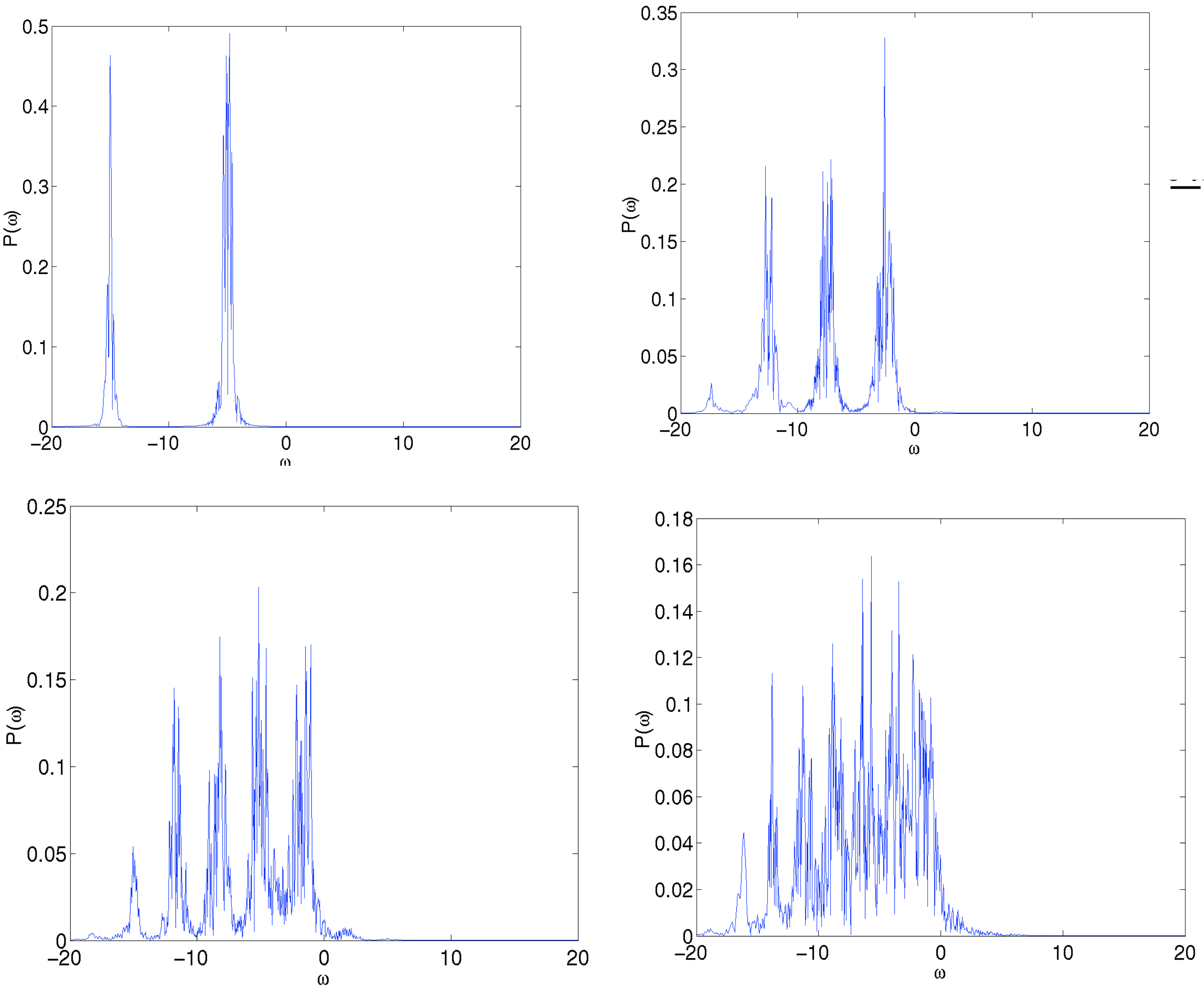

z= 0
x 10^-6
|A|^2
z_1
P(ω)
ω



P(ω)
ω

Conclusions

- Classical description of SASE valid IF
- IF one has quantum SASE: the gain bandwidth decreases as $\sqrt{\ }$ and $\sqrt{\ }$ ⇒ line narrowing, temporal coherence.
- Multiple lines Spectrum:
  - separation , linewidth $\sqrt{\ }$
- Classical limit: increasing separation linewidth → continuous spiky classical spectrum.

For experimental setup see R.B., NIM A 546 (2005) 634,
and proceedings FEL conf 2005

Quantum Free Electron Laser

QFEL

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C. Natoli, L. Palumbo, A. Schiavi[^], A. Tenore
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[^] Dipartimento di Energetica, Universita' di Roma "La Sapienza"

