Quantitative density fluctuation measurements utilizing quadrature reflectometers on DIII-D

G. Wang1,5, W.A. Peebles1, T.L. Rhodes1, G.J. Kramer2, E.J. Doyle1, G.R. McKee3, R. Nazikian2, N.A. Crocker1, X. Nguyen1, L. Zeng1, S. Kubota1 and M.A. VanZeeland4

1 Department of Electrical Engineering and PSTI, University of California, Los Angeles, California, USA
2 Princeton Plasma Physics Laboratory, Princeton, NJ, USA
3 University of Wisconsin, Madison, Wisconsin, USA
4 Oak Ridge Institute for Science Education, Oak Ridge, Tennessee, USA
E-mail: wangg@fusion.gat.com

Received 5 October 2005, in final form 11 April 2006
Published 15 August 2006
Online at stacks.iop.org/NF/46/S708

Abstract
Fixed-frequency reflectometers utilizing quadrature phase detection are employed on DIII-D to study both coherent and turbulent density fluctuations. For coherent fluctuations, the reconstructed phase information successfully identifies MHD/coherent mode activity, e.g. tearing modes, compressional Alfvén eigenmodes, etc. For an $m = 3/n = 2$ tearing mode, a basic 1D phase screen model is applied to infer fluctuation levels, and the result is consistent with the beam emission spectroscopy (BES) measurement. For turbulent fluctuation studies, a 2D full-wave code (Valeo E.J., Kramer G.J. and Nazikian R. 2002 Plasma Phys. Control. Fusion 44 L1) is applied to interpret reflectometry data obtained in the H-mode edge pedestal region to deduce density fluctuation levels. The measurement is simulated using realistic geometry, plasma conditions and antenna patterns. Comparison of the fluctuation level deduced from the 2D code with BES measurement shows reasonable agreement.

PACS numbers: 52.70.Gw, 52.25.Gj, 52.35.Ra

1. Introduction
It has long been recognized that reflectometer density fluctuation measurements have numerous advantages, e.g. highly localized measurements, highly sensitive, flexible spatial coverage, non-perturbative, and they are now anticipated to play a much more enhanced role in the harsh burning plasma environment of next generation devices like ITER. However, although much progress has been made [1], reflectometry measurements have not yet reached their full potential. In particular, extracting an absolute density fluctuation level from the measured microwave electric fields is still an area of active research.

The focus of this paper is to compare density fluctuation levels and spectra derived from recent measurements using a fixed-frequency quadrature reflectometer (65 GHz) with those from beam emission spectroscopy (BES) on the DIII-D tokamak. For coherent fluctuations, the reconstructed phase information successfully identifies MHD/coherent mode activity, e.g. tearing modes, compressional Alfvén eigenmodes (CAE), etc. For an $m = 3/n = 2$ tearing mode, a basic 1D phase screen model is applied to infer fluctuation levels, and the result is consistent with BES measurements. For turbulent fluctuation studies, a 2D full-wave code [2] is applied to measurements in the H-mode edge pedestal region to deduce density fluctuation levels by simulating the reflectometry measurement using realistic geometry, plasma conditions and antenna patterns. Comparison of the fluctuation level deduced from the 2D code with BES measurement shows reasonable agreement.

2. Description of experiment
Two fixed-frequency reflectometers (42 and 65 GHz) utilizing quadrature phase detection have been recently installed on the DIII-D tokamak to study density fluctuations. Figure 1 illustrates a schematic of one of the two similar systems. Half of the microwave power from the source, a Gunn oscillator, is launched into the plasma after passing it through a directional
Received microwave from the plasma boundary. The antennae are standard-gain polyethylene lenses. The measured beam profiles in the E-plane are close to Gaussian distributions. Their radiation patterns are discussed in section 4.

Fluctuations that lie within a wave number range $k_\perp \approx 0–3$ cm$^{-1}$ and has a radial localization of $\sim 1$ cm. In this paper, BES measurements at a similar location will be compared with reflectometer measurements.

