Vortices and Flux Ropes in Electron MHD Plasmas II.

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Laboratory experiments are reviewed which continue to describe novel properties of three-dimensional (3-D) vortices in electron MHD (EMHD) plasmas. These include the collision of vortices which produces no nonlinear interaction since their electromagnetic fields are force-free \((J \times B + neE \simeq 0)\). Nonlinearities arise when the vortex field exceeds the background magnetic field and produces magnetic null points where EMHD breaks down. Dc magnetic fields are produced by vortices of alternating polarities. Magnetic reconnection at 3-D null points changes the field topology on the fast whistler time scale. In a high-beta plasma \((\beta > 1)\) pressure-gradient driven instabilities near the lower hybrid frequency are observed to form elongated magnetic vortices (flux ropes) coupled to flute-like density striations.

I. INTRODUCTION

In Part I we have demonstrated the existence of 3-D vortices in EMHD plasmas and described their topology, helicity, propagation and reflection. In Part II we continue the review with vortex collisions, nonlinear vortices with 3-D magnetic null points, and the observation that EMHD flux ropes can be produced by pressure-gradient driven instabilities in high-\(\beta\) plasmas where \(\beta = n k T_e / (B_0^2/2 \mu_0)\) is the normalized electron pressure. Since the experimental device and measurement techniques have been discussed in Part I we start with the experimental results.

II. EXPERIMENTAL RESULTS

A. Collisions of Vortices

We now consider the collision of two EMHD vortices, which propagate in an unbounded uniform plasma on the same field line \(B_0\) against each other. Since they carry opposite helicities either their toroidal or poloidal field components cancel during the collision. Fundamental questions concern the energy conservation during annihilation/reconnection of opposing fields as well as any nonlinear wave-wave interactions which may give rise to fluid turbulence [1]. The collision of 3-D MHD vortices has been studied earlier and billiard-ball like collisions were observed [2].

As an example, we show the head-on collision between two identical vortices with opposite toroidal magnetic fields. Figure 1 shows the measured toroidal field component \(B_\theta (= \pm B_x\) for \(x = 0\) versus time and axial propagation distance in three cases: (a) When vortex 1 is excited alone and propagates with \(B_x < 0\) in along \(B_0\); (b) when vortex 2 is excited alone and propagates with \(B_x > 0\) opposite to \(B_0\); (c) When both vortices are excited together. In this case, at the time of collision \((t \simeq 0.4 \mu s)\), the toroidal field vanishes at all \(z\) positions, and a two-dimensional (2-D) vortex remains with a poloidal magnetic field created by a toroidal current or electron drift.

After the collision, the toroidal field recovers and the vortices continue to propagate as they did individually. The interaction is entirely linear since the observed fields are essentially identical to a linear superposition of the fields of both individual vortices (Fig. 1d). This result is observed even when the vortex field strength approaches a large fraction of the dc field. It can be understood from the fact that EMHD electromagnetic fields are highly force free, i.e., \(J \times B + neE \simeq 0\) [3]. In contrast to MHD, parallel currents do not attract in EMHD, consistent with the observation that narrow current sheets along a guide field do not tear [4]. The vortex collision also conserves energy. The opposing toroidal fields are not irreversibly annihilated, but the loss of toroidal field energy creates an increase in the poloidal field energy, which is reversed after the vortices separate. In EMHD reconnection the inductive electric field accelerates electrons, which produces new currents and magnetic fields enabling an energy transfer from toroidal to poloidal fields and vice versa. Total zero helicity remains conserved.

The glancing collision between EMHD vortices propagating on adjacent field lines has also been studied. Again, the vortices pass through each other without interaction or deflection.

