Electron magnetohydrodynamic turbulence in a high-beta plasma. I. Plasma parameters and instability conditions

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The interaction of a dense discharge plasma with a weak external magnetic field has been studied experimentally. The electron pressure exceeds the field pressure and forms a magnetic hole in the plasma interior. The ions are unmagnetized, while the electrons are in a transition regime from none to full magnetization. The electron confinement changes from Boltzmann equilibrium to magnetic confinement. The pressure balance equation does not describe the diamagnetism because ambipolar \( \mathbf{E} \times \mathbf{B} \) drifts oppose the diamagnetic drift. The net drift exceeds the sound speed by an order of magnitude and produces a strong two-stream cross-field instability. Although its spectrum is close to the lower hybrid instability, there are significant differences from the classical lower hybrid instability, e.g., the presence of strong magnetic fluctuations. These fall into the regime of electron magnetohydrodynamics (EMHD) with unmagnetized but mobile ions. While the EMHD turbulence is the main focus of the two following companion papers, this first paper describes the plasma diamagnetism and basic parameters that lead to the instability. © 2000 American Institute of Physics [S1070-664X(00)03111-6]

I. INTRODUCTION

The interaction of plasmas and magnetic fields is of fundamental interest in space and laboratory plasma physics. Dense hot plasmas exhibit a significant diamagnetism, which changes the magnetic field inside the plasma. A quantitative measure for this effect is the plasma “beta,” which is the ratio of the particle pressure to the external magnetic field pressure, \( \beta = n k T / (B_0^2 / 2 \mu_0) \). However, the notion of a “high”-\( \beta \) plasma may depend on the phenomenon of interest: Wave phenomena can change for \( \beta = m_e / m_i \ll 1 \), the economics of magnetic fusion devices requires \( \beta \sim 10\%\), magnetic holes can be formed in space plasmas with \( \beta \) of order unity, and extremely high-\( \beta \) plasmas such as the solar interior (\( \beta > 10^5 \)) can completely deform or confine magnetic fields. In the present work, we deal with laboratory plasmas where \( \beta > 1 \) and describe diamagnetism and associated instabilities in the presence of pressure and field gradients. Major findings are that electron magnetohydrodynamics (EMHD) can govern the plasma physics near a magnetic bubble, i.e., the diamagnetism essentially demagnetizes the ions, while leaving the electrons still magnetized. In this parameter regime, a strong instability develops due to the electron diamagnetic drift through the unmagnetized slow ions. Such current-driven cross-field instabilities, observed earlier in plasmas with \( \beta < 1 \), produce large-amplitude density fluctuations near the lower hybrid frequency that propagate at approximately the sound speed in the direction of the electron diamagnetic drift. However, due to the high-\( \beta \) which couples density to magnetic field fluctuations, and the dependence on the temperature ratio \( T_e / T_i \gg 1 \), the instability is not described by a classical lower hybrid drift wave, or an \( \mathbf{E} \times \mathbf{B} \) drift wave instability with unmagnetized ions or magnetized ions. However, it does have the basic characteristics of a drift dissipative instability.