SASE

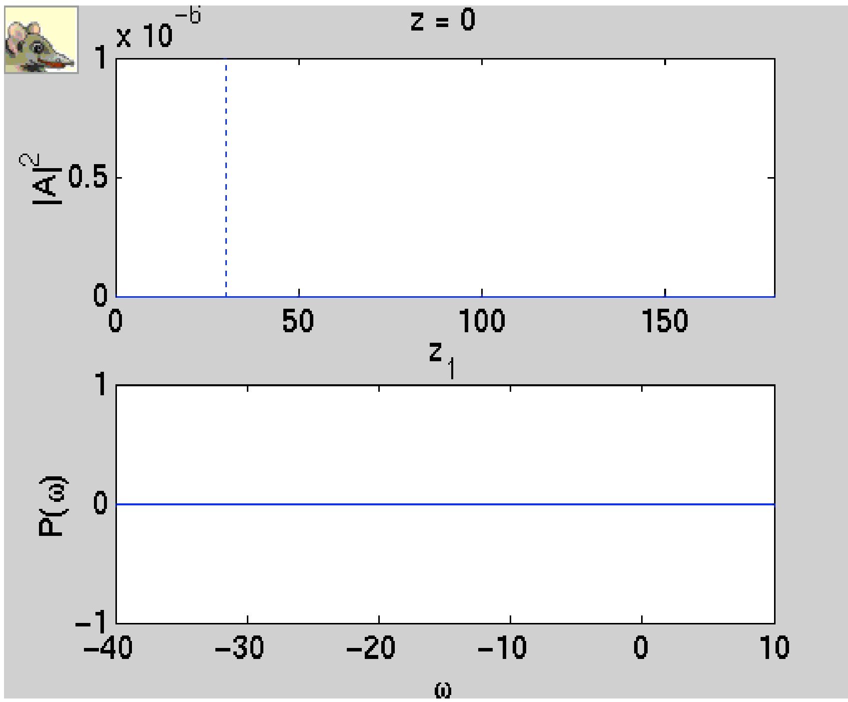
Quantum

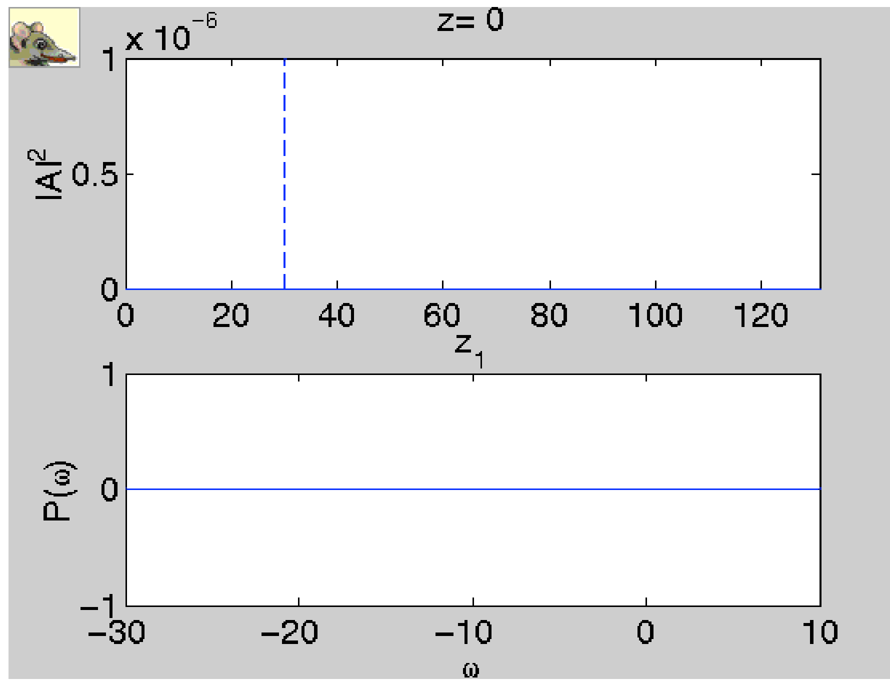
= 0.05

Classical

= 5

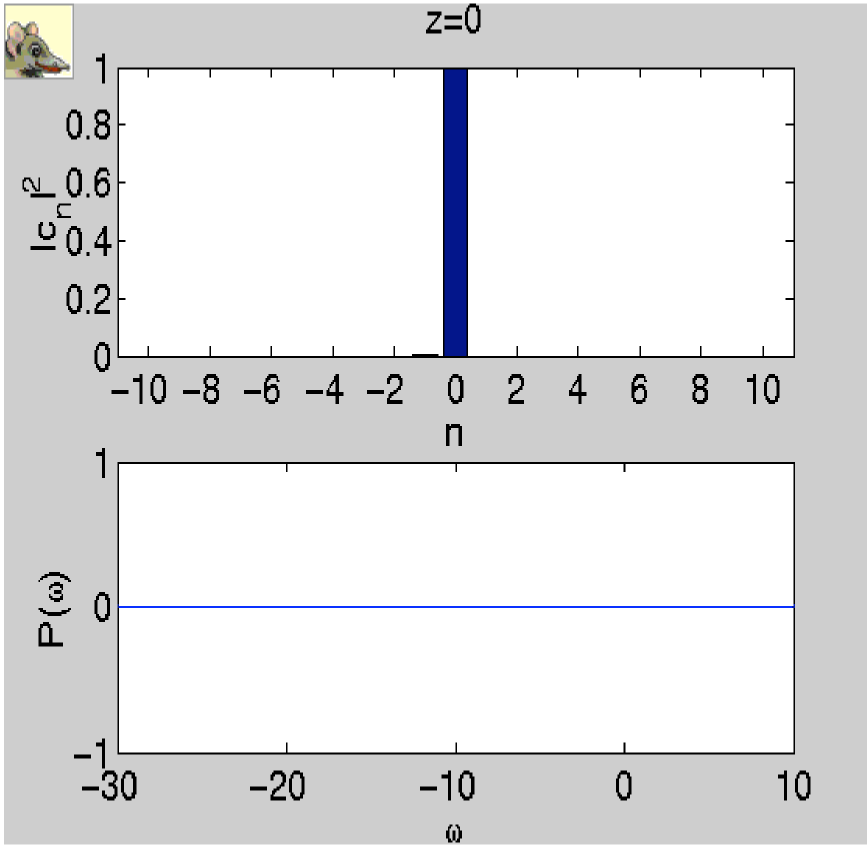
Evolution of radiation time structure in the electron rest frame



