Electron heating by nonlinear whistler waves

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Abstract
Nonlinear whistler-mode phenomena observed in laboratory plasmas will be addressed. Nonlinearities arise when strong waves modify the density, temperature and magnetic field, all of which affect the wave propagation. A brief review of thermal filamentation will be given. The main focus is on the magnetic nonlinearity of whistler modes whose wave magnetic fields exceed the ambient magnetic field $B_0$. Such intense waves are launched from loop antennae with axial fields along $B_0$ and form in one polarity whistler spheromaks and whistler mirrors in the opposite polarity. Spheromaks propagate slower, mirrors faster than linear whistlers. Spheromaks show soliton-like properties. In a whistler vortex (spheromak), the toroidal electron current ring with $J\parallel E$ converts magnetic energy into electron heat. In contrast, linear whistlers and whistler mirrors are supported by electron Hall currents and produce negligible heat ($J\cdot E = 0$). The collision of counter-propagating spheromaks is inelastic, forming a stationary, helicity-free field-reversed configuration, while linear whistler modes pass through each other without interaction. These results are important for the understanding of strong whistler turbulence and reconnection.

1. Introduction

In electromagnetism, nonlinear phenomena usually arise from the properties of matter since Maxwell’s equations are linear. The conductivity is rarely a constant in plasmas, which affects many phenomena such as electromagnetic waves. Most whistler waves involve the interaction of electromagnetic fields with magnetized electrons and unmagnetized ions. The electromagnetic forces on the electrons are in general nonlinear when the fields of the electrons are included. Much of the literature on whistler waves is based on fluid theory and small amplitude waves, which allows linearization of the equations [1,2]. This approach predicts the dispersion relation of plane whistler waves as a function of plasma parameters and frequency. For example, the phase velocity of parallel whistlers scales as $v_{\text{phase}} \propto (B/n)^{1/2}$, where $B$ and $n$ are background magnetic field and density, which are not affected by small amplitude waves. However, the linearization breaks down at large amplitudes, and the wave properties become amplitude-dependent or nonlinear. The three most critical parameters which cause
nonlinearity are magnetic field, density and electron temperature or in more general, electron distribution function.

Many investigations have been done on wave–particle interactions [3,4]. A certain group of electrons in the distribution function can get trapped in the wave and exchange energy with the wave. When gaining energy, these resonant particles create damping through Landau or cyclotron resonance [3]. When resonant electrons transfer energy to the wave, thereby relaxing a non-Maxwellian distribution, the wave grows and instabilities arise [5–7]. These phenomena have been studied extensively in magnetospheric plasmas [2].

A second body of the literature deals with density perturbations created by large-amplitude waves [8–10]. Such modulational nonlinearities are produced by two mechanisms: (i) the ponderomotive force of a nonuniform electric field, $-\nabla|E|^2$, acting on the electrons (radiation pressure), creates a space charge electric field, which accelerates the ions and (ii) electron heating in a nonuniform electric field, which produces a pressure gradient, a space charge electric field and a reduction in density in the heated region since electrons and ions are coupled via space charge forces. The change in density modifies the wave propagation. If a broad channel of enhanced density is formed whistlers with $\omega > \omega_{ce}/2$ can be ducted [11, 12]. This may further enhance the density perturbation and give rise to a filamentation instability. Thermal filamentation occurs when a density depression or trough is formed by the wave. Laboratory observations have shown that an initially diverging whistler beam becomes highly collimated by electron heating which forms a field-aligned density trough or duct [13–15]. Since the duct diameter can be smaller than the parallel whistler wavelength, the guided whistler forms a new eigenmode of the duct rather than performing a snake-like refraction path within the duct.

