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Whistler modes in highly nonuniform magnetic fields. III. Propagation near mirror and cusp fields

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The properties of helicon modes in highly nonuniform magnetic fields are studied experimentally. The waves propagate in an essentially unbounded uniform laboratory plasma. Helicons with mode number \( m = 1 \) are excited with a magnetic loop with dipole moment across the dc magnetic field. The wave fields are measured with a three-component magnetic probe movable in three orthogonal directions so as to resolve the spatial and temporal wave properties. The ambient magnetic field has the topology of a mirror or a cusp, produced by the superposition of a uniform axial field \( B_0 \) and the field of a current-carrying loop with the axis along \( B_0 \). The novel finding is the reflection of whistlers by a strong mirror magnetic field. The reflection arises when the magnetic field changes on a scale length shorter than the whistler wavelength. The simplest explanation for the reflection mechanism is the strong gradient of the refractive index which depends on the density and magnetic field. More detailed observations show that the incident wave splits when the \( k \) vector makes an angle larger than \( 90^\circ \) with respect to \( B_0 \) which produces a parallel phase velocity component opposite to that of the incident wave. The reflection coefficient has been estimated to be close to unity. Interference between reflected and incident waves creates nodes in which the whistler mode becomes linearly polarized. When the magnetic field topology is that of a reversed field configuration (FRC), the incident wave is absorbed near the three-dimensional (3D) magnetic null point which prevents wave reflections. However, waves outside the separatrix are not absorbed and continue to propagate around the null point. When waves are excited inside the FRC, their polarization and helicon mode are reversed. Implications of these observations on research in space plasmas and helicon sources are pointed out. Published by AIP Publishing.

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I. INTRODUCTION

Helicon modes are low frequency whistler modes with parallel, perpendicular, and azimuthal wavenumbers. Their phase fronts and field lines are helical. Historically, they have been associated with waves in bounded solid state slabs where boundary reflections lead to radial standing waves.\(^1\) This model has been adapted for whistler wave studies in gaseous plasma columns.\(^2\) The interest in helicons grew by their application to produce dense rf plasma sources.\(^3\) Discrepancies with linear wave theory arose: The perpendicular wavenumber was not due to boundary reflections but radial density gradients.\(^4\) Helicon waves have also been observed in unbounded plasmas without density gradients\(^5\) where the antenna determines the perpendicular wavenumber. Radial wave propagation has been frequently observed\(^6,7\) which contradicted the theoretical prediction of “paraxial” wave propagation, \( f(k_r,r) \exp(i(m\phi + k_z z - \omega t)) \), where \( f(k_r,r) \) describes a radial amplitude profile of a standing wave with radial wavenumber \( k_r \), \( m = k_\phi r \) is the azimuthal eigenmode number, \( k_z \) is the axial wavenumber, and \( \omega \) is the radial frequency.

Waves excited from antennas resemble more closely helicon modes than plane waves. When the antenna imposes a perpendicular magnetic or electric field, it usually excites a rotating \( m = 1 \) helicon mode.\(^8\) The significant, yet unexplored, property of helicons is their angular field momentum, which can interact with electrons or waves. Landau and Doppler-shifted cyclotron resonance exist not only for the linear field momentum but also for the angular orbital momentum.\(^9\) The transverse Doppler shift has so far received little attention.

The propagation of whistler modes in nonuniform plasmas has been studied extensively. For density gradient scales which are large compared to the wavelength, ray tracing has been applied to whistler wave trapping in density ducts in space plasmas.\(^10\) The guiding of whistler modes in narrow density troughs has been observed experimentally.\(^11\) Since the refractive index depends not only on density but also on the magnetic field, the refraction of whistlers also occurs in nonuniform magnetic fields. It usually assumes large gradient scale lengths where the Wentzel-Kramers-Brillouin (WKB) approximation is valid.\(^12\) Ray tracing studies have been done for wave propagation in the Earth’s magnetic dipole field\(^13\) and for wave injection along expanding field lines of plasma thrusters.\(^14\) Experiments on cyclotron absorption have been done in several laboratory experiments.\(^15-17\)

However, the topic of wave propagation in highly nonuniform magnetic fields (\( B/|\nabla B| < \lambda \)) has received little attention since it is not amenable to ray tracing. It has been addressed in the present work and its preceding Papers I and II.\(^18,19\) In the present paper, the nonuniform field topologies are an axially symmetric mirror field and a field reversed configuration (FRC), both with short gradient scale lengths.
The major new finding is the reflection of whistler modes from a mirror field. It can be explained by the abruptly changing refractive index. But a more detailed explanation is the splitting of the phase front when the incident wave vector turns beyond 90° with respect to \( \mathbf{B}_0 \), which produces a reverse phase and group velocity along \( \mathbf{B}_0 \). Since the latter is more field aligned than the phase velocity, a small \( k_0 \) can cause a significant deflection of the wave amplitude. The interference between incident and reflected waves is observed to produce locally a linear polarization of whistler modes. This leads to a significant temporal amplitude modulation which is absent for circularly polarized whistlers.

As we report in this paper, when a whistler mode propagates against a null point of an FRC, the wave is absorbed in regions where \( \omega / \omega_c \geq 1 \). Absorption eliminates the chance for wave reflection. For an FRC smaller than the lateral wave dimension, the outer part of the wave does not encounter cyclotron resonance and keeps propagating, typically on the separatrix and open field lines. When the wave is excited inside and on the axis of an FRC, the waves become trapped on closed field lines. The field reversal changes the rotation of the \( \mathbf{B} \) vectors inside the separatrix vs that on open field lines. The wave topology also rotates differently inside the separatrix compared to outside since azimuthal phase rotation and polarization rotate together in electron Hall physics. The direction of wave propagation remains the same with and without field reversal; hence, the reversal of the phase rotation changes the spatial twist of the helical phase surface to a right-handed one.