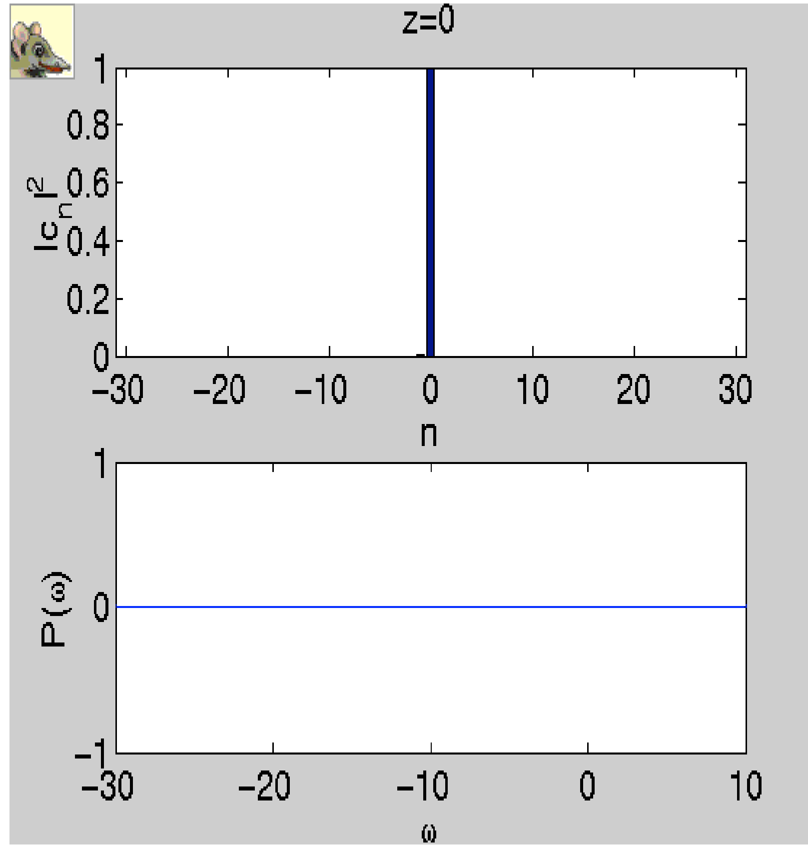


QFEL Model: Momentum distribution and spectrum

Quantum regime

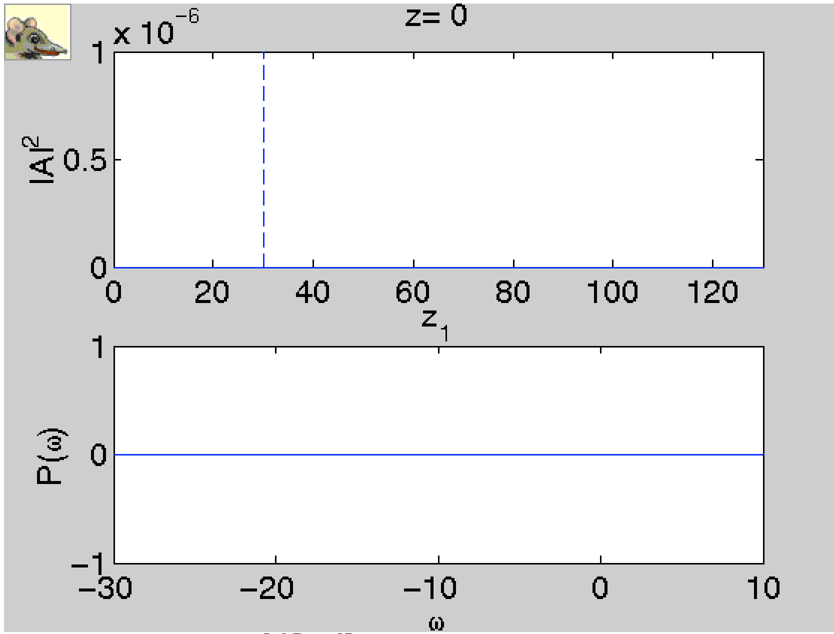


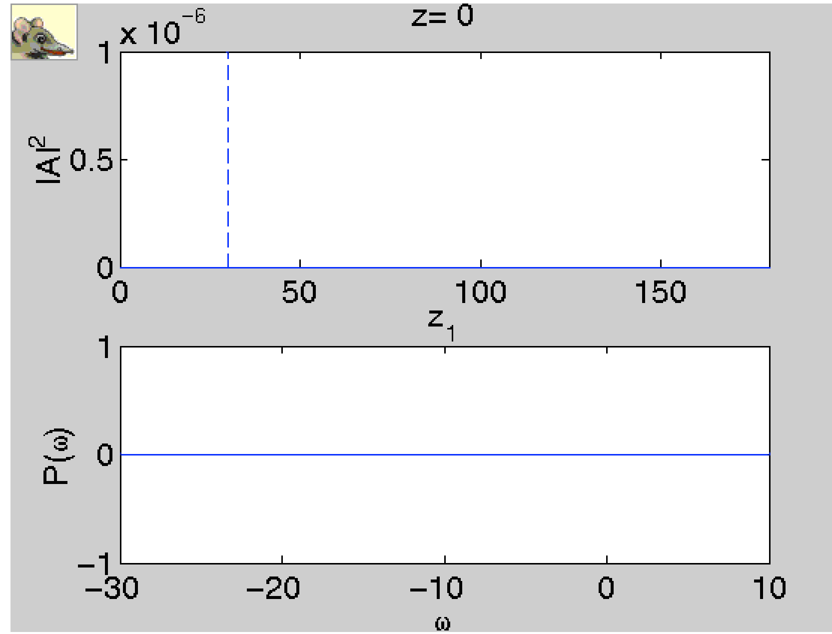
Classical regime

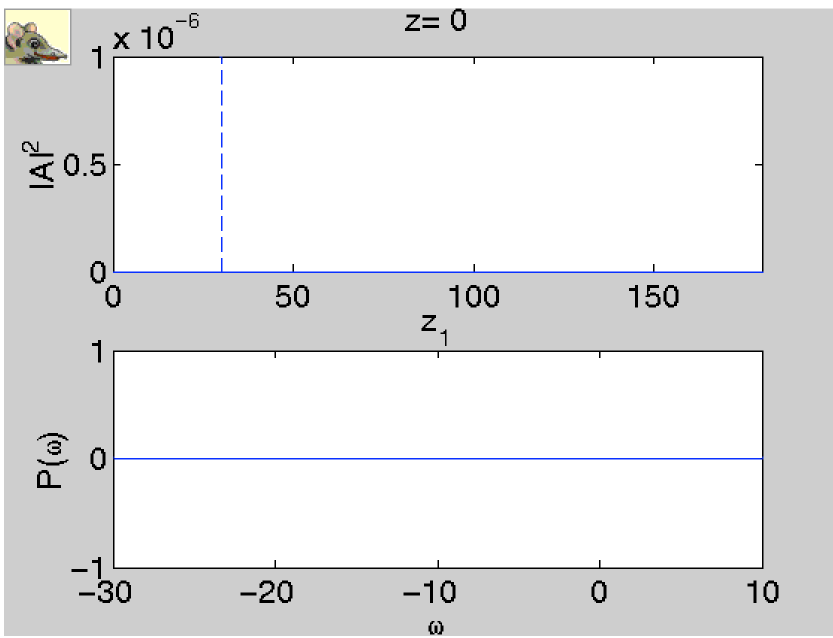


Classical behaviour : both n<0 and n>0 occupied

Quantum behaviour : sequential SR decay, only n<0


z= 0
x 10^-6
|A|^2
1
0.5
0
0 20 40 60 80 100 120
z_1
P(ω)
1
0
−1
−30 −20 −10 0 10
ω


