Magnetic Fluctuation-Induced Particle Transport
and Zonal Flow Generation in MST

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Magnetic and Current Density fluctuations play an important role in plasma relaxation in the Reversed Field Pinch as well as tokamak configurations.

All processes coupled through $\delta J$ and nonlinear mode interactions.
q Profile and Core Magnetic Fluctuation Spectrum

$T_e \sim T_i \sim 500 \text{ eV}$

$\frac{r B_T}{R B_P} = q$

Tearing Modes and broadband magnetic turbulence

Tearing modes and broadband magnetic turbulence
Magnetic Fluctuation-Driven Particle Flux

**Fluctuation-Induced flux**

\[
\Gamma_{i,e} = \Gamma_{i,e}^{es} + \Gamma_{i,e}^{em} = \frac{\langle \tilde{n}_e \tilde{E}_\perp \rangle}{B_0} \pm \frac{\langle \tilde{j}_{\parallel i,e} \tilde{b}_r \rangle}{eB_0}
\]

- Electrostatic
- Magnetic

**Radial Charge Transport**

\[
\Gamma_q = \Gamma_i - \Gamma_e = \frac{\langle \tilde{j}_{\parallel i} \tilde{b}_r \rangle}{eB_0}
\]

\[
j_r = e\Gamma_q
\]

**non-ambipolar flux**
Fast polarimeter measures core mean and fluctuating $B$ & $J$

Faraday rotation angle

$\Psi \sim \int nB \cdot dl$

$\delta \Psi = c_F \int n_0 \delta \overline{B} \cdot d\overline{l} + c_F \int \delta n \overline{B}_0 \cdot d\overline{l}$

$m=1$ activity

$x=-17$ cm

$\delta B \approx 33$ [Gauss]

11-chord FIR laser

32 magnetic coils toroidal array
Ampere's Law: \[ \oint_L \mathbf{dB} \cdot d\mathbf{l} = \mu_0 \delta I \]

Faraday Rotation Fluctuation:
\[ \delta \Psi = c_F \int n_0 \mathbf{dB} \cdot d\mathbf{l} \approx c_F \bar{n}_0 \int \mathbf{dB} \cdot d\mathbf{l} \]

\[ \oint_L \mathbf{dB} \cdot d\mathbf{l} \approx \left[ \int \delta B_z dz \right]_{x_1} - \left[ \int \delta B_z dz \right]_{x_2} \]
\[ \approx \mu_0 \delta I \phi = \frac{\delta \Psi_1 - \delta \Psi_2}{c_F \bar{n}_0} \]

Loop between polarimeter chords is equivalent to a Rogowski coil measurement

Ding, Brower et al. PRL (2003)
Measured Magnetic and Current Density Fluctuation Profiles

\[ \frac{\delta j_\varphi}{J_0} \sim 6\% \]

\((m,n)=(1,6)\) tearing mode

spatially localized in core, peaks at resonant surface

surges at sawtooth crash

\[ \frac{\delta B}{B} \sim 1\% \]

\(r=r_{q(1,6)}\)
Measured Charge Flux at sawtooth crash in MST

\[
\Gamma_q = \frac{\langle \tilde{j}_r \tilde{b}_r \rangle}{eB} = \frac{1}{eB nB} R (k \cdot \tilde{B}) \frac{1}{r} \tilde{b}_r \frac{\partial}{\partial r} r \tilde{b}_\theta = \frac{1}{eB} B_r \left( 1 - \frac{m}{nq(r)} \right) \langle \tilde{j}_r \tilde{b}_r \rangle
\]

where \( \nabla \times \delta \tilde{B} = \mu_0 \delta \tilde{J} \) and \( \frac{|r - r_s|}{r_s} \ll 1 \) and \( \langle \ldots \rangle \) denotes flux surface average.

\[
\frac{1}{\mu_0} < -\frac{1}{r} \tilde{b}_r \frac{\partial}{\partial r} r \tilde{b}_\theta > = \frac{1}{\mu_0} \langle \tilde{j}_r \tilde{b}_r \rangle
\]