The transmission between two antennas is shown and discussed. Wave propagation along the symmetry axis through an FRC is not possible due to the two null points; likewise, propagation into and out of an FRC is cut off as previously reported.\(^{20}\) Incident waves with large lateral dimensions propagate on and beyond the separatrix around the FRC, which complicates the comparison of cyclotron damping experiments with plane wave theory. Inside the FRC, waves can propagate but with reversed polarization and wave rotation compared to outside the separatrix. The difference between polarization and phase rotation is shown here, which is relevant to helicon mode theory and experiments. The importance of orbital angular field momentum to new wave-particle interactions is also pointed out.

Hodograms are a standard tool in space plasma to determine the direction of wave propagation.\(^{21,22}\) A hodogram is the curve which the tip of a three-component wave electric or magnetic field vector traces out in time. For plane parallel whistlers, the hodogram rotates anti-clockwise or forms a right-handed rotation in time around \( \mathbf{B}_0 \). For plane but oblique whistlers, the field vector rotates around the \( \mathbf{k} \)-vector but not around \( \mathbf{B}_0 \).\(^{23,24}\) which is confirmed by observations. The wave field lies in the plane of the hodogram which indicates the local phase front of the plane wave. The normal to the hodogram is parallel or antiparallel to the wave vector \( \mathbf{k} \), depending upon the direction of propagation with respect to \( \mathbf{B}_0 \) and the sign of the field polarization around \( \mathbf{B}_0 \). In either case, the normal satisfies Maxwell’s equation in real or wavenumber space, \( \nabla \cdot \mathbf{B}(r) \propto \mathbf{k} \cdot \mathbf{B}(\omega) = 0 \).

Single spacecraft data do not provide sufficient spatial resolution of phase fronts which prevents the distinction of wave packets from plane waves or the direct verification of wave reflection and refraction. Since the refractive index depends on both the density and the magnetic field, a nonuniformity in either parameter leads to wave refraction. Propagation in density nonuniformities such as ducts has been studied in laboratory experiments,\(^{11,25–27}\) by theories\(^{28–31}\) and by numerical ray tracing models.\(^{10,28,32}\) Comparatively little is known on how whistler modes propagate in strongly nonuniform magnetic fields.\(^{33}\) The Earth’s magnetic field is weakly nonuniform, such that ray tracing theory is justified.\(^{33,34}\) It predicts that multiple reflected whistlers gradually refract away from the Earth. Such predictions cannot be made for whistler modes in lunar crustal magnetic fields\(^{35}\) or for low frequency whistlers at the bow-shock\(^{36}\) or near reconnection regions.\(^{37,38}\)

Nonuniform magnetic fields also exist in the exhaust region of helicon thrusters\(^{14,16,39}\) or toroidal plasma devices\(^{39–42}\) with the focus on plasma production rather than wave refraction. Refraction in nonuniform magnetic fields destroys the cylindrical geometry of helicons. Wave propagation in nonuniform and time-varying magnetic fields has also been studied in a large laboratory device.\(^{43–45}\) In the present laboratory experiment, we establish controlled nonuniform magnetic fields of known topology and strength and measure the wave propagation under various conditions.

This paper is organized as follows: After describing the experimental setup and measurement procedure in Sec. II, the observations and evaluations are presented in Sec. III in several subsections. The findings are summarized in Conclusion, Sec. IV.

II. EXPERIMENTAL SETUP AND DATA EVALUATIONS

The experiments are performed in a pulsed dc discharge plasma of density \( n_e \approx 10^{11} \) cm\(^{-3}\), electron temperature \( kT_e \approx 2 \) eV, neutral pressure \( p_n = 0.4 \) mTorr Ar, and uniform axial magnetic field \( B_0 = 3 \) G in a large vacuum chamber (1.5 m diam, 2.5 m length), shown schematically in Fig. 1(a). A range of densities is obtained by working in the afterglow of the pulsed discharge (5 ms on, 1 s off). The afterglow plasma has Maxwellian distributions and is free of instabilities. The discharge uses a 1 m diam oxide coated hot cathode, a photograph of which is shown in Fig. 1(b).

Whistler modes are excited with magnetic loop antennas (4 or 5 cm diam, 4 turns). The axis of the dipole is perpendicular to \( \mathbf{B}_0 \) which excites an \( m = +1 \) helicon mode.\(^{18}\) The antenna is driven by 5 MHz rf bursts (20 rf periods duration, 5 \( \mu s \) repetition time) whose turn-on reveals phase and group velocity during the wave growth and decay. A small field amplitude (\( B < 0.1 \) G) is applied in order to avoid nonlinear effects.

The wave magnetic field is received by a small magnetic probe with three orthogonal loops (6 mm diam) which can be moved together in three orthogonal directions. The spatial field distribution is obtained by moving the probe through orthogonal planes and acquiring at each position an average...
over 10 shots which improves the digital resolution and reduces spurious noise. Plasma parameters are measured with Langmuir probes also attached to the movable probe. All signals are acquired with a 4-channel digital oscilloscope with an 8-bit amplitude resolution and a 10 ns time resolution.

In order to produce nonuniform magnetic fields, we apply a current through a circular loop (16 cm diam, 4 turns) whose dipole moment is aligned along \( \vec{B}_0 \). The applied current \( I_{\text{loop}, \text{max}} = 640 \text{ A-turns} \) has a sinusoidal waveform of a single period \( T \approx 175 \mu s \) which is long compared to the rf period \( 0.2 \mu s \). Thus, for each burst, the magnetic field is nearly constant but it varies from burst to burst. Figure 1(c) shows the waveforms of the loop current and the rf current bursts. Since the magnetic field perturbation is applied in the afterglow, it does not modify the plasma production during the discharge, i.e., the density profile remains uniform.

