Lower-hybrid turbulence in a nonuniform magnetoplasma

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An experimental study of a pressure-gradient-driven instability in a large discharge plasma (1 m diam, 2.5 m length, ne ≈ 10^{12} cm^{-3}, B ≈ 14 G) is presented. When the electron diamagnetic drift v_d = \nabla n \times B / n e B^2 exceeds the sound speed c_s = (kT_e / m_e)^{1/2}, ion-acoustic-like waves (\omega_k \ll \omega_{in} / c_s) are driven unstable. The growth rate maximizes near the lower-hybrid frequency \omega_{ih} = (\omega_{ex} + \omega_{eh})^{1/2} and the waves propagate essentially across B (k \parallel \ll k \perp). The sound waves grow to large amplitudes (\delta n / n \geq 50\%) and saturate by wave steepening (\lambda \perp \ll \lambda \parallel) and refraction (\nabla T_e \neq 0) away from the destabilizing drift v_d. Magnetic fluctuations result from electron diamagnetic currents and opposing Hall currents associated with the wave density fluctuations. Ions are essentially unmagnetized (v_i / v_d \ll 1) and slow compared to the magnetized electrons, v_i / v_d \approx (m_e / m_i)^{1/2} < 1. In spite of the large amplitude waves little acceleration of electrons or ions is observed. The experiment employs a new technique of conditional averaging with digital oscilloscopes.

I. INTRODUCTION

Instabilities associated with drifts across magnetic fields are of general interest in many areas of plasma physics.\(^1\) Cross-field drifts can arise from gradients in potentials, fields, and plasma parameters (\nabla \phi, \nabla B, \nabla n, \nabla T, \nabla f\). A great variety of modes can be destabilized, such as fluid modes (drift waves,\(^2\) flutes,\(^3\) lower-hybrid waves,\(^4\) ion-acoustic waves\(^5\)) and kinetic modes (electron and ion cyclotron waves\(^6\)). Instabilities can arise in collisionless and collisional plasmas (drift-dissipative instabilities\(^7\)) at different plasma pressures [\beta = n kT / (B^2 / 2\mu_0) \ll 1 \text{ to } \gg 1], at various temperature ratios (\nabla T / T \ll 1 \text{ to } \gg 1), and for magnetized and unmagnetized ions [\omega_i / (B / \nabla B) \ll 1 \text{ to } \gg 1]. Drift instabilities have been studied in space plasmas (ionospheric irregularities,\(^8\) magnetic reconnection,\(^9\) bow shock,\(^10\) active experiments\(^11\)), fusion plasmas (tokamaks,\(^12\) mirrors,\(^13\) pinches\(^14\)), and various laboratory plasmas (Q machines,\(^15\) discharge plasmas\(^16\)). There are common points of interest in studying gradient-driven instabilities in such diverse areas of plasma physics. Among them are growth rates, nonlinear saturation mechanisms, and the effect of the wave turbulence on transport properties.

The present work presents an experimental study of pressure-gradient-driven waves in a linear discharge plasma.\(^17\) The parameter regime is characterized by magnetized electrons (v_i \ll \omega_{ce}), unmagnetized cold ions (v_i > \omega_{ci}, T_i \ll T_e), significant electron pressure (\beta_e \approx 0.5), radial gradients in electron density, temperature and magnetic field but no electric fields (E \times B \approx 0).

In the background of nearly stationary ions the electron fluid exhibits a drift, the diamagnetic drift v_d = \nabla (n kT_e) \times B / n e B^2, which exceeds the sound speed, c_s = (kT_e / m_e)^{1/2} < v_d. This gives rise to a current-driven, cross-field ion-acoustic instability.\(^18\) The waves propagate essentially perpendicular to B (k_\parallel \gg k_\perp, \omega / k = c_s), build up to large amplitudes (\delta n / n \geq 50\%) with a frequency spectrum peaking near the lower-hybrid frequency, \omega \approx (\omega_{ex} + \omega_{eh})^{1/2}. The present finite-beta sound instability involves low-frequency, long wavelength modes (\omega \ll \omega_{ci}, k \ll \lambda) similar to those occurring in lower-hybrid drift instabilities.\(^1\)\(^9\) In fact, Hsia et al.\(^4\) have theoretically shown that for finite k, the lower-hybrid-drift instability and the modified two-stream instability belong to a common class of instabilities. The present instability does not involve the high-frequency (\omega \approx \omega_{ci}) short wavelength (k \ll \lambda) oblique ion-acoustic instability discussed, e.g., by Lashmore-Davies and Martin.\(^5\) There are also noteworthy differences between the present instability and the classical lower-hybrid drift instability, e.g., the observed strong dependence on the temperature ratio T_e / T_i, the difference between drift and phase velocities (v_d \approx \omega / k), and the high collisionality v_i \approx \nu_{in}. The main contributions of the present work lie in the areas of saturation mechanisms and wave-particle interactions. The former arises from wave steepening across B to k_\parallel \gg 1 and from wave refraction due to radial electron temperature gradients. Surprisingly little particle acceleration is observed in spite of the strong lower-hybrid mode turbulence. Further interesting observations are made on the correlations between fluctuations in density, electric, and magnetic fields. Wave \delta E \times B \text{ drifts oppose wave diamagnetic } \delta B \times B \text{ drifts resulting in smaller magnetic fluctuations than expected by pressure balance, } \delta B / B \approx (B / 2) (\delta n / n). \text{ In the present experiment a novel diagnostic technique of on-line conditional averaging is developed. It permits the mapping in space and time of coherent structures in turbulence.}\(^20\)

The paper is organized as follows. In Sec. II the experimental setup is described, followed by an outline of the measurement techniques in Sec. III. The experimental results are presented in Sec. IV in various subsections, while in Sec. V the work is summarized.
II. EXPERIMENTAL SETUP

