Trivelpiece-Gould modes in a uniform unbounded plasma

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Trivelpiece-Gould (TG) modes originally described electrostatic surface waves on an axially magnetized cylindrical plasma column. Subsequent studies of electromagnetic waves in such plasma columns revealed two modes, a predominantly magnetic helicon mode (H) and the mixed magnetic and electrostatic Trivelpiece-Gould modes (TG). The latter are similar to whistler modes near the oblique cyclotron resonance in unbounded plasmas. The wave propagation in cylindrical geometry is assumed to be paraxial while the modes exhibit radial standing waves. The present work shows that TG modes also arise in a uniform plasma without radial standing waves. It is shown experimentally that oblique cyclotron resonance arises in large mode number helicons. Their azimuthal wave number far exceeds the axial wave number which creates whistlers near the oblique cyclotron resonance. Cyclotron damping absorbs the TG mode and can energize electrons in the center of a plasma column rather than the edge of conventional TG modes. The angular orbital field momentum can produce new perpendicular wave-particle interactions. Published by AIP Publishing.

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I. INTRODUCTION

Whistler modes are important in space and laboratory plasmas. In nearly unbounded space plasmas, the waves are assumed to be plane waves whose theory is well developed.1–3 In bounded plasma columns of cylindrical geometry, a theory of cylindrical eigenmodes has been formulated.4,5 It follows the theory of electromagnetic waves in waveguides, even though plasmas are not isotropic media. The excitation of eigenmodes with antennas is still a topic of current research in helicon discharges.5 The wave theory for helicons is complicated by nonlinear effects and nonuniformity and boundary effects. Since the wave damping could not be explained by collisions and Landau damping,6,7 anomalous resistivity9,10 and coupling to Trivelpiece-Gould (TG) modes11 have been proposed without definitive conclusions. TG modes are slow waves which can be easily absorbed and heat electrons, albeit near the plasma boundary. In spite of many theoretical investigations, TG modes have not been clearly observed. The only evidence is found from current density measurements,12 but no short wavelength magnetic or electric modes have been observed so far.

Helical phase fronts are the salient features of helicon modes. Such modes exist not only in bounded plasma columns but also in unbounded plasmas.13 The present work deals with helicon wave properties in uniform plasmas which offer the simplest conditions. Linear whistler modes are excited from antennas inside a preformed, uniform laboratory plasma of dimensions large compared to the helicon wavelength. The field topology of the waves is measured in 3D space with full time resolution. Wave bursts are generated to observe phase and group velocities. Depending on the loop orientation with respect to the ambient field \( \mathbf{B}_0 \), the excited waves have the same radial field dependence as \( m = 0 \) and \( m = 1 \) helicons, where \( m \) is the azimuthal mode number. This leads to the conclusion that in uniform plasmas, the antenna determines the helicon wave topology, not a helicon eigenmode.

Linear superposition of loop antenna fields allowed the study of wave excitation by phased antenna arrays.14 These can excite nearly plane waves, conical waves, and helicon waves of either rotation up to high mode numbers. The properties of orbital angular momentum of vortex waves have been pointed out for helicons.15 The present experiment employs a real physical antenna array which excites the same helicons as earlier predicted by field superposition.

A circular antenna array is used to excite \( m = 8 \) helicon modes. The antenna consists of 16 loops of alternating dipole directions parallel to \( \mathbf{B}_0 \), forming an azimuthal standing wave. This approach is much simpler than phasing a large array. The azimuthal distance between two loops defines half an azimuthal wavelength. The axial wavelength depends on the plasma density which is variable in the afterglow of a pulsed discharge plasma. Thus the propagation angle can be varied so as to excite helicon modes approaching cyclotron resonance, i.e., TG modes. The difference to TG modes in small columns is that the dominant perpendicular wavenumber is in azimuthal direction, not in radial direction. Furthermore, we do not require an eigenmode theory to explain the TG mode. It is simply a whistler mode of an unbounded plasma which rotates around \( \mathbf{B}_0 \) with a short wavelength compared to the parallel wavelength. When the propagation angle exceeds the resonance cone angle, given by \( \cos \theta_{res} = \omega / \omega_c \), the wave becomes highly electrostatic and strongly damped. An \( m = 0 \) helicon mode does not damp significantly under the same conditions.

