Vortices and Flux Ropes in Electron MHD Plasmas I.

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Laboratory experiments are reviewed which demonstrate the existence and properties of three-dimensional vortices in Electron MHD (EMHD) plasmas. In this parameter regime the electrons form a magnetized fluid which is charge-neutralized by unmagnetized ions. The observed vortices are time-varying flows in the electron fluid which produce currents and magnetic fields, the latter superimposed on a uniform dc magnetic field \( \mathbf{B}_0 \). The topology of the time-varying flows and fields can be described by linked toroidal and poloidal vector fields with amplitude distributions ranging from spherical to cylindrical shape. Vortices can be excited with pulsed currents to electrodes, pulsed currents in magnetic loop antennas, and heat pulses. The vortices propagate in the whistler mode along the mean field \( \mathbf{B}_0 \). In the presence of dissipation, magnetic self-helicity and energy decay at the same rate. Reversal of \( \mathbf{B}_0 \) or propagation direction changes the sign of the helicity. Helicity injection produces directional emission of vortices. Reflection of a vortex violates helicity conservation and field-line tying. Part I of two companion papers reviews the linear vortex properties while the companion Part II describes nonlinear EMHD phenomena and instabilities.

I. INTRODUCTION

Vortices are common phenomena in fluids, gases, and plasmas. They often arise from velocity shear such as occur in flows past obstacles (for example, von Karman vortex streets [1], Kelvin-Helmholtz instabilities [2], drift waves [3], ion temperature gradient driven vortices [4], black auroras [5]) or differential rotations (cyclones [6], rotating pure electron plasmas [7]). Two-dimensional vortices form fluid rotations around one axis, three-dimensional vortices exhibit two linked rotations around orthogonal axes (e.g., Hills vortex [8, 9]). Vortices in MHD plasmas (e.g., spheromaks [10], Alfvérons [11], flux ropes [12]) are more familiar than in electron MHD plasmas where their existence has been theoretically discussed [13, 14] but only recently studied experimentally [15, 16]. The EMHD vortices are time-dependent three-dimensional (3-D) flows in an electron fluid which is embedded in a stationary ion fluid and a uniform dc magnetic field \( \mathbf{B}_0 \). The vortices occur on time scales shorter than an ion cyclotron period or spatial scales smaller than an ion Larmor radius such that the ions are effectively unmagnetized. The time-dependent electron flows produce currents, \( \mathbf{J} = ne\mathbf{v} \), and perturbed magnetic fields, \( \nabla \times \mathbf{B}(\mathbf{r}, t) = \mu_0 \mathbf{J} \), which propagate as whistler eigenmodes [17] along the dc magnetic field. EMHD vortices can be excited either from controlled sources or arise self-consistently in plasma instabilities such as pressure-gradient driven instabilities in high-\( \beta \) plasmas where \( \beta = n_kT_e/(B_0^2/2\mu_0) \) is the normalized electron pressure. After a brief description of the laboratory experiment the basic properties of whistler vortices such as topology, helicity, propagation and reflection are reviewed in Part I while vortex collisions, nonlinear phenomena, reconnection at 3-D null points and helicity generation by instabilities are reviewed in Part II.

II. EXPERIMENTAL ARRANGEMENT

The experiments are performed in a large laboratory plasma device sketched schematically in 1. A 1 m diam \( \times 2.5 \) m long plasma column of density \( n_e < 10^{12} \) cm\(^{-3}\), electron temperature \( kT_e < 5 \) eV, Argon gas pressure \( p \approx 3 \times 10^{-4} \) Torr, is produced in a uniform axial magnetic field \( B_0 < 10 \) G with a pulsed dc discharge (50 V, 600 A, \( t_{\text{pulse}} \approx 5 \) ms, \( t_{\text{rep}} \approx 1 \) s) using a large oxide-coated cathode. High-\( \beta \) (> 1) experiments are performed in the active discharge, low-\( \beta \) studies in the quiescent, uniform, current-free afterglow plasma. The controlled excitation of EMHD vortices is accomplished with pulsed currents to electrodes [18], pulsed magnetic fields to antennas (loops, toroids) [19], and heat pulses which produce localized thermal currents [20]. The time-varying magnetic fields associated with the plasma currents are measured with a triple magnetic probe, recording three orthogonal vector components versus time at a given position. By repeating the highly reproducible discharges and mov-

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FIG. 1: Experimental setup and basic plasma parameters.
ing the probe to many positions in a three-dimensional volume, the vector field $\mathbf{B}(r, t)$ is obtained with high resolution ($\Delta r \approx 1$ cm, $\Delta t \approx 10$ ns). This allows us to calculate at any instant of time the current density $\mathbf{J}(r, t) = \nabla \times \mathbf{B}/\mu_0$ without making any assumptions about field symmetries or using $\nabla \cdot \mathbf{B} = 0$. The plasma parameters are obtained from a small Langmuir probe which is also movable in three dimensions.