To first order, whistler waves are cold-plasma waves whose dispersion does not depend on electron temperature. However, the damping depends on temperature in a collisional plasma, especially when due to Coulomb collisions ($\nu \propto T_{e}^{-3/2}$). In a cold, dense plasma dominated by classical collisions, a small-amplitude whistler wave excited by a loop antenna rapidly spreads and damps [16]. However, a large-amplitude whistler wave propagates with little damping since heating increases the conductivity. Since the hot electrons are confined to the field lines, a filamentary radiation pattern arises. However, this nonlinearity does not require density changes and occurs much faster than a filamentation instability, typically within one rf cycle. It is also important for transient currents which are transported by whistlers [17].

A very strong nonlinearity arises when the wave changes the magnetic field since this parameter modifies the wave propagation. Surprisingly, this nonlinearity has received little attention so far, perhaps because it requires whistler waves with wave magnetic field comparable to the ambient field. The present paper will mainly focus on this nonlinearity. It will be shown that whistlers can even reverse the magnetic field and propagate with magnetic null points even though locally the wave is evanescent near the null points. After describing the experimental arrangement, the field topology, propagation and damping will be discussed. Nonlinear wave–wave and wave–particle interactions will be presented.

1.1. Experimental setup

The experiments are performed in a large (1 m diam, 2.5 m length) pulsed dc discharge plasma generated with a 1 m diam oxide coated cathode shown schematically in figure 1. The parameter regime ($n_{e} \lesssim 10^{12}$ cm$^{-3}$, $kT_{e} \simeq 2$ eV, $B_{0} = 5 \ldots 10$ G) is that described by electron MHD (magnetized electrons, unmagnetized ions) [18]. Insulated magnetic loop antennae (4 turns, 15–20 cm diam) are inserted into the plasma centre and a charged (1200 V) capacitor (0.1 $\mu$F) is discharged into each loop using a fast power transistor. This results in a damped oscillatory
current \((I_{\text{max}} \simeq 140 \text{ A})\) which decays with period \(T = 2\pi(LC)^{1/2} \simeq 5 \mu\text{s}\) and decay time \(\tau \simeq 28 \mu\text{s}\) in vacuum. Basic circuit analysis yields the circuit inductance \(L \simeq 8.6 \mu\text{H}\) and series resistance of \(R_{\text{vac}} = 2.3 \Omega\) in vacuum and \(R_{\text{pl}} \simeq 7.1 \Omega\) in plasma. The increase in resistance, \(\Delta R = R_{\text{pl}} - R_{\text{vac}} \simeq 4.8 \Omega\), allows one to calculate the power deposited into the plasma, \(I^2 \Delta R / 2 = 40 \text{ kW}\), a significant fraction of the applied power of 155 kW. In vacuum, the magnetic field in the centre of the loop at the peak current is given by \(B = 32 \text{ G} \gg B_0\). Depending on current polarity, the resultant field is either reversed or enhanced as shown schematically in figure 1.

The local magnetic field is measured with a single magnetic probe containing three orthogonal small loops (5 mm diam) which can be moved in three orthogonal directions. From highly repeatable \((\delta n/n < 3\%\)\) discharges at a fast repetition rate \((1 \text{ Hz})\) the space-time dependence of the field is measured with a four-channel digital oscilloscope \((10\text{-bit, } 2 \text{ ns resolution})\) and stored and digitally processed. We measure the fields at each probe position in vacuum and in plasma with different current polarity. This allows us to distinguish the fields created by plasma currents, \(B_{\text{plasma}}\), from those created by antenna currents, \(B_{\text{coil}}\), and to demonstrate important differences in the axial field direction relative to \(B_0\). A Langmuir probe is also attached next to the magnetic probe so as to measure the plasma parameters in space and time. We also perform light emission measurements with a photomultiplier tube with good temporal \((<1 \mu\text{s})\) and spatial \((<2 \text{ cm } \perp \text{ line of sight})\) resolution.