Maxwell Stress

\[
\Gamma_q = \langle \delta j \delta b_r \rangle / eB
\]

\[
\frac{\Gamma_q}{\Gamma_{\text{Particle}}} \leq 1\%
\]

Charge flux is radially localized and changes sign across resonant surface.
Charge Transport and Radial Electric Field

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \tilde{J} = 0, \quad \nabla \cdot \tilde{E} = \frac{\rho}{\varepsilon_0} \quad \Rightarrow \quad \varepsilon_0 \frac{\partial E_r}{\partial t} = e(\Gamma^i_r - \Gamma^e_r) \]

\[ \frac{\langle \tilde{j}_r \cdot \tilde{b}_r \rangle}{B} \rightarrow 1 \sim 4 \text{ [A/m}^2\text{]} \text{ at the core (FIR Faraday)} \]

\[ \Delta \tilde{E}_r = \int \frac{\langle \tilde{j}_r \cdot \tilde{b}_r \rangle}{\varepsilon_0 B} dt \]

Leads to a huge electric field, \( \sim 50 \text{ MV/m in core} \)

However, shielding occurs due to ion polarization current

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \tilde{J} = 0, \quad \nabla \cdot \tilde{E} = \frac{\rho}{\varepsilon_0 \varepsilon_\perp} \]

\[ \varepsilon_0 \varepsilon_\perp \frac{\partial E_r}{\partial t} = \frac{\langle \tilde{j}_r \cdot \tilde{b}_r \rangle}{B} \]

\[ \varepsilon_\perp = 1 + \left( \frac{c}{V_A} \right)^2 \]

\[ \Delta E_r = \left( \frac{1}{1 + \frac{c^2}{V_A^2}} \right) \int \frac{\langle \tilde{j}_r \cdot \tilde{b}_r \rangle}{\varepsilon_0 B} dt \approx \left( \frac{V_A}{c} \right)^2 \int \frac{\langle \tilde{j}_r \cdot \tilde{b}_r \rangle}{\varepsilon_0 B} dt \]

At reconnection, a radial electric field is established due to non-ambipolar transport, but electric field is reduced by \(10^4\) due to shielding by the ion polarization drift.
Localized Radial Electric Field and ExB Flow

Charge flux generates a local $E_r$ with spatial scale ~5 cm that changes sign across resonant surface.

1. ExB generates flow and flow shear (which may strongly damp the mean flow)
2. Flow is toroidally and poloidally symmetric ($m=0, n=0$) *zonal flow*
driven by resistive tearing modes
Measurements indicate the following coupled relaxations:

- Lorentz force $\langle \delta J \times \delta B \rangle$
- Hall dynamo on electrons
- Current relaxation (electron velocity)
- Torque on ions
- Ion force balance
- Momentum relaxation (ion velocity)
- Ion viscous damping
- Electric Field and Flow shear
- Zonal flow

Tearing mode: $\nabla J_{||}(r) \rightarrow \delta B, \delta J$

Charge Transport $\langle \delta j_{||}/b_{r} \rangle$
Evidence of potential structure?

Potential measurement by Heavy Ion Beam Probe (HIBP)

Possible Effect of Electrical Field on Plasmas (Open Questions)

A potential well is formed near the reconnection layer.

Questions not clear to me:

(1) Where does electric field energy come from?

(2) Do ions and electrons gain energy from the field?