Another important result is that the pressure balance equation used in MHD, \( B_{\text{plasma}}^2 / 2 \mu_0 + n k T_e = B_0^2 / 2 \mu_0 \) = const, does not hold when \( B_{\text{plasma}} \rightarrow 0 \) and the electron confinement changes from magnetic force balance to Boltzmann equilibrium. In this transition regime, the diamagnetic drift is opposed by an \( \mathbf{E} \times \mathbf{B} \) drift which reduces the diamagnetic field change. If the electric field equals the ambipolar field, \( \mathbf{E} = - \nabla (n k T_e / m_e) \), the net drift, hence the diamagnetism, would vanish. Observations show that the average magnetic field inside the plasma column is fully expelled for \( \beta \sim 5 \) rather than for \( \beta = 1 \). Likewise, the ratio of normalized magnetic and density fluctuations is smaller than expected from the pressure balance equation, \( \delta B / B < \beta / 2 \delta n / n \). These basic features of a bounded high-\( \beta \) plasma are described in this work, Part I of three companion papers. Basic instability properties such as frequency spectra, correlations, and amplitude distributions are also presented. Part II describes in detail the features of the instability as obtained from single-point measurements. These include cross-correlations between density and magnetic field fluctuations, amplitude distributions showing wave steepening and current sheet formation, and magnetic hodograms showing that the magnetic fluctuations have the properties of low frequency oblique whistlers. Finally, in Part III the three-dimensional (3D) space–time properties of the instability are described from conditionally averaged multipoint measurements. The average direction and speed of propagating density and magnetic field fluctuations are obtained. It is shown that the density fluctuations form flutelike perturbations propagating along the mean isobars at sonic speeds, that the magnetic...
field and current density form flux ropes of negative self helicity, and that it is possible to reconstruct the “instantaneous” magnetic field lines, current density lines, and single particle orbits in a turbulent high-$\beta$ plasma.

The present Part I is organized as follows: After describing in Sec. II the experimental setup and measurement techniques, the experimental results are presented in Sec. III, divided into various subsections. The conclusion, Sec. IV, points out the relevance of the present findings to related observations and applications.

II. EXPERIMENTAL ARRANGEMENT

The experiments are performed in a large linear plasma device (1 m diam $\times$ 2.5 m length) schematically shown in Fig. 1. A uniform axial dc magnetic field $B_0=5$ G is applied. A pulsed discharge ($V_{\text{dis}}=60$ V, $I_{\text{dis}}=750$ A, $t_{\text{puls}}=4$ ms, $t_{\text{rep}}=1$ s) of density $n \leq 8 \times 10^{11}$ cm$^{-3}$, electron temperature $kT_e \leq 5$ eV is produced in Argon at a pressure $p_n \approx 0.3$ mTorr using a 1 m diam oxide-coated cathode. The cathode emission is made nonuniform so as to produce a strong radial density and temperature gradient.

The plasma parameters ($n,kT_e,\phi_{pl}$) are measured with a small Langmuir probe (1 mm $\times$ 1.5 mm) movable in 3D space. Other Langmuir probes movable in one dimension (1D) are also used to record electron/ion saturation currents and their fluctuations. The validity of Langmuir probe measurements in weakly magnetized plasmas has been checked earlier via independent diagnostics with electron plasma waves, ion acoustic waves, and whistler waves. Time-varying magnetic fields are measured with a magnetic probe consisting of three orthogonal, electrostatically shielded loops (1 cm diam, 30 turns), which provide $\langle B_z, B_x, B_y \rangle$ simultaneously versus time ($\sim 10$ ns resolution) at a given position $r$. The probe voltages ($V_{\text{probe}} \propto -dB/dt$) are integrated either by analog techniques (operational amplifier) or digitally (digital oscilloscope/computer). The probe is calibrated in a known magnetic field configuration. The spatial distribution of $B(r,t)$ is obtained by repeating the pulsed discharges and slowly varying the probe position in a 3D volume ($\sim 1$ cm resolution). Fluctuations are analyzed by forming statistical averages over an ensemble of highly reproducible discharge pulses. Using conditional averaging, the typical space–time behavior of the turbulence...
TABLE I. Basic plasma parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron density</td>
<td>( n_e = 5 \times 10^{11} \text{ cm}^{-3} )</td>
</tr>
<tr>
<td>Electron temperature</td>
<td>( kT_e = 4 \text{ eV} )</td>
</tr>
<tr>
<td>Electron plasma frequency</td>
<td>( \omega_{pe} / 2 \pi = 6.4 \text{GHz} )</td>
</tr>
<tr>
<td>Electron cyclotron frequency</td>
<td>( \omega_{ce} / 2 \pi = 14 \text{MHz} )</td>
</tr>
<tr>
<td>Lower hybrid frequency</td>
<td>( \omega_{hyd} = (\omega_{pe}, \omega_{ce})^{1/2} / 2 \pi = 52 \text{kHz} )</td>
</tr>
<tr>
<td>Electron cyclotron radius</td>
<td>( r_{ce} = 1.4 \text{ cm} )</td>
</tr>
<tr>
<td>Electron-ion collision frequency</td>
<td>( \nu_{ei} = 1.8 \times 10^6 \text{ s}^{-1} )</td>
</tr>
<tr>
<td>Electron mean free path</td>
<td>( \lambda_{ei} = 66 \text{ cm} &lt; \lambda_{ri} )</td>
</tr>
<tr>
<td>Electron cyclotron radius</td>
<td>( r_{ce} = 1.4 \text{ cm} )</td>
</tr>
<tr>
<td>Normalized electron collision frequency</td>
<td>( \nu_{ei} / \nu_{ri} = 0.02 )</td>
</tr>
<tr>
<td>Ion cyclotron frequency</td>
<td>( \omega_{ci} / 2 \pi = 190 \text{ Hz} )</td>
</tr>
<tr>
<td>Ion cyclotron radius</td>
<td>( r_{ci} = 120 \text{ cm} )</td>
</tr>
<tr>
<td>Ion-neutral charge-exchange collision frequency</td>
<td>( \nu_{ne} = 5 \times 10^3 \text{ s}^{-1} ) (( kT_e = 0.4 \text{ eV} ))</td>
</tr>
<tr>
<td>Ion mean free path</td>
<td>( \lambda_{ci} = 28 \text{ cm} )</td>
</tr>
<tr>
<td>Normalized ion collision frequency</td>
<td>( \nu_{ei} / \nu_{ci} = 4 )</td>
</tr>
</tbody>
</table>