z= 0
x 10^-6
|A|^2
1
0.5
0
0 50 100 150
z_1
P(ω)
1
0
−1
−30 −20 −10 0 10
ω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|A|^2
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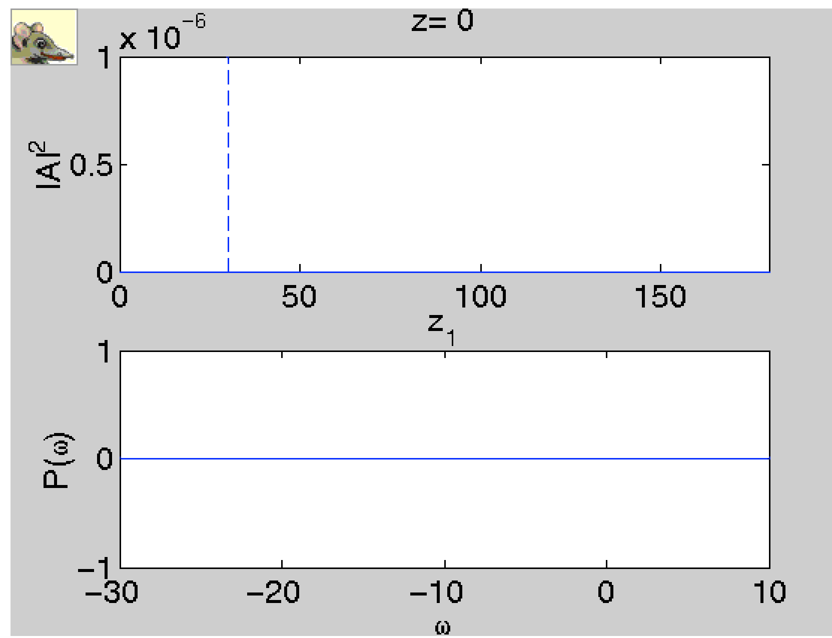
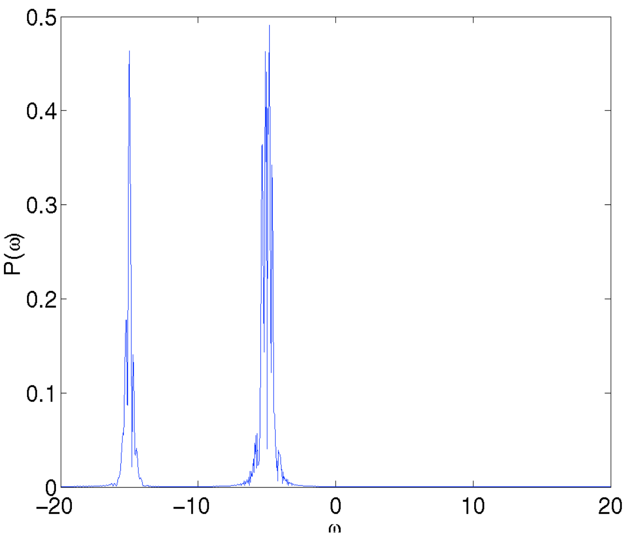
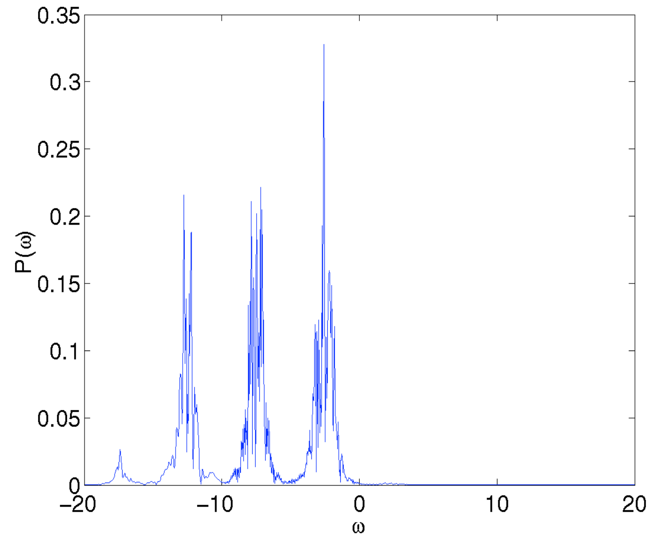
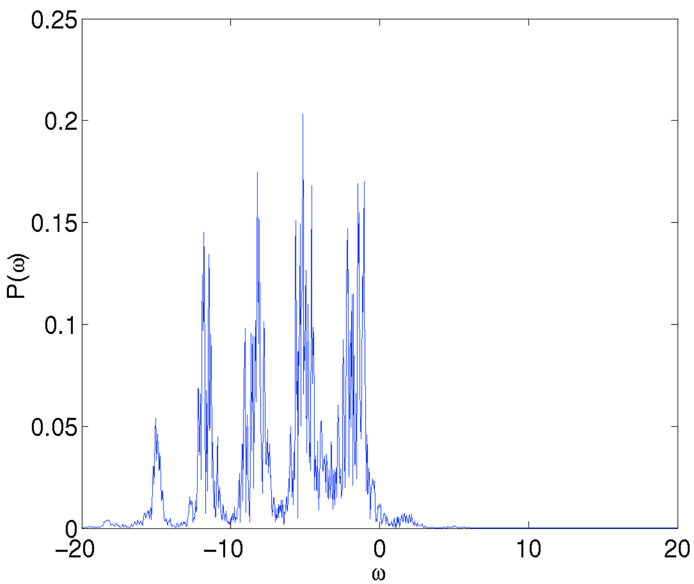
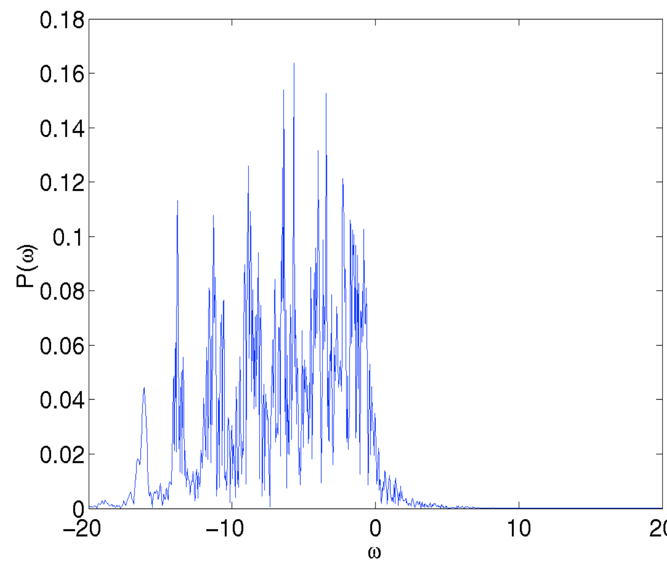
0.5
0
0 50 100 150
z_1
P(ω)
1
0
−1
−30 −20 −10 0 10
ω