B. Nonlinear EMHD Phenomena

Nonlinear phenomena generally arise when the plasma current modifies those parameters, which determine the penetration of the magnetic field into the plasma. These include the basic plasma parameters such as the density and electron temperature and the net magnetic field. The ion density is difficult to perturb by EMHD phenomena, but the electron temperature can readily change on EMHD time scales. An example is the observation of a current filamentation instability created by Joule heating, which produces a field-aligned channel of high con-
Magnetic nonlinearities arise since the propagation of whistlers depend on magnetic field strength and direction. Alternatively, the penetration of magnetic fields is governed by \( \partial \mathbf{B} / \partial t = \nabla \times (\mathbf{v}_c \times \mathbf{B}) \), which is nonlinear in \( \mathbf{B} \) since \( \mathbf{v}_c \propto \mathbf{J} \propto \nabla \times \mathbf{B} \). However, EMHD vortices \( \mathbf{B}(r,t) \) in a stronger background magnetic field \( \mathbf{B}_0 \) satisfy \( \mathbf{J} \times \mathbf{B}(r,t) = 0 \), which linearizes the \( \mathbf{v}_c \times \mathbf{B} \) term. The cross product creates a weak nonlinearity which explains why EMHD vortices are remarkably linear structures. In contrast to electrostatic waves, whistler vortices can transport linearly field energies in excess of the particle energy density. However, the linearity does break down when the vortex fields exceed the dc field. In this case magnetic null points are formed where the EMHD equations are no longer valid. The EMHD approximation breaks down first by large Larmor radius effects \( r_{ce} \geq |B/\nabla(B)| \), then by collisions \( \nu_{ei} \geq \omega_{ce} \) and inertial effects \( \omega \geq \omega_{ce} \).

One of the manifestations of magnetic nonlinearities is the generation of dc magnetic fields and harmonics from alternating vortex fields [8]. These observations are made in a high-\( \beta \) plasma where the plasma currents are large enough to produce vortex fields exceeding the dc field. When the applied dipole field alternates between parallel and antiparallel to \( \mathbf{B}_0 \) the measured toroidal field shows a time-average dc component as well as harmonics not present in the applied spectrum. The plasma nonlinearity effectively “rectifies” applied ac fields to produce dc magnetic fields.

Since the nonlinearity mainly originates from the failure of EMHD near magnetic null points their properties have been investigated in more detail. In these studies, a current pulse (150 A, 6 \( \mu \)s) is applied to a shielded loop antenna (12 cm diam, 4 turns) arranged so as to produce a strong dipole field opposite to \( \mathbf{B}_0 \). Figure 2 shows a snapshot of the total magnetic field lines traced through the two cusp-type null points created on axis where the dipole field cancels \( \mathbf{B}_0 \). The field lines lie on a 3-D separatrix surface, which divides “closed” dipolar field lines from “open” field lines similar to an idealized planetary magnetosphere in a uniform interplanetary field [9]. The field lines enter/leave the null points along a “spine” and a “fan” [10]. The field lines near the null point can be expanded into a Taylor series, \( \mathbf{B} = \mathbf{M} \cdot \mathbf{r} \), where the properties of the matrix \( \mathbf{M} \) characterize the shape of the 3-D null point. In vacuum, the potential field has a spine orthogonal to the surface of the fan. In the presence of plasma, the induced currents prevent the penetration of the dipole field into the plasma during turn-on and de-
lay its disappearance during turn-off of the loop current. The field penetration and relaxation is controlled by the propagation of a nonlinear whistler vortex. During turn-off it propagates with two null points away from the loop antenna. The null point topology changes to that of improper spiral nulls [10], an observation of which is shown in Fig. 3. The plane of the fan is inclined to the spine due to currents orthogonal to the spine (Fig. 3a), and the field lines in the fan spiral due to currents along the spine (Fig. 3b). The vortex also exhibits spiral nulls in the electron flow or current density. As the vortex propagates, the closed flux inside the separatrix decreases, the separatrix becomes distorted and tilted due to nonlinear propagation effects $[\nu_\parallel = f(B)]$. The change of topology from a 3-D vortex to a uniform field implies that reconnection takes place on EMHD time scales. Similarly, the creation of a vortex during switch-on involves reconnection.

### III. EMHD TURBULENCE IN HIGH-BETA PLASMAS

The previous experiments demonstrated that EMHD vortices can be excited by pulsed stimulations in a quiescent plasma. The present observations will show that magnetic flux ropes can also naturally evolve from plasma instabilities. In particular, we consider plasmas of high electron beta where $\beta = n kT_e/(B_0^2/2\mu_0)$ where density fluctuations lead to magnetic field fluctuations. Earlier, similar cases have been studied theoretically [11].