The plasma parameters indicate that the peak electron pressure \( nkT_e = 5 \times 10^{-7} \text{ J/cm}^3 \) greatly exceeds the external magnetic field pressure \( B_0^2/2\mu_0 = 1 \times 10^{-7} \text{ J/cm}^3 \), suggesting that for \( \beta = 5 \) the electron diamagnetism entirely expels the external magnetic field. This is confirmed by magnetic loop measurements presented in Fig. 3 as ensemble averages over 200 shots. For reference, Fig. 3(a) shows the time dependence of the pulsed plasma production and decay as indicated by the ion saturation current to a Langmuir probe \( (I_{i,sat} \approx n_e \sqrt{\pi kT_e}) \). The plasma parameters achieve steady-state values at \( t \approx 2 \text{ ms} \) into the 4 ms long discharge pulse, while in the afterglow the electron pressure is lost within a fraction of 1 ms (further details presented below). The diamagnetic loop signal in the column center is shown in Fig. 3(b). The induced voltage \( (V_{loop} = -d\Phi/dt) \) indicates a flux change \( \Phi = B_z (dx) \) during growth and decay of the plasma pressure. Integration of the loop signal yields the change in the magnetic field \( \Delta B_z = \int V_{loop} dt / (N \pi r^2) \) or 1 G/4.5 \( \mu \text{V s} \) for \( N = 4 \) turns and \( r = 6 \text{ cm} \). When added to the initial value \( (B_0 = 5 \text{ G}) \), the time behavior of the internal axial magnetic field inside the plasma, \( B_z = B_0 + \Delta B_z \), is obtained and plotted in Fig. 3(c). As expected, the interior of this high-\( \beta \) plasma is essentially field-free. A further check of the validity of the diamagnetic measurements is provided by a loop rotation. Figure 3(d) shows that the measured magnetic field varies sinusoidally with angle \( \alpha \) between loop axis and \( B_0 \).