These observations cast a new light on helicon wave theory and possibly suggest new heating schemes. It may also suggest new active experiments in space plasmas. The injection of waves with high angular momentum causes a significant transverse Doppler shift which can give rise to new wave-particle interactions.
The paper is organized as follows. After describing the experimental setup and measurement procedure in Section II, the observations and evaluations are presented in Section III in several subsections. The findings are summarized in the Conclusion (Section IV).

II. EXPERIMENTAL SETUP AND DATA EVALUATIONS

The experiments are performed in a pulsed dc discharge plasma of density $n_e \approx 10^{10} - 10^{12} \text{ cm}^{-3}$, electron temperature $kT_e \approx 1 - 4 \text{ eV}$, neutral pressure $p_n = 0.4 \text{ mTorr Ar}$, uniform axial magnetic field $B_0 = 5 \text{ G}$ in a large device (1.5 m diam, 2.5 m length), shown schematically in Fig. 1. The variation in plasma parameters is due to the choice of time of the pulsed discharge and its afterglow (5 ms on, 1 s off). The discharge uses a 1 m diam. oxide coated hot cathode, a photo of which is shown in Fig. 2.

Whistler modes are excited with two types of antennas: a simple loop (4 cm diam) and a circular array of 16 loops (2 cm loop diam, 15 cm ring diam). The antenna axes are parallel to $B_0$. The dipole moment of the array elements alternates between adjacent loops such that an oscillating standing wave of 8 wavelengths per circumference is formed. This antenna excites an $m = \pm 8$ helicon mode, while the single loop excites an $m = 0$ helicon wave. The bifilar winding of the array antenna produces no net azimuthal current, hence does not excite an $m = 0$ mode. The antennas are energized with 4.5 MHz wave bursts repeated every 50 ms throughout the discharge and afterglow. The burst generator is a weakly damped $L$-$C$ ringing circuit, triggered with a phase locked function generator. Long wave bursts are also produced with a 5 MHz function generator. This wave burst excitation reveals group and phase velocity properties of helicon modes.

The wave magnetic field is received by a small magnetic probe with three orthogonal loops (6 mm diam) which can be moved together in three orthogonal directions. The spatial field distribution is obtained from repeated pulses, averaged over 10 shots by moving the probe through orthogonal planes. The field in vacuum is also measured and can be subtracted from the total measured field, so as to obtain only the wave field produced by plasma currents. The vacuum field drops off rapidly on the scale of the loop radius ($B_{\text{vac}} \propto r_{\text{loop}}^{-3}$). Plasma parameters are measured with Langmuir probes also attached to the movable probe. In order to obtain the local plasma parameters vs time, the probe current is recorded at a dc voltage which is incremented in small steps so as to obtain $I(V)$ at any time. All signals are acquired with a 4-channel digital oscilloscope.

We used linear superposition principles in order to investigate advanced antenna arrays. In a uniform plasma, waves excited by a single loop placed at different positions can be superimposed so as to obtain the wave from more than one antenna. Loops have been arranged in an array of circular rings with increasing radii so as to generate a planar circular array. When the loops are phased azimuthally, a rigid-rotor like field rotation is produced which excites helicon modes of integer $m$-values. Conical waves are excited when the loops are phased radially. When the amplitudes are varied radially, Bessel function profiles can be produced to model helicon “eigenmodes.” Many of these results have been published earlier.