Coherent density fluctuation measurements

Coherent density fluctuations have been successfully measured using the derived phase information, including low frequency MHD activity (e.g. tearing modes and toroidicity-induced Alfvén eigenmodes in the frequency range of tens to a couple of hundred kilohertz) and high frequency modes (e.g. CAEs with frequencies of several megahertz). Figure 2 is an example of the detection of high frequency CAEs. Figure 2(b) is a contour plot of the magnetic fluctuation $B$ signal, which measures the time rate-of-change of the magnetic field) spectrum measured by a magnetic loop mounted inside the vacuum vessel close to the plasma, showing the existence of several bursts of bands of CAEs between 3400–3700 kHz in the time window of 870–970 ms. This bursty behaviour is associated with the modulation of the neutral beam injection power. Figure 2(c) is a contour plot of the magnetic fluctuation spectrum measured by a 65 GHz 65 GHz reflectometer using X-mode polarization, (b) $\dot{B}$ from a magnetic loop and (c) neutral beam power time trace.

![A schematic plot of the quadrature reflectometers on DIII-D.](image)

**Figure 1.** A schematic plot of the quadrature reflectometers on DIII-D.

![A contour plot of fluctuation spectra (log scale) from (a) 65 GHz reflectometer using X-mode polarization, (b) $\dot{B}$ from a magnetic loop and (c) neutral beam power time trace.](image)

**Figure 2.** Contour plot of fluctuation spectra (log scale) from (a) 65 GHz reflectometer using X-mode polarization, (b) $\dot{B}$ from a magnetic loop and (c) neutral beam power time trace.

\[
\delta \phi = 2 \sin^{-1} \left( \frac{x_y y_{x+1} + y_y x_{x+1}}{\sqrt{(x_x + x_{x+1})^2 + (y_y + y_{y+1})^2}(x_{x+1}^2 + y_{y+1}^2)} \right),
\]
reflectorometer using X-mode polarization with the reflection layer varying in time in the range of $\rho \sim 0.4$–0.6, where $\rho$ is the normalized minor radius. The figure shows that the system detected the mode activity at these locations. Some differences between the mode characters in figures 2(a) and (b) can be noted, e.g. different numbers of ‘fingers’ in the last burst after 960 ms. This could be related to the fact that they are different measurements of $B$ and density fluctuations, which will be investigated in the future and discussed elsewhere.

Figure 3 is an example of low frequency MHD activity showing the detection of a tearing mode. Figure 3(a) is a contour plot of the poloidal magnetic fluctuation ($B$ signal) spectrum measured by a magnetic loop, showing the existence of an $m = 3/n = 2$ tearing mode ($m$ and $n$ are the poloidal and toroidal mode numbers, respectively) between 20–30 kHz in the time window of 4000–4200 ms, together with its second harmonic between 45–60 kHz. The safety factor of $q = 1.5$ surface was located at $\rho \sim 0.24$. This is an H-mode discharge with frequent ELMing activity as indicated by the $D_\alpha$ signal in figure 3(c). Figure 3(b) is a contour plot of the phase fluctuation spectrum measured by a 65 GHz reflectometer operating with O-mode polarization and the reflection layer at $\rho \sim 0.84$. The figure clearly illustrates detection of the mode. It can be observed that during edge localized modes (ELMs), broadband phase fluctuations increased significantly. The partial reason for this is that as an ELM occurs, the edge density profile is modified and the detection location moves radially inwards/outwards. In addition, edge turbulent fluctuations are significantly modified during an ELM, as seen by the spectrum of $B$ signal shown in figure 3(a).