### III. EXPERIMENTAL RESULTS

#### A. Properties of Linear Whistler Vortices

The penetration of the applied magnetic field into a plasma is theoretically described by Faraday’s law and Ohm’s law, which for an ideal uniform plasma dominated by the Hall effect yields $\partial \mathbf{B}/\partial t = \nabla \times (\mathbf{v} \times \mathbf{B})$ when pressure gradients are absent. Here, $\mathbf{v} = -\mathbf{J}/ne = -\nabla \times \mathbf{B}/ne\mu_0$ is the electron fluid velocity and $\mathbf{B} = \mathbf{B}(r, t) + \mathbf{B}_0$ the total magnetic field. Displacement currents are negligible compared to conduction currents, $J_{\text{dis}}/J_{\text{cond}} \approx (\omega/\omega_p)^2 \approx 10^{-7}$. Fourier analysis of the equation yields the dispersion of low-frequency whistlers, $\omega \approx \omega_c(kc/\omega_p)^2$. For small field perturbations, $B(r, t) \ll B_0$, propagating with wave velocity $\pm v_0 = \partial z/\partial t$ along the dc magnetic field, the linearized solution of the differential equation yields $\mathbf{J}/ne = \pm v_0 \mathbf{B}(r, t)/B_0$. The perturbed field has a positive (negative) self-helicity density $\mathbf{J} \cdot \mathbf{B}(r, t)$ for propagation along (opposite to) the dc magnetic field. The same holds for the magnetic self-helicity density $\mathbf{A}(r, t) \cdot \mathbf{B}(r, t)$ and the kinetic helicity density $\mathbf{v} \cdot \omega \times \mathbf{J} \cdot (\nabla \times \mathbf{J})$. The unique property of EMHD vortices that their helicity densities depend on propagation direction is the result of the Hall effect, $\mathbf{E} = \mathbf{J} \times \mathbf{B}/ne$. The latter also shows that the electromagnetic fields are force-free and frozen into the electron fluid. When electron inertia is included, which becomes relevant for scale lengths shorter than the electron inertial length $c/\omega_pe$, the generalized vorticity $\mathbf{\Omega} = \nabla \times \mathbf{v} - (e/m)\mathbf{B}$ is frozen into a collisionless electron fluid [21].

When a positive voltage pulse is applied to an electrode the injected current in the plasma flows in the form of a spiral as shown in Fig. 2. The helical current flow can be thought of as a superposition of the field aligned current and an electron Hall current. The latter is produced by a radial electric field which is due to the collection of electrons at the electrode. Note that for $B_0 > 0$ the Hall current $J_{\text{Hall}} = B_0 \times E/ne$ produces a right-handed helix and for $B_0 < 0$ a left-handed helix. The front of the spiralling current system propagates at the whistler speed along $\mathbf{B}_0$. Since the current must be closed ($\nabla \cdot \mathbf{J} = 0$), the current density lines at the front return as outer helices to the negative return electrode. The length of the current tube is determined by the applied pulse length and propagation speed. For short pulses a current vortex is formed which detaches from the electrodes and propagates through the plasma. It exhibits knotted current density lines[18].

Figure 3 shows an experimental verification of a short 3-D vortex in the perturbed magnetic field. It is excited by a current step applied to a toroidal magnetic antenna with $\mathbf{B}_\theta \perp \mathbf{B}_0$. Snapshots of (a) the toroidal vector magnetic field $\mathbf{B}_\theta = (B_x, B_y)$ and (b) the linked poloidal or dipolar field $(B_{\Phi}, B_z)$ show a speromak-like field topology. The current density or electron fluid velocity $\mathbf{v} = -\mathbf{J}/ne = -\nabla \times \mathbf{B}/ne\mu_0$ show a similar topology. The vortex can be viewed as a whistler wave packet consisting of a half cycle oscillation propagating predominantly along the larger guide field $\mathbf{B}_0$. As the vortex propagates, it gradually becomes $V$-shaped since the off-axial obliquely propagating fields are slower than central field propagating along $\mathbf{B}_0$. Dispersion of whistlers can produce secondary vortices ahead and behind the main vortex leading to nested vortices.