z= 0
x 10^-6
|A|^2
1
0.5
0
0 50 100 150
z_1
P(ω)
1
0
−1
−30 −20 −10 0 10
ω

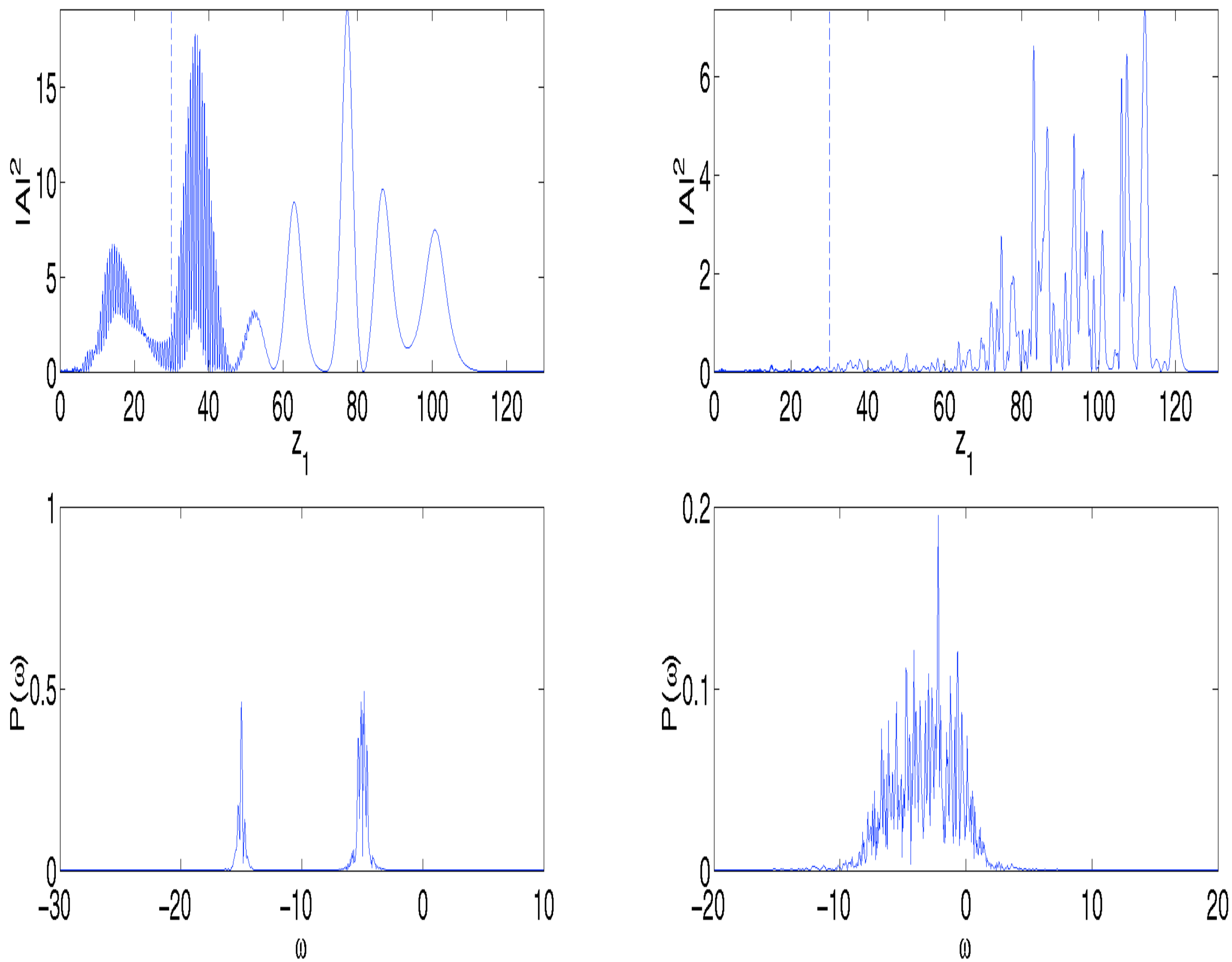

P(ω)
ω
0
0.1
0.2
0.3
0.4
0.5
−20
−10
10
20


P(ω)
ω
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
−20
−10
10
20


P(ω)
ω
0
0.05
0.1
0.15
0.2
0.25
−20
−10
10
20

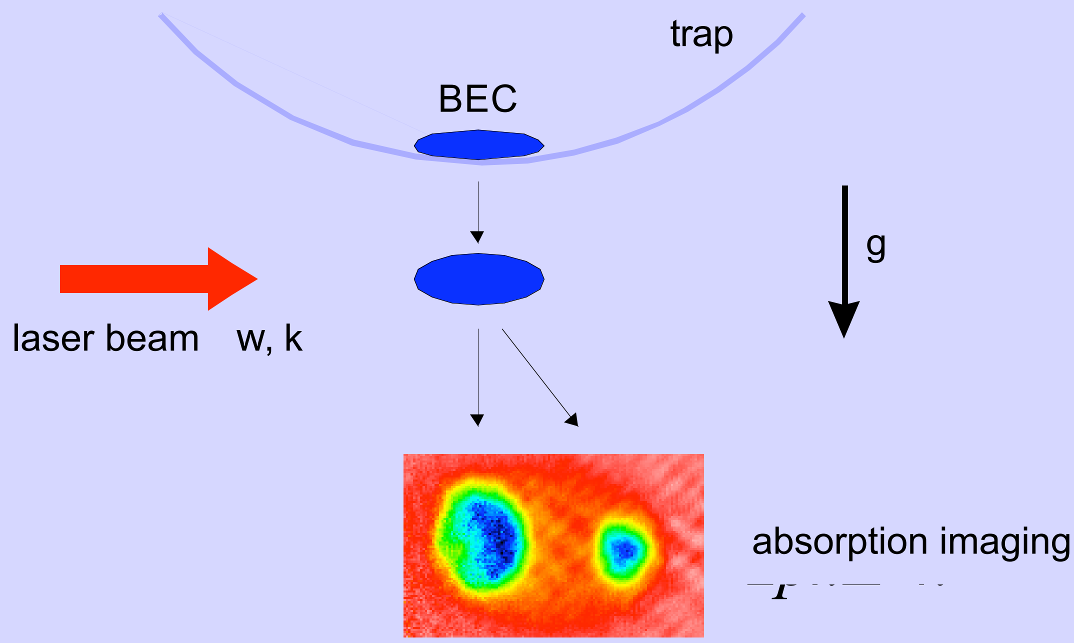

P(ω)
ω
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
−20
−10
10
20

Quantum Classical



Experimental Evidence of Quantum Dynamics – The LENS Experiment

- **Production of an elongated ^{87}Rb BEC in a magnetic trap**
- **Laser pulse during first expansion of the condensate**
- **Absorption imaging of the momentum components of the cloud**



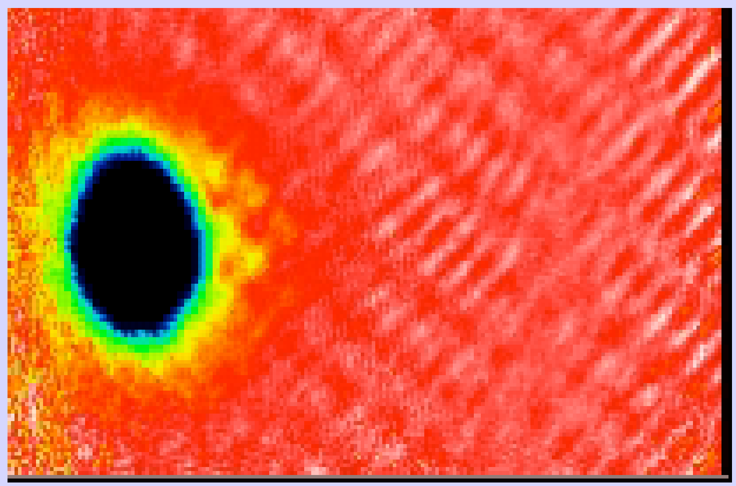
Experimental values:

Δ = 13 GHz
w = 750 μm
P = 13 mW

R. B., F.S. Cataliotti, M.M. Cola, L. Fallani, C. Fort, N. Piovella, M. Inguscio J. Mod. Opt. **51**, 785 (2004) and Optics Comm. **233**, 155(2004) and Phys. Rev. A 71, 033612 (2005)

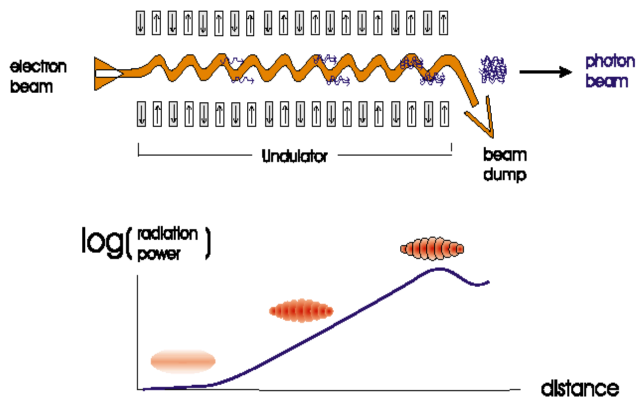
The experiment

Temporal evolution of the population in the first three atomic momentum states during the application of the light pulse.



pump light

n=0	n=-1	n=-2
p=0	p=-2hk	p=-4hk

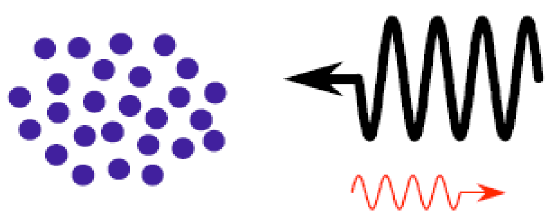


Classical FEL SASE experiments (DESY, SLAC):

- GeV linac (Km) and long undulators (100 m)
- Radiation spectrum broad and chaotic (spikes)
- High cost (10^9 U$) and large dimensions

Quantum FEL SASE:

- quantum purification (monocromatic spectrum)
- must use a laser undulator
- reduced cost (10^6 U$) and compact devise (m)