The experiment is performed in a controlled laboratory plasma of simple linear geometry with typical parameters listed in Table I. As shown in Fig. 1 an argon discharge plasma is generated with a large cathode and adjacent grounded mesh anode. The plasma is immersed in a uniform axial magnetic field $B = 14 \, \text{G}$ that is sufficiently strong to magnetize the electrons but not the ions ($v_{ei} > \omega_c$). The 2 m long plasma column terminates on a floating endplate on the opposite side of the cathode. Thus, the axial current flow is limited to the cathode-grid region. There exists a radial density and temperature gradient but no dc potential gradients are observed. The electron pressure gradient $\nabla p = \nabla (nkT_e)$ gives rise to a fluid drift, the electron diamagnetic drift $\mathbf{v}_d = \nabla p \times \mathbf{B} / neB^2$. The unmagnetized ions are lost radially and replaced by volume ionization which establishes steady-state conditions ($\partial n / \partial t = 0$). The relative drift between the electron and ion fluids gives rise to the observed instability. The diamagnetic current also produces a significant reduction of the axial magnetic field inside the plasma.

A variety of diagnostic tools is used to analyze waves and particles. Basic plasma properties are obtained from Langmuir probes ($n_e, kT_e, \phi_p$) and a directional velocity analyzer$^{22}$ ($f(n), kT_e$), while waves are detected with electrostatic rf probes ($\delta \phi$), an electric dipole probe$^{23}$ ($\delta E$), and magnetic probes ($\delta B$) movable in three orthogonal directions.$^{24}$ Particle drifts are obtained from differential flux measurements with one-sided plane probes, and from propagating test ion-acoustic waves with and against drifting ions.

III. MEASUREMENT TECHNIQUES

The dc discharge is operated in pulsed mode ($t_{on} = 5 \, \text{msec}, t_{off} = 1 \, \text{sec}, 0.5\% \, \text{duty cycle}$), which reduces the average discharge power ($I_{dis} = 1000 \, \text{A}, V_{dis} = 40 \, \text{V}, P_{avg} = 40 \, \text{kW} \times 0.5\% = 200 \, \text{W}$) and allows the study of growth and saturation of the instability which is a function of the electron temperature. Figure 2 shows the typical time sequence of discharge pulses employed. A first discharge pulse ($t_{d1} \leq 4 \, \text{msec}$) is applied to produce a quasi-steady-state plasma ($\partial / \partial t = 0$) that is typically accomplished after $\Delta t \approx 2 \, \text{msec}$. The upper trace displays the ion saturation current $I_{(l)} \propto n(kT_e)^{1/2}$ on the axis of the column. When the probe is located off axis large fluctuations $\delta I$, are observed which are the subject of the present investigation. Upon switch-off of the first discharge pulse the fluctuations rapidly decay in the afterglow. The decay is characterized by a slow density and a rapid electron temperature decrease ($\tau_n \approx 2 \, \text{msec}, \tau_T \approx 300 \, \text{microsec}$). In order to observe the growth of the instability a second discharge pulse is applied in the afterglow. The double pulse sequence is repeated in order to perform statistical averages of the fluctuations.

The fluctuations are analyzed with a dual-channel digital oscilloscope (LeCroy 9400) as shown in Fig. 3(a). A fluctuating quantity, e.g., $\delta n$, from a stationary reference probe is passed through a gate switch and applied to one channel of the oscilloscope. The gate window serves to define a time interval during which the noise properties are to be analyzed. The oscilloscope is triggered at a desired level of fluctuations.
FIG. 2. Time dependence of density and fluctuations. (a) Probe ion saturation current versus time at the center of plasma (top trace) and at the non-uniform edge (lower trace) showing the dependence of fluctuations on $\nu(nkT_i)$. The discharge consists of two consecutive pulses repeated with $t_{rep} = 1$ sec. The first pulse is used to generate a quiescent afterglow plasma; the second, shorter pulse is used to heat the electrons ($T_e > T_i$) in order to study the growth of the instability. (b) Expanded trace $I_{sat}$ during the second discharge pulse showing growth, saturation, and decay of fluctuations.

The fluctuations, e.g., at 75% of the peak value. Whenever the fluctuation exceeds this condition for the first time within a gate pulse, the oscilloscope is triggered and the waveform is stored. An ensemble average is formed, sometimes called "conditional" average because of the threshold condition,

$$\langle \delta n(t) \rangle = \frac{1}{N} \sum_{i=1}^{N} (\delta n_i)_{\delta n > \delta n_{min}}.$$

A second fluctuating signal is applied to the second channel and averaged under the same condition. When the second signal (usually not gated) is identical to the first the resultant ensemble average yields information about the temporal coherence of the waveform. A highly coherent wave train would yield many oscillations upon averaging while a poorly coherent fluctuation would rapidly converge to zero mean away from the trigger. The second signal is typically obtained from a movable probe displaced by $\Delta r$ from the fixed reference probe. In this case the conditional averages between both channels will be delayed in time when the fluctuation is due to a propagating wave. If the "signal" probe is moved in three orthogonal directions the magnitude and direction of wave propagation can be obtained from

$$v = \omega/k = (v_x^{-2} + v_y^{-2} + v_z^{-2})^{-1/2},$$

where $v_x = \Delta x/\Delta t$, $v_y = \Delta y/\Delta t$, and $v_z = \Delta z/\Delta t$ are obtained from time-of-flight data. Furthermore, the amplitude decay with distance

FIG. 3. Measurement principle for performing on-line conditional averages of fluctuations. (a) Fluctuations are detected with two probes. The signal of the fixed reference probe is gated and used to trigger a digital oscilloscope (LeCroy 9400) at a desired threshold $V_{min}$. The signal of the second movable probe is ensemble averaged subject to the condition that the reference signal exceed the trigger level. (b) Typical waveforms of the gate pulse and the gated fluctuations $\delta V_{float}$ in the floating potential of a Langmuir probe (bottom trace). Oscilloscope triggers whenever the gated noise exceeds the threshold level for the first time. (c) Example of a single trace (left) of a gated density fluctuation $\delta n_i$, and of an ensemble average (right), $\langle \delta n(t) \rangle = \langle 1/N \rangle \sum_{i=1}^{N} \delta n_i, N = 100$. The fluctuations exhibit little temporal coherence beyond one oscillation.
gives a measure for spatial coherence, growth, and decay. Finally, the second fluctuation may be derived from a different physical parameter, for example, a magnetic fluctuation. In this case the conditional average establishes correlations between $\delta n$ and $\delta B$. In summary, with the present averaging technique one can "lock" on to a "typical" fluctuation and map its propagation in space, its coherence in time, and the correlation with different physical properties ($\delta n$, $\delta \phi$, $\delta E$, $\delta B$, $\delta J$). Numerous examples will be shown in the following section.

