High frequency instability of a magnetized spherical electron sheath

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A positively biased spherical electrode in a magnetized plasma exhibits a ring of energetic electrons in the equatorial plane where the sheath electric field is normal to the magnetic field. High frequency waves are excited which propagate with the average \( \mathbf{E} \times \mathbf{B} \) drift and form toroidal eigenmodes. Up to 20 harmonic eigenmodes are observed in the spectrum. Injected test waves are amplified. The drift wave can excite whistler modes. Electron inertia produces the instability. © 2010 American Institute of Physics. [doi:10.1063/1.3437398]

I. INTRODUCTION

Sheaths are a fundamental phenomenon in plasma physics which have been studied extensively for a long time. Sheaths arise at interfaces between a plasma and a solid or plasmas of different properties and serve to control fluxes of electrons and ions. Sheaths can also produce instabilities, particularly when electron rich. Long cylindrical sheaths or electron layers are known to excite magnetron and diocotron instabilities. Current-driven instabilities arise from disturbing the ambient plasma by particle drifts. Ionization phenomena and particle transit time effects also produce instabilities. A well-known high frequency instability arises from electron inertia in diodes. In a Debye sheath the transit time of electrons, which is of order of the electron plasma frequency, creates a 180° phase shift between rf current and electric field, i.e., a negative differential resistance, which destabilizes the sheath-plasma resonance. An analogous instability can also arise in ion-rich sheaths.

In magnetic fields anisotropic particle motions result in complicated sheaths around finite-size electrodes. Recently much attention has been focused on ion-rich sheaths at tokamak divertors. Electron-rich sheaths in magnetized plasmas have been extensively studied for spherical collectors modeling tethered satellites in space. Part of the sphere collects electrons along the field, part across the field resulting in a nonspherical sheaths. A population of trapped electrons in the equatorial plane perpendicular to the field has been predicted in theory, observed in simulations and in earlier experiments. The stability of such trapped particles has not yet been studied and is the topic of the present investigation. It is shown that the sheath is unstable to high frequency oscillations below the electron plasma and cyclotron frequencies. The instability excites electron drift waves which form azimuthal eigenmodes. The instability is thought to arise from the electron inertia of energetic electrons in the electron belt around the sphere. The cyclotron motion of nonadiabatic electrons results in a long transit time through the sheath which resonates and destabilizes electron drift eigenmodes. The instability is partly electromagnetic and can excite whistler modes. Amplification of test waves has been demonstrated. Such instabilities are important for probes in plasmas, tethered satellites in space, antennas in plasmas, and may explain the onset of instabilities of spacecraft charged to positive potentials.

The paper is organized as follows: after describing the experimental setup in Sec. II the observations of the instability properties are presented in Sec. III. A physical picture of the instability mechanism is presented in Sec. IV and contrasted with related high frequency sheath instabilities.

II. EXPERIMENTAL ARRANGEMENT

The experiments are performed in a linear dc discharge device (40 cm diameter, 120 cm length) shown schematically in Fig. 1. Typical parameters are a plasma density \( n_e = 10^9 \) cm\(^{-3}\), electron temperature \( kT_e = 3 \) eV, uniform axial magnetic field \( 0 < B_0 < 40 \) G, argon gas pressure \( p = 10^{-4} \) Torr, discharge voltage \( V_{\text{dis}} = 100 \) V, and current \( I_{\text{dis}} = 0.25 \) A. The spherical electrodes (1.5 and 3.2 cm diameter) are biased at a constant or pulsed voltage \( V_{\text{sph}} < 250 \) V drawing typically \( I_{\text{sph}} = 60 \) mA. Unlike in many previous experiments, the plasma is not produced by ionization near the electrode. Plasma parameters are obtained from Langmuir probe traces.

Coax-fed rf probes are used to measured fluctuations in the sheath and vicinity. One rf probe can rotate around the polar angles \( \theta \) and \( \phi \); the other can move radially. A shielded magnetic loop (8 mm diameter, five turns) is used to measure rf magnetic field fluctuations. The rf signals are fed via tuned (10–500 MHz) or broadband rf amplifiers into a digital oscilloscope (2 GHz, 2500 samples). Test waves are excited with a cw signal generator connected to one rf probe.