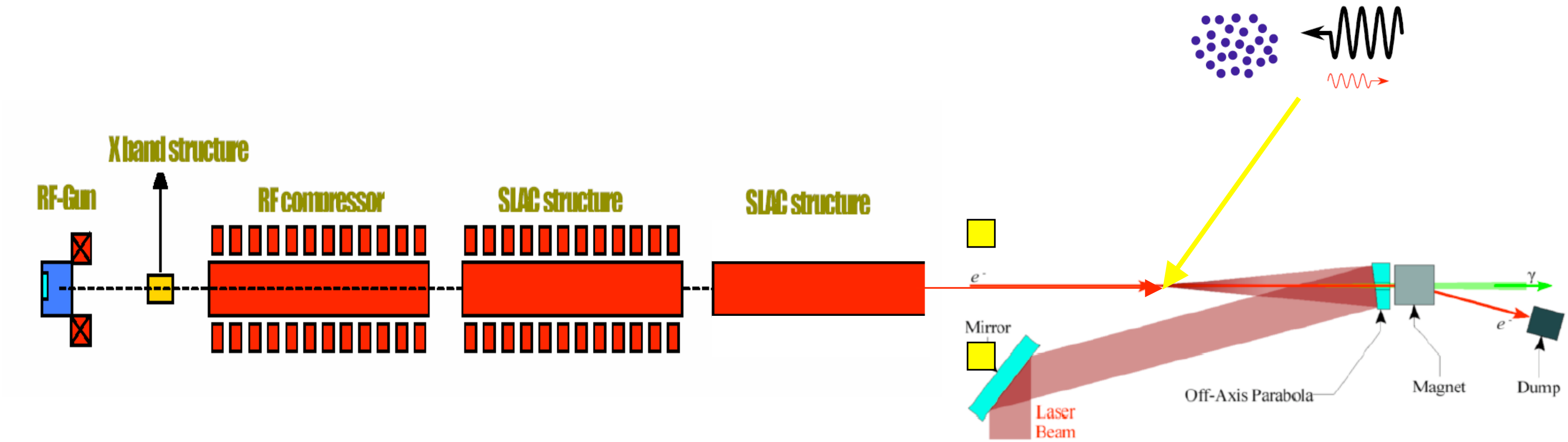
SPARC + FLAME ==> PLASMON X + QFEL

Ingredients of QFEL Project:

- electron beam 15-100 MeV, 100 A , $\varepsilon_n < 2$ mm mrad
- Laser wiggler at 0.8 micron , 10-100 TW (Ti:Sa)

Both under development for SPARC/PLASMON_X

Preliminary parameters list for QFEL



Electron beam

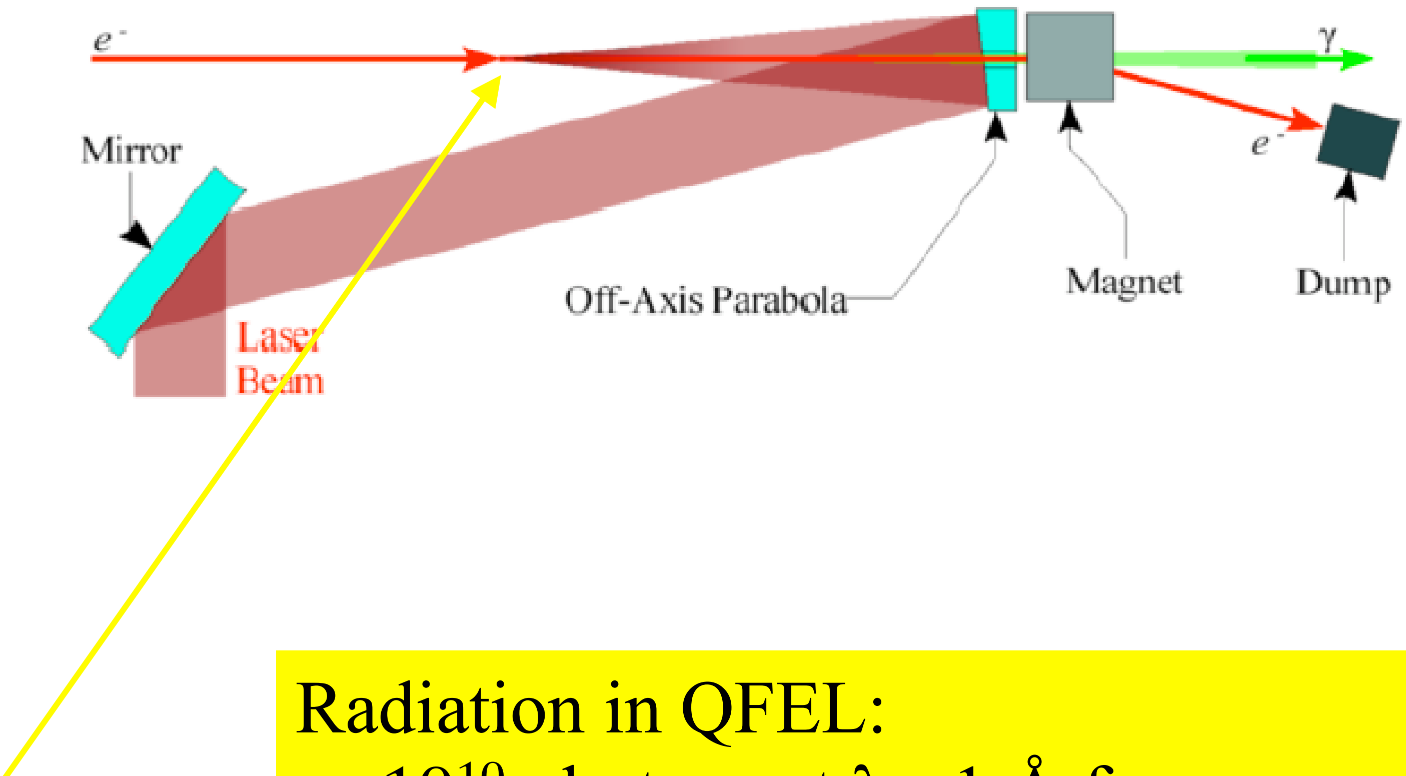
E [MeV]	20
I [A]	40
ε_n [μm]	1
$\delta\gamma/\gamma$ [%]	0.03
β^* [mm]	0.5-1

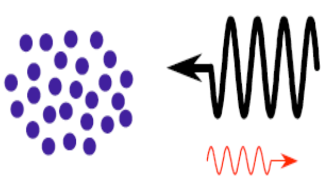
Laser beam

λ [μm]	0.8
P [TW]	1
E [J]	4
w_o [μm]	5-10
Z_r [μm]	80-300

QFEL beam

λ_r [A]	1.7
P_r [MW]	0.3





Radiation in QFEL:

- $\sim 10^{10}$ photons at $\lambda \sim 1$ Å for some ps
- monocromaticity ($\Delta\lambda/\lambda < 10^{-4}$)

QFEL

Preliminar studies are based on a 1D quantum model.

It is necessary to extend the analytical/numerical study of the 1D model to a 3D quantum model in order to demonstrate the feasibility of a Quantum SASE experiment at INFN-LNF