The superposition of a uniform axial magnetic field \( \vec{B}_0 \) and the field from the current loop is displayed in Fig. 2. When the loop field opposes the weaker axial magnetic field, an FRC field is produced [Fig. 2(a)]. It has two 3D null points on axis where the axial loop field cancels the uniform field. A separatrices divides field lines closing around the loop from the “open” field lines produced by an external solenoid. The total field strength is calculated from Biot-Savart’s law and displayed by contours of \( B_{\text{total}} \) for a \( y-z \) plane at \( x = 0 \). The field is symmetric around the axis \( (0, 0, z) \).

When the loop field adds to the axial field, a mirror-type magnetic field is produced [Fig. 2(b)]. The field lines constrict toward the coil with a large mirror ratio, \( B_0(0, 0, 0)/B_0(0, 0, 40 \text{ cm}) \approx 10 \). There also exists a null point located in the plane of the loop at \( r > r_{\text{loop}} \). It occurs all around the loop and hence is a 2D null line. The topology varies during the slowly varying loop current from uniform to cusps mirror and back to uniform magnetic field. The wave propagation depends not only on the field topology but also on the gradient scale length of the field compared to a whistler wavelength. Refraction explains wave propagation in the regime of \( L = B_0/|\nabla B_0| \gg \lambda \), and reflection may arise for short gradient scale lengths. Figures 2(c) and 2(d) show contour plots of the gradient scale length for both cases. The smallest scale lengths arise at the null points and near the loop wires. Waves approaching the null point will encounter cyclotron resonance \( \omega/\omega_e = 1 \), while there is no resonance at the mirror point \( (0, 0, 0) \). It is worth nothing that refraction and ray tracing concepts are no longer valid when \( L < \lambda \).

III. EXPERIMENTAL RESULTS

A. Wave reflections of helicon modes at a magnetic mirror

An \( m = 1 \) helicon mode is excited by a loop antenna located at \( z \approx 44 \text{ cm} \) and launching waves toward a strong mirror point. Figure 3(a) displays contours of \( B_z(0, y, z > 0) \) which show the wave amplitude and phase front (contours of \( B_z = \text{const} \)). The wave propagates against \( \vec{B}_0 \) with \( V \)-shaped phase fronts which are cuts through an \( m = 1 \) helical phase front in 3D (see Fig. 6 in Ref. 46). As the wave propagates toward the increasing strength of the converging field, the parallel wavelength increases and the propagation angles of the oblique wings become highly oblique. The latter is due to the rapid change of the \( \vec{B}_0 \) topology rather than a change of the wave phase front.

A new feature is observed: the formation of waves with phase fronts which are inclined opposite to those of the central wave packet. In time, these waves propagate opposite to the incident wave packet and hence must be reflected from the mirror region. There are no density gradients to produce the reflection. Since there is no reflection in a uniform magnetic field, the reflection cannot be produced by the conducting coil.

Counter-propagating waves are known to produce linear polarization (see Figs. 5 and 6 in Ref. 18). It arises at
FIG. 2. Magnetic field topologies for studying whistler mode propagation in highly nonuniform fields. (a) Field lines and field strength of a cusp configuration. A 3D null point is formed on axis. (b) A strong mirror field is formed when the coil field adds to the axial field \( B_0 = 3 \) G. Magnetic field gradient scale length, \( L = B_0 / |\nabla B_0| \), for (c) the magnetic cusp configuration and (d) the magnetic mirror. When \( L < \lambda \), refraction and ray tracing concepts are no longer valid.

FIG. 3. Observation of wave reflection from a magnetic mirror field. (a) Contours of \( B_x \) showing waves propagating from an loop antenna toward a rapidly increasing magnetic field. It creates a reflected wave with reversed V-shaped phase fronts partly visible in the lower half of the plane. (b) Ellipticity of magnetic hodograms showing periodic locations of nearly linear polarization (\( \epsilon \approx 0 \)) in the interference zone between incident and reflected waves (see black arrows). The incident wave has a nearly circular polarization with \( \epsilon \approx 1 \) at the nodes. (c) Amplitude modulation peaks at the interference nodes thereby also identifying the locations of linear polarization. (d) Examples of hodograms showing circular polarization in the middle of the incident wave (point A) and linear polarization at an interference node (point B). Views from two orthogonal directions show a circular disk in (d) and a pencil shaped hodogram in (e).
interference nodes where two field components vanish while the third remains finite. With a single component left, the field has a linear polarization. The latter can be shown from magnetic field hodograms. The hodogram ellipticity, $\epsilon = \frac{B_{\min}}{B_{\max}}$, assumes a minimum for linear polarization, while circularly polarized waves have an ellipticity of $\epsilon = 1$.

Figure 3(b) shows a contour plot of the ellipticity $\epsilon$. The incident wave has a high ellipticity, i.e., the wave polarization is nearly circular. There is a string of ellipticity minima in the interference region between incident and reflected waves implying linearly polarized whistler modes. Since the ellipticity results from a time average, the minima do not propagate but are standing wave phenomena.

An alternate feature of the wave interference is the amplitude dependence on time. For circularly propagating waves with two orthogonal components ($b \cos \omega t, b \sin \omega t$), the total amplitude is constant $|B(t)| = (b^2 \cos^2 \omega t + b^2 \sin^2 \omega t)^{1/2} = b = \text{const}$. For linear polarization, the amplitude varies in time as $B(t) = (b^2 \cos^2 \omega t)^{1/2} = b(1 + \cos 2\omega t)/2)^{1/2}$. We define an amplitude modulation by a suitably normalized difference between the root-mean-square (rms) value and the time averaged amplitude, $\Delta B_{\text{rms}} = (\langle B^2 \rangle - \langle B \rangle^2)^{1/2}$, where $\langle B^2 \rangle = \frac{1}{T} \int |B(t)|^2 \, dt$ is the time averaged value of the absolute value $|B(t)|$. For example, for linear polarization, $\Delta B_{\text{rms}} = (b^2 \cos^2 \omega t)^{1/2} = b^2/2$ and $\langle B \rangle = b\langle \cos \omega t \rangle = b/2$, resulting in $\Delta B_{\text{rms}} = b^2/2$. For circular polarization, $\Delta B_{\text{rms}} = \langle B \rangle = b$ and $\Delta B_{\text{rms}} = 0$.