Visual images of spherical sheaths are shown in Fig. 2. In a uniform field-free plasma a positively biased sphere produces a visible shell of light at the surface of the sphere [Fig. 2(a)]. The light is produced by excitation of neutrals in collisions with electrons accelerated to energies \( U > 15 \) eV. The visible sheath starts at a potential of \( > 10 \) V above the ambient plasma, implying that it contains essentially no ions (\( kT_i < 1 \) eV). Outside the sheath the electrons do not have enough energy to produce light, but their density is higher than inside the sheath.

In a magnetized plasma [Fig. 2(b)] the sheath exhibits a luminous ring in the equatorial plane of the sphere \( (\theta = \pi/2) \) normal to the field \( \mathbf{B}_0 \). The ring contains trapped electrons also seen in earlier experiments and...
Initial observations of sheath instabilities were made on a ionization produces a “fireball” [Fig. 2(c)], which has very different properties from an electron-rich sheath.8,23 The present experiment is performed at low pressures without ionization phenomena where the light of the sheath is much fainter than that of a fireball. The mean free path for elastic collisions of 3 eV electrons at 10−4 Torr is \( \lambda_e \approx 500 \) cm. Initial observations of sheath instabilities were made on a small sphere of 1.5 cm diameter [Fig. 2(b)] but all present results were obtained with a larger sphere of 3.3 cm diameter [Fig. 2(d)] where small rf probes could be inserted into the sheath with little perturbation. The same probes were also used to infer electron density and electric fields inside the sheath.

### III. EXPERIMENTAL RESULTS

#### A. Instability onset

The appearance of an instability onset becomes obvious when the electrode voltage is pulsed sufficiently positively. Figure 3(a) shows waveforms of the voltage and current of the sphere and the probe rf signal in the sheath. The current overshoot is due to ion expulsion during the formation of an electron-rich sheath. The instability starts after the ions are expelled, i.e., in an electron-rich sheath, and grows within a few rf cycles to saturation. However, the instability amplitude can have large fluctuations even though the plasma parameters are constant.

A fast Fourier transform (FFT) of the rf waveform between 6 μs < \( t < 18 \) μs is shown in Fig. 3(b). A fundamental frequency \( f_0 = 12 \) MHz is excited and a stronger second harmonic \( 2f_0 \) is observed. The excited frequencies lie below the electron plasma frequency (\( f_p \approx 300 \) MHz) and the electron cyclotron frequency (\( f_c = 84 \) MHz), but well above the corresponding ion frequencies, hence are electron modes. In order to interpret the instability spectrum of Fig. 3(b), FFTs have been formed for successive time intervals (\( \Delta t = 1.28 \) μs), which are short compared to the duration of the emission but long compared to the rf period. The time-resolved spectrum of Fig. 3(c) reveals that the instability is fairly narrowband but the frequency decays in time which explains the broad linewidth when averaged over a long time span. Next one observes that the instability starts at the second harmonic, which implies that it is not created from the fundamental by a nonlinear process. It will be shown below that both lines are electron drift eigenmodes which are destabilized.

The instability has a well-defined onset as a function of voltage on the sphere as demonstrated in Fig. 4. Below \( V_{sph} \approx 50 \) V, there is no instability. Just above threshold the oscillations appear as intermittent wave bursts. These merge into a continuous wavetrain with increasing voltage [Fig. 4(a)]. Due to the intermittent waveform, the instability amplitude is here defined by the rms value over \( N \) samples in time after onset, \( V_{rms} = (N^{-1}\Sigma V_{rms}^2)^{1/2} \). Figure 4(b) shows that the rf amplitude rises approximately linear with \( V_{sph} \) partly because the duty cycle increases and partly due to an amplitude increase. When the sphere is biased negatively there is no visible sheath and no rf instability.