A nonuniform dense discharge plasma ($n \approx 10^{12}$ cm$^{-3}$, $kT_e \approx 10kT_i \approx 5$ eV, 2 m length) is created in a weak external magnetic field ($B_0 = 5$ G). The radial profiles of the average electron pressure $\langle p_e \rangle = \langle nkT_e \rangle$ and axial magnetic field $\langle B_z \rangle$, shown in Fig. 4, demonstrate that the center of this high-\(\beta\) plasma ($\beta_{\text{max}} \approx 5$) is essentially unmagnetized. In the transition region the elec-

![FIG. 3: Measured topology of 3-D null points after turn-off of the loop current. Magnetic field lines viewed normal to (a) the spine and (b) the fan. The expansion matrix identifies the configuration as that of a improper non-potential spiral null. Note that the helicity density changes sign across the null point.](image)

![FIG. 4: Electron pressure and internal magnetic field vs radius in a high-\(\beta\) plasma column. Averages are formed over fluctuations driven by pressure gradients.](image)
electron pressure gradient is balanced by both electric and magnetic forces \( \nabla n k_Te \simeq J \times B + ne E \), implying that the MHD pressure balance equation \( n k_Te + B_z^2/2\mu_0 = \text{const} = B_0^2/2\mu_0 \) is not applicable. The unmagnetized ions are not confined and steady-state requires plasma production to balance losses.

A strong instability is associated with the electron diamagnetic drift \( v_e = -\nabla p_e \times B/B^2 \) through the essentially stationary ions \( v_e > c_s = \sqrt{kT_e/m_i} \). Figure 5a shows the time dependence of the density, as inferred from the ion saturation current to a Langmuir probe, of the pulsed discharge \( (t_{\text{pulse}} = 4 \text{ ms}) \). Strong density fluctuations \( \langle \delta n/n \rangle \simeq 20\% \) are observed during the discharge but not in the afterglow. The magnetic field also shows strong fluctuations as shown for the axial component \( \delta B_z \) in Fig. 5b. Fluctuations also exist in the other components, \( \delta B_x, \delta B_y \), as well as in the plasma potential, \( \delta \Phi_{\text{pl}} \). Fourier analysis (Fig. 5c) reveals a frequency spectrum with a broad peak below the local lower hybrid frequency \( f_{\text{lh}} \simeq \sqrt{\nu_e c_i} \simeq 50 \text{ kHz} \). Autocorrelations of the fluctuations show coherence only over one to cycles, cross-correlations between density, potential and magnetic field fluctuations are high \( (\geq 0.75) \). Spatially, the fluctuations amplitude maximize in regions of largest pressure gradients.

In order to determine the space-time behavior of the fluctuations, the method of on-line conditional averaging is used [12]. With one reference probe, the fluctuations are applied to a digital oscilloscope which triggers (i.e., stores a waveform) only when a specified condition \( (e.g., \text{a fluctuation threshold in a time window}) \) is satisfied. An ensemble average is formed over many \( (\geq 100) \) repeated discharge pulses which yields the "typical" properties of a fluctuation, in particular, its temporal coherence, \( \langle \delta n(t) \rangle \). With a multi-channel oscilloscope the fluctuations in density and magnetic field are recorded simultaneously and conditionally averaged. By moving the magnetic probe relative to the reference Langmuir probe propagation and spatial coherence of the fluctuations is obtained. From the spatial information of \( \delta \mathbf{B} = (\delta B_x, \delta B_y, \delta B_z) \), the current density \( \delta \mathbf{J}(r, t) = \nabla \times \delta \mathbf{B}/\mu_0 \) is obtained.

Figure 6 shows a snapshot of the density perturbation \( \langle \delta n(r) \rangle \propto \langle \delta I_{e,\text{sat}} \rangle = \text{const} \) in 3-D space. The fluctuations consist of field-aligned perturbations which propagate in the direction of the electron diamagnetic drift but at a slower speed which is close to the sound speed, \( v \simeq \sqrt{kT_e/m_i} \approx 2 \times 10^5 \text{ cm/s} \). Their spatial coherence extends only over few oscillations, implying that strong spatial turbulence rather than coherent azimuthal eigenmodes characterize the instability.