The radial dependence of the ensemble-averaged electron pressure \( (nkT_e) \) and axial magnetic field \( (B_z) \), measured during the steady-state discharge, is displayed in Fig. 4. As expected, the field is reduced in the high density region of the plasma column. From the radial magnetic field variation one can calculate the current density \( J_z = \mu_0 \omega_e \partial B_z / \partial y = 0.16 \text{ A/cm}^2 \) and the electron drift velocity \( v_d = -J_z / ne = 2 \times 10^8 \text{ cm/s} \). It is interesting to note that the field increases at the plasma edge above the dc field \( B_0 = 5 \text{ G} \) indicated by a straight line. Since on the present time scale the total flux inside the highly conducting plasma chamber is conserved, a reduction of the flux in the column center implies a field increase at the plasma edge. A second interesting observation is that the magnetic field is not fully expelled where \( \beta = 1 \) (see dashed line), implying that the pressure balance equation, \( B_z^2/2\mu_0 + nkT_e = B_0^2/2\mu_0 \), does not hold. This aspect has been further investigated by measuring the dependence of \( B_z \) in the column center for different values of peak plasma \( \beta \).

The plasma \( \beta \) in Fig. 5 is varied in two ways: (i) the electron pressure is varied with discharge power \( (nkT_e) \) at a fixed external magnetic field \( B_0 = 5 \text{ G} \) and (ii) the field \( B_0 \) is varied at a constant plasma pressure. In either case the magnetic field is not completely expelled at \( \beta = 1 \) as predicted by the pressure balance equation (see dashed line).
The reason is that the pressure balance equation applies to a single-fluid MHD plasma where the pressure gradient is balanced only by the magnetic force, \( \nabla p = \mathbf{J} \times \mathbf{B} \). An electric field does not exert a net force on a charge-neutral fluid and \( \mathbf{E} \times \mathbf{B} \) drifts produce no net currents to alter the magnetic field. However, these conditions do not hold when the ions are unmagnetized while the electrons remain magnetized (EMHD). Now the electron pressure is balanced by both electric and magnetic forces. As \( \beta \) increases and \( B \to 0 \), a transition from magnetic force balance, \( \nabla p = \mathbf{J} \times \mathbf{B} \), to Boltzmann equilibrium, \( \nabla \rho_e = -n_e \mathbf{E} \), occurs. The electric field \( \mathbf{E} \perp \mathbf{B} \) is directed so as to produce an \( \mathbf{E} \times \mathbf{B} \) drift opposing the \( \nabla \rho_e \times \mathbf{B} \) drift. This is readily seen in Fig. 2 where \( \mathbf{E} = -\nabla \phi_p \) [Fig. 2(c)] opposes \( \nabla p = \nabla nkT_e \) [Figs. 2(a)–2(b)]. Thus the Hall current opposes the diamagnetic current, and the magnetic field inside the plasma is not as low as predicted by the pressure balance equation. Vice versa, \( \beta > 1 \) is required to completely expel the magnetic field from the plasma. The unmagnetized ions are not confined and drift radially outward along the ambipolar electric field. As long as plasma production can replace the ion loss, a steady-state can be maintained. But if the ion outflow exceeds the neutral inflow the density will decrease, which is noticeable in Fig. 3.

**B. Basic instability properties**

The presence of an instability manifests itself in large density and magnetic field fluctuations seen on single shot traces. Figure 6(a) displays the ion saturation current \( I_{i,\text{sat}} = n_i \) for a single discharge pulse (noisy waveform) as well as for an ensemble average of \( N = 100 \) shots (solid smooth line). Likewise, a single trace of the diamagnetic loop voltage [Fig. 6(b)] indicates strong magnetic fluctuations in \( B_z \), which are not observed in an ensemble average [e.g., Fig. 3(b)]. The fluctuations exhibit no phase coherence from shot to shot, i.e., they are not transient phenomena of the pulsed discharge but an instability. The fluctuations are largest during the discharge and decay rapidly in the afterglow. Details of the decay will be discussed below.