The electron collection by a spherical probe in a magnetoplasma has received much attention in the past.18,22,24 Although it is not the main focus of this work the dc current-voltage characteristics, shown in Fig. 4(c), will be briefly discussed.

Since the ions are unmagnetized the ion saturation current \( I_{sat} \approx 1.5 \) mA at the plasma potential \( \Phi_p = -23 \) V yields a reliable measure of the density \( n \approx 1 \times 10^9 \) cm\(^{-3}\), based on the electron temperature of \( kT_e = 4.3 \) eV in the low current electron retardation regime. The negative plasma potential is due to a resistive oxide layer on the aluminum chamber wall. The electron saturation regime shows discrep-
around the sphere. Electrons moving along which is close to the observed radial extent of the blue halo density depletion is evident from the current waveform in pulse mode. The relatively low electron current is partly due to a density depletion in the flux tube of the sphere and partly due to the reduced electron collection across $B_0$. A density depletion is evident from the current waveform in pulsed mode [Fig. 3(a)], which shows an initially much higher saturation current that decreases on the time scale of short successive time intervals.

On the basis of conservation of energy and canonical angular momentum, Parker and Murphy predicted an upper bound electron current given by the random electron flux through a field-aligned channel of radius $R_{PM} = R_{sph}[1 + (8V_{sph}/\omega_c B^2_{sph})^{1/2}]$. In the present experiment, the radius of the sphere is $R_{sph} = 1.6$ cm, its potential is $V_{sph} = 100$ V, the field is $B = 30$ G, and $\omega_c / 2\pi = 84$ MHz, which yield $R_{PM} = 2.75$ cm or a sheath thickness $s = R_{PM} - R_{sph} = 1.2$ cm, which is close to the observed radial extent of the blue halo around the sphere. Electrons moving along $B_0$ into the sheath are energized; their Larmor radius increases, which brings them closer to the sphere such that they are collected. Simulations of Singh et al. showed that the uppermost current collection occurs when the electrons are completely demagnetized, i.e., their final Larmor radius exceeds the sphere radius, which occurs at a potential $V_{sph} = (m/2\epsilon)(\omega_c R_{sph})^2 \approx 200$ V for the present parameters.

The instability onset ($V_{sph} = 50$ V) lies in the electron saturation regime. The maximum Larmor radius of 50 eV electrons is $r_e \approx 0.8$ cm, implying that some electrons traverse the sheath across $B_0 = 30$ G. The instability grows the more electrons are demagnetized. Thus, as suggested by the reviewer, it is tempting to associate the demagnetization
of the electrons with the mechanism of the instability, which is qualitatively also supported by a lower threshold voltage for lower magnetic fields and smaller spheres.

B. Instability waveforms and spectra

The instability waveforms and spectra depend strongly on $V_{\text{sph}}$ as shown in Fig. 5. At low voltages ($V_{\text{sph}}=80$ V) the bursty emissions are nearly monochromatic at $f=2f_0$. As the voltage is raised ($V_{\text{sph}}=140$ V) the fundamental emission $f_0$ appears with some delay relative to $2f_0$. The latter drifts downward and upward in time. The spectra have more lines at still higher voltages ($V_{\text{sph}}=200$ V). The $2f_0$ emission gradually shifts upward with $V_{\text{sph}}$ while the $f_0$ line does not. Frequency hopping occurs on a time scale of a few microseconds, shortening with increasing $V_{\text{sph}}$ and occurs also for dc voltages.

Frequency properties on shorter time scales can be seen directly inferred from rf waveforms in time. Figure 6(a) shows that the instability grows within several cycles at a nearly constant frequency $2f_0$. The observed growth rate is not determined by the voltage waveform since at the onset time $V_{\text{sph}}=180$ V is constant and the current $I_{\text{sph}}$ decays on a time scale slow compared to the rf period [see Fig. 3(a)]. The growth rate increases with $V_{\text{sph}}$. Figure 6(b) describes the rf waveform during switch off of $V_{\text{sph}}$. Note that a switching transistor disrupts the current rapidly ($\Delta t<100$ ns) while the voltage $V_{\text{sph}}$ floats and decays slowly compared to the rf period. Thus there is a decaying electric field without electron collection. Under these conditions the $2f_0$ decays rapidly while the $f_0$ emission persists for a few cycles with decreasing frequency. These are important clues for the instability mechanism discussed in the final section.