2. Field topology

Figure 2 shows field properties in a transverse \(y-z\) plane through the centre of the antenna coil \((x = 0)\) at one instance in time. Figure 2(a) presents contours of the axial component \(B_z\) of the total magnetic field \((B_{\text{tot}} = B_0 + B_{\text{coil}} + B_{\text{plasma}})\). The two large regions of negative \(B_z\) on axis are the spheromaks where the positive guide field has been reversed. The two small regions off-axis outside the coil are of no interest since here the field is due to the coil’s near-zone field. At the time shown, the coil field on axis is parallel to \(B_0 (B_z, \text{coil} > 0)\), hence the return field outside the coil is opposite to \(B_0\) and creates a net field reversal. There is also an out-of-plane toroidal magnetic field component \(B_x\), shown as a contour plot in figure 2(b). It links with
Figure 2. Field topology of whistler spheromaks emitted from a loop antenna. (a) Contours of the axial field component $B_z$. (b) Contours of the toroidal (out-of-plane) field component $B_x$. The linkage between poloidal and toroidal fields produces helicity which is negative for spheromaks propagating against $B_0$, positive with $B_0$. (c) Vector field $(B_y, B_z)$ showing the poloidal field topology with two radial null points on axis and two $O$-type nulls off-axis. (d) Current density component $J_x$ which is a cut through the toroidal current $J_\theta$.

The poloidal field $(B_y, B_z)$ shown as a vector field in figure 2(c). The coil field is self-evident. Of primary interest are the two adjacent field-reversed regions. Each contains a null point on axis and a toroidal null line. Near the null line flows a toroidal electron current, $J_x$, shown in figure 2(d) as a contour plot. The net field lines in the field-reversed regions are nested helices as in a spheromak. For the left spheromak, the toroidal and poloidal field linkage is left-handed (negative helicity), and the helicity sign is reversed for the right spheromak. Magnetic helicity density and current helicity density, $j \cdot B_{\text{tot}}$, have the same sign.

Before describing the propagation of the magnetic structure, we will contrast the topology of such whistler spheromaks [19] with that of whistler mirrors which are excited when $B_z$, coil is parallel to $B_0$. In this case the wave increases the net field strength in the centre of the wave packet as shown in figure 3. There are no magnetic null points (except near the coil where a toroidal X-line is formed due to the highly localized coil current). The mirror field in the plasma is produced by a toroidal current layer carried by electron Hall currents, $J_\theta \simeq -neE_z/B_0$. There is also a toroidal field component, $B_x$, which produces negative helicity.

3. Wave excitation

The antenna current induces electron currents which move away from the antenna as wave currents. The propagating field structures are large amplitude, low-frequency whistler wave packets in a high electron-beta plasma. The ejection process can be explained by the magnetic force: when the antenna current rises, Lenz’s law implies an opposing plasma current. Opposing currents repel, hence the light electron current ring is ejected away from the fixed
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Figure 3. Field components of a whistler mirror, (a) $B_z$, (b) $B_x$, showing that whistler mirrors have the same helicity properties as whistler spheromaks.

Figure 4. Nonlinear properties of large amplitude whistlers. (a) Peak amplitude and width of a whistler spheromak versus applied antenna current. The normalized product of width and amplitude is nearly constant as in solitons. (b) Peak wave intensity of a whistler spheromak and mirror versus antenna current.

The smaller $J \times B_0$ force causes a small radial inward force. No plasma current is induced when the antenna current peaks. When the antenna current decays, the induced current has the same direction as the antenna current. It remains close to the antenna until the antenna current reverses sign and the $J \times B$ force repels the induced current ring. In case of the spheromak, the toroidal current forms an $O$-type null in the magnetic field. In case of the mirror, it spreads out into an elongated Hall current layer of approximately half a wavelength. If the antenna current is switched off at the first zero crossing (one-half rf cycle) no propagating pair of spheromaks is formed, but a field-reversed configuration (FRC) is formed. The FRC elongates at the whistler speed in both directions along $B_0$ but remains centred at the antenna. For small-amplitude whistlers, the current distribution is independent of polarity and forms an elongated solenoid of maximum length, $\lambda/2 \simeq 50$ cm.