IV. EXPERIMENTAL RESULTS

First, the basic properties of the plasma are shown, next the fluctuations are analyzed, and finally particle acceleration and transport studies are presented.

A. Basic plasma properties

Figures 4 and 5 show the radial variations of plasma properties. Figure 4(a) displays a set of Langmuir probe traces at different radial positions from which electron density $n$, temperature $kT_e$, and plasma potential $\phi_p$ are derived. The plasma potential is essentially constant, i.e., there is no dc radial electric field in the plasma. A possible explanation is the line-tying effect of the conducting boundaries (cathode and opposing endplate) that form equipotential surfaces across $B$.

Figure 4(b) shows the ion saturation current versus time at different radial positions. Both ensemble averages ($I_{sat}$) and single traces ($I_{sat}$) are shown for each position, the latter indicating the fluctuation levels. Radial gradients in $I_{sat}$ exist both during the discharge and in the afterglow. The largest fluctuations are observed during the discharge at the outer radial positions where pressure gradients are strongest.

Figure 5 summarizes the radial dependence of $n$, $kT_e$, and $\delta n/n$ during the discharge. Typical gradient scale lengths are $n/\nabla n \approx 25$ cm, $T_e/\nabla T_e \approx 50$ cm. The peak-to-peak relative density fluctuations are of order $\delta n/n \approx 50\%$ in the edge region of the column, i.e., a strong plasma instability is generated. Axial gradients are at least one order of magnitude smaller than radial gradients.

Figure 6 describes the diamagnetic properties of the plasma. Magnetic probe signals shown in Fig. 6(a) as $dB_t/\tau$ (top trace) and $B_{z(t)}$ (bottom trace) indicate a significant reduction of the external axial magnetic field $B_0 = 14$ G in the plasma interior ($B_p \approx 10$ G on axis) during the discharge. The density fluctuations of the instability produce fluctuations in the axial magnetic field. Figure 6(b) shows the variation of the axial magnetic field in the plasma, $B_p$, with electron density and temperature. The relation is as expected from pressure balance, $B_p^2/2\mu_0 + nkT_e = B_0^2/2\mu_0 = const$. The figure also presents the normalized electron pressure, $\beta = nkT_e/(B_0^2/2\mu_0)$.

Variations in selected plasma parameters are produced...
FIG. 6. Modification of the magnetic field inside the plasma, $B_p$, by the electron pressure $nkT_e$. (a) Magnetic probe signal near axis versus time without integration (top trace, note fluctuations) and with time integration (lower trace, note large drop in axial field during discharge). (b) Dependence of magnetic pressure $B_p^2/2\mu_0$ inside the plasma ($r=0$) versus electron pressure. Normalized pressure $\beta = nkT_e/(B_p^2/2\mu_0)$ is shown on the right-hand scale.

by pulsing the discharge. By observing the dependence of the fluctuations on these parameters one can draw certain conclusions on the instability mechanism. Figure 7 shows the time dependence of basic parameters during the heating pulse applied at $t_c = 0.8$ msec in the afterglow of the main discharge pulse. Cathode current and voltage (top traces) yield density and energy of the injected primary electrons ($4\mu_0^2 \leq 16 \text{ eV}$, $n_{\text{hot}}/n_{\text{cold}} \approx 3 \times 10^{-3}$). The floating potential (middle trace) decreases due to electron injection and exhibits fluctuations as a result of the instability. The potential fluctuations $\delta V_f$ are obtained by subtracting the ensemble-averaged potential from a single potential waveform. The fluctuations are only observed at increased electron temperatures ($kT_e \sim 1.5 \text{ eV}$). Density gradients also exist in the afterglow plasma. By not raising the cathode voltage much above the ionization potential (15.4 V) the density variations are minimized while the electron temperature is increased significantly (400%). Thus, the instability depends not only on the existence of a radial density gradient but also on hot electrons, i.e., a significant electron pressure gradient, $\nabla (nkT_e)$. The visible light emission indicates the presence of $>10 \text{ eV}$ discharge electrons which are required to heat the bulk electrons.

Prior to onset of the instability the propagation of a test ion-acoustic wave has been studied. The purpose of this experiment is to check for possible ion drifts, to obtain a direct measure of the sound speed, and to demonstrate sound wave propagation across $B$. A test pulse was launched from a fixed
rf probe and received by a similar probe movable in \( \pm x, y, z \) directions. Figure 8(a) indicates the timing of the test pulse with respect to the onset of the instability while Fig. 8(b) displays the received density perturbation \( \delta n_{(i)} \) at different radial positions \( \Delta r \) at \( r \approx 26 \) cm. The propagation delay with respect to the excitation pulse yields one component of the phase velocity, \( \omega/k_x \). By comparing the propagation velocities in the \( \pm \Delta r \) direction (\( v_x = c_s \pm v_d \)) radial ion drifts can be identified. Such time-of-flight measurements have been performed in three orthogonal directions, the results of which are summarized in Fig. 9. The average phase velocity

\[
\frac{v_x}{c_s} \approx 0.105 \times 10^5 \text{ cm/sec}
\]

points mainly in the azimuthal direction along the electron diamagnetic drift. The average phase velocity in azimuthal direction \( \frac{(v_x + v_\perp y)}{c_s} \approx 0.26 \times 10^5 \text{ cm/sec} \) agrees to within \( \sim 10\% \) with the calculated sound speed \( c_s = \left[ \frac{k(T_e + T_i)}{m_i} \right]^{1/2} \approx 2.2 \times 10^5 \text{ cm/sec} \) at \( k(T_e + T_i) \approx 2 \text{ eV} \).