The fluctuation amplitude varies with radial position as shown in Fig. 7. For reference, the radial profile of the electron pressure, \( nkT_e \), is also shown. It is evident that the rms fluctuations in the loop voltage, \( V_{\text{loop, rms}} = \left\langle (V_{\text{loop}} - \langle V_{\text{loop}} \rangle)^2 \right\rangle^{1/2} \), maximize in the region of the strongest pressure gradients. Density fluctuations show the same behavior. This feature suggests that the instability is excited by the electron diamagnetic drift, \( v_{\text{dia}} = \nabla \rho_e \times \mathbf{B}/neB \), through the unmagnetized ions. From Fig. 3 one finds \( |n/\nabla n| \approx 40 \) cm and \( |T_e/\nabla T_e| \approx 50 \) cm, which yields a pressure gradient scale length \( L_p = |p_e/\nabla p_e| = (1/40 + 1/50)^{-1} \approx 22 \) cm and a diamagnetic drift speed, \( v_{\text{dia}} = kT_e/(eBL_p) = 4.6 \times 10^6 \) cm/s for \( kT_e = 4 \) eV and \( B_z = 4 \) G. This drift is opposed by a smaller \( \mathbf{E} \times \mathbf{B} \) drift \( (E/B \approx 2 \times 10^6 \) cm/s) resulting in a net drift close.
to that derived from the magnetic field measurement. This net drift greatly exceeds the ion sound speed such that the condition for a current-driven cross-field instability is satisfied. Later wave measurements will show that the fluctuations propagate at $v_d' = c_s$ in the electron diamagnetic drift direction.

Frequency spectra and autocorrelations have been determined for magnetic fluctuations measured with the 3D probe, whose dimensions are small compared to the scale length of the fluctuations. Figure 8(a) shows a single-shot trace of $\delta B_y(t)$ versus time, while Fig. 8(b) shows its autocorrelation function, $A(\tau) = \int \delta B_y(t) \delta B_y(t-\tau) \, dt$, normalized to its peak value $A(0)$, during the steady-state period of the instability ($1.5 \leq \tau \leq 3.5$ ms). In order to improve the statistics, the correlation is taken over an ensemble of 25 similar time waveforms. One observes that the coherence in time is lost after $\tau \approx 50$ $\mu$s, which is just one period of a typical fluctuation. Thus the instability must have a broad frequency spectrum, as is confirmed in Fig. 9.

Figure 9(a) shows a Fast Fourier Transform (FFT) of the magnetic fluctuations, $\delta B_y(\omega)$, during the steady-state period of the discharge for the same waveforms employed in Fig. 8(b). The spectrum exhibits a broad peak at $f = 18$ kHz, but extends to $f = 60$ kHz, which is far below the electron cyclotron frequency at $B_0 = 5$ G ($f_{ce} = 14$ MHz), far above the ion cyclotron frequency ($f_{ci} = 175$ Hz), but in the range of the lower hybrid frequency $f_{LH} = (f_{ci}f_{ce})^{1/2} = 51$ kHz. Since the latter varies with radial position, $f_{LH} \approx B_0(y)$, the spectrum changes radially as displayed in the contour plot of Fig. 9(b). One observes that toward the column center ($y \rightarrow 0$) the peak in the spectrum shifts to lower frequencies ($B_0 \rightarrow 0$). For reference, the lower hybrid frequency at the local magnetic field is indicated by a dashed line. In earlier measurements at higher magnetic fields, the instability has also been observed to occur below but near the lower hybrid frequency. The radial variation of the instability amplitude has already been described in Fig. 7.

C. Plasma afterglow

After the turn-off of the discharge current, the plasma parameters decay, hence the diamagnetism and the instability vanish. These processes are summarized in Fig. 10, which starts with a semilog plot of the electron density and temperature, normalized to their steady-state values in the discharge, at the column center versus afterglow time $t_a$. The density decays slowest with an e-folding time of $\tau_n$.
Density decay is slower than electron temperature decay, which in turn is much slower than the loss of light-producing energetic electrons. The axial field recovers $\langle B_z \rangle > B_0$, and the fluctuations decrease with decaying electron energy, although $\nabla n$ persists.