(3) Where does flow energy go?
Momentum transport and nonlinear torque

\[ \langle \delta J \delta B \rangle_{/>} \]

with \( k_3 (m=0,n=1) \)

without \( k_3 (m=0,n=1) \)

\[ \vec{k}_1 \pm \vec{k}_2 = \vec{k}_3 \]

\[
\begin{pmatrix}
1 \\
7
\end{pmatrix}
- 
\begin{pmatrix}
1 \\
6
\end{pmatrix} =
\begin{pmatrix}
0 \\
1
\end{pmatrix}
\]

\[ \langle \delta J \delta B \rangle \text{ and charge transport observed when three modes are coupled.} \]
What dissipates the electric field or dissipates ExB flow after the sawtooth crash?
Perpendicular Momentum Balance Equation

\[ \rho \frac{\partial V_{E \times B}}{B \partial t} - \frac{\mu^*}{B} \nabla^2 V_{E \times B} = \frac{\langle \tilde{j}_\parallel \tilde{b}_r \rangle}{B} \]

\( \rho \) is mass density and \( \mu^* \) is the classical viscosity coefficient

\[ \frac{\rho}{B^2} \frac{\partial E_r}{\partial t} - \frac{\mu^*}{B^2} \nabla^2 E_r = \frac{\langle \tilde{j}_\parallel \tilde{b}_r \rangle}{B} \]

ion viscosity \( \mu^*_\perp = \frac{3n k T_i}{10 \omega_{ci}^2 \tau_i} \)

\[ \Gamma_x^i - \Gamma_x^e = \langle \tilde{j}_z \tilde{B}_x \rangle \frac{eB_0}{eB_0} + c \frac{\partial}{\partial x} \nabla_{xys} = \frac{1}{4 \pi e} \frac{\partial E_{0x}}{\partial t} + \frac{n_i m_i c^2}{eB_0^2} \frac{\partial E_{0x}}{\partial t} \]

See: R. E. Waltz, Phys. Fluids, 25, 1269(1982);

\[ \varepsilon_0 (1 + \left( \frac{c}{V_A} \right)^2) \frac{\partial E_r}{\partial t} \approx \varepsilon_0 \left( \frac{c}{V_A} \right)^2 \frac{\partial E_r}{\partial t} = \frac{\langle \tilde{j}_\parallel \tilde{b}_r \rangle}{B_0} + \frac{\mu^*}{B_0} \nabla^2 V_{E \times B} \]
Electric Field Dynamics (with Collisional Dissipation)

![Graph showing electric field dynamics](image_url)
Effect of mode-mode interaction on $<\delta J x \delta B>$ force and charge transport result from nonlinear mode-mode interaction

\[ q = \frac{r B_T}{R B_p} \]

\[ \vec{k} \cdot \vec{B} = 0 \quad \text{(resonant surface)} \]

\[ \vec{k}_1 \pm \vec{k}_2 = \vec{k}_3 \]
MST Reversed-Field Pinch (RFP) is toroidal configuration with relatively weak toroidal magnetic field $B_T$ (i.e., $B_T \sim B_p$)

$$q(r) = \frac{r \frac{B_T}{R}}{B_p} < 1$$

$R_0 = 1.5 \text{ m}$, $a = 0.51 \text{ m}$, $I_p < 600 \text{ kA}$

$B_T \sim 3-4 \text{ kG}$, $n_e \sim 10^{19} \text{ m}^{-3}$, $T_{e0} < 1.3 \text{ keV}$

$\tau_E \sim 10 \text{ ms}$, $\beta = \langle p \rangle / B^2(a) = 15\%$
Measured Core Magnetic Fluctuations by Faraday rotation

Faraday Rotation $\Psi = c_F \int n \vec{B} \cdot d\vec{l}$

$\Psi = \Psi_0 + \delta \Psi$, $\vec{B} = \vec{B}_0 + \delta \vec{B}$, $n = n_0 + \delta n$

$\delta \Psi = c_F \int n_0 \delta \vec{B} \cdot d\vec{l} + c_F \int \delta n \vec{B}_0 \cdot d\vec{l}$

$c_F \int \tilde{n} B_{z_0} dz = c_F \int \tilde{n} B_{\theta} \cos \theta dz$

$\leq c_F \int (\mu_0 J(0) \frac{r}{r} - \tilde{n} dz = c_F \frac{\mu_0 J(0)}{2} r x \int \tilde{n} dz \to 0$