**B. Fluctuation properties**

First, the fluctuations are analyzed in the time and frequency domains, then the wave propagation is measured in three dimensions, and finally the correlations between fluctuations in density, potential, electric field, magnetic field, and current density are demonstrated.

Figure 10(a) displays density fluctuations versus time \( \left( \delta n_{(i)} \right) \), single trace at top) and versus frequency \( \left( \delta n_{(i)} \right) \), ensemble average, lower trace). The temporal waveform during saturation is highly nonsinusoidal, exhibit-
The frequency spectrum peaks at \( f \approx 35 \text{ kHz} \), which is of order of the calculated lower-hybrid frequency, \( f_{\text{lb}} = (f_{\text{th}}/f_{\text{ma}})^{1/2} \approx 120 \text{ kHz} \) for \( B_p \approx 12 \text{ G} \). The high-frequency tail of the spectrum is produced by the short rise time of the oscillations, i.e., the high-frequency components are harmonics of the fundamental. Figure 10(b) shows the dependence of the instability frequency at maximum amplitude \( f_{\text{peak}} \) versus density (top scale) and external magnetic field \( B \) (bottom scale). While the frequency is nearly independent of density it increases proportional to \( B_0 \), i.e., it scales like the lower-hybrid frequency.

Since it will be shown below that the temporal oscillation \( \delta n_{(t)} \) is caused by a propagating wave the sharp rise corresponds to a steep wave front. Figure 11 shows a detailed picture of the wave steepening process as observed by a plane probe with surface aligned normal to the direction of wave propagation. The expanded view of the waveform \( \delta n_{(t)} \) (bottom trace) indicates a rise time \( t_r \approx 0.5 \text{ µsec} \) compared with an oscillation period \( t_p \approx 20 \text{ µsec} \). Based on the typical propagation speed \( v_p \approx 2.2 \times 10^5 \text{ cm/sec} \), the thickness of the wave front has decreased by an order of magnitude from \( \Delta z / 2 \approx 1.1 \text{ mm} \) to \( \Delta z \approx 0.11 \text{ mm} \). As the wave propagates across \( B \) the wave front becomes narrower than an electron Larmor radius, \( r_e \approx 3.5 \text{ mm} \), implying that cross-field drifts are strongly modified by finite Larmor radius effects. The wave steepening does not proceed to wave breaking characterized by oscillatory structures near the steep wave front.

The spatial properties of the fluctuations have been investigated both during growth and saturation of the instability. Figure 12 shows conditionally averaged fluctuations of magnetic probe signals \( \langle \delta B_z \rangle \) versus time at different probe displacements along three orthogonal directions. The observations are made during the growth of the instability where the temporal coherence is good, i.e., well-defined wave trains are excited. An obvious propagation delay is noticeable in the azimuthal direction \( [\Delta y = r_\Delta \theta, \text{Fig. 12(a)}] \). The azimuthal phase velocity \( v_\varphi = \Delta y / \Delta \theta \approx 2.6 \times 10^5 \text{ cm/sec} \) points in the direction of the electron diamagnetic drift, \( v_d = V_d \times B / n_e B^2 \), but its magnitude is closer to the sound speed \( c_s \approx 2.5 \times 10^5 \text{ cm/sec} \) than to the calculated drift speed \( v_d \approx 8 \times 10^5 \text{ cm/sec} \) for \( kT_e = 1.5 \text{ eV}, B = 14 \text{ G}, n_p / V_d = 17 \text{ cm} \). The wave is highly coherent in the azimuthal direction. In the radial direction \( [\Delta x = \Delta r, \text{Fig. 12(b)}] \) the instability onset is delayed toward the plasma center, while the phases are delayed radially outward. However, the phase velocity \( v_\varphi = \Delta x / \Delta t \) varies with time and radius which will be shown in more detail below. Finally, in the axial direction along \( B \) [Fig. 12(c)] the instability onset, oscillation amplitudes, and phases show little variation over substantial distances \( 0 < \Delta z < 50 \text{ cm} \). Thus, the fluctuations consist of nearly field-aligned waves propagating close to the sound speed mainly along the electron diamagnetic drift.
A detailed picture of the radial wave propagation is presented in Fig. 13. Density and magnetic field fluctuations are coupled in a high-beta plasma ($\delta n/n \propto -\delta B / B$), and hence have the same propagation characteristics. One observes that waves growing spontaneously out of the density profile have phase fronts in the radial direction (no radial delay), but within a few oscillations the waves begin to propagate radially outward ($v_r = \Delta x/\Delta t \approx 3 \times 10^5$ cm/sec). Thus, the waves refract from the azimuthal toward the radial direction.

After having described the wave propagation at instability onset the characteristics of the saturated waves will now be shown. It is instructive to start with the temporal coherence properties shown in Fig. 14. Fluctuations in the ion saturation current are shown at an increasing number of events $N$ for ensemble averages. In Fig. 14(a) the trigger is chosen to be the first significant oscillation at instability onset. The temporal coherence extends over several small oscillations but not over the large saturated waves that average out to zero. When the gate is delayed so as to trigger from the saturated fluctuations [Fig. 14(b)] the coherence reduces to just one oscillation in time.

