AN OVERVIEW OF THE NIMROD CODE

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Modern tokamak plasmas are hot enough to be virtually collisionless. Analysis of the nonlinear dynamics of these plasmas for long time scales, for example the evolution of neoclassical magnetic islands and their effect of β-limits, therefore requires extension of the usual resistive MHD model to include additional "non-ideal" effects. These effects include the Hall term, anisotropic thermal transport, different ion and electron temperatures, finite electron inertia, neo-classical contributions to the stress tensor, pressure contributions from energetic species, etc. The details of the non-circular geometry of these devices are also essential for accurate modeling.

One way to formulate such a model is to directly solve the nonlinear, two-fluid dynamical equations simultaneously with Maxwell's equations. The only assumption is that the ions and electrons have Maxwellian distribution functions and can thus be described as interacting fluids. This approach misses purely kinetic effects such as those related to Landau damping and finite Larmor radius, but includes all the cold plasma wave branches such as cyclotron, hybrid, and whistler waves, as well as low frequency Alfvén and sound waves[1]. Both collisionless and resistive reconnection are described by this model. The NIMROD code is presently being developed to solve the equations of such a model in three-dimensional toroidal or linear periodic geometry and time.

In addition to developing a core physics kernel that solves the two-fluid equations in space and time as an initial value problem, the NIMROD Project aims to produce an integrated, user-friendly code system that includes a grid generator, pre- and post-processing analysis capability (including three-dimensional and animated graphical output), and provides interfaces to many of the existing equilibrium, transport, and analysis codes used in the fusion community. User access to NIMROD is accomplished through a flexible, multi-layered graphical user interface (GUI). The NIMROD Project is a self-directed, multi-institutional, interdisciplinary effort that employs modern team management techniques[2].

The NIMROD physics kernel simultaneously solves the two-fluid equations and Maxwell's equations, ignoring the displacement current. We also assume quasi-neutrality, so that $n = n_e = Zn_i$. In this limit, the momentum equation for each species can be added to yield the single fluid momentum equation

$$\rho\left(\frac{\partial \mathrm{v}}{\partial t} + \mathrm{v}\cdot\nabla\mathrm{v}\right) = -\nabla\cdot\mathrm{P}' + \mathrm{J}\times\mathrm{B} \quad , \tag{1}$$

and subtracted to yield the generalized Ohm's law

$$\begin{aligned} \mathrm{E} = &-\underbrace{\mathrm{v}\times\mathrm{B}}_{\text{Ideal MHD}} + \underbrace{\eta\mathrm{J}}_{\text{Resistive MHD}} + \underbrace{\frac{1}{ne}\frac{1-\nu}{1+\nu}\mathrm{J}\times\mathrm{B}}_{\text{Hall Effect}} \\ &-\underbrace{\frac{1}{ne(1+\nu)}\nabla\cdot\left(\mathrm{P}'_e - \nu\mathrm{P}'_i\right)}_{\text{Pressure Effects}} + \underbrace{\frac{1}{\varepsilon_0\omega_{pe}^2(1+\nu)}\left[\frac{\partial \mathrm{J}}{\partial t} + \nabla\cdot\left(\mathrm{vJ}+\mathrm{Jv}\right)\right]}_{\text{Electron Inertia}} \quad . \end{aligned} \tag{2}$$

In Equations (1) and (2), v is the center of mass velocity, ν is the mass ratio, $\omega_{pe}^2 = ne^2/\varepsilon_0 m_e$ is the plasma frequency, and P'_α is the stress tensor for species α written in the center of mass frame, which is related to the individual species pressure by $P'_\alpha = P_\alpha + n_\alpha m_\alpha (v_\alpha - v)(v_\alpha - v)$. The species pressure tensors are decomposed as $P_\alpha = p_\alpha I + \Pi_\alpha$, where p_α is the scalar pressure and Π_α is the stress tenor, which may include viscous and neo-classical effects. The scalar pressure for each species satisfies the energy equation

$$\frac{\partial p_\alpha}{\partial t} + v_\alpha \cdot \nabla p_\alpha = -\frac{3}{2} p_\alpha \nabla \cdot v_\alpha - P_\alpha : \nabla v_\alpha - \nabla \cdot q_\alpha + Q_\alpha \quad , \tag{3}$$

where q is the heat flux and Q_α is a source or sink of energy. The individual fluid velocities appearing in Equation (3) are related to the center of mass quantities by simple transformations. Finally, the density satisfies the continuity equation

$$\frac{\partial n}{\partial t} = -\nabla \cdot (n v_e) = -\nabla \cdot (n v_i) \quad . \tag{4}$$