Vortices can also be excited with a simple loop antenna with dipole moment along $\mathbf{B}_0$ which excites the poloidal vortex field. The toroidal field develops self-consistently. It is worth noting out that, in contrast to an isotropic conductor, the toroidal current is not directly driven by the toroidal inductive electric field of the loop antenna, but it is a Hall current due to a radial space charge electric field associated with the adiabatic compression of electrons. The incompressible electrons stream along the dc field which produces current/fold linkage similar to the case of electrode excitation. Space charge and inductive electric fields have been determined separately [22].

#### B. Helicity Injection and Directionality of Propagation

The helicity of a vector field $\mathbf{A}$, defined as $K = \int \mathbf{A} \cdot (\nabla \times \mathbf{A}) \, dV$, is of fundamental interest for 3-D vor-
FIG. 3: Three-dimensional magnetic vortex excited by a toroidal antenna. (a) Vector plot of the toroidal field component $\tilde{B}_{\text{tor}} = (\tilde{B}_x, \tilde{B}_y)$ in an $x$-$y$ plane at the center of the vortex ($z = -10$ cm). (b) Poloidal field components $\tilde{B}_{\text{pol}} = (\tilde{B}_y, \tilde{B}_z)$ in a $y$-$z$ plane along the axis of the vortex ($x = 0$). Note the left-handed linkage of $\tilde{B}_{\text{tor}}$ and $\tilde{B}_{\text{pol}}$ due to vortex propagation against $B_0$.

Helicity conservation between source and plasma is generally satisfied. For example, when a current pulse is applied to a loop antenna as shown in Fig. 4 two vortices are excited, one propagating along $B_0$ with positive self-helicity density $A(r, t) \cdot B(r, t)$, the other one opposite to $B_0$ with negative self-helicity density. Thus, the total self-helicity is zero, consistent with the fact that the loop does not inject net helicity. This also holds for the mutual magnetic helicity, $\int A_0 \cdot B(r, t) \, dV = 0$, where $\nabla \times A_0 = B_0$. The mutual helicity refers to the linkage of the vortex field $B(r, t)$ with the mean field $B_0$, and the total helicity is the sum of the self helicity and mutual helicity [25]. There is no mutual current helicity, $\int B_0 \cdot J(r, t) \, dV = 0$. Two vortices of opposite self-helicities are also excited by a torus antenna, which excites the toroidal magnetic fields of the vortices. While the self-helicity vanishes, the mutual helicity does not vanish since the applied toroidal field links with the uniform field so as to inject net helicity. Both vortices have the same $B_\theta$, hence carry the same mutual helicity, as expected from helicity conservation.

A loop placed on the axis of a torus can be used to apply helical fields to a plasma, i.e., to inject helicity. The sign of the applied helicity depends on the relative direction of loop and torus currents. When positive helicity is injected a vortex with positive self-helicity is excited which travels along $B_0$, for negative helicity the propagation direction reverses. Thus, helicity conservation implies directional radiation of EMHD fields. This new concept has been tested in a computer simulation [26] and verified experimentally [27]. Figure 5 shows a snapshot of magnetic field components in two orthogonal planes demonstrating that a loop-torus antenna radiates predominantly one vortex along $B_0$ which has the same positive helicity as that of the antenna. The completely symmetric loop-torus antenna exhibits a high directivity of 20 dB (power ratio $P_{\text{right}}/P_{\text{left}} = 100$). The antenna directivity also holds for receiving vortices. By measuring the helicity of unknown vortices, their direction of
propagation can be determined. Transmission between two antennas is non-reciprocal and unidirectional, where the direction can be selected by the antenna helicity or the direction of \( B_0 \). These interesting helicity properties can lead to useful antenna applications, and explain the asymmetric plasma production in helicon devices.

C. Reflections of Vortices from a Conducting Boundary

In this experiment we consider the conservation of helicity and energy as a vortex specularly reflects from a conducting boundary. It is expected that helicity is conserved; however, helicity must change sign since reflection changes the direction of propagation along \( B_0 \). The observation shows that the latter is the case, i.e., helicity is not conserved.

A vortex is excited with a torus antenna and propagates in the direction opposite to \( B_0 \) against a conducting plate whose surface normal is along \( B_0 \). Figure 6 shows magnetic field components for both the incident and reflected vortex in an \( x - y \) plane at \( z \simeq 13 \) cm in front of the plate. The poloidal field component \( B_z \) shown in contours does not change sign upon reflection while the toroidal field \( B_\theta \) does reverse. Thus both the magnetic self-helicity and the mutual helicity, hence total magnetic helicity, are reversed. The boundary conditions imply that the normal magnetic field and the tangential electric field vanish at the conductor. Near the plate the magnetic field lines are predominantly radial due to induced toroidal currents in the conductor. The plate draws no axial current from the plasma since \( B_\phi \simeq 0 \) at the surface. During the pulse reflection the axial magnetic flux near the plate grows and decays which causes the toroidal inductive electric field to reverse sign. This implies a sign reversal of the radial Hall current and axial current, i.e., the poloidal current system, which explains the reversal of the toroidal magnetic field component.