Compared with the half-wavelength of a linear whistler wave, the axial amplitude profile of a spheromak is contracted while that of a mirror is lengthened. Figure 4(a) shows the peak amplitude and $-3$ dB width (full width at half energy density) versus applied antenna current for the spheromak. The nonlinear decrease in width times the increase in amplitude, suitably normalized to the applied field, is nearly constant. The nonlinear features of wave contraction coupled with amplitude enhancement are characteristic for solitary waves. Thus,
A nonlinear whistler spheromak can be considered a form of \textit{whistler soliton}. It is not an envelope soliton [20] but each cycle produces a solitary wave. It is produced by an amplitude-dependent propagation velocity described next. In contrast, a whistler mirror spreads out with increasing amplitude resulting in a wave intensity much lower than that of a spheromak at the same antenna field/current (see figure 4(b)).

4. Nonlinear wave propagation

Once the induced current is ejected from the antenna, it propagates as a whistler wave packet predominantly along the ambient magnetic field $B_0$, even when $B_{wave} > B_0$. For the present parameters ($\omega/2\pi \simeq 0.2$ MHz, $\omega_{ce}/2\pi \simeq 22$ MHz for $B_0 = 8$ G, $\omega_{pe}/2\pi \simeq 6000$ MHz), the parallel group velocity of a linear whistler is given by $v_{group} \simeq 2v_{phase} \simeq c(\omega/\omega_{ce})^{1/2}/\omega_{pe} \simeq 22$ cm $\mu$s$^{-1}$. We display in figure 5(a) the wave energy density $B_{wave}^2/2\mu_0$ as contours in the $z$--$t$ plane. The slope of the contour crest yields the axial propagation speed which is observed to vary with amplitude and polarity. The first wave is in response to the current turn-on and excites a wave with $B_{z,wave} \parallel B_0$ propagating into the unperturbed magnetic field $B_0$ at essentially the expected propagation speed of a linear whistler. The second wave is a whistler spheromak which propagates slower than a linear whistler since the wave lowers the average magnetic field ahead of the wave. The third wave is a whistler mirror ($B_{z,wave} \parallel B_0$), which moves faster than a linear whistler since the net field is enhanced by the wave. At some distance the mirror will catch up with the preceding spheromak leading to wave-wave interactions to be discussed below.

By varying the antenna current, the dependence of propagation speed on wave magnetic field is measured. Figure 5(b) shows that whistler spheromaks slow down with increasing wave amplitude while whistler mirrors speed up, relative to linear whistlers. Slower spheromaks have larger wave energy density than faster mirrors. Due to the axial magnetic field gradient, the front of the spheromak should propagate slower than the centre which could account for wave steepening. The spheromak’s trailing side may be compressed by the faster propagating mirror. Wave collisions will be discussed below.
5. Wave-particle interactions and heating

In the present case, we are dealing with whistler waves whose peak energy density, $B_{\text{wave}}^2/2\mu_0 \simeq 2.5 \times 10^{-7}$ J cm$^{-3}$, exceeds the electron energy density, $nkT_e \simeq 1.6 \times 10^{-7}$ J cm$^{-3}$. Thus the primary energy flow is from the wave to the particles. The wave propagates slower than the electron thermal velocity ($v_{\text{phase}}/v_{\text{thermal}} \simeq 0.12$). Few low energy electrons would satisfy the conditions for inverse Landau damping ($v_e = v_{\text{phase}} \ll v_{\text{thermal}}$); few high energy electrons would satisfy the conditions for cyclotron damping [$v_e = (\omega_c - \omega)/k_\parallel \simeq (\omega_c/\omega)v_{\text{phase}} \gg v_{\text{phase}}$]. For large amplitude waves, the cyclotron frequency varies locally within one wavelength, hence the nature of the interaction is more stochastic than coherent and resonant. Resonant interactions with ions are irrelevant ($v_{\text{phase}}/v_{\text{ion,thermal}} > 100$).

Experimentally, we find that for comparable antenna currents a whistler spheromak produces copious electron heating while a whistler mirror does not. Heating is qualitatively inferred from visible light emission and quantitatively obtained from Langmuir probe traces. Light is produced by excitation of neutrals and ions by energetic electrons ($>10$ eV).