The instability has mainly been studied with steady-state voltages. One interesting aspect is an increased harmonic content with increasing $V_{\text{sph}}$. Figure 7(a) shows a rf waveform with increased time resolution. Amplitude and frequency are changing slowly compared to the rf period and on a time scale of the ion plasma period ($f_{\text{pi}}^{-1}=1$ $\mu$s) which may indicate sheath thickness variations.

Figure 7(b) shows a single-shot waveform on a submicrosecond time scale (0.2 ns resolution) which reveals a highly nonsinusoidal but periodic rf oscillation. The FFT of

FIG. 5. (Color online) Time-resolved spectra of the instability waveforms [see Fig. 4(a)] at different voltages on the sphere which has a step function waveform [see Fig. 3(a)]. Spectra for $V_{\text{sph}}=80$ V show bursts of emissions at $2f_0$, for $V_{\text{sph}}=140$ V a delayed onset of the $f_0$ line as well as frequency shifts of $2f_0$, and for $V_{\text{sph}}=200$ V line splitting and frequency hopping on microsecond time scales.

FIG. 6. (Color online) Waveform at (a) onset of the instability and (b) switch off of the current to the sphere. The instability grows in the $2f_0$ mode even at a high $V_{\text{sph}}=180$ V. At switch off ($t=0.2$ $\mu$s), the electron drift velocity decays; hence the frequency decreases.
this waveform [Fig. 7(c)] shows a rich spectrum of at least 22 harmonics on a logarithmic scale. Such features are lost on long-term or averaged FFTs due to phase and frequency drifts. They reveal that the instability mechanism involves highly impulsive electron currents. The wide frequency span also shows that there is no emission at the cyclotron frequency.\( f_c=84 \text{ MHz} \) or the unmagnetized sheath-plasma resonance, \( f_{sph, res} = f_r (1 + r_{sheath}/r_{sph})^{1/2} \approx 235 \text{ MHz} \).\[^{12}\]

C. Electron drift eigenmodes

The spatial properties of the rf oscillations have been investigated with two rf probes using conditional averaging techniques.\[^{26}\] A fixed probe signal is applied to a tuned rf amplifier whose output is used to trigger a multichannel digital oscilloscope. A movable probe signal is equally amplified and displayed on the oscilloscope. Figure 8(a) shows the rf oscillations tuned to the second harmonic \( f/2f_0 = 22 \text{ MHz} \) of the instability versus time as the probe is rotated by \( \Phi = 2\pi \) around the equator in \( 10^\circ \) increments. The peak rf amplitude changes little around the sphere. The time delay shows that the oscillation propagates in the \( \mathbf{E} \times \mathbf{B} \) direction with a wavelength \( \lambda = \pi r \), where \( r = r_{sph} + 5 \text{ mm} \approx 21 \text{ mm} \). A polar plot of the oscillation at a fixed time shows that the second harmonic produces an \( m=2 \) azimuthal eigenmode [Fig. 8(b)].

As shown in Fig. 7 all Fourier components belong to a fairly coherent periodic waveform hence propagate at the same velocity \( v_r = \lambda = 1.45 \times 10^5 \text{ cm/s} \) and have wave-numbers \( m = f/ f_0 \). With the reasonable assumption that these are electron drift waves, the phase velocity corresponds to a drift velocity \( v_\phi = E_r / B_0 \) which for \( B_0 = 30 \text{ G} \) requires an electric field \( E_r = 44 \text{ V/cm} \). Probe measurements shown below indicate that such fields occur within the sheath but that there is also velocity shear since \( E_r \neq \text{const} \). Thus, other factors such as a frequency-dependent growth rate must also determine the frequency of the instability.