The contour plot $\Delta B_{\text{rms}}$ in Fig. 3(c) shows a string of localized $\Delta B_{\text{rms}}$ peaks where $\epsilon$ had minima, both describing locations of linearly polarized whistler modes. Without interference, the wave amplitude of the circularly polarized incident wave is nearly constant in time ($\Delta B_{\text{rms}} \approx 0$).

The lower half of Fig. 3 displays selected hodograms confirming the polarization properties of interfering whistler modes. In the center of the incident wave packet [location A, Fig. 3(d)], the $B$-field rotates with a constant radius in a right-hand circle (left plot, view along its normal). The side view (right plot, view at right angle to its normal) shows that the vector rotates in a plane with normal predominantly in the $z$-direction. At this location, the hodogram describes a slightly oblique whistler mode. At a location of wave interference [location B, Fig. 3(e)], the hodogram is nearly linear with $\epsilon = B_{\min}/B_{\max} \approx 0$. The views are the same as in (d) and confirm that the hodogram is a narrow cylinder. The major axis describes the direction of linear polarization. The field components normal to the major axis vanish due to destructive interference of incident and reflected oblique whistler modes.

The direction of wave propagation along $B_0$ can be identified from the sign of the helicity density $J \cdot B$. Figures 4(a) and 4(b) display instantaneous contours of $B_x$ and $J_y$, respectively, for cw conditions. Both contours are highly similar although the sign of $B_x$ and $J_y$ differs for the incident wave (see black dots) but is equal for the reflected wave (see white dots). Recall that in ideal electron magnetohydrodynamics (EMHD), $J = \nu_0 \sigma_0 B$, where $\nu_0$ is the phase velocity parallel to $B_0$ and $\sigma_0 = ne/B_0$ is the Hall conductivity. Thus, one has $J \cdot B > 0$ for $\nu_{\text{phase}} \cdot B_0 > 0$ and vice versa. The incident wave propagates against $B_0$, and hence, $J \cdot B < 0$, while the reflected wave has $J \cdot B > 0$, as shown in Fig. 4(c). Since $J \propto B$, the helicity is proportional to the magnetic energy density and the periodicity is half a wave length.

The reflected wave is best seen at the end of the wave burst. After the incident wave has propagated beyond the mirror point, the reflected waves still propagate to the right away from the mirror point. No interference occurs. Figures 4(d)–4(f) show contours of the three field components $B_x$, $B_y$, and $B_z$, respectively. All show $V$-shaped phase contours of alternating polarity, spaced $\lambda$ apart. The wave propagation is oblique and points radially outward, similar to the incident waves.

B. Space-time evolution and reflection coefficient

Rf bursts allow us to observe incident and reflected waves separately. Figure 5 displays a time series of contours of a component of the helicity density, $J_x B_x$, at (a) the beginning of the rf burst, (b) the middle of the burst, and (c) the end of the rf burst. For each case, a sequence of six consecutive snapshots is presented so as to observe the direction of wave propagation. Superimposed are white lines showing the direction of the background magnetic field $B_0$.

At the beginning of the rf burst, only an incident wave is observed [Fig. 5(a)]. No reflected wave is generated until the incident wave arrives as the mirror field $(\Delta t' \approx 2T = 2f)$. From the axial displacement of the phase fronts, one obtains a parallel phase velocity of the incident wave, $\Delta z/\Delta t' \approx 66 \text{ cm} / \mu \text{s}$. Closer to the mirror point, where the field increases by an order of magnitude [see Fig. 2(b)], the axial wavelength increases and the $k$ vector becomes highly oblique. When the wave reaches the mirror point $(\Delta t \approx 2T = 2f)$, the reflection starts. Near the axis, the large incident wave covers up the smaller reflected waves, but radially outward, the reflected wave begins to dominate.

In the middle of the rf burst, both the reflected and incident waves are visible [Fig. 5(b)]. The waves appear to be stationary during the steady state of the rf burst, but the phase fronts propagate by half a wavelength each half rf period. With the increasing distance, the reflected wave contours vanish due to wave decay and discrete contour levels. The reflection coefficient depends on the axial position due to wave damping and reaches unity near the reflection point, as is shown below.

At the end of the rf burst, the incident wave moves out of the measurement plane so that the phase fronts of the reflected wave become visible [Fig. 5(c)]. The phase fronts are also $V$-shaped but wider than those of the incident wave, as there is no wave energy along the axis of the reflected wave. Connecting the centers of the wave amplitude peaks yields the direction of the group velocity which is oblique and radially outward. The normal to the phase fronts points in the direction of the phase velocity, which is even more oblique than the group velocity. Extrapolating the group velocity backwards might give the impression that the origin
for the reflection is near the center of the loop, but the lack of wave energy on axis suggests otherwise. The large waves appear to originate near the wire of the loop where the largest gradient scale length and the steepest curvature of the field lines are located [see Fig. 2(c)]. In this region, $\mathbf{k} \cdot \mathbf{B}_0$ reverses sign causing the wave to reflect.

