Antenna radiation patterns in the whistler wave regime measured in a large laboratory plasma

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The radiation pattern of electric dipole and magnetic loop antennas is determined experimentally in the parameter regime $0.1 < \omega/\omega_c < 1$, $1 < \omega_p/\omega_c < 30$ ($\omega_c$, $\omega_p$ = electron plasma, cyclotron frequency, respectively). The measurements are performed in a uniform, dense magneto-plasma (density $10^{10} < n_e < 10^{12}$ cm$^{-3}$, temperature $0.2 < kT_e < 3$ eV, magnetic field $B_0 < 100$ G) of very large dimensions (50 cm diameter, 300 cm length) where boundary effects are negligible even for many far-field patterns. When the antenna dimensions are small compared to the parallel whistler wavelength ($k \parallel L_{w} < 1$) the radiation pattern is characterized by resonance cones. For finite sized antennas ($k \parallel L_{w} \gg 1$) no cones are observed, but a dipole-like pattern with a field-aligned narrow lobe. Amplitude and phase distribution in the near zone and far zone are investigated. The antenna orientation with respect to the static magnetic field is varied. At large amplitudes ($B_r/B_0 = 1\%$) the radiation pressure modifies the plasma density and the wave self-focuses.

INTRODUCTION

For many years the study of antennas in plasmas has been a topic of great interest. Particular attention has recently been given to VLF and ELF radiation due to its importance in spacecraft applications. Many authors [Bunkin, 1957; Kuehl, 1962; Arbel and Felsen, 1963; Wu, 1963; Kogelnik and Motz, 1963; Mittra and Duff, 1965; GiaRusso and Berge- son, 1970; Wang and Bell, 1972] have worked on the theory of the antenna radiation problem. Experimentally, antenna impedances have been studied by a number of investigators [Balmain, 1964; Duff and Mittra, 1970; Koons and McPherson, 1974; Vernet et al., 1975]. Rather few observations of antenna radiation patterns are given, most of which are restricted to near-zone phenomena [Fisher and Gould, 1971; Gonfalone, 1974] and longitudinal waves [Shen et al., 1970; Nakamura et al., 1973; Ishizone et al., 1974]. Recently, resonance cone field measurements have also been extended into the far field [Boswell, 1975; Bernabei et al., 1975]. The experimental difficulties of determining an entire antenna radiation pattern in space plasma are formidable, requiring complicated "mother-daughter" satellite experiments. Although laborato-

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Since in many practical situations the applied power levels may be large ($B_{ax}/B_0 > 1\%$) nonlinear effects due to the modification of the plasma properties by the wave may considerably change the antenna radiation characteristics. The general assumption that instabilities deteriorate the radiation efficiency is not borne out in this experiment. On the contrary, a self-focusing process is observed which produces a pencil-shaped field-aligned radiation pattern. In ionospheric plasmas these processes may occur at relatively small power levels [Stenzel, 1976a].

**EXPERIMENTAL SETUP AND MEASUREMENT TECHNIQUES**

The experiments are performed on a large magnetized plasma device shown schematically in Figure 1. A uniform plasma column of diameter $d \gg 40$ cm and length $L = 300$ cm is produced by a dc discharge with a large ($\sim 50$ cm dia.), indirectly heated, oxide-coated cathode and adjacent mesh anode. The plasma is radially confined by a uniform axial magnetic field ($0 < B_0 < 150$ G), temporal fluctuations $\delta B/B_0 \ll 10^{-4}$, spatial variations $\delta B/B_0 = \pm 0.5\%$) produced by a set of eight external solenoidal coils (110 cm dia.) driven with a transistor-regulated power supply (15 V, 2000 A). Axial end losses are reduced with a surface magnetic field [Limpaecher and MacKenzie, 1973] produced with arrays of permanent magnets of alternating polarity (Samarium-cobalt, $B_{max} = 4$ kG). Due to the magnetic field confinement and the efficient electron emission, a large volume (\~450 liter), high density ($n_e = 10^{12}$ cm$^{-3}$), nearly collisionless plasma (krypton; pressure $p < 5 \times 10^{-4}$ torr; collision frequency $v_{ei}/\omega_c \approx 10^{-2}$ in afterglow) is produced at moderate input powers (cathode 9 kW; discharge 40 V x 250 A = 10 kW). Density and magnetic field are independently variable over wide ranges; the electron temperature ranges from $kT_e = 2$ eV ($=10 kT_i$) during the active discharge to $kT_e = 0.2$ eV = $kT_i$ in the late afterglow of a pulsed discharge. In pulsed operation (typically 5 msec on, 500 msec off), one has a choice of different electron distribution functions: during the discharge there exists a small population of energetic primary electrons (1/2 $mv^2 = 40$ eV $\gg 1/2 mv^2$; relative concentration $n_p/n_e \approx 10^{-3}$); in the afterglow ($t_a > 0.5$ msec) the electron distribution is essentially Maxwellian unless perturbed by large amplitude waves.