The toroidal field component produces a twist in the total field line \( B_0 + B(r,t) \). Since the field line in the conducting boundary is stationary while the one in the plasma reverses its twist, the field lines cannot connect across the boundary, i.e., field line tying does not occur. The concept of a continuous field line breaks down in the electron-depleted sheath at the boundary since the field is not frozen into a conducting matter. Thus, global helicity need not be conserved. However, energy is highly conserved since the dissipation in the plate is negligible. No fields are transmitted through the plate which is large compared to the vortex and thick compared to the resistive skin depth. The plasma does have losses which cause a decay of the vortex but there is no change due to the reflection.
Figure 7 shows contours of the magnetic helicity per axial length, $dH/dz$, in a time-position diagram for (a) the self-helicity and (b) the mutual helicity. The self-helicity is determined from the product of the two linked vortex fluxes, $H_{\text{self}} = \int \mathbf{A} \cdot \mathbf{B} dV \simeq 2 \Phi_{\text{tor}} \Phi_{\text{pol}}$, where $\Phi_{\text{tor}} = \int B_0 dr dz$ and $\Phi_{\text{pol}} = \int B_0 2\pi r dr$, selecting only one sign of $B_0$ so as to avoid counting the return flux. The mutual helicity is obtained from $H_{\text{mut}} = \int \mathbf{A}_0 \cdot \mathbf{B} dV = \int A_0 B_0 dV$, where $A_0 = r B_0 / 2$, hence $H_{\text{mut}} = \pi B_0 \int r^2 B_0 dr dz$. The toroidal antenna located at $z = 0$ launches two vortices of opposite self-helicities and equal mutual helicities propagating in opposite directions to $B_0$. The left-propagating vortex reflects from the conducting plate at $z \simeq -23$ cm which reverses the sign of both the self helicity and the much larger mutual helicity.

A comparison of the axial decay of magnetic energy and helicity is presented in Fig. 8. The peak values along the propagation characteristics $z - v_{\parallel} t$ are plotted vs $z$ for (a) $dU_m/dz = \int (B^2 / 2\mu_0) 2\pi r dr$ and (b) $dH/dz$. The properties of the incident vortex are obtained from measurements without the plate, those of the reflected vortex from the difference between measurements with and without plate. The incident vortex, propagating in $-z$ direction, dissipates magnetic energy on a scale length $L_z = U_m / |dU_m / dz| = \frac{1}{2} B_0 / |dB / dz| \simeq 13$ cm. No energy is lost at the conducting boundary ($\sigma_A / \sigma_0 \simeq 10^5$). The reflected vortex, propagating in $+z$ direction, decays at the same rate as the energy. Of course, the sign the helicity reverses upon reflection. The only exception is the decay of the mutual magnetic helicity which scales linearly with $B_0$, i.e., decays with $e$-folding length $L_z \simeq 26$ cm. Since the total magnetic energy, $\frac{1}{\mu_0} \int |B_0 + B|^2 2\pi r dr dz = \frac{1}{2\mu_0} \int B_0^2 2\pi r dr dz + \frac{1}{2\mu_0} \int B^2 2\pi r dr dz$, changes at the same rate as that of the vortex alone ($L_z \simeq 13$ cm), the total magnetic helicity decays at about half the rate of the total magnetic energy, a result coincidentally in agreement with Taylor’s conjecture [23] for the relaxation in the presence of localized dissipation (e.g. in current sheets), which does not apply to the present case.

The present result has interesting consequences on the reflection of ducted whistlers in the magnetosphere. Their helicity should reverse upon reflection from the ionosphere, implying a lack of field line tying or frozen-in field lines at the reflecting interface.
IV. SUMMARY AND CONCLUSIONS

Basic laboratory experiments have shown that helicity is a fundamental property of fields and currents in EMHD plasmas. Our main findings are that transient EMHD fields form 3-D vortex topologies which propagate in the whistler mode, have a unique sign of helicity depending on propagation direction, and conserve helicity except for a sign change upon reflection at a conducting boundary. Further properties will be described in the companion review, Part II.

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