Wrinkling, crumpling, folding, and all that:

Pattern formation on thin sheets

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The complex morphologies of thin sheets consist of wrinkles, crumples, folds, creases, and blisters. These descriptive words may sound lucid – but do they carry any quantitatively distinguishable content?

Following the classical approach of pattern formation theory, we seek to impart a universal meaning to these modes of deformation as distinct types of symmetry-breaking instabilities of a flat, featureless sheet. This idea motivates us to consider the general problem of an *axisymmetric stretching* of a sheet. A familiar realization of this problem is the "map maker's conflict": projecting a flat sheet onto a foundation of spherical shape. Another representative realization is the Lame' set-up: exerting a radial tension gradient on a sheet, which may be free-standing or resting on a solid or liquid foundation.

Capillary forces provide a natural tool to study these and other realizations of the axisymmetric stretching problem. Furthermore, the singular behavior of sheets as their thickness becomes exceedingly small appears to generate a new playground for unexplored capillary effects. In this talk I will describe some key experiments in which capillary forces are used to probe the basic instabilities of stressed sheets, and some lessons drawn from our observations. I will introduce a set of *morphologically-relevant parameters* that underlie the development of complex patterns in these experiments, and will show how wrinkling, crumpling, and possibly other deformation types can be understood as primary and secondary instabilities in a universal phase space spanned by those parameters.