The plasma diagnostics consists of a 70 GHz (4 mm) microwave interferometer for absolute path-integrated density measurements, a microwave resonator probe [Stenzel, 1976b] for local density measurements and of various Langmuir probes for spatially resolved measurements of $n_e$, $T_e$ and the electron distribution function $f_e(v)$. All diagnostic data are time resolved by sample-and-hold techniques so as to supply information about the plasma build-up, the steady-state condition, the afterglow decay and the perturbations due to large amplitude waves.

Whistler waves are excited and detected with various antennas which are inserted into the center of the plasma column through one axial and two orthogonal radial ports. The axial and the horizontal radial ports are equipped with large gate valves and a decompression chamber with a separate pump station so that probes can be exchanged while the machine is under operation. There are linear motorized probe drive systems installed on the radial and axial ports while the vertical port has a circular motor drive system which rotates the bellow-mounted probe shaft by $\phi = \pm 180^\circ$ around the vertical axis. Thus the receiver antenna at the end of the vertical probe shaft describes a circle whose radius $\rho$ is varied by inclining the probe shaft with respect to the vertical and whose plane of rotation is adjusted by moving the probe shaft through a sliding seal.

The antennas used in the present experiment are shown in Figure 2. A balun-fed electric dipole of length $L_d = 1.1$ cm or $L_d = 4.4$ cm (free-space wavelength $\lambda_0 = 200$ cm) is mounted on the axial probe. Although the dipole balancing with a $\lambda/2$-line is frequency dependent it eliminates the need for...
as shown in Figure 3. A constant amplitude rf signal (pulsed or steady state, $50 \leq f \leq 300$ MHz, $0 < P_d \leq 100$ W) is applied to the exciter antenna and the spatial field distribution is mapped with a receiver antenna movable on a circle of radius $\rho$ ($0 < \rho < 25$ cm) around the exciter. The received signal is applied to a spectrum analyzer tuned manually to the incident line. The analyzer output which is proportional to the rf amplitude is sampled at the desired time during the discharge or afterglow, averaged over many discharge pulses and plotted versus position $\rho$, $\phi$.

**EXPERIMENTAL RESULTS**

The radiation patterns of two electric dipole antennas of different lengths and a magnetic loop antenna are shown in Figures 4-6. The received
Fig. 4. Radiation pattern $|E_{\phi}|$ vs. $\phi$ of the long dipole antenna at different normalized densities, $\omega_p^2/\omega^2$. At low densities oblique resonance cones are observed, at high densities the radiation is confined to a single, field-aligned narrow lobe. Discharge plasma, $kT_e = 2$ eV, $\nu_m/\omega_c = 4 \times 10^{-4}$, $\nu_p/\omega_c < 6 \times 10^{-3}$.

Fig. 5. Antenna radiation pattern of the short dipole at different values $\omega_p/\omega$. The receiver gain in the bottom two traces is decreased by 10 dB, otherwise constant. In comparison with Figure 4, the shorter dipole exhibits at the same density more pronounced resonance cones which extend into the far zone ($\rho > 4\lambda_1$, top trace). Early afterglow, $kT_e = 1$ eV, $\nu_m/\omega_c \ll \nu_p/\omega_c < 1.8 \times 10^{-2}$. 
wave amplitude is displayed vs. rotation angle $\phi$ of the receiver antenna ($\rho = \text{const}$) at different normalized densities $\omega^2 / \omega_{\rho}^2$ ($\omega_{\rho}, \omega = \text{const}$). Two distinctly different radiation patterns are seen: (i) at low densities the double humped distribution indicates a resonance cone pattern which will be verified further below; (ii) at high densities a single peaked distribution is observed corresponding to a narrow antenna lobe in the direction of the static magnetic field. With increasing density, there is a continuous transition from one characteristic pattern to the other.