The spatial coherence and propagation properties of saturated density fluctuations are shown in Fig. 15. Conditionally averaged density fluctuations $\langle \delta n(t) \rangle$ are shown at different azimuthal positions [Fig. 15(a)] and radial positions [Fig. 15(b)]. Azimuthally, the propagation speed and amplitude are essentially constant. Radially, however, the velocity $v_r = \Delta x/\Delta t$ decreases from $v_r \approx \infty$ near the center of $v_r = 4 \times 10^5$ cm/sec at the outer radial positions. The amplitude $\langle \delta n \rangle$ is small near the axis where the electron drift is small, maximizes off axis where $V(nkT_e)$ is large, and decreases toward larger radii as a result of poor coherence but not due to a lack of growth [compare Figs. 4(b) and 5]. By measuring in a transverse plane ($15 < x < 45$ cm, $|\Delta y| < 5$ cm) both the radial phase velocity ($v_r = \omega / k_r$) and the azimuthal velocity ($v_\phi = \omega / k_\phi$), the total phase velocity $v = \omega / |k| = (v_r^{-2} + v_\phi^{-2})^{-1/2}$ is obtained as well as the direction of wave propagation, $\tan \theta = k_r / k_\phi = v_\phi / v_r$, where $\theta$ is the angle between $\mathbf{v}$ and the azimuthal direction of the magnetic fluctuations $\langle \delta B_r \rangle$ near the onset of instability. (a) Magnetic probe signal $\delta B_r(t)$ at different azimuthal positions showing wave propagation in the direction of the azimuthal electron diamagnetic drift. (b) Growth and wave propagation at different radial positions, $\Delta r = \Delta x$. Instability starts earlier at outer radial positions. Growing waves begin to propagate radially outward. (c) Fluctuations at different axial positions show simultaneous growth and no phase delays, i.e., $k_\phi = 0$. 

FIG. 12. Spatial properties of magnetic fluctuations $\langle \delta B_{r(\phi,z)} \rangle$ near the onset of instability. (a) Magnetic probe signal $\delta B_{r(\phi,z)}$ at different azimuthal positions showing wave propagation in the direction of the azimuthal electron diamagnetic drift. (b) Growth and wave propagation at different radial positions, $\Delta r = \Delta x$. Instability starts earlier at outer radial positions. Growing waves begin to propagate radially outward. (c) Fluctuations at different axial positions show simultaneous growth and no phase delays, i.e., $k_\phi = 0$. 

FIG. 13. Conditionally averaged density fluctuations near the onset of instability at different radial positions. Vertical lines through constant phase points indicate refraction of phase fronts from azimuthal into radial directions.

FIG. 14. Comparison of temporal coherence between growing and saturated density fluctuations. (a) Ion saturation current versus time, ensemble average over $N = 1, 10, 100,$ and 1000 events (discharge pulses). Trigger is at the onset of instability. First few oscillations are coherent, subsequent large fluctuations average out to zero. (b) When the gate window is shifted into saturated instability so as to trigger off large amplitude fluctuations, the coherence is limited to about one oscillation, i.e., wave train consists of independent density perturbations.

electron drift $v_d$. Figure 15(c) summarizes the propagation characteristics in the transverse $x$-$y$ plane. A typical instantaneous phase front $(\langle \delta n \rangle_{\text{max}})$ has been constructed from the measurements of $(\langle \delta n \rangle)_{\text{sat}}$. Normal to the phase front are vectors showing the local phase velocity $v_p$. The velocity varies in magnitude and direction. The magnitude is close to the local sound speed $(k\bar{T}_e/m_e)^{1/2}$ and decreases radially due to $v_T$. Waves are destabilized when the diamagnetic drift $v_d$ exceeds the azimuthal phase velocity $v_p = c_p \cos \theta$. Refraction ($\theta = \pi/2$) can establish stability ($v_p < v_d$), and hence provide for a saturation mechanism.
changes from nearly azimuthal at small radii to nearly radial at large radii.

A possible interpretation for the observed wave front is the refraction of sound waves in a nonisothermal plasma. A growing ion-acoustic wave that is initially propagating alongside the azimuthal diamagnetic drift refracts radially outward because of a lower phase velocity at larger radii and a higher phase velocity at smaller radii. A consequence of the refraction is a reduction in the growth rate because with increasing angle $\theta$ the drift may no longer exceed the phase velocity component $v_r = u/\cos \theta$. Thus, wave refraction presents a saturation mechanism for the instability. Because of wave refraction azimuthal eigenmodes are not formed, i.e., waves are swept outward well before closing azimuthally. New waves grow while others refract and decay, thereby establishing a stationary saturated turbulence level.

After the description of the temporal and spatial wave properties the relation between fluctuations in various physical parameters will now be presented. Figure 16(a) compares simultaneous traces of ion density fluctuations $\delta n(t)$ and floating potential fluctuations $\delta \phi(t)$ versus time. The two probes are located on the same flux tube but displaced axially ($\Delta z = 3$ cm, $\Delta z/k_q = 0$). Density and potential fluctuations are positively correlated. Although qualitatively in agreement with Boltzmann’s relation, $n = n_0 \exp(e\delta \phi/kT)$, quantitatively one finds $\delta n/n > e\delta \phi/kT$. Thus the ion waves with $k_r > k_q$ are not satisfying all of the criteria of ion-acoustic waves. Possible reasons will be discussed in Sec. V.

The presence of potential fluctuations implies the existence of fluctuating electric fields. These have been directly measured with a dipole electric field probe. Figure 16(b) shows simultaneous traces of density fluctuations $\delta n(t)$ and electric field fluctuations $\delta E_x(t)$. The electric field maximizes in the direction of wave propagation and is predominantly electrostatic since $\delta E(t)$ follows $(\partial / \partial t) \delta \phi(t) = -v_r \partial \delta \phi / \partial y$, hence $E_x = -\partial \delta \phi / \partial y$. The electric field has large spikes at the steepened wave fronts and zeros at density maxima and minima. Based on the peak electric field strength $E_{\text{max}} = O(1 \text{ V/cm})$, one would expect significant particle acceleration to occur, which will be discussed below. While dc electric fields across B were essentially absent [Fig. 4(a)] ac electric fields $\delta E \| B$ are imposed due to axial density and potential gradients.