Figure 6(a) shows typical Langmuir probe traces which yield the local density, temperature and plasma potential versus time. Note that the probe voltage is kept constant during each shot but increased in 200 increments between ensembles of 10 repeated shots. Strong heating and energetic electron tails are observed in whistler spheromaks. The background temperature of approximately 2 eV can increase locally up to 15 eV with tail temperatures of 30 eV. Heating in mirrors is relatively small (a few electronvolts). The tail electrons produce copious light in the otherwise dark afterglow plasma. Electron temperature and light emission are shown versus time in figure 6(b). For comparison, the antenna current waveform is also displayed. Heating and light are highly correlated with each other and the wave magnetic field. Heating starts when a spheromak approaches and ends when the field has propagated past the probe or line of sight ($\Delta z = 2$ cm). With increasing distance from the antenna, the peak in $kT_e$ or light shifts to later time. The source of light propagates much slower than the electron thermal speed. Thus, the hot electrons are not produced at the antenna and simply stream away. Instead, heating occurs
locally in the toroidal current layer of the propagating spheromak. The lack of heat confinement and radiation losses explain the rapid drop in temperature after the passage of the wave.

The whistler mirror has a large positive plasma potential in its current layer, which produces a strong radial electric field and toroidal Hall current. In ideal EMHD, the radial magnetic force and electric force are balanced \((-neE + J_\times B = 0)\). For the whistler spheromak, the large outward pressure gradient is balanced by magnetic and electric forces pointing into the toroidal current ring. On the time scale of a wave period density modifications are negligible.

The scaling of electron heating with various parameters has also been investigated. There is a lower threshold for the wave magnetic field \((B_{z,\text{wave}} > B_0)\) because a field reversal with toroidal null line has to be formed for a significant heating to occur. Vice versa, for a large \(B_{z,\text{wave}}\), heating vanishes as \(B_0\) is increased since the magnetic nulls vanish. When the ambient field is lowered to \(B_0 = 0\), little electron heating occurs near the antenna on every half cycle which produces the same field topology without null lines. For decreasing plasma density the wave current becomes insufficient to produce field reversals and weak heating occurs only near the antenna on every half cycle. When the loop antenna is rotated heating vanishes since the toroidal null line degenerates into 3D null points and the wave current becomes predominantly a Hall current.

The strong electron heating in a whistler spheromak can be understood as follows: the dominant current of a spheromak is localized near the toroidal magnetic null line where the electrons are essentially unmagnetized or stream along the weaker toroidal magnetic field. The toroidal current is driven by a toroidal inductive electric field produced by the decay of the wave magnetic field, \(E_{\text{toroidal}} \propto -dB_z/dt\). Thus, electrons are accelerated along \(-E_{\text{toroidal}}\) and magnetic energy is dissipated, \(J \cdot E > 0\). The Larmor radius of the energetic electrons is large enough to demagnetize the electrons around the null line. However, the electrons must be scattered since simple acceleration would produce currents and recreate magnetic fields. Collisions and microturbulence scatter the electrons, the latter being produced by current-driven ion sound turbulence [21]. Simple acceleration of electrons does not convert magnetic energy since the streaming electrons would produce currents and fields. The localization of the wave current near the null line produces a much larger current density than for a whistler mirror whose Hall current is spread out axially over half a wavelength. There is little heating in whistler mirrors which are produced by crossed electric fields and currents for which \(J \cdot E = 0\).

6. Wave-wave interactions

When a wave modifies those plasma parameters which affect its own propagation there will also be a nonlinear interaction with other similar waves. This leads to wave–wave interactions, an important effect in strong turbulence. For example, Alfvén waves modify the plasma density and magnetic field which determine the dispersion of this mode \([v_{\text{Alfvén}} \propto (B/nm_i)^{1/2}]\) and thereby couple frequency and wavevectors of different waves. Low frequency whistlers have a similar dispersion \([v_{\text{whistler}} \propto (B/nm_e)^{1/2}]\) but the main difference is that the ion density change is small \((n_i \simeq n_e \simeq \text{const.})\) at higher frequencies, hence the electrons are also incompressible \((\nabla \cdot v_e \simeq 0)\) and the primary nonlinear effect arises from the wave magnetic field. Here we consider only strong nonlinearities within one oscillation, not ponderomotive and thermal effects creating density modifications on the time scale of many oscillations.