The density increase changes two important parameters. First, the electric length of the dipole, $L_d / \lambda_\parallel$, or correspondingly, the circumference of the loop, $2\pi R_l / \lambda_\parallel$, increases with density. The antenna dimensions are normalized to the wavelength of a plane right-hand circularly polarized electromagnetic wave (whistler mode) whose dispersion is given by

$$\frac{k_\parallel^2 c^2}{\omega^2} = \left(\frac{\lambda_\parallel}{\lambda_\parallel}ight)^2 = 1 - \frac{\omega_p^2}{\omega (\omega - \omega_c)}$$  \hspace{1cm} (1)

Second, with increasing density the normalized distance between receiver and exciter antenna, $\rho / \lambda_\parallel$ increases. At low densities the near zone ($\rho / \lambda_\parallel \ll 1$) of a short ($L_d / \lambda_\parallel \ll 1$) antenna is mapped, at high densities the far zone of a finite-sized antenna is observed.

The question whether the resonance cone pattern vanishes due to the increase in antenna length or in antenna separation is resolved in Figures 4 and 5 which compare the patterns of two dipoles of different lengths. At high densities, corresponding to far-zone conditions ($\rho / \lambda_\parallel > 2$), the short dipole (Figure 5) still exhibits a cone structure while the long dipole (Figure 4) has the single lobe pattern. Thus, the resonance cone is the characteristic radiation pattern of small antennas. Furthermore, it is not only confined to the near zone of an oscillating point charge [Fisher and Gould, 1971], but also exists in the far-zone electromagnetic radiation field as predicted by Kuehl [1962]. Figure 6 shows that the magnetic dipole exhibits characteristics similar to those of the electric dipole. The observation that resonance cones can be excited with magnetic loop antennas has recently been pointed out by Boswell and Goncalone [1975], who also observed warm plasma interference structures.

The density change in Figures 4-6 also affects
a third parameter, the coulomb collision frequency. Since the data are taken during the discharge and early afterglow where $v_{el}/\omega_c$ is small, collisional damping cannot explain the disappearance of the cones at high densities. Comparison between Figures 4 and 5 shows in fact more pronounced cones in the case of relatively higher collision rates.

The two characteristic radiation patterns have been investigated in more detail. The properties of the resonance cones excited by a magnetic loop antenna are shown in Figure 7. By scanning the receiver loop antenna at different radii $\rho$ around the exciter loop, a cross section through the cone along its axis parallel to $B_0$ is obtained (Figure 7(a)). The constant cone angle indicates the uniformity in density and magnetic field. The half width of the peaked field distribution is constant and approximately equal to the projection of the loop diameter in the direction of radiation along $\rho$. The field amplitude (Figure 7(b)) decreases with distance approximately as $E \propto \rho^{-1}$. The interference structure due to warm plasma modes [Kuehl, 1973] is not observed in the electric field measurements presumably because of the large size of the receiver antenna compared to the fine structure spacing.

According to cold-plasma theory, the cone angle $\theta_c$ approaches a limiting value at large densities given by

$$\sin \theta_c = \left[ \frac{\omega^2}{\omega_c^2} \right]^{1/2} \frac{\omega}{\omega_c}$$

(2)
The high density limit is easily approached in this experiment. Figure 8(a) shows the cone angle as a function of density and Figure 8(b) gives the dependence of \( \theta_c \) on \( \omega/\omega_c \) at large densities consistent with (2).

Resonance cones are excited with shielded magnetic loop antennas as well as with electric dipoles, and the cone structure is detected with either type of antenna. Thus, both electric and magnetic field components are present. The polarization of these field components has been investigated by varying the orientation of the receiver antenna as shown schematically in Figure 9(a). Amplitude maxima or minima are observed depending on whether the dipole is parallel or perpendicular to the local electric field. In vacuum, perfect nulls are obtained; in the plasma, on the resonance cone, only amplitude minima are observed. These occur when the dipoles are crossed, i.e., at \( \beta = 90^\circ \) for a rotation of the receiver dipole in the x-y plane perpendicular to \( \mathbf{B}_0 \) (Figure 9(b)) and, for a rotation in the x-z plane at \( \alpha = 90^\circ - \theta_c \), i.e., when the dipole axis is tangential to the cone surface (Figure 9(c)). The maximum amplitude is detected when the receiver dipole axis is in the x-z plane at \( \alpha = -\theta_c \), i.e., normal to the cone surface. Thus, in general, three field components with different phases are present on the resonance cone. The rf magnetic field also exhibits elliptic polarization.

