Numerical Challenges for FEL Simulations

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Undulator Modeling

- ☐ Advanced Elements:
  - Apple-Type Undulator.
  - Electron Bypass (Compressor, optical klystron, bunch shifter)
  - Radiation Bypass (focusing elements, monochromators)
  - RF undulators
  - Waveguide modes for long-wavelength FELs
  - Oscillator configuration
- ☐ Adaptive Integration step sizes to allow for exact configuration (element length)
- ☐ Dual field modeling for heterogeneous undulator types (apple-type, mix of planar and helical).

Standard Time-discretization

- ☐ Par-axial Equation:

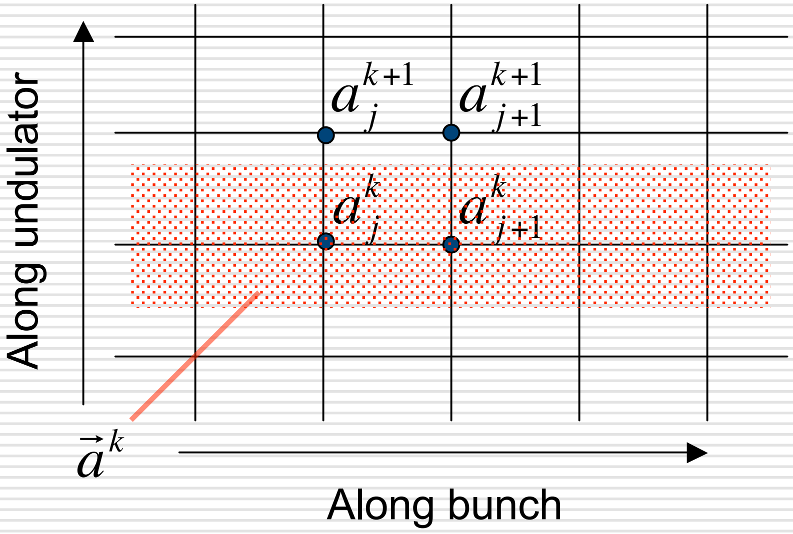
$$\left[\vec{\nabla}_\perp^2 + 2ik\left(\frac{\partial}{\partial z} + \frac{\partial}{c\,\partial t}\right)\right]\vec{A} = \vec{S}$$

- ☐ Discretization in *z* and *t*

$$\frac{\partial}{\partial z}\vec{A} \Rightarrow \frac{a_j^{k+1} - a_j^k}{\Delta z}$$

$$\frac{\partial}{c\,\partial t}\vec{A} \Rightarrow \alpha\frac{a_{j+1}^k - a_j^k}{\Delta t} + (1-\alpha)\frac{a_{j+1}^{k+1} - a_j^{k+1}}{\Delta t}$$

$$\vec{\nabla}_\perp^2\vec{A} \Rightarrow \alpha L\vec{a}^k + (1-\alpha)L\vec{a}^{k+1}$$



Steady-state model: $a_j^k = a_{j+1}^k$

Standard-discretization (cont')

- To avoid to have the whole field in memory to solve the equations, the problem is solved in a quasi-steady-state method.
  - Assume quasi-periodic boundary condition: ($a_j \sim a_{j+1}$)
  - Use steady-state solver ($a^k \rightarrow a^{k+1}$)
  - Advance radiation field ($a_j \rightarrow a_{j+1}$)
- Limitation:
  - Gain reduction for frequencies close to Nyquist frequency
  - No backward propagation of information
  - No coherent start-up effects for short bunches

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CPU and Memory Demands

Radiation field sampled every few wavelengths	~ 20000 @ 1Å
About 100 modes per transverse dimension	~10000 for 2D Grid
2(n+1) particle per 'beamlet' for quite loading and correct shot noise up to *n*th harmonic	4 for fundamental
~1024 beamlets to achieve smooth distribution.	~ 4096 at 1 Å