For the tearing mode shown in figure 3, figure 4(a) plots the RMS level of magnetic fluctuations integrated over the time window of 4000–4200 ms (it was averaged Thomson scattering density profile measurement history is a sliding average over a 10 ms time window with a time resolution of 1.28 ms, which displays a modulation as a function of time. The relative density fluctuation level associated with the mode can be estimated from the phase information according to a basic 1D phase screen model [1] such as $\delta n/n \approx (\delta \phi/(k_0 L_n))$, where $\delta \phi$ is the RMS phase fluctuation level, $k_0$ is the vacuum wave number of the launched wave and $L_n$ is the density scale length. The result is shown as a thick solid line in figure 4(b). Each data point in the curve represents a reflectometer measurement in a time window of 1.3 ms, within which no strong ELM activity (as shown in figure 3(c)) is present. For each point, $\delta \phi$ in each time window was obtained by integrating the mode spectrum shown in figure 3(b) over 22–28 kHz and subtracting the turbulent density fluctuation background which is calculated by integrating the spectrum over adjacent frequencies, i.e. 28–34 kHz. $L_n$ was calculated using the averaged Thomson scattering density profile measurement during ELM-free time slices from 4000–4200 ms (it was measured with 12.5 ms resolution). It can be observed that the overall time evolution (the low frequency modulation) of $\delta n/n$ measured via reflectometry is consistent with the evolution of the RMS level of magnetic fluctuations, i.e. both amplitudes dip at around 4110 and 4210 ms. Direct comparison with BES is also plotted in figure 4(b). These BES measurements were obtained with the recently upgraded BES system [6]. BES data plotted in figure 4(b) were measured using two adjacent channels with locations of $\rho \sim 0.82$ and 0.85, and the time history is a sliding average over a 10 ms time window with a 5 ms overlap for adjacent windows. It is clear that BES data show a similar amplitude modulation as a function of time. In addition, BES data indicate that the tearing mode amplitude has a strong radial dependence, i.e. it is much higher at $\rho \sim 0.85$ than at 0.82. It is seen that there is reasonable agreement between the BES and reflectometer.
(from \( \rho \sim 0.84 \pm 0.02 \)) density fluctuation levels derived from the simple phase screen model, although more sophisticated approaches can be adopted, e.g. [7], 2D modelling, etc.

4. Turbulent density fluctuation measurements

One case of the broadband density fluctuation measurements is studied here in the H-mode edge pedestal region measured with the 65 GHz system with O-mode polarization. Figure 5(a) shows the density profile measured by Thomson scattering mapped to the outboard midplane using the EFIT equilibrium solver [9]. In an ELM-free time window of 4000–4002 ms, the measured raw data, the real versus imaginary part of the microwave electric field, is plotted in figure 5(b). The figure shows a fairly well-defined annulus with a substantial width which is thought to be due to two-dimensional scattering effects discussed earlier. Figure 5(c) is the power spectrum of the phase extracted from the data plotted in figure 5(b). Figure 5(c) shows a broadband fluctuation in the phase as well as a coherent mode at \( \sim 20–30 \text{kHz} \) which is illustrated in figure 3(b). The spectrum decays with an index of \(-3.3\) in the frequency range 85–800 kHz.

A 2D full-wave code [2] is utilized here to simulate the reflectometer measurement with realistic geometry, plasma conditions and antenna radiation patterns. Using the measured density and electron temperature profiles at 4000 ms by Thomson scattering and magnetic field equilibrium from EFIT, the distributions of the density, electron temperature and the magnetic field strength can be reconstructed in the 2D cross section of the measurement plane (see figure 1). The code assumes a Gaussian antenna radiation pattern with the flexibility of tuning the beam waist locations. As shown in figure 6(a), the incident beam in the simulation is chosen to match that measured in the 65 GHz system in the plasma region (the dashed line marks the location of the outer tile and the antenna is located at \( R = 3.872 \text{m} \) as marked in the figure).

In the code the density fluctuation is simulated by a 2D density distribution (in \( x, y \)) with the following spectral properties:

\[
\frac{1}{n^2}(\tilde{n}_1 \tilde{n}_2) = (\delta n / n)^2 \exp \left\{ x \cdot \frac{\Delta k}{2} \right\} \cos(x \cdot k_m),
\]

where \( \delta n / n \) is the density fluctuation level, \( \Delta k \) is its spread, and \( x = (x_1 - x_2, y_1 - y_2) \) with \( x_1 - x_2 \) the radial and \( y_1 - y_2 \) the poloidal displacement. For a given choice of \( (\delta n / n, k_m, \Delta k) \), several hundred runs are made to form a statistical ensemble by varying the relative phase of the spectral components randomly.