SPARC

INFN Istituto Nazionale di Fisica Nucleare

ENEA

ULTRAS INFM
NATIONAL LABORATORY FOR ULTRAFAST AND ULTRAINTENSE OPTICAL SCIENCE

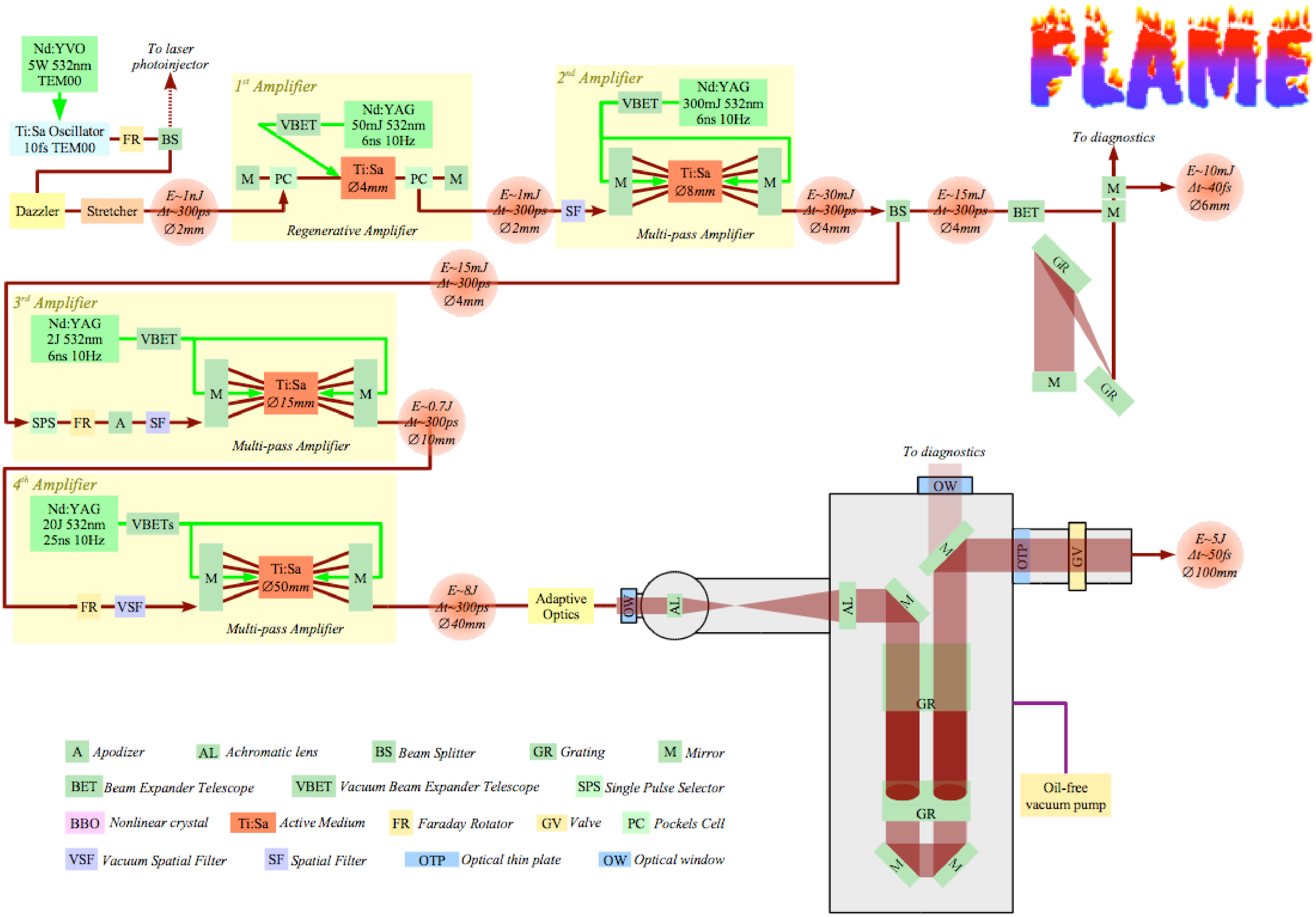
La Sapienza
Università degli Studi di Roma

Stanford Linear Accelerator Center

UCLA

elettra
Synchrotron Light Laboratory

The Frascati Laser for Acceleration and Multidisciplinary Experiments



laser pulses: 50 fs, 800 nm >100 TW @10 Hz


Thank you and see you in my office in Brazil

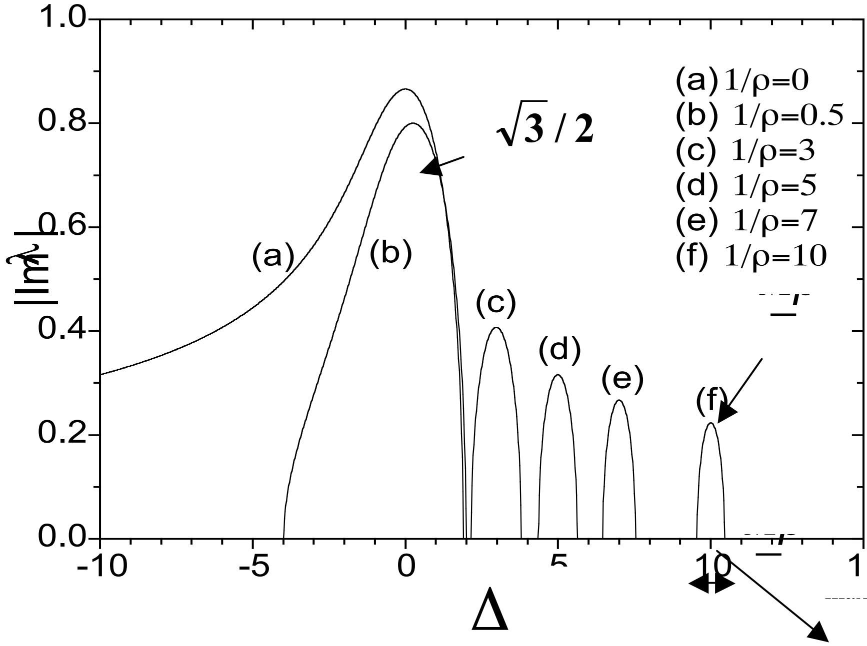
About angular momentum and linear momentum

Carruthers and Nieto, Rev. Mod. Phys. 40, 4411 (1968)

Linear Theory

Instability in the linear regime Q.M.