D. Source location

Since the rf probes are movable in all directions, the spatial dependence of \( V_{rf} \) has also been mapped along the remaining polar coordinates \( r \) and \( \theta \). The signal is passed through a broadband amplifier to observe variations in amplitude and frequency rather than phase.
Figure 9(a) shows the instability amplitude versus polar angle, which clearly indicates that the source lies in the blue sheath in the equatorial plane. Figure 9(b) shows a slight frequency upshift as the probe is rotated out of the equatorial electron ring. Figure 9(c) shows $V_{\text{rf}}$ versus radius showing the instability peaks in the blue sheath (see sketch inserted). Figure 9(d) shows $f_0$ even outside the sheath where $E/B = 0$ ($V_{\text{in}} = 180 \text{ V}, B_0 = 30 \text{ G}, V_{\text{dis}} = 100 \text{ V}, I_{\text{dis}} = 0.15 \text{ A}, 10^{-4} \text{ Torr Ar}$).

E. rf magnetic field

Boundary conditions at the conducting sphere require that the electric field has to be radial. With $E_x(\phi, \theta)$ one has $\nabla \times E \neq 0$ and there must be a rf magnetic field $B_{\text{rf}}$. Alternatively, $B_{\text{rf}}$ can be explained by an oscillating electron Hall current $J_\theta$ which acts like a magnetic loop antenna. The radiation pattern is more complicated than that of a loop antenna because of induced currents in the conducting sphere and rf currents into the sphere and closing to ground.

Using a shielded magnetic probe, a rf magnetic field has been observed. In order to minimize the perturbations of the magnetic probe on the drifting electrons, the measurements have been performed outside the sheath ($r = 2.5 \text{ cm}, \theta = \pi/2$). Figure 10(a) shows the magnetic probe signal vs time as the radial probe shaft is rotated. Negligible phase shift indicates a linear polarization. A plot of the peak rf voltage at $t = 14 \mu\text{s}$ versus probe angle $\theta$ [Fig. 10(b)] shows that nulls are in the radial direction and peaks in the axial direction ($\theta = \pm \pi$), i.e., the rf magnetic field at this location is axial. This polarization is consistent with azimuthal rf...
currents in the sheath. It cannot be produced by axial rf currents in the feed wire to the sphere. The probe is much closer to the sheath than the feed wire.

The rf magnetic field has also been observed at larger distances from the sphere (Δz > 5 cm), albeit with decreasing amplitudes and more elliptical polarization. Since ω < ωe and ωp > ωe, the sheath instability can excite electron whistler modes. Unfortunately, the present device is not large enough to observe wavelengths such that the radiation problem cannot be properly investigated.

F. Parameter dependence

The dependence of the instability frequency on various parameters has been investigated. For an electron drift eigenmode, one would expect f ∝ (E/B) for a fixed wavenumber k = 1/r_{sph}. From Fig. 5 one observes a less-than-proportional frequency increase with V_{sph}. Possibly because V_{sph} increases the sheath thickness such that E_r = const.

Attempts have been made to measure the electric field in the sheath using the rf probes as cylindrical Langmuir probes (0.18 mm diameter, 4 mm length). The interpretation of the I-V characteristics in a non-Maxwellian non-neutral plasma is not as straightforward as in the ambient plasma.

Figure 11 shows I-V characteristics of the probe for different radial probe positions y through the luminous ring. The sphere is biased at a constant voltage V_{sph} = 100 V. Close to the sphere the I-V characteristics exhibit an inflection point near V_{probe} = 90 V, determined by curve fitting and numerical derivatives. It has been verified that this potential shifts proportional to V_{sph}. Since electron collection starts at V_{probe} = 20 V, the tail electron energy is (1/2)mv^2 = 110 eV. With increasing distance from the sphere the inflection point becomes less pronounced until the sheath edge is reached (y ≈ 1 cm) where the plasma potential V_{plasma} = −10 V can be identified. The electric field has an average value E_r = 100 V/cm but appears higher at the sphere and vanishes at the sheath edge. The observed phase velocity of the waves required an electric field of E_r = 44 V/cm which is located in the outer sheath. But the sheath is not a rigid rotor and the concept of rotation even breaks down when one considers the electron orbits in the inner sheath.

