STOCHASTIC METHOD FOR THE SIMULATION OF PLASMA KINETIC WITH COULOMB COLLISIONS.

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Collective kinetic effects and Coulomb collisions are essentially important in a great variety of plasma physics problems such as laser and particle beam interaction with plasma, shock waves and plasma expansion, plasmas heating, and many other phenomena. At present the simulation of nonlinear plasma phenomena in hydrodynamic or collisionless limits is a conventional and widely used approach, but the simulation methods for electromagnetic processes in a collisional plasma have not been developed sufficiently. The problem is that the Fokker-Planck equation for the collisional plasma dynamics is very difficult as regards either analytic or direct numerical investigation.

An effective approach to overcome these difficulties is the method of stochastic differential equations [1]. It is known that the equation of Fokker-Planck and Langevin equation are the alternative methods for the description of Markovian diffusion process $\vec{v}(t)$. Thus, the starting point of our work is the stochastic equivalence of these methods, i.e. we are looking for the nonlinear Langevin equation: $dv_i/dt = F_i + \Delta_{ik}\xi_k$, which describe the particle motion in collisional plasma. Here F_i, Δ_{ik} are the deterministic functions and $\vec{\xi}(t)$ is the random white noise with the following characteristics: $<\vec{\xi}(t)> = 0$, $<\xi_i(t+\tau), \xi_k(t)> = \delta_{ik}\delta(\tau)$. So, our purpose is to derive this equation and to generalize the well known PIC-method for the case of collisional plasma.

The Langevin approach can be simply applied when the integral of collision can be written as a function of velocity. But, as for the description of Coulomb collisions by the Landau integral of collisions, the kinetic equation for the f_α has an integral dependence on the f_β (where f_α and f_β are the distribution function of particles of species α and β respectively). In general case the linearization of integral of collisions was proposed in Ref.[1].

The method developed is applied directly for the diffusion model of collision integral:

$$St[f_\alpha] = \frac{\partial}{\partial v_i}\nu\left[v_i f_\alpha + \frac{T_\beta}{m_\beta}\frac{v_i v_k}{v^2}\frac{\partial}{\partial v_k} f_\alpha\right],$$

where $v_i = v_i - u_i$; $T = T_b(\vec{x}, t)$ and $\vec{u}(\vec{x}, t)$ are the temperature and average velocity of particles β, ν is the collision frequency. The corresponding stochastic differential equation is derived and numerical algorithm is developed and tested.

The method is also applied directly when the approximate dependence of f_b on the velocity is known. So, the Langevin equation was derived for the "quasi-Maxwellian" distribution [1]: $f_b(v) = n_b(m_b / 2\pi T_b)^{3/2} \exp\left(-m_b(\vec{v} - \vec{u})^2 / 2T_b\right)$, where $n_b(\vec{x}, t)$, $T = T_b(\vec{x}, t)$ and $\vec{u}(\vec{x}, t)$ were calculated independently. The equation derived is exact in the asymptotic cases (for example, in the case, when the collisions with a background are dominant), and in general case this equation can be regarded as an improved model. The test problems of beam velocity relaxation and temperatures relaxation in the two component plasma were considered and a good agreement with a theoretical solutions was obtained.

There is a wide range of plasma physics problems for which the integral of collisions can be written as an explicit function of velocity. This is the case when the collisions between the electrons and relatively cold ions are dominant (this assumption corresponds to the so-called Lorentzian plasmas). In this case the integral of collisions has the form:

$$St[f] = \frac{a}{2} \frac{\partial}{\partial v_i} \frac{v^2 \delta_{ik} - v_i v_k}{v^3} \frac{\partial}{\partial v_k} f \qquad (1)$$

where: $a = Z \Lambda \omega_{pe}^4 / 4\pi n_e$, ω_{pe} and n_e are the plasma frequency and electron concentration, Ze is the ion charge, Λ is the Coulomb logarithm. The corresponding Langevin equation is [2]:

$$\frac{d\vec{v}}{dt} = -\Omega(v)\left[\vec{v} \times \left[\vec{v} \times \vec{x}\right]\right] + \vec{F}_L \qquad (2)$$

where $\Omega(v) = \sqrt{\frac{a}{|v|^5}}$ and $\vec{F}_L = \frac{e}{m}\left(\vec{E} + \frac{1}{c}\left[\vec{v} \times \vec{B}\right]\right)$.

The account of ions mobility leads to the obvious substitution $\vec{v} \rightarrow \vec{v} - \vec{u}_{ion}$ in the stochastic term of Eq.(2), where $\vec{u}_{ion}$ is the hydrodynamic velocity of ions. The stochastic equations of plasma particles motion (2) and $d\vec{x} / d\vec{t} = \vec{v}$ along with the Maxwell equations make up a complete system which is alternative to the plasma description in terms of distribution function. Based on the symmetrical stochastic integral we can write the following expressions for finite differences:

$$\vec{v}^{+} = \vec{v}^{n} + \frac{e\Delta t}{2m} \vec{E}^{n+1/2}$$

$$\vec{v}^{n+1} = \vec{v}^{n} + \frac{e\Delta t}{m} \vec{E}^{n+1/2} + \left[\frac{\vec{v}^{n} + \vec{v}^{n+1}}{2} \times \left[\frac{e\Delta t}{mc} \vec{B}^{n+1/2} - \Omega(v^{+})\left[\vec{v}^{+} \times \Delta\vec{h}\right]\right]\right]$$

where $\Delta\vec{h} = \int \vec{x}(t)\,dt = \vec{g}\sqrt{\Delta t}$ is an increment of diffusion process in the time step Δt and the random value $\vec{g}$ has a normal distribution with mean zero and standard deviation one. We used these expressions for the collisional PIC-code creation and solved the test problem of Lorentz plasma conductivity in which a good agreement with the theoretical dependence of conductivity on plasma parameters was obtained.

Using the approach developed, we considered the problem of the short laser pulses interaction with overdense Lorentzian plasmas [2]. The temperature dependence of the absorption coefficient is calculated in the wide range of plasma parameters including both collisional and collisionless regimes. For the case of strongly nonlinear regime of laser-plasma interaction the strong longitudinal electric field at the plasma boundary is observed. The generation of a high frequency radiation as well as the formation heated electrons is investigated. It is shown that the electron-ion collisions behave like a filter - they decrease the fraction of heated particles with relatively low energies and practically do not influence the more heated electrons.

We conclude by mentioning that this approach is applicable to various problems in collisional plasma simulations.

[1] Ivanov M.F., Shvets V.F. USSR J. of Comp. Math. and Math. Phys. 20 (1980) 682.

[2] Cadjan M.G., Ivanov M.F. Phys.Lett.A 236 (1997) 227.