The separation of incident and reflected waves at the end of the rf burst allows one to estimate the reflection coefficient, here defined as the energy ratio $R = \frac{U_{\text{refl}}}{U_{\text{inc}}}$, where $U' = \frac{du}{dz} = \int B^2/(2\mu_0)2\pi dr$ is the wave energy per axial length. Figure 6(a) shows traces of $U'(t,z)$ at the end of the rf burst for the mirror field. The early large value corresponds to the incident wave energy, and the second peak is the reflected wave. The family of curves shows the dependence on the axial position. Each wave decays in its direction of propagation, i.e., in the $-z$ direction for the incident wave and $+z$ direction for the reflected wave. The propagation delay of the reflected wave confirms that the wave propagates in the $+z$-direction. In the absence of damping, the reflection coefficient would be independent of position, but to avoid damping effects, the reflection ratio should be measured at the point of reflection which is near

**FIG. 4.** Helicity properties and components of the reflected wave magnetic field. (a) Instantaneous contours of $B_x$ during quasi cw conditions (middle of long rf burst). (b) Contours of the current density $J_x$ which are nearly identical to those of $B_x$ but with reversed sign for the incident wave (see black dots) and equal sign for the reflected mode (see white stars). (c) Contours of $J_x B_x$, one component of the current helicity density. Its sign depends on the direction of wave propagation along $B_0$. Its magnitude is proportional to the energy density. The contours indicate the phase fronts whose spatial periodicity is $2\pi$. As the wave propagates into the constricting mirror field, the wavelength increases and the wave becomes highly oblique. The energy decreases since the propagation velocity increases. The energy density of the reflected wave is much smaller than that of the incident wave. (d), (e), and (f) Contours of $B_x$, $B_y$, and $B_z$, respectively, for the reflected wave which is visible after the incident wave has left the measurement plane at the end of the rf burst. The reflected wave has also $V$-shaped phase fronts which trace back to near the mirror coil where the reflection originates.
As the incident wave travels from 40 cm to 20 cm, its energy decays from 1 to $10^{-1}$. Extrapolating to $z = 0$, its energy would decay to $10^{-1}$. When the reflected wave travels from 40 cm to 20 cm, its energy increases from $10^{-3}$ to $10^{-2}$. Thus, the extrapolated reflection coefficient is $R' = 1$ at the loop which is probably underestimated since the wave penetrates to the other side of the loop [see Fig. 7(a)]. Nevertheless, the wave reflection from a strong magnetic field is very significant.

The same evaluation has been done for the cusp field topology. Figure 6(b) shows no reflected wave. Likewise, no reflection is expected or seen in a uniform field (not shown).

### C. Wave reflection mechanism in highly nonuniform magnetic fields

Reflection and refraction of plane waves are usually explained by the properties of the refractive index $n = c/v_{\text{phase}}$. Reflection occurs when the refractive index changes abruptly, while refraction applies to a gradually changing index where reflection is negligible. The refractive index of low frequency whistler modes depends on the density and magnetic field, $n \approx \omega_p / (\omega_0 \gamma) \frac{1}{2} \propto (n_e / B_0)^{1/2}$. It is well known that density discontinuities reflect waves, but much less is known about reflections from magnetic field
The phase velocity increases predominantly on axis where wave propagation as it travels to and through the large loop. Diagnostic antenna and the large loop. Figure 7(a) shows the aligned. The model is based on observations of the space-highly oblique whistlers, the group velocity is closely field a significant reflection of the wave energy since for near the loop where the reflection occurs. They are approximately equal, implying $R \leq 1$ near $z \geq 0$.

Gradients. This effect has been shown and explained in a recent letter. In the present work, we observe in detail the motion of the incident phase front in an ambient magnetic field whose curvature changes on a scale smaller than a wavelength. The off-axial field lines bend faster than the phase front such that the incident wave changes from oblique to perpendicular and beyond. When $\mathbf{k} \cdot \mathbf{B}_0 < 0$, the wave propagates opposite to $\mathbf{B}_0$, i.e., it is reflected. This forms a turning point for the phase and group velocities which can lead to wave reflection from the sides of the mirror field. Even a small negative $k_y$ produces a significant reflection of the wave energy since for highly oblique whistlers, the group velocity is closely field aligned. The model is based on observations of the space-time evolution of the wave burst shown in Figs. 5 and 7(a).

1. Wave propagation through mirror and cusp fields

In order to better observe the wave propagation at the mirror point, the measurement plane has been extended to both sides of the large loop. The source antenna is now positioned closer to the large loop. There is an unavoidable data gap of $\Delta z = \pm 2$ cm width to prevent collisions between the diagnostic antenna and the large loop. Figure 7(a) shows the wave propagation as it travels to and through the large loop. As the wave travels toward the mirror point, the parallel phase velocity increases predominantly on axis where $B_0(r)$.

 Peaks. This causes the $V$-shaped phase fronts to become elongated such that the $\mathbf{k}$ vector becomes highly oblique to the $\mathbf{B}_0$ field. Furthermore, the advancing wave encounters a rapidly changing magnetic field as it travels along the $\mathbf{B}_0$ lines. When the field lines become steeper to the $V$-shaped phase fronts, the angle between $\mathbf{k}$ and $\mathbf{B}_0$ exceeds $90^\circ$. This results in wave reflection.

These conditions occur only on the side of the incident wave. On the left side of the mirror loop, the transmitted wave fronts gradually return to $V$-shapes and the helicon mode continues to propagate in the $-z$ direction. It shows that the reflection must be smaller than unity.

The reflection mechanism has some similarity to that proposed for whistler reflections in the magnetosphere. When a whistler wave propagates toward the poles, the wave encounters the lower hybrid resonance where $\mathbf{k} \perp \mathbf{B}_0$. This condition creates a turning point for electron whistler waves. The main difference is that in the present experiment, the propagation direction changes within less than a wavelength, while in space, the reflection is a classical ray bending model.