$\delta \Psi \approx c_F \int n_0 \delta \vec{B} \cdot d\vec{l}$

$\delta \vec{B} = 33$ [Gauss]
(1) Measured Current Profile Relaxation

At crash, current profile flattens

\[ E_{\parallel} \approx E_T(r) = \frac{V_L}{2\pi R} - \int_r^a \frac{\partial}{\partial t} B_P(r',t)\,dr \]

At crash, electric field increases

\[ E_{\parallel} \gg \eta J_{\parallel} \]

\[ \text{Brower, Ding, et al PRL, 88, 185005 (2002)} \]
Hall Dynamo is balanced by Inducted Electric Field

\[ \eta_{||} \langle J \rangle_{||} = \langle E \rangle_{||} + \langle \vec{\delta} \times \vec{\delta} B \rangle_{||} - \frac{\langle \vec{\delta} \vec{J} \times \vec{\delta} B \rangle_{||}}{n_e e} \]

MHD dynamo

Hall Dynamo

Parallel Mean Field Ohm’s Law from 2-Fluid Theory

\[ \frac{\langle \delta J \times \delta B \rangle_{||}}{n_e e} \approx \frac{B_p}{B_T} \left(1 + \left(\frac{B_T}{B_p}\right)^2\right) \langle \delta j \varphi b_r \rangle \]

\[ E_{||}, <\delta J \times \delta B >_{||}/n_e e \]

Ion Momentum (Torque) over Sawtooth Crash

\[
\rho \frac{\partial \tilde{V}}{\partial t} + \rho \tilde{V} \cdot \nabla \tilde{V} = \langle \delta J \times \delta B \rangle_{\parallel} + \mu_{\perp}^* \langle \nabla^2 \tilde{V} \rangle_{\parallel}
\]

Huge Imbalance !!

\[
\langle \rho \frac{\partial \tilde{V}}{\partial t} \rangle_{\parallel} + \rho \tilde{V} \cdot \nabla \tilde{V} \rangle_{\parallel} = \langle \delta J \times \delta B \rangle_{\parallel} + \mu_{\perp}^* \langle \nabla^2 \tilde{V} \rangle_{\parallel}
\]

\[
\begin{align*}
\sim 5 \text{ N/m}^3 & \quad \delta \tilde{V} \leq 1 \text{ km/s} \\
\Delta r \geq 1 \text{ cm} & \quad \sim 60 \text{ N/m}^3 \\
\rho \langle \tilde{V} \cdot \nabla \tilde{V} \rangle \leq 3 \text{ N/m}^3 & \quad \mu_{\perp} \frac{V}{a^2} \sim -10^{-2} \sim 10^{-3} \text{ N/m}^3
\end{align*}
\]

Classical dissipation
At sawtooth crash ion momentum in the core drops much faster than classical viscous time.
Fluctuation-Induced Radial Charge Flux

\[
\Gamma_q = \frac{\langle \tilde{j}_\parallel \tilde{b}_r \rangle}{eB} = \frac{\langle \tilde{j}_{\parallel,i} \tilde{b}_r \rangle}{eB} - \frac{\langle \tilde{j}_{\parallel,e} \tilde{b}_r \rangle}{eB}
\]

charge flux

On MST for a specified (m,n) mode

\[
\langle \tilde{j}_\parallel \tilde{b}_r \rangle = \langle \tilde{j}_\theta \frac{B_p}{B} + \tilde{j}_\phi \frac{B_T}{B} \rangle \tilde{b}_r \rangle
\]

\[
\nabla \times \delta \vec{B} = \mu_0 \delta \vec{j}
\]

\[
\vec{k} \cdot \vec{B} = \frac{m}{r_s} B_\theta + \frac{n}{R} B_\phi
\]

\[
\frac{R}{nB} (\vec{k} \cdot \vec{B}) < \frac{1}{r} \tilde{b}_r \frac{\partial}{\partial r} r \tilde{b}_\theta \rangle = \frac{R}{nB} (\vec{k} \cdot \vec{B}) < \tilde{j}_\phi \tilde{b}_r \rangle \]

\[
\frac{|r - r_s|}{r_s} \ll 1
\]