At the sheath edge electrons with kT_e = 4 eV have a Larmor radius r_{ce} = v/ω_{ce} = 2.3 mm and perform normal cycloidal E × B drifts. Deeper into the sheath the motion becomes nonadiabatic since the electrons gain more energy and orbit in a nonuniform radial electric field. When electrons gain an energy mv_e^2/2 = 77 eV their cyclotron radius exceeds the sheath thickness, r_{ce} = v/ω_{ce} > 1 cm, and they are collected. Their actual orbits depend on pitch angles and distance of the guiding center line from the sphere as they approach the sphere along B_0. Both computer simulations and theories have discussed such particle orbits. For the purpose of understanding the sheath instability, it is important to note that the transit time of electrons across the
magnetized sheath is much longer than in an unmagnetized sheath which will be discussed further below.

The probe saturation current decreases toward the sphere. Since the velocity increases radially \( v \approx V_{\text{plasma}} \sqrt{r} \) and the current is conserved \( I \approx r^2 n v \), the electron density in the sheath decreases by a factor \( n_{\text{sphere}} / n_{\text{edge}} = (2.6 \text{ cm} / 1.6 \text{ cm})^2 = 100 \) eV/100 eV\(^{1/2} \approx 0.53 \). 

The dependence of the instability on magnetic field and density is summarized in Fig. 12(a). The magnetic field has been varied in the range 10 G < \( B_0 < 30 \) G with observing only small frequency changes with \( B_0 \). The instability vanishes below 10 G. At low fields only the fundamental frequency \( f_0 \approx 10 \) MHz is excited. With increasing field more and more harmonics appear.

It should be pointed out that changes of the magnetic field also affect the density. With increasing field the primary electrons from the distant cathode are better confined such that the ionization rate, hence plasma density, increases. At higher density the sheath thickness decreases which increases \( E \) such that \( E/B = \text{const} \).

Figure 12(b) addresses the frequency dependence on density. In a dc discharge, the density is proportional to the discharge current which can be varied via the cathode temperature, leaving all other parameters fixed. The instability frequency \( \omega \approx f \approx \sqrt{2} \) mode is found to be proportional to the square root of the discharge current, i.e., proportional to the plasma frequency. The latter is about an order of magnitude higher than the instability frequency \( f_p \approx 280 \) MHz, and also well below the electron density in the sheath \( n_{\text{sheath, min}} \approx 280 \times 0.53^{1/2} = 203 \) MHz. Thus the frequency dependence on \( f_p \) arises from other effects such as the variation in sheath thickness \( s_{\text{sheath}} \approx \lambda_{\text{Debye}} / \omega \). A thinner sheath increases the electric field; hence drift velocity and frequency increases with \( f_p \). It also reduces the electron transit time across the sheath which shifts the growth rate to higher frequencies \( (\omega \tau = \text{const}) \).

The density has also been varied by changing the neutral gas pressure, resulting in the same dependency \( f_{\text{inst}} \approx f_p \). However, at high neutral pressures and voltages, the energetic electrons ionize the gas, which modifies the sheath into a fireball\(^{8,33} \) [see Fig. 12(c)]. The rf instability disappears since the applied potential drop shifts from the sheath to a double layer and an ion-rich sheath at the chamber wall. Earlier experiments relied on sheath ionization to produce the plasma, hence did not observe the present instabilities\(^{19,20} \).

Finally, the instability produced by two spheres of different diameters has been compared for otherwise identical parameters. As expected, the small sphere excites higher frequencies, although the frequency ratio is smaller than predicted by the inverse ratio of radii.