In the high density regime, far-zone antenna patterns have been measured at different frequencies. Figure 10 shows results for the magnetic loop antenna polarized such that the rf magnetic field is perpendicular to the static magnetic field. In the present frequency regime the 3 dB width of the antenna pattern is observed to decrease toward lower frequencies. This behavior is consistent with the theoretical prediction [Helliwell, 1965] that oblique whistlers can propagate only when the ray angle \( \theta - \alpha \) with respect to \( \mathbf{B}_0 \) is smaller than

\[
\theta_c = \arcsin (\omega/\omega_c), \quad \text{for } 0.189 < \omega/\omega_c < 1
\]

\[
|\theta - \alpha| < \frac{1}{2} \arctan \frac{\left[ (1 - \omega^2/\omega_c^2)^{1/2} - 3^{1/2} (\omega/\omega_c) \right]^{3/2}}{2^{3/2} (1 - \omega^2/\omega_c^2)^{3/4}}, \quad \text{for } 0 < \omega/\omega_c < 0.189
\]

Thus, with decreasing frequency the ray cone narrows.

Fig. 9. Electric field polarization on the resonance cones. (A) Schematic view of antenna arrangement. The receiver dipole can be rotated by an angle \( \alpha \) in the x-z (or \( \mathbf{B}_0 \)) plane and by an angle \( \beta \) in the x-y plane (\( \perp \mathbf{B}_0 \)). (B) Electric field amplitude on the cone vs. angle \( \alpha \). (C) \( |E_\parallel| \) vs. \( \beta \). The cone fields have elliptic polarization; the major field component is normal to the cone surface in the plane of the exciting dipole and \( \mathbf{B}_0 \). \( kT_e = 2 \) eV.
Fig. 10. Radiation patterns of the magnetic loop antenna at different frequencies in the high density regime. The 3 dB width narrows toward lower frequencies consistent with the behavior of the group velocity cone for whistlers. The receiver gain is varied so as to normalize the field maxima. Afterglow plasma, \( kT_e = 0.4 \text{ eV} \).

(2) and are thus not resonance cone effects. At low frequencies \( (\omega/\omega_c < 0.5) \) sidelobes may arise from the superposition of the three possible oblique whistler modes as indicated by the calculated VLF patterns for infinitesimal dipoles by Wang and Bell [1972].

Antenna polarization effects are shown in Figure 11. The exciter magnetic loop is rotated such that the angle of the rf magnetic field with respect to the static magnetic field varies from 0 to 90°. For \( \gamma = 0 \) (\( B_{rf} \perp B_0 \)) a single large-amplitude lobe along \( B_0 \) is observed; for \( \gamma = 90° \) (\( B_{rf} \parallel B_0 \)) the radiation pattern is broadened, exhibiting a null on axis but the energy flow is still predominantly in the direction of \( B_0 \). At no other polarization angles does the radiation pattern have a major lobe significantly oblique to \( B_0 \).

A rather detailed picture of the antenna radiation pattern can be obtained from interferometer measurements which yield both amplitude and phase distributions. Axial interferometer traces are taken.
Fig. 12. Phase and amplitude distribution in the r-z plane of the radiation from the magnetic loop antenna indicated to scale at the origin. (A) Contours of constant phases \( \phi_{\text{rf}} \). Dashed lines indicate the theoretical group velocity cone \( \pm |\theta - \alpha|_{\text{max}} = \pm \arcsin (\omega / \omega_c) = \pm 53^\circ \) and the complementary phase velocity cone, \( \pm \theta_{\text{max}} = \pm 37^\circ \). Note that the significant wave amplitudes are confined to the former and the wave normals to the latter. Concave phase fronts are the result of the refractive index surface for \( \omega / \omega_c > 0.5 \). (B) Radial density profile in the experimental region. Axially, the density is constant. (C) Contours of constant relative wave magnetic field amplitude, \( |B_{\text{rf}}(r,z)| \).

at different radial positions so as to map the field distribution in two dimensions. Two examples at different frequencies \( (\omega / \omega_c = 0.8; 0.285) \) are shown in Figures 12 and 13. The exciter antenna is the magnetic loop antenna shown to scale at the origin which is approximately 9 cm displaced from the column axis for purpose of ducting experiments [Stenzel, 1976c]. Receiver and exciter antennas are identical. The radial density profile in the experimental region is shown in Figure 12(b). Figure 12(a) displays the contours of constant phase \( \phi_{\text{rf}}(r,z) = k \cdot r \), i.e., maxima and minima of the interferometer signal, and Figure 12(c) depicts the contours of constant amplitude \( |B_{\text{rf}}(r,z)| \) in relative
For frequencies $\omega > \omega_c / 2$ (Figure 12) the refractive index surface [Helliwell, 1965] is concave such that wave normal and ray direction lie on opposite sides with respect to the magnetic field direction. Since the wave energy diverges with distance from an exciter, the wave normals have to converge which gives rise to the observed concave phase fronts in Figure 12(a). The amplitude distribution shows the characteristics of a small source, i.e., with increasing distance the energy spreads over a larger cross section such that the intensity decreases rapidly. Collisional damping of a plane wave would be negligible compared to the observed amplitude decay. The Poynting flux is confirmed to lie within the ray cone whose axis is parallel to $B_0$ and whose opening angle is given in (2) or (3) by $|\theta - \alpha|_{\text{max}} = \theta_c = \arcsin (\omega/\omega_c) = 53^\circ$. In spite of the uniform medium the amplitude distribution exhibits structure across $B_0$ which previously appeared as sidelobes in the radiation pattern measurements (Figure 10). The structure develops with distance from the exciter and lies well within the resonance cone. No simple explanation for the fine structure has been found so far.