Of particular interest are the properties of the magnetic field fluctuations \( \delta \mathbf{B}(r, t) \) some of which are displayed in Fig. 7. A snapshot of an isosurface of positive \( \delta B_z(r, t = \text{const}) \) is displayed in Fig. 7a together with a

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**FIG. 5:** Properties of fluctuations driven by electron pressure gradients. (a) Density fluctuation observed in the ion saturation current vs time for a pulsed discharge. Solid line represents the ensemble average over 100 repeated discharge pulses. (b) Fluctuations in the axial magnetic field component caused by the electron diamagnetism. (c) Frequency spectrum of the fluctuations exhibits a broad peak below the local lower hybrid frequency, \( f_{\text{lh}} \simeq \sqrt{\nu_e c_i} \simeq 50 \text{ kHz} \).

**FIG. 6:** Spatial distribution of density fluctuations obtained by conditional averaging. Surfaces of \( \delta n(r) = \text{const} \) form field-aligned flutes which propagate at the sound speed in the direction of the electron diamagnetic drift.
FIG. 7: Snapshot of magnetic fluctuations and associated currents in 3-D space. (a) Isosurface of the axial field component, contours of $\delta B_z$ in the end $x-y$ planes, and vector field of the transverse components $(\delta B_x, \delta B_y)$ in the mid $x-y$ plane. The latter indicates that the fluctuating field has the topology of a flux rope. (b) Field lines of $\delta J(r,t) = \nabla \times \delta B/\mu_0$ form left-handed spirals.

vector field of $(\delta B_x, \delta B_y)(x,y)$ in the mid-plane of the rectangular measurement volume. The axial field component is due to the electron diamagnetic currents, $\delta J_\theta$, the observed azimuthal field, $\delta B_\theta$, implies the presence of field-aligned currents. The perturbed magnetic field forms flux ropes with a left-handed twist. The current density also has linked axial and azimuthal components which result in spiralling lines of $\delta J(r)$ some of which are displayed in Fig. 7b. The field lines of $\delta B(r)$ and $\delta J(r)$ have a left-handed twist irrespective of the sign of $\delta n$ or $\delta B_z$.

Both the magnetic field and current density field exhibit negative helicity. Figure 8a shows contours of the current helicity density $\delta J \cdot \delta B(x,y)$ which has the same distribution as the magnetic energy density $\delta B^2/2\mu_0$. The magnetic structures are essentially relaxed force-free field configurations. Based on the properties of EMHD vortices they may be interpreted as elongated low-frequency whistler vortices with group velocity opposite to $B_0$. The preferred sign of helicity or direction of wave propagation is thought to be related to one-sided location of the plasma/energy source, i.e., the cathode.

FIG. 8: (a) Helicity density $(\nabla \times \delta B/\mu_0) \cdot \delta B$ and (b) energy density $(\delta B)^2/2\mu_0$ showing the same spatial distribution or a constant ratio, i.e., the scale length of a force-free flux rope \{$L \simeq 2(\delta B)^2/2\mu_0)/|\delta J \cdot \delta B| \simeq 2.5 \text{ cm}$\}.

IV. SUMMARY AND CONCLUSIONS

Further laboratory experiments on EMHD vortices have shown that collisions produce no nonlinear interactions between their force-free fields. However, this robust linearity breaks down when the vortex field exceeds the background field and produces magnetic null points where EMHD is no longer valid. The structure of spiral null points has been studied as well as changes in the field topology due to magnetic reconnection. In a high-$\beta$ plasma flux ropes of negative helicity are generated by pressure-gradient driven instabilities. These examples indicate the importance of EMHD phenomena in high-$\beta$ plasmas and in magnetic reconnection in the vicinity of magnetic null regions where the ions are unmagnetized ($r_{ce} < |B/\nabla(B)| < r_{ci}$).

Acknowledgments

The authors gratefully acknowledge support for this work by the National Science Foundation under grant PHY-9713240.