We now come to the wave propagation against a cusp magnetic field obtained by reversing the loop current. The cusp field has comparable gradient scale lengths to that of a mirror field [see Figs. 2(a) and 2(c)], such that one might also expect a wave reflection. However, Fig. 7(d) shows that no reflection is observed. The incident wave encounters cyclotron resonance and is strongly absorbed in regions around the null point where $\omega/\omega_0 > 1$. A reflected wave cannot propagate out of the null point region. This has been tested by placing the antenna on the null point and observing no wave excitation (not shown). Since the phase front of the wave is wider than the null region, the $V$-shaped wings of the wave packet continue to propagate along the separatrix albeit with small amplitudes. The phase normal on the separatrix indicates a nearly parallel whistler mode.

Highly nonuniform fields imply small spatial scales, as is shown in Figs. 7(b) and 7(e), where the gradient scale length, $L = B_0/|\nabla B_0|$, is shown for the mirror and cusp cases, respectively. They interact with part of a larger wave packet such that only a fraction of the wave energy is reflected or absorbed. The interaction may be better described as a scattering process rather than a refraction process of a plane wave because there, as shown in Figs. 7(c) and 7(f), the measured wavelength is smaller than the gradient scale length.

Figure 8 shows the exciter antenna located at $x = 0$, $y = 5$ cm, and $z = 18$ cm which is in the right hemisphere of the 16 cm diam loop. At this location, the loop can radiate from near the cusp null point. Furthermore, wave propagation toward and away from the nonuniform $B_0$ field can be compared. The $B_0$ field lines are shown by white lines. The receiving probe records the three wave components, one of which $B_z$ is displayed by contours in the central $y$–$z$ plane ($x = 0$), except in gaps near the rf antenna and the large loop.

Without loop current, the field is uniform and small, $B_0 = 3$ G. The $m = +1$ helicon mode propagates symmetrically to both sides of the rf antenna with equal amplitudes but odd signs in the current density, magnetic helicity, and phase.
helicity. Note that this is a relatively high frequency helicon mode \((\omega/\omega_c = 0.6)\). It has the same \(V\)-shaped phase contours as other low frequency mode studies (e.g., \(\omega/\omega_c = 0.35\) for \(B_0 = 5\) G, Fig. 4(a) in Ref. 18).

Waves can transmit through the field constriction in a mirror field [Fig. 8(b)]. Surprisingly, the constriction does not enhance the wave amplitude. The wave speed increases with increasing \(B_0\), whereby the conservation of the energy flux \(S = v_{\text{group}} B_0^2/2\mu_0\) requires a decrease in field strength. Conversely, the waves propagating along expanding field lines have a larger amplitude on the right side of the antenna.

In a weak FRC [Fig. 8(c)], the cyclotron resonance region is small compared to the lateral extent of the helicon wave packet. The center of the wave packet is damped while the outer wings do not experience cyclotron damping and continue to propagate around the FRC. This should also apply to plane waves whose transverse dimensions are always larger than an FRC. To the right side of the rf loop, the waves propagate similar to that in a nearly uniform field.

In a strong FRC, the null point shifts close to the rf antenna \((\Delta z = 2\) cm) so as to excite waves inside an FRC or waves traveling away from a mirror point. Figure 9 indicates the antenna position and the field topology for the three cases: uniform, mirror, and cusp \(B_0\). The contours of \(J_x B_x\) yield the direction of wave propagation from the sign of the helicity and give a measure for the wave energy density from the contour levels.

Finally, we move the rf antenna close to the large coil \((\Delta z = 2\) cm) so as to excite waves inside an FRC or waves traveling away from a mirror point. Figure 9 indicates the antenna position and the field topology for the three cases: uniform, mirror, and cusp \(B_0\). The contours of \(J_x B_x\) yield the direction of wave propagation from the sign of the helicity and give a measure for the wave energy density from the contour levels.

In a uniform field [Fig. 9(a)], the wave propagates along \(B_0\) with the largest signals along the \(z\)-axis of the antenna \((x = y = 0)\).

In a mirror field [Fig. 9(b)], the waves propagate again along \(B_0\) and spread radially outward since the field lines diverge away from the loop. No wave reflections are seen when the wave leaves the peak mirror field.

In a strong FRC field [Fig. 9(c)], the waves propagate to the right side against \(B_0\), and hence, \(J_x B_x < 0\). To the left side of the rf antenna (not shown), the wave would propagate along \(B_0\) and have a positive helicity density \((J_x B_x > 0)\). Waves propagating along the spine are absorbed near the 3D null point and hence cannot escape axially. Waves have curved phase fronts, yet propagate nearly parallel along the diverging \(B_0\) lines.

Figures 9(d) and 9(e) show two hodograms on the antenna axis \((y = 2 \) cm and \(z \approx 10\) cm), one in a mirror and another in a cusp field. In the mirror field (b), \(k \cdot B_0 > 0\) such that the polarization is right-handed with respect to \(B_0\).
The surface normal $\mathbf{n}$ points in the direction of the wave vector $\mathbf{k}$ which is nearly parallel to $\mathbf{B}_0$. It confirms that the wave propagates closely in the $+z$ direction with nearly circular polarization.

When the field is reversed [Fig. 9(e)], the hodogram also rotates right-handed in time around the normal which now points in the direction opposite to the $\mathbf{k}$ vector. The hodogram is nearly circular when viewed along $\mathbf{n}$. In the FRC topology, $\mathbf{k}$ remains unchanged while $\mathbf{B}_0$ is reversed. The right-handed polarization around $\mathbf{B}_0$ is preserved, but the spatial phase helicity becomes right-handed in the $z$-direction. Thus, the hodogram normal, $\mathbf{n}$, and the $\mathbf{k}$ vector point in opposite directions when $\mathbf{k}/C_1 < 0$, which is a well-known sign ambiguity for hodograms. Since a simple loop excites helicon modes with phase rotation in the same direction as the polarization, the phase rotation is in the $-\phi$ direction which shows it is an $m = +1$ mode in a uniform field or mirror field.