$-n[\partial n/\partial t] = 900 \, \mu s$, while the electron temperature decays at a faster rate, $\tau_T = T_e[\partial T_e / \partial t] = 160 \, \mu s$. In a discharge plasma there are also energetic tail electrons ($eV_{\text{dis}} = 50 \, \text{eV}$), which produce visible light detected with a photodiode and displayed in Fig. 10. These unconfined electrons decay at the fastest rate, $\tau_{\text{tail}} = I_{\text{light}}/[\partial I_{\text{light}} / \partial t] = 27 \, \mu s$. The axial magnetic field in the column center rises from $B_0 = 5 \, \text{G}$ at an initial rate comparable to the loss rate of the energetic electrons. The bulk plasma $\beta$ still exceeds unity at $t = 150 \, \mu s$, yet the diamagnetism has essentially vanished. Note that the rise of $B_z$ is delayed in the afterglow ($\sim 30 \, \mu s$), which, as shown below, is the result of magnetic field diffusion through the plasma. Finally, the pressure-driven instability, as indicated by the rms fluctuations on the loop voltage, decays at a rate $\tau_{\text{instab}} = V_{\text{loop}}/[\partial V_{\text{loop}} / \partial t] = 74 \, \mu s$, which is mainly determined by the electron temperature decay and the magnetic field increase, $v_{\text{dia}}/c_s \propto T_e^{1/2}/B_z$, but not by changes in the density gradients.

The decay of the diamagnetism and the fluctuations at different radial positions is shown in Fig. 11. The ensemble-averaged diamagnetic loop voltage, $\langle V_{\text{loop}}(y,t) \rangle$ [Fig. 11(a)], shows that the field rises inside the column $\langle V_{\text{loop}}(y > 0) \rangle$, decays outside it $\langle V_{\text{loop}}(y > 0) \rangle < 0$, and that the change in $B_z$ diffuses/propagates from the edge of the plasma column to the center. The observed delay time $\tau_{B_z} = 50 \, \mu s$ is much faster than the resistive diffusion time $\tau = \mu_0 \sigma / L^2 = 680 \, \mu s$ based on the Spitzer conductivity $\sigma_0 \approx 60 \, \Omega^{-1} \text{cm}^{-1}$ and radial scale length $L = 30 \, \text{cm}$. The transient EMHD field is likely to be transported by an oblique whistler into the plasma. The net field $\langle B_z \rangle(y,t)$ is obtained by integrating the loop voltage and adding the dc magnetic field, and is displayed in Fig. 11(b). Outside the column, the field rapidly decays to $B_0 = 5 \, \text{G}$, while inside the column it rises more slowly to $B_z = 5 \, \text{G}$. The rms fluctuations on the loop voltage maximize off-axis ($|y| = 30 \, \text{cm}$) and decay on a similar time scale as the average magnetic field [Fig. 11(c)]. Nevertheless, at $t = 150 \, \mu s$, the normalized plasma pressure at the column center still yields $\beta(0) = 2$, yet instability and diamagnetism have essentially vanished.

**IV. CONCLUSIONS**

The properties of a dense discharge plasma in a weak external magnetic field have been measured. The electron diamagnetism expels the field from the plasma interior. In the presence of unmagnetized ions, the electron pressure is balanced by both magnetic and electric forces. The pressure balance equation is not applicable and values of plasma $\beta > 1$ are required for complete field expulsion. The cross-field electron drift exceeds the sound speed and drives a strong instability in the region of pressure gradients. Coupled density and magnetic field perturbations are excited near the lower hybrid frequency. The fluctuations propagate in the
direction of the diamagnetic drift but at the much slower sound speed. The saturated instability exhibits short correlation times. In the afterglow plasma, the instability dies out well before the density gradients vanish.

The present observations should be of interest to other plasma configurations with similar parameters. For example, natural magnetic holes have been observed near the magnetopause with strong lower hybrid wave activity at the hole boundary. Although classical pressure balance and the lower hybrid drift instability have been invoked, the present laboratory observations could modify this interpretation. The present results could also apply to other high-$\beta$ plasmas such as diamagnetic cavities produced in Barium cloud releases in space.4

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3T. J. Dolan, Fusion Research (Pergamon, New York, 1982), Vol. 1, p. 34.