In the present implementation of NIMROD, the individual species pressures are related by $p_e = p_i = p/2$, where the single fluid pressure satisfies the adiabatic equation

$$\frac{\partial p}{\partial t} + v \cdot \nabla p = -\gamma p \nabla \cdot v \quad . \tag{5}$$

The combined system of Equations (1-5), along with Maxwell's equations, are very stiff. They contain responses ranging from electron cyclotron frequencies ($\Omega_{ce} \sim 1.7 \times 10^{11} sec^{-1}$ in a proposed tokamak) to resistive diffusion ($\tau_R^{-1} \sim 1.4 \times 10^{-5} sec^{-1}$). The low frequency modes of interest to the fusion program evolve on the order of seconds, and in a hot plasma the behavior of these modes can be affected by all the terms in Equation (2). Thus some sort of implicit method must be used to solve these equations.

All the terms on the right hand side of Equation (2) are symmetric, with the exception of the Hall term. We treat this term separately. Equation (1) is explicitly time-differenced as

$$\left(I - f_{visc} \Delta t \nabla \cdot \nu \nabla I\right) \cdot \frac{v^{n+1}}{\Delta t} = \frac{v^n}{\Delta t} - (v \cdot \nabla v)^* - \frac{1}{\rho_0} \nabla p^n + \frac{1}{\rho_0} J^n \times B^n + \nabla \cdot \nu \nabla v^n, \tag{6}$$

and the symmetric part of Equation (2) is implicitly time-differenced as

$$E = -v^{n+1} \times B^* + \eta\left(J^n + f_\eta \Delta J\right) - \frac{1}{ne(1+\nu)} \nabla \cdot \left(P'^n_e - \nu P'^n_i\right) + \frac{1}{\varepsilon_0 \omega_p^2 (1+\nu)} \left[\frac{\Delta J}{\Delta t} + \nabla \cdot (vJ + Jv)^*\right] \quad , \tag{7}$$

$$= E_{imp} + E_{exp} \quad , \tag{8}$$

where $\Delta J = J^{n+1} - J^n$, $0 \le f_\eta \le 1$, and E_{imp} and E_{exp} are the implicit and explicit parts of the electric field, respectively. The implicit electric field can be written as

$$E_{imp} = \frac{\mu_0}{\Delta t} \mathbf{Z}_{imp} \cdot \Delta J \quad , \tag{9}$$

where

$$\mathbf{Z}_{imp} = \left(\frac{c^2}{\omega_p^2} \frac{1}{(1+\nu)} + \frac{\eta f_\eta \Delta t}{\mu_0}\right) I \quad , \tag{10}$$

is the implicit impedance tensor. It contains electron inertia and resistive effects.

Note that the ideal MHD response is contained in the explicit part of the electric field. We treat these terms with a semi-implicit method[3-5] based on the linearized ideal MHD response. We thus write Faraday's law as

$$\frac{\Delta \mathrm{B}}{\Delta t} = -\nabla \times \left(\mathrm{E}_{imp} + \mathrm{E}_{exp} + \mathrm{E}_{SI} \right) \quad , \tag{11}$$

where E_{SI} is a "semi-implicit electric field" given by

$$\mathrm{E}_{SI} = \frac{\mu_0}{\Delta t} \mathbf{Z}_{MHD} \cdot \Delta \mathrm{J} = \frac{1}{\Delta t} \mathbf{Z}_{MHD} \cdot \nabla \times \Delta \mathrm{B} \quad , \tag{12}$$

where

$$\mathbf{Z}_{MHD} = \frac{s_{MHD}\, \Delta t^2}{\mu_0 \rho_0} \left(B_0^2 \mathrm{I} - \mathrm{B}_0 \mathrm{B}_0 \right) \quad , \tag{13}$$

and s_{MHD} is a semi-implicit coefficient. The resulting implicit field equation to be solved is then

$$\nabla \times \left[\left(\mathbf{Z}_{imp} + \mathbf{Z}_{MHD} \right) \cdot \nabla \times \Delta \mathrm{B} \right] + \Delta \mathrm{B} = -\Delta t \nabla \times \mathrm{E}_{exp} \quad . \tag{14}$$