Fluctuations in the axial magnetic field $\delta B_z(t)$ [Fig. 16(c), top trace] are anticorrelated with density fluctuations (middle trace). Simple pressure balance $B^2/2\mu + n kT = \text{const}$ predicts $\delta B_z/B_z = - (\delta n/n) x (n kT)/B_z^2/\mu_0 = -(\delta n/n)\beta/2(1 - \beta)$, where $\beta = n kT/(B_z^2/2\mu_0)$. Although the sign is in agreement the observed relative fluctuations $\delta B_z/B_z \approx 0.2 \text{ G}/12 \text{ G} \approx 1.7\%$ are much smaller than those predicted by pressure balance alone ($\delta n/n \approx 0.4$, $\beta = 0.4$ at $r = 25$ cm, $\delta B_z/B_z \approx 13\%$). The reason for the difference is that electron diamagnetic drifts are opposed by $\delta E \times B$ drifts which are not considered in the pressure balance equation.

The time derivative of the magnetic field fluctuation $(\partial / \partial t) (\delta B_z) = \delta J_z$, [Fig. 15(c), bottom trace] is proportional to the fluctuating cross-field current density $\delta J_z$, since the time derivative is given by a convective derivative

$$\left(10 \mu\text{sec/div}\right)$$

**FIG. 16. Correlation between fluctuations of different physical parameters.**

(a) Density and potential are positively correlated although the Boltzmann relation is quantitatively not satisfied, $e\delta \phi/kT < \delta n/n$. (b) Comparison of density and electric field fluctuations, $\delta E_x = -\nabla \delta \phi$. Because of propagation of fluctuations, temporal and spatial derivatives are related by $\partial / \partial t = -v_r \partial / \partial y$. Wave steepening causes strong, spiky electric fields. (c) Simultaneous traces of density fluctuations $\delta n(t)$, (middle trace), magnetic fluctuations $\delta B_z$, and current density fluctuations $\delta J_z$, (lower trace). Density and magnetic field are anticorrelated as expected from plasma diamagnetism. The time derivative of the magnetic fluctuation is proportional to the radial current density, $\delta J_z = (\nabla \times B)/\mu_0 = (\partial / \partial y) \delta B_z/\mu_0 = -\delta B_z/\mu_0 v_r$. The current is caused by electron diamagnetic drifts associated with the wave pressure, $\delta B_z = \nabla (\delta p) \times B/\mu_0$, and smaller opposing electron $\delta E \times B/B^2$ drifts.

$$\left(10 \mu\text{sec/div}\right)$$

$$\left(50 \text{ mAm/cm}^2\right)$$

$$\left(0.1 \mu\text{G}\right)$$

$$(\partial / \partial t = -\nabla \nabla)$$

$$\delta J_z = \frac{(\nabla \times \delta B)_z}{\mu_0} = \frac{\partial \delta B_z}{\partial y}/\mu_0 = -\frac{\partial \delta B_z}{\partial t}/\mu_0 v_r.$$
The measurements of fluctuating fields and plasma parameters are summarized in a physical picture shown in Fig. 17. In the transverse x-y plane the instantaneous contours of constant pressure are meandering along the azimuthal direction, i.e., along the unperturbed diamagnetic drift $\nabla (n k T_e) \times B / ne B^2$. In time the flutelike contours propagate with phase velocity $v_{ph} = c$, oblique to the drift. Regions of density enhancements/depressions have a positive/negative plasma potential, respectively. The wave electric field $E$ is opposite to the wave pressure gradient, $\nabla p = - \nabla \delta \phi \times B / ne B^2$. The resultant electron drifts would cancel if potential and density perturbations would obey the Boltzmann's relation

$$\frac{\delta E \times B}{B^2} = - \nabla (\delta \phi \times B) / B^2 = - k T_e \frac{\nabla \delta n \times B / ne B^2}{n e B^2} = - \nabla (\delta \rho \times B) / ne B^2.$$

However, the observation clearly indicates that the diamagnetic current is stronger than the Hall current.

Although not directly measured, the time-varying magnetic field should give rise to an inductive electric field, $\nabla \times E = - (\partial / \partial t) \delta B$. It can be estimated to be $\delta E_{ix} = k_T e / \omega B < c, \delta n / n B$, and shown to be negligible compared with the electrostatic field

$$\frac{\delta E_{iz}}{\delta E_{iy}} = \frac{e B_e}{c} k T_e / \omega B = \frac{c n e}{\omega} \left( \frac{m_e}{m_i} \right)^{1/2} \ll 1.$$

The dominant current is produced by the electron drifts across $B$. The unmagnetized ions acquire a velocity $\delta v_p \approx e B_e / m, \omega$, which is negligible compared to the electron $\delta E \times B$ drift,

$$\frac{\delta v_p}{\delta v_e} = \frac{e B_e}{m, \omega} = \frac{\omega e}{\omega e} \frac{m_e}{m_i} \left( \frac{m_i}{m_e} \right)^{1/2}.$$

Two factors modify the electron cross-field drifts: (i) Coulomb collisions ($\omega < \nu_e < \omega_{ce}$) generate an ac electron current along $\delta E_p$, given by $\delta v_e = \sigma_e \delta E_p$, where

$$\sigma_e = \frac{\omega e (\nu_e - \omega)^2}{(\nu_e - \omega)^2 + \omega_{ce}^2} \approx \frac{\omega e}{\omega_{ce}^2}.$$

The corresponding electron drift

$$\delta v_{ac} = \frac{\sigma_e}{n e} \frac{e B_e}{\omega_{ce}} \frac{\nu_e (\nu_e - \omega)}{\nu_e - \omega} \approx \frac{\nu_e}{\omega_{ce}} \frac{\delta E_{iy}}{B}$$

is small compared to the Hall drift $\delta E_{iy} / B$ or the diamagnetic drift $\nu_e$. (ii) Steepening of the wave front to $k_T e r_e > 1$ requires finite Larmor radius corrections$^{26}$ for cross-field drifts reducing the drifts by a factor $J_{(k_T e r_e)} \sim 1 - k_T e r_e$.