The present work validates the superposition results by measurement from an actual antenna array. A single circular array is simple to build and to energize without phasing. All 16 loops are formed from a single wire with the sense of rotation varied from loop to loop. All loops are fed with the same current. The loop reversal is equivalent to a 180° phase shift between the array elements. The antenna field is an oscillating azimuthal standing wave. It excites whistler modes with the same azimuthal standing wave topology.
which can be decomposed into equal $m = +8$ and $m = -8$ helicon modes. The latter are modes whose phase rotates azimuthally in the opposite direction as the polarization of the magnetic field. Unless there is a need for specifying the sign of the mode number, this simple antenna eliminates the complexity of phasing a multi element array.

**III. EXPERIMENTAL RESULTS**

**A. Helicon excitation by superposition and direct measurements**

First, we show the topology of a high order helicon mode, created by two different approaches. One method relies on superposition of waves excited from loop antennas at different positions. In this case, the field from a single loop has been measured. Since density and magnetic field are uniform, the same wave pattern is produced when the antenna position is changed. By superposition, new radiation patterns of the fields of two or more antennas are obtained. Here, we generate the radiation from a circular array of 24 phased loops on a 16 cm diam circle indicated by a white dashed line. The phase of adjacent antennas differs by 120° which results in a clockwise-rotating $m = -8$ helicon mode.

The resultant wave pattern is shown in Fig. 3(a). The snapshot of contours of $B_z(x,y,z = -15 \text{ cm})$ clearly shows the helicon mode structure. It rotates clockwise in time, i.e., in $-\phi$ direction. There are 8 wavelengths per circumference, $2\pi r = 82$. With decreasing radius, the local azimuthal wavelength becomes shorter or the local $k_\phi$-vector increases, $k_\phi = m/r$. The perpendicular $k$-vector, $k_\perp = (k_\phi, k_\perp)$, becomes much larger than the parallel $k$-vector, resulting in a propagation angle $\theta = \arctan(k_\perp/k_\phi)$ which reaches and exceeds that of the oblique cyclotron resonance, $\theta_{\text{res}} = \arccos(\omega/\omega_\phi)$. Thus, the helicon mode runs into oblique cyclotron resonance with decreasing radius where it becomes a TG mode. Such modes are highly electrostatic and easily damped both of which are reasons for the disappearance of wave magnetic fields in the center of the helicon.

The present experiment employs a real antenna array of 16 loops as shown in Fig. 2. Since phasing an array of many loops is rather involved, we chose to apply an azimuthal standing wave which is much easier to implement than phasing. Every other loop is reversed and all loops are connected in series. An oscillating standing wave can be decomposed into counter propagating $m = +8$ and $m = -8$ modes both of which can and do propagate as whistler modes.

Figure 3(b) shows a snapshot of contours for $B_z(x,y,z)$ under similar conditions as in Fig. 3(a). Again, the $m = 8$ helicon mode structure is very pronounced at the radius of the antenna. Since the measurement is taken at a distance of $\Delta z = 15 \text{ cm}$ from the antenna, the field is not the near-zone field of the antenna but that of a propagating wave. Helicon fields rotate like a rigid rotor; hence, the $m = 8$ mode structure should persist radially since the wave propagates both radially inward and outward. However, the azimuthal wave structure is lost radially inward and the wave amplitude drops rapidly toward the axis of the helicon. The reason is that the azimuthal wavelength approaches the electron inertial length, which is the shortest wavelength of whistler modes. Alternatively, the propagation angle approaches that for oblique cyclotron resonance such that the mode can be called a TG mode. This direct experiment validates the results of the earlier superposition method.

**B. Helicon propagation and damping**

The snapshots of Figs. 4(a) and 4(b) did not display the time dependence of the fields. This is now shown in Figs. 4(c) and 4(d) by time-of-flight diagrams of $B_z(z,t)$ which demonstrate that the fields propagate axially. The slope of the dashed line indicates the axial phase velocity.