At mode resonant surface charge flux is zero,

but can be non-zero locally (near the resonant surface) to form charge filamentation

Phase between current and magnetic fluctuation

\[
< \delta j_\phi(r_s) \delta b_r(r_s) > \\
= | \delta j_\phi(r_s) || \delta b_r(r_s) | \cos \Delta
\]

\[
\Delta = ph_\_ \delta j_\phi(r_s) - ph_\_ \\
= ph_\_ \delta j_\phi(r_s) - ph_\_ \\
\]

Perpendicular global magnetic fluctuation has a constant phase
Magnetic Fluctuation Spectrum

Tearing Modes

magnetic turbulence

Tearing modes and broadband magnetic turbulence

standard 400ka
ppcd 400ka

noise

$P(f) \, [\text{Gs}^2/\text{kHz}]$

$f \, [\text{kHz}]$
RFP Safety Factor Profile

$T_e \sim T_i \sim 500 \text{ eV}$

$q = \frac{r B_T}{R B_P}$

$(0,1)$
Charge Transport and Radial Electric Field

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0, \quad \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \quad \Rightarrow \quad \varepsilon_0 \frac{\partial E_r}{\partial t} = e(\Gamma_r^i - \Gamma_r^e)
\]

\[
< \tilde{j}_{//} \tilde{b}_r > \quad B
\]

10⁻¹² [A/m²] at the edge (probe, Neal Crocker, 2001)

1⁻⁴ [A/m²] at the core (FIR Faraday)

If Charge flux induced only by magnetic fluctuation

\[
\Delta \tilde{E}_r = \int \frac{< \tilde{j}_{//} \tilde{b}_r >}{\varepsilon_0 B} dt
\]

560 [MV/m] ~ 6.7 [GV/m] at the edge (dt=0.5ms) >> T_e/a

56 ~ 224 [MV/m] at the core (dt=0.5ms) >> T_e/a
Ion Polarization Drift

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = 0, \quad \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0 \varepsilon_\perp} \]

\[ \varepsilon_0 \varepsilon_\perp \frac{\partial E_r}{\partial t} = \frac{\langle \tilde{j}_{//} \tilde{b}_r \rangle}{B} \]

\[ \varepsilon_\perp = 1 + \left( \frac{c}{V_A} \right)^2 \]

Due to ion polarization current

\[
\Delta E_r = \left( \frac{1}{1 + \frac{c^2}{V_A^2}} \right) \int \frac{\langle \tilde{j}_{//} \tilde{b}_r \rangle}{\varepsilon_0 B} dt \approx \left( \frac{V_A}{c} \right)^2 \int \frac{\langle \tilde{j}_{//} \tilde{b}_r \rangle}{\varepsilon_0 B} dt
\]

At reconnection, a radial electric field is established due to non-ambipolar transport, but electric field is reduced by $10^4$ due to shielding by the ion polarization drift.

See: R. E. Waltz, Phys. Fluids, 25, 1269 (1982);
Magnetic fluctuations play an important role in magnetic reconnection in the laboratory plasma and astrophysical plasmas.

\[ \eta_{\parallel} \langle J \rangle_\parallel = \langle E \rangle_\parallel + \langle \delta \vec{v} \times \delta \vec{B} \rangle_\parallel - \langle \delta \vec{J} \times \delta \vec{B} \rangle_\parallel / n_e e \]
(1) Hall Dynamo: \[ \frac{<\delta J \times \delta B>}{ne} \]

(2) Ion Momentum Balance: \[ <\delta J \times \delta B> \]

(3) Magnetic Fluctuation-Induced Particle Transport;
   - Maxwell Stress \[ \frac{<\delta J \parallel b_r>}{eB} \]
   \[ \frac{1}{r} \frac{\partial}{\partial r} (r \tilde{b}_r \tilde{b}_\theta) \]

*All three processes are coupled through current density fluctuations*

(4) Nonlinear mode-mode interaction

*Identify the role of magnetic and current density fluctuations in particle and momentum transport*