C. Heating and transport

Ion and electron acceleration is often associated with strong lower-hybrid waves. Possible electron heating has been investigated with Langmuir probes swept rapidly enough to resolve the plasma parameters within a fluctuation. Figure 18 shows density fluctuations $\delta n_{(t)}$ and simultaneous Langmuir probe currents $I_{(t)}$ on two time scales, a slow scale (10 $\mu$sec/div) to display several oscillations as well as the timing of the probe trace, and an expanded scale (0.5 $\mu$sec/div) for evaluating the probe trace and to check for density variations during the sweep. The Langmuir probe voltage is varied linearly in time with a triangular waveform. In Fig. 18(a) the probe trace is swept out during a density maximum, in Fig. 18(b) during a minimum. The knee of the trace (enhanced by dashed lines) indicates the difference in plasma potential ($A\delta \phi = 0.6$ V $= 0.26 k T_e / e$), and in density ($n_{max} / n_{min} = 1.4$), but the retardation regions ($I_{(t)} \sim e^{V_{(t)} / k T_e}$) show only small changes in the electron temperature ($k T_{e_{max}} = 2.35$ eV, $k T_{e_{min}} = 2.22$ eV). No significant electron tails are produced. Time-averaged Langmuir probe traces [Fig. 4(a)] showed no temperature enhancements in regions of strong turbulence (Fig. 5). The temporal increase of the electron temperature (Fig. 7) is not due to the growth of the instability but the injection of energetic discharge electrons.

While the absence of electron heating may not be surprising since the instability is driven by gradients in the electron pressure $n k T_e$, there exists the possibility of ion heating by waves with $\delta \phi \approx k T_e / e$. Using a directional ion velocity analyzer a measurement analogous to Fig. 18 has been performed. Figure 19 shows the ion flux $I_{(V)}$ in the direction of wave propagation swept both during a density maximum, $\delta n_{max}$ and minimum, $\delta n_{min}$. The fluctuations $\delta n_{(t)}$ are shown both on a slow time scale (5 $\mu$sec/div) and an expanded scale (0.5 $\mu$sec/div) indicating that $n_{(t)} = \text{const}$.

FIG. 17. Schematic summary of the microphysics for the fluctuations as derived from observations. In the transverse x-y plane the original isobars (dashed line) are modified into flutelike contours (solid line) propagating oblique to the azimuthal direction. The instantaneous electron diamagnetic drift follows the meandering isobars. At density maxima (minima) the potential is positive (negative). The electrostatic field $E$ gives rise to a $\delta E \times B$ drift, weakening the wave diamagnetic drift, $\nabla \phi \times B / ne B^2$, such that the magnetic perturbations are small, $|\delta E_{iy} / B| \ll (\delta \phi / 2) \delta n / n$. Ion drifts $\nu_i$ and collisional electron drifts $\nu_{ei}$ are much smaller than electron cross-field drifts.
while sweeping the analyzer characteristics. Potential and density changes are evident but no significant ion heating compared with the wave potential or any significant energetic ion tails are found. The ion temperature just before onset of the instability differs little from that during the instability ($kT_e \lesssim 0.4$ eV). Because of the relatively large potential fluctuations ($\delta \phi \approx kT_e$) the velocity analyzer traces have to be analyzed with care. For example, simple time averaging over $I-V$ traces fluctuating between those for $\delta n_{\text{max}}$ and $\delta n_{\text{min}}$ results in a broadened trace and erroneously high ion temperatures. Proper ensemble averaging where the sweep is triggered off a fluctuation maximum or minimum (conditional averaging) shows no significant ion temperature variations. The lack of wave–particle interactions may be ascribed to the fact that the phase velocity of the sound wave is much higher than the ion thermal speed.

Finally, ion and electron transport in the presence of the turbulence has been investigated by flux measurements to a plane, one-sided probe. The probe can be rotated by 180° so as to collect current along and opposite to the azimuthal direction at different radial positions. Figure 20(a) shows typical time-averaged $I-V$ traces near the floating potential which minimizes the perturbations of the plasma. Both the ion and electron fluxes are larger in the direction of wave propagation ($+y$) than opposite to it ($-y$). Current differences $I_{-y} - I_{+y}$ are evaluated for ions in the saturation
FIG. 19. Test for ion acceleration by large amplitude waves. Analogous to Fig. 18 the \( I-V \) characteristics of a directional ion velocity analyzer is swept rapidly during both a density maximum and minimum (see \( \delta n_{\text{on}} \) on slow time scale). Density and potential shifts are apparent (\( \Delta \rho = 0.64 \text{ V} \), \( n_{\text{max}}/n_{\text{min}} = 1.4 \)) but little ion heating (\( k_{\text{cmtn}} = 0.35 \text{ eV} \), \( k_{\text{cmtn}} = 0.45 \text{ eV} \)) or tail formation is observed.

FIG. 20. Direct measurements of particle drifts with a one-sided planar probe. (a) Current-voltage characteristics of the probe near the floating potential for particle collection in + y direction (larger flux) and - y direction (smaller flux). A current difference exists for both electrons and ions. Both particle/fluid drifts are in the direction of the electron diamagnetic drift. (b) Radial dependence of density \( n_{\text{on}} \) and flux differences of ions and electrons. The drifts maximize in regions of large pressure gradients.

regime and for the electrons at the floating potential and plotted versus radius in Fig. 20(b). While the electron current represents the diamagnetic current, \[ \Delta J_e = (\Delta J_e)_{0} \left( \frac{J_{\text{sat}}}{J_{\text{sat}}} \right) \]
\[ = (\Delta J_e)_{n} \times \left( \frac{m_i/m_e}{2\pi} \right)^{1/2} \]
\[ = 100 (\Delta J_e)_{n} < 200 \text{ mA/cm}^2; \]
\[ J_{\text{ion}} = -n k_{B} T_e / B_0 = 100 \text{ mA/cm}^2, \]
the directed ion flux could be due to collisional drag by electrons and/or waves. Steepening of ion-acoustic waves is thought to give rise to a net ion flow with the waves.\(^7\)

The dominant effect of the instability on enhanced transport appears to be the radial \( \delta E \times B \) drift of electrons. It allows electrons to drift along the phase front radially in and out and to exchange heat across B.