**FIG. 3.** Comparison of two $m = 8$ helicons. (a) Superposition of the fields of properly phased 24 loops, placed on a circular array indicated by the white-dash circle, producing an $m = -8$ helicon. Only one loop field has been measured, the others are time and space shifted copies. Note the empty field region in the center. (b) Measured field from a physical array antenna of 16 alternating loops. It forms an $m = 8$ helicon mode with standing waves in azimuthal direction and wave propagation in axial direction. Note the loss of waves toward the center where the azimuthal wavelength decreases and the wave runs into oblique cyclotron resonance, i.e., becomes an azimuthal TG mode.
The propagation angle of a helicon mode with respect to B₀ depends on k₀, determined by the antenna, and k∥, determined by the density for a given frequency and magnetic field. The density decays in the afterglow of the pulsed discharge. The helicon propagation has been observed at different afterglow times which is shown in Fig. 5(a) for the m = 0 mode and in Fig. 5(b) for the m = 8 helicon mode. Displayed are contours of the wave energy density, \( B^2 = (B_x^2 + B_y^2 + B_z^2) \) at a fixed time for three different afterglow times. The single loop antenna excites axially propagating waves throughout the measurement plane with little radial energy spread. As the density decreases, the wave energy drops but the damping remains unaffected. At the very late afterglow, the large contour values are again due to the antenna near-zone field rather than the wave field intensity.

Under the same experimental conditions, the m = 8 array antenna also excites axially propagating waves with peak intensities in the flux tubes of the array elements. But the wave damping increases as the density decays. At the lowest density, the near-zone field intensity dominates but no traveling wave propagates away from the antenna. The change of wave damping must depend on the difference in field topologies since the plasma parameters are the same. The high m = 8 mode has a large k∥ while the m = 0 mode has k∥ = 0 both of which are imposed by the antenna. Other low mode helicons (m = 1–3) readily penetrate to the axis where B₀ switches sign. The axial wavenumber k∥ decreases in time leading to a large propagation angle \( \theta \approx \arctan(k∥/k₀) \), especially radially inward from the antenna array, where \( k_φ = m/r \gg k_r \approx k_z \). Cyclotron resonance occurs when \( \theta_{res} = \arccos(5 \text{ MHz/14 MHz}) = 69° \) which implies that \( \tan \theta_{res} \approx k_φ/k_z = 2.6 \). At the antenna radius of \( r = 7.5 \) cm, one has \( \lambda_φ = 5.9 \) cm and cyclotron damping would arise when...
\[ \lambda_z = \frac{\lambda_\phi}{\tan \theta_{res}} > 15 \text{ cm} \]

The values are approximations since they apply to plane wave theory while helicons are not plane waves. Nevertheless they show that cyclotron damping can explain the observed energy decay and the fact that no waves can be supported inside the high order helicon which is demonstrated next.

Returning to the high order helicon field profiles in the transverse plane (Fig. 3), we now show their dependences on density or afterglow time. Figure 6(a) displays contours of one field component, \( B_z(x, y, z = -15 \text{ cm}) \), at three different times in the afterglow. The wave maps the field pattern of the antenna array, 16 alternating poles, peaked near the location of the antenna array indicated by a white circle. However, with decreasing radius the azimuthal wavelength should decrease, \( \lambda_\phi = 2\pi r/m \), but instead the wave structure and amplitude vanish. On the other hand, for increasing radius, the wavelength increases and the wave structure remains with significant amplitudes. As the density decreases, the field amplitude decreases, the azimuthal wave number remains unchanged, but the central area of the helicon becomes nearly field-free. These features are consistent with the earlier interpretation of wave absorption by oblique cyclotron resonance. The mode number and radial position determine \( k_\phi \), the density determines \( k_z \), which leads to TG modes inside a high mode number helicon at low densities.