In the simulation, the output of the receiver antenna is calculated by \( M \equiv \int dz E_{AR}(z)E_{AI}(z) \), where \( E_{AR}(z) \) and \( E_{AI}(z) \) designate the complex electric field amplitudes of the receiver antenna and reflected wave at the antenna plane, respectively and \( z \) denotes the axis along the electric field direction. It represents an integration of the reflected electric field at the antenna plane coupled with the antenna radiation pattern, which can be compared with experimental measurements. Figure 6(b) indicates that the receive antenna pattern used in the simulation is matched with the measured values. The coherent reflection [10] \( G \), defined by \( G \equiv \langle |M|^2 \rangle / \sqrt{\langle |M|^4 \rangle} \), where \( \langle \cdot \rangle \) denotes ensemble average, can be used as an indicator of the density fluctuation level. At a lower density fluctuation level, less microwave power is scattered...
and a stronger specular reflection is obtained. Therefore $G$ increases to one when the density fluctuation level decreases to zero. Simplified analytical expressions of $G$ can be obtained from reflectometer theories, e.g. a phase screen model [1], and others (e.g. [7]).

Plotted in figure 7 is $G$ versus $\delta n/n$ for different choices of $(k_m, \Delta k)$ with 300 runs for each point. In figure 7 $\Delta k$ (assume $\Delta k_x = \Delta k_y = \Delta k$) was scanned at 0.5, 1.0 and 2.0 cm$^{-1}$, and $k_m$ (assume $k_x = 0, k_y = k_m$) was scanned at 0 and 1 cm$^{-1}$. The two curves of $\delta n/n$ scanned at $\Delta k = 1$ cm$^{-1}$, $k_m = 0$ and $\Delta k = 1$ cm$^{-1}$, $k_m = 1$ show that $G$ decreases as $\delta n/n$ is increased, and their difference is quite uniform at each $\delta n/n$ level. At a fixed $\delta n/n = 1.0\%$, $G$ values were calculated for several different ($k_m, \Delta k$) choices. From these simulations it is concluded that $G$ is rather insensitive to these variations. It has been previously observed on DIII-D [11,12] that $\Delta k$ is of the order of 1 cm$^{-1}$ and $k_m < 1$ cm$^{-1}$. So the uncertainty of $G$ at a given $\delta n/n$ is basically given by this scan.

By comparing $G$ values calculated from the experiment with those obtained from the code output described above, the density fluctuation level can be deduced. The lower dashed line in figure 7 with $G = 0.6$ is computed using the experimental data shown in figures 5(b) and (c) but with the coherent mode in the frequency range 20–30 kHz filtered. Considering the aforementioned uncertainty of the $G$ value, we get $\delta n/n \sim (0.8 \pm 0.2)\%$ for this measurement at $\rho \sim 0.84$.

For this particular plasma discharge, BES obtained $\delta n/n \sim 0.33\%$ integrated in the frequency range 60–200 kHz averaged over 4000–5000 ms at $\rho \sim 0.85$. Since we obtained quite similar frequency spectra for the reflectometer measured electric field and from BES, as demonstrated in figure 8, we removed frequency components in the reflectometry data other than 60–200 kHz (also −200 to −60 kHz) and the specular reflection (−2 to 2 kHz), and $G = 0.81$ is obtained as plotted in the upper dashed line in figure 7, which indicates $\delta n/n \sim (0.5 \pm 0.2)\%$. This result is in reasonable agreement with BES measurement, given the fact that these two diagnostics differ in several ways in this particular measurement, e.g. differences in measurement location, different averaging time windows, potentially wider range of $k_\perp$ sensitivity for reflectometer [13] than BES, etc.

5. Summary and conclusion

In summary, fixed-frequency reflectometers utilizing quadrature phase detection have been employed on DIII-D to study both coherent and turbulent density fluctuations. For coherent fluctuations, the reconstructed phase information successfully identifies MHD/coherent mode activity, e.g. tearing modes, CAEs, etc. For an $m = 3/n = 2$ tearing mode, a basic 1D phase screen model is applied to infer fluctuation levels, and the result is consistent with BES measurement. For turbulent fluctuation studies, a 2D full-wave code [2] is utilized to interpret the reflectometry measurement in the H-mode edge pedestal region to deduce density fluctuation levels by simulating the measurement with realistic geometry, plasma conditions and antenna patterns. Comparison of the deduced fluctuation level from the code with BES measurement shows reasonable agreement.

Acknowledgments

The authors would like to thank M. Austin, B. Bray, K. Burrell and T. Leonard for useful discussions. This work is supported by US DOE Grant Nos. DE-FG03-01ER54615, DE-FG03-96ER54373, SC-JP333701, DE-AC02-76CH03073 and DE-AC05-76OR00033.

References