The three different field topologies occur during each discharge pulse. It is worth showing the time dependence of the received probe signal which depends on many parameters such as $I_{\text{loop}}$, $z_{\text{rf}}$, $r_{\text{probe}}$, $B_0$, $\omega$, and $n_e$. In Figs. 9(f)–9(h), we show just one parameter variation, the position of the receiving probe along the $z$-axis, while all other parameters remain constant. Closest to the antenna [$z \approx 2$ cm in Fig. 9(f)], the received signal $B_x(t)$ is large and can be seen for uniform $B_0$ ($I_{\text{loop}} = 0$), mirror $B_0$ ($I_{\text{loop}} > 0$), and cusp $B_0$ ($I_{\text{loop}} < 0$). Only for small $I_{\text{loop}}$, when the null point coincides with the rf loop position, the signal vanishes. When the receiving probe is located in the middle of the measurement plane [$z = 10$ cm, Fig. 9(g)], the signal is received for $I_{\text{loop}} \geq 0$, albeit with a smaller amplitude due to damping and wave spread. However, for $I_{\text{loop}} \leq 0$, the signal reaches the probe only for large FRC fields where both the exciting and receiving antennas are inside the separatrix.

Finally, when the receiving probe is located at the far right-hand side of the plane [$z \approx 18$ cm, Fig. 9(g)], no signal is observed because the position of the null point coincides with that of the receiving probe. Since the transmitting antenna is inside the separatrix ($z \approx -2$ cm), there are waves inside the separatrix but they are absorbed before reaching the receiver probe. Thus, one antenna on a null point is sufficient for total blackout between the transmitter and the receiver.

2. Polarization and phase helicity

Polarization and helicity of whistler modes are different quantities. Polarization refers to the rotation of a field vector which is usually described by a hodogram. The polarization of electron whistlers ranges from linear to circular, generally right-hand elliptical. The phase surface of helicons is helical. The theoretical model for helicon modes assumes paraxial wave propagation, $\mathbf{B} \propto \exp \left( i(m\phi + k_zz - \omega t) \right)$, which describes axial and azimuthal phase propagation with radial
standing waves. Paraxial propagation leads to helical phase surfaces with a constant radius. This theoretical model does not explain observations where \( k_r \) is determined by density profiles or antenna properties but not by reflections from radial boundaries. Even in unbounded uniform plasmas, spiraling whistler modes have been observed.\(^5\)

There is radial phase propagation in an unbounded plasma but with little radial energy flow, resulting in weakly
conical phase surfaces. At a fixed instant of time, the phase front of a parallel propagating helicon ($k_B^0$ in the $z$-direction) is described by $m/\omega + k_z z = \text{const.}$ The phase surface forms a left-handed helix, $d\phi/dz = m/\omega < 0$. If either $m$ or $k_z$, but not both, is reversed, the helix is right-handed or the phase helicity is positive.

When time increases, the helical phase surface translates along $z$ but does not rotate, $m \phi + k_z \Delta z - \omega \Delta t = 0$. When the helix passes through a plane $z = \text{const}$, the phase contours in the plane rotate around the $z$-axis or $B_0$. The sense of rotation $d\phi/d\Delta t = \omega/m$ is positive or ccw for $m > 0$, irrespective of $k_z$. Thus, when a loop antenna with the axis across $B_0$ excites waves to both sides, they have the same sense of rotation but opposite spatial phase helicities. This is not always agreed upon (see Fig. 7 in Ref. 51).

Since the wave field lines lie approximately on the phase surfaces (see Fig. 7(g) in Ref. 46), a twisted phase front implies twisted field lines, i.e., magnetic helicity $A^1(B_0)^2$, where $A$ is the vector potential. Magnetic helicity is a conserved quantity in ideal electron magnetohydrodynamics (EMHD). 52 If the antenna injects helicity, the wave will acquire the same helicity and travel only in one direction.53 The radiated wave is much stronger when the antenna twist matches that of a helicon mode which can only occur on one side of the antenna. The directionality switches sides when the $B_0$ field is reversed. This has sometimes been associated with different properties of $m = -1$ and $m = +1$ modes, but the effect is due to the antenna directionality. A single loop does not inject helicity, and hence, the oppositely propagating waves must have opposite helicities and equal amplitudes to conserve zero total helicity.
In the present experiment, we can readily change the direction of \( \mathbf{B}_0 \) with the FRC configuration. The \( m = 1 \) loop is placed inside the separatrix and its wave propagation is measured on axis. For comparison, the wave properties are also measured in mirror field topologies and uniform fields. Of particular interest is the helicon field rotation which is seen by a sequence of successive snapshots separated by \( \Delta t = T/4 \).

Figures 10(a) and 10(b) show the field rotation of \( B_z(x, y, z = 8 \text{ cm}) \) when \( k \) is in the same direction as \( \mathbf{B}_0 \) which is either a uniform field or in a mirror configuration. In both cases, the phase rotates in the \( +\phi \) direction. By the above definition, a temporal rotation in the \( +\phi \) direction (\( \Delta \phi/dt = \omega/m \geq 0 \)) identifies it as an \( m = +1 \) helicon mode.

When \( \mathbf{B}_0 \) is reversed, while the wave propagation direction remains along \( z \), the sense of phase rotation of \( B_z \) in the \( x-y \) plane is found to reverse in the \( -\phi \) direction defining it as an \( m = -1 \) mode [Fig. 10(c)]. The amplitude also remains the same since the antenna is not phased. Thus, the \( m = -1 \) mode propagates just as well as the \( m = +1 \) mode. The reversed rotation and the same \( k \), imply that the helical phase surface has a right-handed twist. The phase rotates in the same direction as the polarization, i.e., right-handed with respect to the reversed \( \mathbf{B}_0 \).