Figure 6(b) displays contours of the total wave energy density, \( B^2 = (B_x^2 + B_y^2 + B_z^2)(x, y, z = -15 \text{ cm}) \). It shows that all field components have similar spatial properties as the \( B_z \) component. Due to the predominantly axial group velocity, the wave energy axially away from the antenna still peaks on the radial location of the antenna array. Virtually no energy is observed inside the helicon. As the density decays, the wave energy decreases, the empty inner region expands, and the energy density completely vanishes on a constant scale for the contour values. The orthogonal planes of Figs. 5(b) and 6(b) lead to a three-dimensional picture of a hollow helicon cylinder.
IV. CONCLUSION

High m-number helicons have been excited with a circular antenna array. Depending on mode number and array radius, the modes can be described as H modes or TG modes. TG modes are formed radially inward, where the increasing $k_r$ creates oblique cyclotron resonance. Whether TG modes arise from radial or azimuthal propagation does not change the physics of oblique cyclotron resonance. In the present experiment, the TG mode is absorbed in the central region of the helicon. The hollow helicon cannot be explained by the dependence of high-order Bessel functions or by the lack of radial energy flow since the field penetration varies with density which determines $k_{\perp}$ and the resultant oblique propagation angle.

The present observation shows that TG modes (defined as quasi electrostatic modes near the oblique cyclotron resonance) are volume modes in uniform unbounded plasmas and not only boundary modes of nonuniform plasma columns as originally discovered and later adopted to helicon sources. In retrospect, our findings may not be surprising since it has already been shown that helicon modes exist in unbounded plasmas and the difference between helicon and TG modes is just the angle of wave propagation. Our method of generating azimuthal TG modes is simple and straightforward while radial TG modes are difficult to excite with standard helicon antennas, hence have rarely been observed.

When the circular array is opened into a straight line array, high $k_{\perp}$ components can also be imposed along the array which produce oblique plane wave near cyclotron
resonance (see Fig. 5 in Ref. 17). In spite of its similar physics, the circular version of oblique whistlers near cyclotron resonance is novel and has not been discussed or observed in uniform plasmas and helicon sources.

The present results have several implications.

Cyclotron absorption in the center of a helicon mode produces an absorptive boundary which prevents the formation of radial standing waves which is the underlying assumption in helicon wave theory. The radial field dependence cannot be described by Bessel functions if part of the plasma column is field-free and presents an absorptive boundary.

The solution of the wave equation for helicon eigenmodes assumes paraxial wave propagation, i.e., radial standing waves and axial phase propagation. It is valid for free-space electromagnetic waves in cylindrical waveguides, but not in anisotropic plasmas. Phase and group velocities point in different directions. For example, in a Gendrin mode with parallel group velocity, the radial propagation transports no energy, hence cannot be described by a Bessel function as in an isotropic medium. In general, oblique phase and group velocities reflect differently. Many observations of helicon modes show radial wave propagation,20,21 Helicon wave theory may be a crude approximation for very small plasma columns, where wave tunneling dominates over wave propagation effects. It certainly cannot be extended to high mode number helicons, to helicons without boundaries, to nonlinear effects in helicon devices, and explain antenna-wave coupling for different antenna configurations.

Although high mode number helicons have not yet been produced in helicon devices, the wave absorption in the center of such modes may be of interest to electron heating and ionization. In space plasmas, high-mode number helicons can produce perpendicular wave-particle interactions not considered previously. For example, wave absorption transfers the field orbital angular momentum to the electrons, scattering them in velocity space, possibly producing a current rings or whistler instabilities by temperature anisotropies.

An axial density gradient can also lead to TG modes in high order helicons. An axial drop in density causes a decrease in $k_{||}$ which for fixed $k_{\perp}$ brings the wave into cyclotron resonance. Alternatively, the density drop increases the electron inertial scale such that $k_{\phi}c/\omega_{p} \rightarrow 1$ which is the limit for whistler wave propagation. The same can happen in a converging magnetic field where the transverse wavelength shrinks to the electron inertial scale.

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