The anti-symmetric Hall term is also treated semi-implicitly by operator splitting and a symmetric semi-implicit operator. The Hall correction to the solution of Equation (14) is

$$\nabla \times \left[\mathbf{Z}_{SI-Hall} \nabla \times \Delta \mathrm{B}^{Hall} \right] + \Delta \mathrm{B}^{Hall} = -\Delta t \nabla \times \left[\frac{1}{ne} \frac{1-\nu}{1+\nu} \mathrm{J}^{MHD} \times \mathrm{B}^{MHD} \right] \quad , \tag{15}$$

where $\Delta \mathrm{B}^{Hall} = \mathrm{B}^{n+1} - \mathrm{B}^{MHD}$, and

$$\mathbf{Z}_{SI-Hall} = \frac{1-\nu}{1+\nu} \frac{s_{Hall} \Delta t}{\mu_0 e n} B_0 \mathbf{I} \quad . \tag{16}$$

The adiabatic pressure equation, Equation (5), is also advanced with a semi-implicit term. The resulting system is unconditionally stable to all normal modes, and the time step is only restricted by the advective CFL condition $|\mathrm{v}| \Delta t / \Delta x < 1$, where v is the flow velocity. This is not severe restriction for fusion problems where the flow speeds are generally smaller than Alfvén or sound speeds.

Spatial discretization in the poloidal plane is accomplished with finite elements and a non-orthogonal Eulerian grid. The toroidal coordinate is treated with a dealiased, pseudospectral representation and Fast Fourier transforms (FFTs). The non-orthogonal poloidal grid consists primarily of unstructured blocks of structured quadrilaterals (RBLOCKs). In the core of the plasma the grid is chosen to be closely aligned with the initial axisymmetric (equilibrium) flux surfaces. (Occasional regridding may be required as the axisymmetric component evolves.) Non-conforming RBLOCKs are joined with blocks of unstructured triangular elements (TBLOCKs). TBLOCKs or degenerate RBLOCKs are also used to represent the region around the magnetic axis, thus avoiding the coordinate singularity inherent in pure flux coordinates. In TBLOCKs, all variables are treated as linear quantities; in RBLOCKs, geometric and equilibrium quasntities are treated as bicubic splines and dependent variables are bilinear.

Extensive numerical analysis of the spatial and temporal discretization described here has been performed. The numerical dispersion relation exactly analogous to the cold plasma dispersion relation in the MHD limit. The formulation is free of the point-to-point "red-black" mode.

The spatial representation used in NIMROD does not assure that the magnetic field remains solenoidal, although a lower bound on the size of $\nabla \cdot \mathrm{B}$ has been calculated as a function of spatial resolution. This effect is controlled by solving the additional equation

$$\left(I - f_{div}\Delta t \nabla \kappa_{div} \nabla \cdot I\right) \cdot \frac{B^{n+1} - B^*}{\Delta t} = \nabla \kappa_{div} \nabla \cdot B^* \quad , \tag{17}$$

where B^* is the solution of Equations (14) and (15), and f_{div} and κ_{div} are user-defined coefficients. It is easy to see that Equation (17) is equivalent to a diffusion equation for $\nabla \cdot B$.

NIMROD has been designed from the beginning to be used on state-of-the-art massively parallel computing platforms. Each RBLOCK or TBLOCK can be assigned to a different CPU. The blocks communicate by message passing. Various choices of pre-conditioner are available for the conjugate gradient (CG) . Favorable performance scalings with increases in problem size and number of CPUs have been obtained on T3E computers.

NIMROD is presently undergoing an extensive validation program. It has been successfully benchmarked against several known linear MHD solutions in toroidal geometry with shaped poloidal cross section, and against stable whistler waves and linear and nonlinear resistive tearing modes in a doubly periodic cylinder. The nonlinear saturation of an n = 1 ideal internal kink mode in toroidal geometry similar to the DIII-D experiment has also been computed. When run with small but finite resistivity, this mode undergoes complete Kadomtsev reconnection and generates secondary islands and extended regions of stochastic magnetic fields. The time evolution of this mode is illustrated in Figure 1a-c.

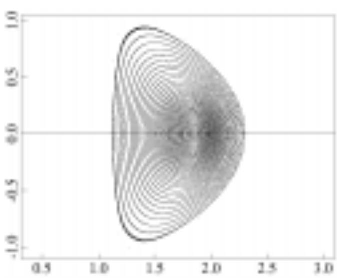

Figure 1a

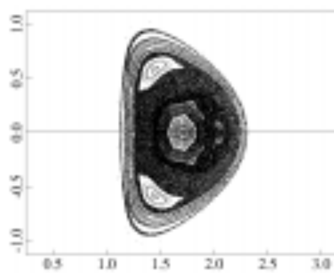

Figure 1b