Figure 13 shows the corresponding measurements at frequency below $\omega = \omega_c / 2$ where the refractive index surface is more complicated [Helliwell, 1965]. For wave normal angles $\theta < \arccos (2\omega/\omega_c)$ the rays and wave normals are on the same side with respect to $B_0$ so that the wave fronts are convex for diverging rays. This is basically observed for distances $\Delta z > 2\lambda (\approx 30 \text{ cm})$ for the exciter. Closer to the source the phase and amplitude distribution are relatively choppy indicating interference effects. Such interference effects can occur for low frequency oblique whistlers in perfectly uniform unbounded media because the refractive index is multivalued. For given parameters $\omega$, $\omega_c$, and $\omega_p$ there are, in general, three different wave normal directions for a given ray direction. Interference of these modes can result in amplitude variations along the direction of the energy flow [Burtis, 1974].

The above-described results were obtained in a plasma of uniform density and magnetic field at small wave amplitudes. In some applications, for example, experiments on wave-particle interactions or long-distance communication in space, it may be necessary to excite large amplitude whistler waves. Nonlinear effects are expected to alter the radiation pattern. In the present experiment we have investigated nonlinear effects when the wave intensity $B^2 / 2\mu_0$ is increased to values approaching
the kinetic energy density of the electrons, $n_e kT_e$.

In the nonlinear regime narrower, less diverging radiation patterns and hence relatively larger amplitude signals are observed than in the linear regime. For example, Figure 14(a) shows that an increase in the applied rf power from $P_{rf} = 0.1 \text{ W}$ to $P_{rf} = 10 \text{ W}$ leads to a decrease in the 3 dB beam width by a factor of $\approx 2$ and to a relative amplitude increase by a factor of $\approx 4$. (Note that the receiver sensitivity for the bottom trace has been decreased by 20 dB.) In the linear regime the beam divergence gives rise to a continuous amplitude decrease with distance from the exciter. The comparison in Figure 14(b) shows that in the nonlinear regime the amplitude decays very slowly and varies periodically in space. At larger distances the spatial oscillations decay and the radiation pattern can approach the shape of a narrow nondiverging filament of half width comparable to the antenna dimensions [Stenzel, 1976a,c]. Such nonlinear effects are observed at wave intensities $S \approx 1 \text{ W/cm}^2$ (or $B_{rf}^2 / 2\mu_0 / (n_e kT_e) \approx 10\%$) over a wide range of frequencies ($0.2 < \omega / \omega_c < 1$). The observed nonlinear effects can be explained by the process of self-focusing and self-trapping [Litvak, 1968; Washimi, 1973]. The radiation pressure of the wave creates a density depression into which the wave energy refracts. This leads to a convective instability whereby a long magnetic-field-aligned density trough is formed which ducts the whistler wave. The investigation of the filamentation instability is a separate topic and will be published in detail elsewhere [Stenzel, 1976c].

CONCLUSIONS

The measurement results represent, to the author's knowledge, the first detailed experimental data on antenna radiation patterns in the whistler wave regime. These measurements could be performed because of the development of a large, uniform, collisionless laboratory plasma.

The observations confirm that electrically short antennas radiate largely in the direction of the resonance cone. While these characteristics were known to hold in the electrostatic near zone the present results extend these properties into the electromagnetic far zone. However, for antennas of finite electric length the resonance cone pattern vanishes and the energy flow is largely in the direction of the static magnetic field. A continuous transition from one pattern to the other occurs for both electric and magnetic dipole antennas. Finite sized antenna patterns have so far received little theoretical analysis.

At large applied power levels the antenna radiation pattern can be significantly different because the intense wave can modify the plasma properties. The dominant nonlinear effect observed in the
The present experiment is due to self-trapping in a field-aligned density trough created by the wave radiation pressure.

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