The azimuthal phase rotation for the three \( \mathbf{B}_0 \) topologies is shown with higher temporal resolution in Fig. 10(d). For a uniform field, the rotation is nearly constant (\( \Delta \phi/dt = \omega/m \) const), while in nonuniform fields, there are jumps in phase which indicate partially linear polarizations such as the antenna vacuum field or that the plane is not everywhere orthogonal to the curved \( \mathbf{B}_0 \) lines.

Since the magnetic field lines are approximately parallel to the phase fronts (see Fig. 3 in Ref. 19), the twist of the phase surface is related to the helicity of the wave magnetic field or the current density since \( \text{EMHD} \) physics rotates the phase fronts (see Fig. 3 in Ref. 19), the twist of the helicon phase surface in a uniform magnetic field differs on either side of the antenna. However, the temporal phase rotation is the same when the different spirals propagate in different directions through \( z = \text{const} \) planes. Thus, the \( m \)-number is the same for helicons emitted to both sides. Reversal of \( k \) with respect to \( \mathbf{B}_0 \) is not reciprocal.

For a single loop, the field rotation is the same as that of the polarization because \( \text{EMHD} \) physics rotates the wave field in the same direction as the polarization (see Fig. 7 in Ref. 46 or Fig. 8 in Ref. 54). With a circular antenna array, phased in the \( \phi \) direction, one can impose the \( m \)-number of helicon modes such that field rotation and polarization differ (see Fig. 7 of Ref. 55). Oppositely propagating waves (\( \pm k \) or \( \pm m \)) can produce axial or azimuthal standing waves where helicons exhibit linear polarization (see Fig. 4 in Ref. 48).

The early antennas for helicon plasma sources were "double saddle" antennas (also known as "Boswell" or "Nagoya Type III" antennas, see Fig. 8 in Ref. 56), which are essentially Helmholtz coils draped over a cylindrical glass tube to produce an rf magnetic field across \( \mathbf{B}_0 \). These antennas excite equal \( m = +1 \) modes to both sides just like a single internal loop (see Fig. 6 in Ref. 3). When such antennas are elongated (length \( \approx \lambda/2 \)) and twisted by \( 180^\circ \) around \( \mathbf{B}_0 \) (see Fig. 9 in Ref. 56 or Figs. 11 and 12 in Ref. 54), the radiation pattern becomes directional. The preferred direction is that where the antenna helicity matches the wave helicity. Directionality arises from the fact that the phase helicity switches sign on either side of the antenna. Directionality can also be obtained from axially phased arrays which match the parallel wave propagation (see Fig. 3 of Ref. 57). The antenna directionality explains the confusion about the existence of \( m = -1 \) helicon modes in plasmas.\(^{56} \) There is no question that negative \( m \)-mode helicons can propagate in plasmas (see Fig. 7 of Ref. 55).

Rotating magnetic field (RMF) antennas have also been used for more efficient coupling to whistler waves.\(^{58,59} \) The \( r-f \) field is perpendicular to \( \mathbf{B}_0 \) and rotates in the \( +\phi \) or \( -\phi \) direction by a \( \pm 90^\circ \) phase shift in the currents of the two orthogonal loops. When the antenna field rotates in the same direction as the field polarization (\( m = +1 \)), the wave excitation is stronger than for the opposite direction of rotation. But the enhancement over a single loop is modest and can be explained by the fact that the rotating field is \( \sqrt{2} \) larger than for the non-rotating field for the same antenna current. However, field rotation opposite to the electron rotation decreases the wave amplitude significantly (see Fig. 13 in Ref. 54). Unlike twisted antennas, the rotating field antennas impose no \( k_x \) and hence exhibit no directionality.

**IV. CONCLUSION**

Experiments on wave propagation in highly nonuniform mirror and cusp magnetic fields have been described. Wave reflection from a strong mirror field is observed and explained. Reflection can arise from strong gradients in the refractive index which depends on the density and magnetic field. Although refraction by density gradients is well known, the complementary effect of reflection by magnetic field gradients has not been demonstrated elsewhere. Strong magnetic field gradients can exist in space plasmas at magnetic shocks or lunar crustal magnetic fields, but the diagnostics for wave reflection would be very difficult. It may also be difficult to observe reflections in helicon devices because of axial density gradients and boundaries which can also produce wave reflections.

The interference between incident and reflected waves produces nodes in which two field components vanish. At these locations, linear polarization arises which is confirmed by hodograms and by amplitude oscillations.

Wave propagation against a magnetic null point of an FRC leads to wave absorption by cyclotron resonance. When the wave is launched inside the separatrix, the wave is trapped. The polarization and phase rotation inside the FRC are reversed compared to those outside the FRC. The helicon phase rotation is that of an \( m = -1 \) mode which propagates as well as an \( m = +1 \) mode.

Propagation against cyclotron resonance can arise in plasma thrusters with expanding magnetic fields.\(^{60} \) Theories for cyclotron damping assume plane waves and 1D field lines which cannot be realized in space or laboratory
plasmas. Simulations are required to model wave propagation in nonuniform magnetic fields.1

This series of papers on whistler modes in nonuniform plasmas demonstrates the discovery of several other new effects. Whistler modes propagating radially across circular magnetic fields are observed. Along circular field lines, counter propagating waves can form standing waves along $B_0$ while also propagating across $B_0$.

Whistler wave packets are inherently 3D structures which require 3D field measurements. The current density can only then be calculated, the field lines and phase surfaces which require 3D field measurements. The relationship between angular momentum and helicity has been pointed out. Novel diagnostics with vector fields of hodograms nor-tubes rather than straight field lines. The relationship between angular momentum and helicity has been pointed out. Novel diagnostics with vector fields of hodograms normals and of Poynting vectors are used to show the flow of phase and energy in highly nonuniform fields.

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