$$A \propto e^{i\lambda\bar{t}}$$

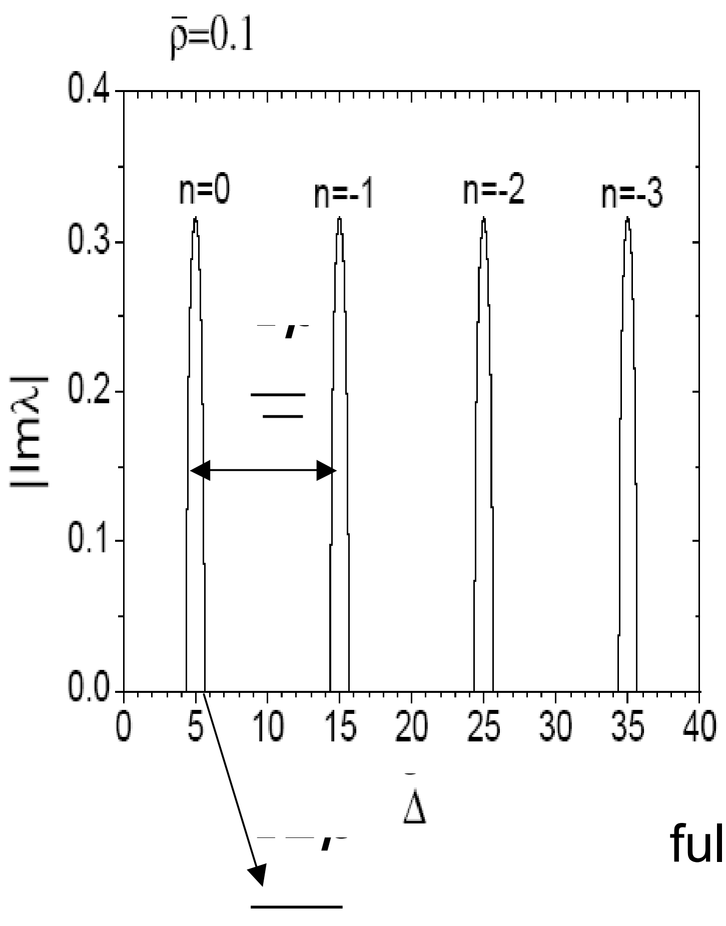


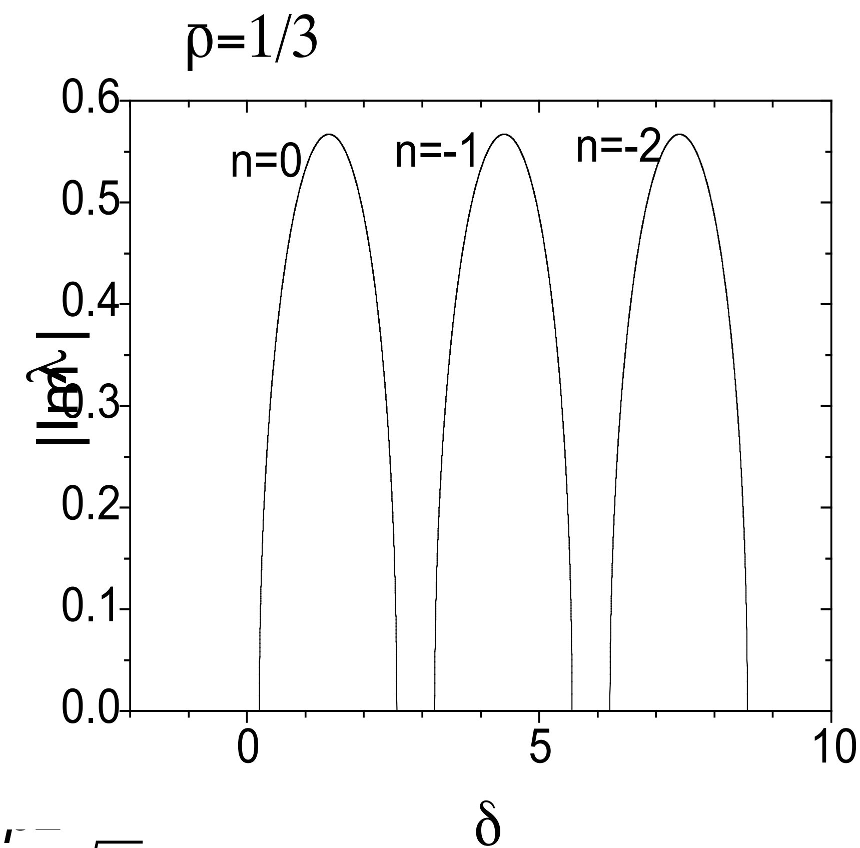
Classical limit (a)

In the quantum regime $\bar{\rho} \leq 1$

Gain bandwidth

$$A \propto e^{i\lambda\bar{t}}$$





full width $\sqrt{\ }$

Continuous limit $\sqrt{\ }$
(unpublished)

Quantum limit : discrete resonances as in a cavity

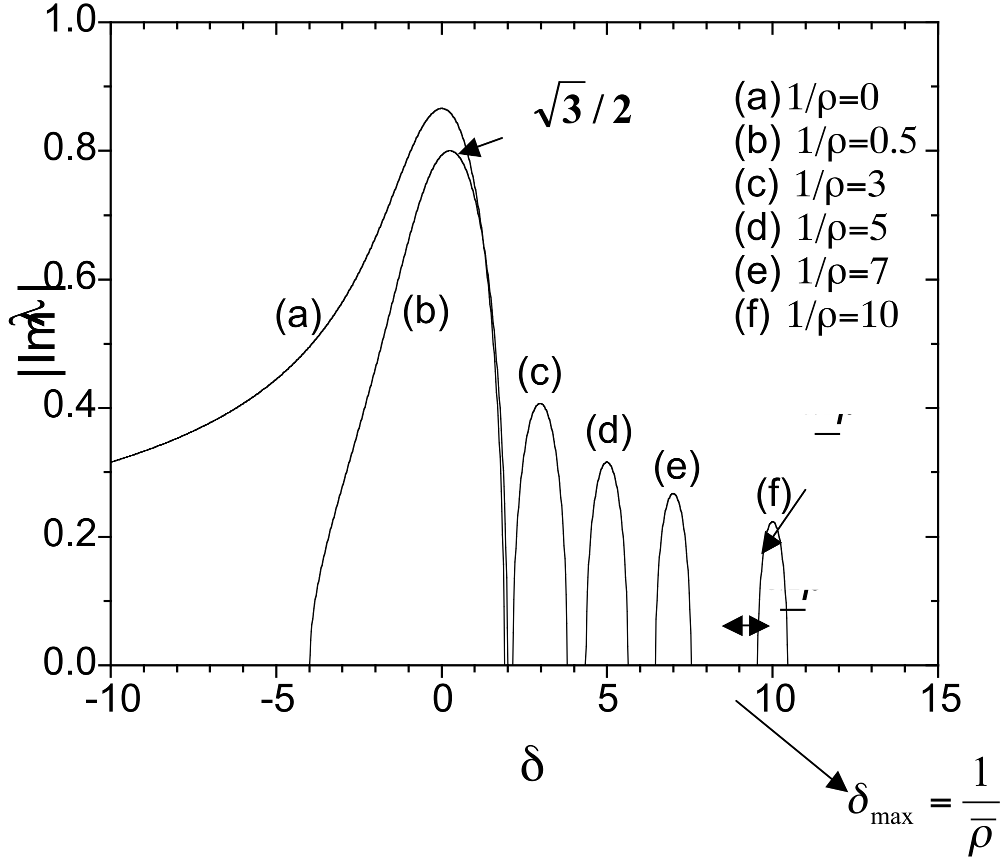
Instability in the linear regime Q.M. $(\lambda-\delta)\left(\lambda^2-\frac{1}{\bar{\rho}^2}\right)+1=0$

$A \propto e^{i\lambda\bar{t}}$

Classical limit (a)

In the quantum regime $\bar{\rho} \leq 1$

Gain bandwidth



$\delta = \frac{mc(\gamma_0-\gamma_r)}{\hbar k\bar{\rho}}$ FEL

$\delta = \frac{\omega_p-\omega}{\bar{\rho}\omega_R}$ CARL

Quantum Purification and Multiple line Spectrum

- In the quantum regime the gain bandwidth decreases as [illegible] → line narrowing.
- Spectrum with multiple lines. When the width of each line becomes larger or equal to the line separation, continuous spectrum, i.e., classical limit. This happens when [illegible] i.e. [illegible]

The Wigner function for linear and momentum variable

The rotational Wigner function

J. P. Bizarro, PRA, 49, 3255-3275 (1994), J. Zac, J. Phys. A, 37, L617 (2004)

$$\eta = \varepsilon_1^2 \varepsilon_2 \varepsilon_n$$

$$N = \tau I/e$$

$$P = h\nu I/e$$

Resonance condition in the quantistic FEL

–

4 2.7 3

10^{10} 10^{10}

$\sqrt{\ }$

Emittance limitations

If radiation beam must contain e-beam:

If e-beam must contain radiation beam:

- Fully quantized model for CRL

- Linear quantum theory and SS instability:
- Photon statistics and entanglement

- Uncertainty relation for p and θ with periodic boundary conditions (wrong and correct)

- Fundamental limitation for bunching and energy spread

- Minimum Uncertainty State for p and θ

- The usual Wigner function and the “rotational” Wigner function
- Open problems

POSSIBLE EXPERIMENTAL PARAMETERS