A basic experimental test on nonlinear interactions is to propagate two whistler waves against each other and check for changes in the propagation of one wave by the presence of the other wave. Two identical loop antennae, separated by 50 cm along \(B_0\), are used to propagate two whistler spheromaks of opposite helicity against each other. Similar experiments with small amplitude whistlers \((B_{z,\text{wave}} \ll B_0)\) have shown no interaction upon head-on collisions.
of wave packets [22]. However, the present large-amplitude whistlers do interact strongly. Figures 7(a)–(d) show snapshots of field lines as two spheromaks collide inelastically and reconnect into a single FRC. Unlike classical solitons, these nonlinear waves do not re-emerge after the collision. The collision preserves total magnetic helicity which is zero: the counter-propagating spheromaks carried opposite helicity and an FRC has no helicity.

The merged FRC tilts back and forth. The oscillation in a plane is the projection of a precessional motion in 3D space. Figure 7(e) shows a hodograph [23] of the $B$-field vector in the centre of the FRC. The field structure precesses since the frozen-in magnetic field lines are convected by the toroidal electron fluid drift.

Since the FRC does not propagate and does not interact with boundaries, the loss of magnetic energy implies true dissipation of magnetic energy. The free magnetic energy of the spheromaks dissipates locally within one-half cycle. Such fast energy conversion is important in the physics of magnetic reconnection. Recently it has been recognized that Hall or electron MHD plays an important role near magnetic null points [18, 24–27], yet little is known about the exact mechanism of electron heating on small scales. Likewise, dissipation in solar wind turbulence occurs at short wavelengths approaching the regime of whistler turbulence [28]. The dissipation processes in 3D whistler turbulence are therefore of great interest.

Much can be learned from controlled experiments of wave-wave interactions. A variety of wave interactions occur in whistler turbulence. So far, we found that head-on collision of two spheromaks produces the strongest wave-wave interaction. Collisions of two oppositely propagating whistler mirrors leads to a standing wave-like structure between the coils with negligible electron heating or light emission. The head-on collision of a whistler spheromak with a whistler mirror leads to a partial cancellation of the opposing axial fields but an addition of the parallel toroidal fields. Glancing collisions between radially displaced spheromaks have also been studied.
Figure 8. Light emission from colliding spheromaks with different radial offsets. The line of sight is radial through the axis midway between the coils spaced axially by 50 cm. Inelastic spheromak collisions cause extended light emissions which are most pronounced for head-on collisions ($\Delta r = 0$), but still occur for glancing collisions until the spheromaks no longer overlap ($\Delta r > 2R_{\text{coil}}$).

Figure 8 shows the light emission from two colliding spheromaks generated by radially displaced loop antennae, separated axially by 50 cm. As long as the spheromaks overlap they interact nonlinearly and produce extended light emission. The waves pass by each other only when the loops are separated by their diameter. The light is then a linear addition of both spheromak emissions.

7. Summary

Properties of nonlinear whistler modes have been reviewed. The new regime of whistlers with wave magnetic field exceeding the ambient field leads to a rich field of nonlinear effects. The most striking observation is that soliton-like whistler spheromaks propagate with magnetic null points along the ambient field slower than linear whistlers. The toroidal current is carried by unmagnetized electrons which are strongly heated. Head-on collisions of whistler spheromaks are inelastic and form a stationary FRC. When the wave magnetic field enhances the background field propagating mirror fields are produced. The interaction of whistler structures with different topologies may form the basis for strong whistler turbulence. Magnetic nulls are responsible for the strongest nonlinear effects and electron heating.

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