V. DISCUSSION AND CONCLUSION

The present experiment has shown the properties of a strong instability involving ion-acoustic waves near the lower-hybrid frequency (\( \omega = \omega_{\text{hy}}, \ k_1 = -k, \ \omega = k_{\perp}, \ T_e > T_i \)). The instability is driven by electron diamagnetic currents associated with density and temperature gradients across B. The instability differs from the lower-hybrid drift instability\(^4\) due to its dependence on \( T_e / T_i \) and differences in phase and drift velocities (\( \omega / k = c_s < v_{d,0} \)). The instability also differs from the ion-acoustic instability of perpendicular shocks\(^5\) which exhibits high-frequency, short wavelength, significantly oblique modes (\( \omega \approx \omega_{\text{ps}}, \ k_{\parallel} \approx 1, \ k_{\perp} \approx 1 \)). The closest theoretical description for the present sound instability is given by Kadomtsev.\(^1\) In the collisionless case the instability arises when the electron cross-field drift exceeds the sound speed. The growth rate \( \gamma \approx k c \), maximizes for \( k_1 / k_\perp = (m_i/m_e)^{1/2} \). In the collisional case (electron mean-free path \( \lambda_e < \lambda_\perp \)) the drift-dissipative sound instability also grows as \( \gamma \approx k c \), but for \( k_1 = k_\perp \ (\nabla n / n) (\nabla \rho / \rho_{d,0}) \). Because of some simplifying assumptions (\( \nabla T_e = 0, \ \nabla B = 0, \ T_e = 0, \ \beta = 0 \)) that are not satisfied in the present case, a close comparison between theory and experiment does not seem justified. Furthermore, a linear instability analysis does not make a prediction on the saturated state which is a main focus of the present investigation.

Two new phenomena have been observed during the evolution of the instability, i.e., wave refraction and steepening. Wave refraction has been ascribed to the nonuniform electron temperature profile \( T_e(r) \) and the dependence of the phase velocity on electron temperature, \( \omega / k \approx T_e^{1/2} \). Surprisingly, previous investigations on sound instabilities with \( \nabla T_e \) have not considered this process\(^5,29\) Self-refraction due to the dependence of phase velocity on wave amplitude\(^30\) is not believed to be the dominant refraction mechanism since it would have caused the sound waves to refract both radially inward and outward. Similarly, the sheared azimuthal ion drift should refract waves radially inward, which is not observed. Two consequences of wave refraction on the instability have been pointed out. (i) As the waves bend away from the azimuthal drift their growth rate decreases, hence refraction causes saturation. (ii) Azimuthal eigenmodes are not
set up since the waves refract radially outward before closing azimuthally.

Wave steepening of sound waves in unmagnetized plasmas has been well documented in theory3,31 and experiments.32 But in magnetized plasmas nonlinear sound waves are theoretically more involved31 and there are few observations. In the present work sound waves are found to steepen across B. No oscillatory structures are found around the shocklike wave fronts, suggesting that nonlinear steepening is balanced by dissipative rather than dispersive effects. The steepening proceeds to a scale length where the electrons become unmagnetized ($k_\perp r_m > 1$). While at small wave amplitudes the electrons respond adiabatically to the lower-hybrid modes ($r_\parallel < k_\perp^{-1} < r_m$), at larger amplitudes finite Larmor radius effects decrease the $\Delta E \times B$ drift, and for highly steepened waves the electrons within the shock front directly respond to the wave electric field $E$. As without B, electron Landau damping limits steepening beyond $k \lambda_D = 1$. Thus, steepening also leads to saturation of the instability.

In the present experiment magnetic fluctuations $\Delta B$ have been observed from which cross-field currents or electron drifts could be obtained. It has been pointed out that if the parallel electron pressure was simply balanced by an electric force ($-eE_{\parallel} = -P_B/e$, Boltzmann relation) the corresponding perpendicular forces ($\nabla_e E_{\perp}$) would give rise to two equal and opposing drifts ($E \times B/B^2$, $v_\parallel p + neE + mn_0 v_\parallel = 0$).
The resultant cross-field drifts should be obtained from kinetic rather than particle or fluid theories. Little wave steepening or spectral broadening was observed on the magnetic probe signal but, unfortunately, the magnetic probe resolution ($\sim 3r_m$) was not comparable to that of the electric probes ($\lesssim r_m$).

Finally, particle acceleration and heating by the lower-hybrid mode turbulence has been investigated. Electron and ion distributions were measured at a maximum and minimum where the particle energies might differ by $e\Delta$. Little bulk heating and no significant energetic tails were observed. The sound wave propagates too fast to accelerate a significant number of ions by wave–particle resonance. The electrons primarily drift perpendicular to the wave electric field, and hence gain no energy. However, due to Coulomb collisions there is a small ac electron current along and in phase with $\Delta E_\parallel$, which leads to the dissipation $J_{ac} = n_m e_0 (\Delta E/B)^2$. Although the turbulence may contribute to redistributing the electron temperature radially, no net electron heating can be expected since the energy source for the instability is $\nabla (n k T_e)$. Because of the relatively weak wave–particle interactions the sound wave amplitude could grow to relatively large amplitudes ($\Delta n/n > 50\%$). Similarly strong sound waves are observed when ions are injected above the sound speed across B into magnetized, stationary electrons.33

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