~10^8 particles and gridpoints

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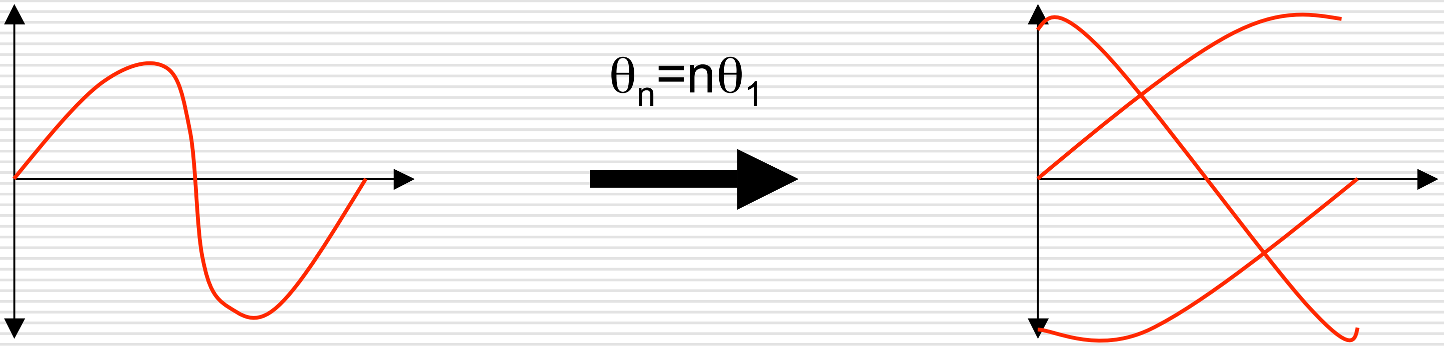
HGHG Simulation I

- ☐ Harmonic up-conversion:



$\theta_n=n\theta_1$

- ☐ Converted phase space distribution has to be cloned to preserve frequency bandwidth:
  - Phase space distribution is not physical (large energy spread)
  - Continuous coupling in model in contrast to uneven filled buckets in reality

HGHG Simulation II

- Frequent switch between FEL interaction and beam transport through chicanes.
- So far only simplified beam optics in FEL codes, no bending system or coherent radiation effects
- Relative shift between radiation beam and electron beam
- Possible optical elements for radiation beam transport.

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Phasespace Loading

- For *M* macro particle, modeling *N* electrons:
  - Correct statistic: $$\left\langle |B|^2 \right\rangle \equiv \left\langle \left| \frac{1}{M} \sum_{m=1}^{M} e^{i\theta_m} \right|^2 \right\rangle = \frac{1}{N}$$
  - No energy correlation: $$\frac{d}{dz}\left\langle |B|^2 \right\rangle = 0$$
  - Higher harmonics: $$\left\langle \left| \frac{1}{M} \sum_{m=1}^{M} e^{in\theta_m} \right|^2 \right\rangle = \frac{n}{N}$$
- Complicated Algorithm to ensure correct statistic, some bound to a reference wavelength, making adjacent slices disjoint (though supporting the quasi-steady-state algorithm of time-dependent simulations)

Spontaneous Radiation

- ☐ SASE Simulations have a cut in frequency and divergence. Though it covers all mode, relevant for FEL, the total spontaneous power is defined by the numerics
- ☐ For complete modeling of the signal the spontaneous spectrum has to be calculated independently, typically based on Lienard-Wichert potentials. For full angular distribution, calculation as time consuming as FEL simulations.
- ☐ Removal of spontaneous part in the FEL simulation and then add the ‘pure’ FEL signal to the spontaneous signal.

External Effects

- Wakefields
  - Externally supplied, because internally calculation would prevent the simulation of bunch subsections.
  - Automatic calculation from particle distributions in start-end simulations.
- CSR in Undulator / Magnetic Chicane
  - With increasing beam current and current modulation this effect might influence the FEL operation.
- Spontaneous Radiation
  - Energy losses easy to calculate
  - Quantum Fluctuation might introduce phase-energy correlation in simulation, thus, introducing errors in the micro bunching.

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Conclusion & Outlook

- ☐ Most challenges arise in the time-dependent calculations and the wide-spread of FEL types (SASE FEL, HGHG, Two stage FEL with manipulation of electron beam and/or radiation field)
- ☐ Advanced methods for phase space loading, though number of macro particles are getting close to number of electrons.
- ☐ A massive parallel approach (~1000 nodes with 1 GByte memory) would render most problems simple to solve. It would also allow to modeled more complex undulators and to use more advance solver.

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