### G. Test wave amplification

An instability not only excites waves spontaneously but also amplifies injected test waves. Such amplification has been observed when the test wave frequency coincides with that of an unstable mode, e.g., when \( f_{\text{tw}} \approx 2f_0 \approx 25 \) MHz. One of the two probes is used to excite the wave, the second one to receive the wave. In order to avoid direct coupling the exciter antenna is rotated by \( \Delta \theta \approx 45^\circ \) upstream from the receiver antenna. The bias to the sphere is pulsed \( (V_{\text{sph}} = 140 \text{ V}, t_{\text{pulse}} = 15 \mu \text{s}) \). Figure 13(a) shows the received signal versus time and Figs. 13(b) and 13(c) show the power spectra in the absence \( (\Delta t_1) \) and presence \( (\Delta t_2) \) of the bias pulse, respectively. Without instability only a small direct-coupled signal is observed. But when the sphere is biased positively \( (6.7 \mu \text{s} < t < 21 \mu \text{s}) \) the test wave is greatly enhanced \( (>20 \text{ dB}) \) and exceeds the natural instability line at \( f_0 \approx 10 \) MHz visible in Fig. 13(c). On an expanded time scale the test wave is observed to grow within a few cycles at turn on of \( V_{\text{sph}} \). It is an absolute instability since after one revolution the spatial pattern of the eigenmode fields remains unchanged while the amplitude grows in time.

Since sheaths are nonlinear potential structures, it is not surprising that strong test wave amplification produces non-
linear effects, some of which are displayed in Fig. 14. With increasing amplification, controlled by $V_{\text{sph}}$, the received signal first shows the appearance of a harmonic of the test wave, $f=2f_{\text{tw}}=4f_0=50$ MHz. Next a subharmonic at $f_0=f_{\text{tw}}/2=12.5$ MHz is created by parametric decay. At $V_{\text{sph}}=123$ V, a multitude of harmonics is seen at $nf_0$, $n=1\ldots10$, of which only $f_{\text{tw}}=2f_0$ was injected. These are all very narrow lines compared to the natural instability spectrum shown at the bottom. Thus the test wave has entrained the instability. In the time domain the electron bunches rotating in the luminous sheath become phase locked to the test wave forming a nonsinusoidal waveform which, unlike in Fig. 7(a), does not change in time.

IV. SUMMARY AND CONCLUSIONS

A new instability has been observed in a magnetized electron-rich sheath of a spherical electrode in a collisionless plasma. The instability excites electron drift waves at frequencies below the electron plasma and cyclotron frequencies. The oscillating current ring also produces a rf magnetic field which can excite whistler modes.

The location of the instability in the luminous sheath suggests that the energetic electron population excites the instability. Electrons in the outer sheath region perform cycloidal motions but approaching the sphere large Larmor radius effects dominate and electrons are eventually collected across $B_0$. The instability depends on the collection of electrons. The electron transit time through the sheath $\tau$ creates a phase shift between current and electric field which can lead to a negative rf differential conductivity at frequencies satisfying $\omega\tau\approx2\pi$. Inertial effects have been studied in vacuum diodes\cite{9,10} and in electron-rich sheaths in unmagnetized plasmas. In the latter case, the electron transit time through the sheath is of order $\omega_0^{-1}$ such that a resonance near $\omega_0$, the sheath-plasma resonance,\cite{14} can be destabilized, which gives rise to the well-known sheath-plasma instability.\cite{12}

In the present case of a magnetized plasma the electron transit time across $B_0$ is much longer than $\omega_0^{-1}$, which requires a lower frequency resonance to produce the instability. This is an electron drift eigenmode $\omega=k_mv_{\text{drift}}$, where $k_m=mn/r_{\text{sph}}$, $m=1,2\ldots$. It leads to an absolute instability whose frequency is determined by both $\tau$ and $\omega$, which in turn depend on $E$, $B_0$, mode number $m$, sheath thickness, and $r_{\text{sph}}$. The instability may select the fundamental mode or the second harmonic. Burstlike electron currents can produce a rich spectrum of harmonics.

The present instability differs from typical diocotron instabilities\cite{2,4} in geometry ($k_i>k_j$) and parameter regime (low field, large Larmor radius regime, $\omega_0 > \omega_e$) where theory\cite{28} and experiments\cite{29} do not show diocotron instabilities.

While all observations indicate that the qualitative instability mechanism is due to electron inertia, it is clear that a kinetic and electromagnetic theory and/or simulation are required to explain the instability quantitatively.

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