Chapter II. Basic Plasma Diagnostics

One of the most important and frequently used plasma diagnostic techniques is the Langmuir probe method. This method, which was first introduced by Langmuir[1] about fifty years ago, can be used to determine the values of the ion and electron densities, the electron temperature, and the electron distribution function. This method involves the measurement of electron and ion currents to a small metal electrode or probe as different voltages are applied to the probe. This yields a curve called the probe characteristic of the plasma.

Another important technique, using microwaves, is frequently employed to measure plasma parameters, especially in situations where it is difficult to insert probes into the medium. An interferometer method is used to determine the phase shift of the microwaves transmitted through the plasma and the average electron density is deduced from the amount of phase shift.

By combining the microwave or ratio frequency method with the probe technique, we can measure the density to better than 1% accuracy. Electromagnetic waves propagating along a density gradient with frequency $_o$ excite electron plasma waves at the critical density layer z_o for which $_o$ = $_p(z_o)$, where $_p$ is the plasma frequency. Since the propagation of these electron plasma waves is a sensitive function of the density profile, a careful mapping of the electron plasma wave propagation characteristics will reveal the density along its propagation path. This more advanced method is described in Chapter V.

1) Langmuir Probe

The fundamental plasma parameters can be determined by placing a small conducting probe into the plasma and observing the current to the probe as a function of the difference between the probe and plasma space potentials. The plasma space potential is just the potential difference of the plasma volume with respect to the vessel wall (anode). It arises from an initial imbalance in electron and ion loss rates and depends in part upon anode surface conditions, and filament emission current.

Referring to the probe characteristic, Figure II-l, we see that in region A when the probe potential, V_p , is above the plasma space potential, V_s , the collected electron current reaches a saturated level and ions are repelled, while in region B just the opposite occurs. By evaluating the slope of the electron I-V characteristic in region B the electron temperature T_e is obtained, and by measuring the ion or electron saturation current and using the T_e measurement, the density can be computed.

The current collected by a probe is given by summing over all the contributions of the various plasma species:

$$I = A\sum_i n_i q_i \bar{v}_i \tag{1}$$

where A is the total collecting surface area of the probe; $\bar{v}_i$ = the average velocity of species I, and $\bar{v}_i = \frac{1}{n}\int v f_i(\vec{v})d\vec{v}$ for unnormalized $f_i(\vec{v})$. It is well known in statistical mechanics that collisions among particles will result in an equilibrium velocity distribution f given by the Maxwellian function:

$$f_\alpha(\vec{v}) = n\left(\frac{2\pi KT_\alpha}{m_\alpha}\right)^{-3/2} Exp\left(\frac{-\frac{1}{2}m_\alpha|\vec{v}|^2}{KT_\alpha}\right) \tag{2}$$

This distribution function is used to evaluate the average velocity of each species.

We will first consider a small plane disc probe which is often used in our experiments. When it is placed in the xy plane, a particle will collide with the probe and give rise to a current only if it has some v_x component of velocity. Thus, the current to the probe does not depend on v_y or v_z. The current to the probe from each species is a function of $V \equiv V_p - V_s$.

$$I(v) = nqA\int_{-\infty}^{\infty} dv_y \left(\frac{2\pi KT_\alpha}{m_\alpha}\right)^{1/2} Exp\left(\frac{\frac{1}{2}m_\alpha v_y^2}{KT_\alpha}\right)\int_{-\infty}^{\infty} dv_z \left(\frac{2\pi KT_\alpha}{m_\alpha}\right)^{1/2} Exp\left(\frac{\frac{1}{2}m_\alpha v_z^2}{KT_\alpha}\right)$$
$$\cdot\left(\int_{v_{\min}}^{\infty} dv_x v_x \left(\frac{2\pi KT_\alpha}{m_\alpha}\right)^{1/2} Exp\left(\frac{\frac{1}{2}m_\alpha v_x^2}{KT_\alpha}\right)\right). \tag{3}$$

The lower limit of integration in the integral over v_x is v_{min} since particles with v_x component of velocity less than $v_{\min} = \left(\frac{2qV}{m_\alpha}\right)^{1/2}$ are repelled, Figure II-2.

The integrals over v_y and v_z in (3) give unity so the current of each species is just

$$I(v) = nqA\int_{v_{\min}}^{\infty} dv_x v_x \left(\frac{2\pi KT_\alpha}{m_\alpha}\right)^{-1/2} Exp\left(\frac{\frac{1}{2}m_\alpha v_x^2}{KT_\alpha}\right) \tag{4}$$

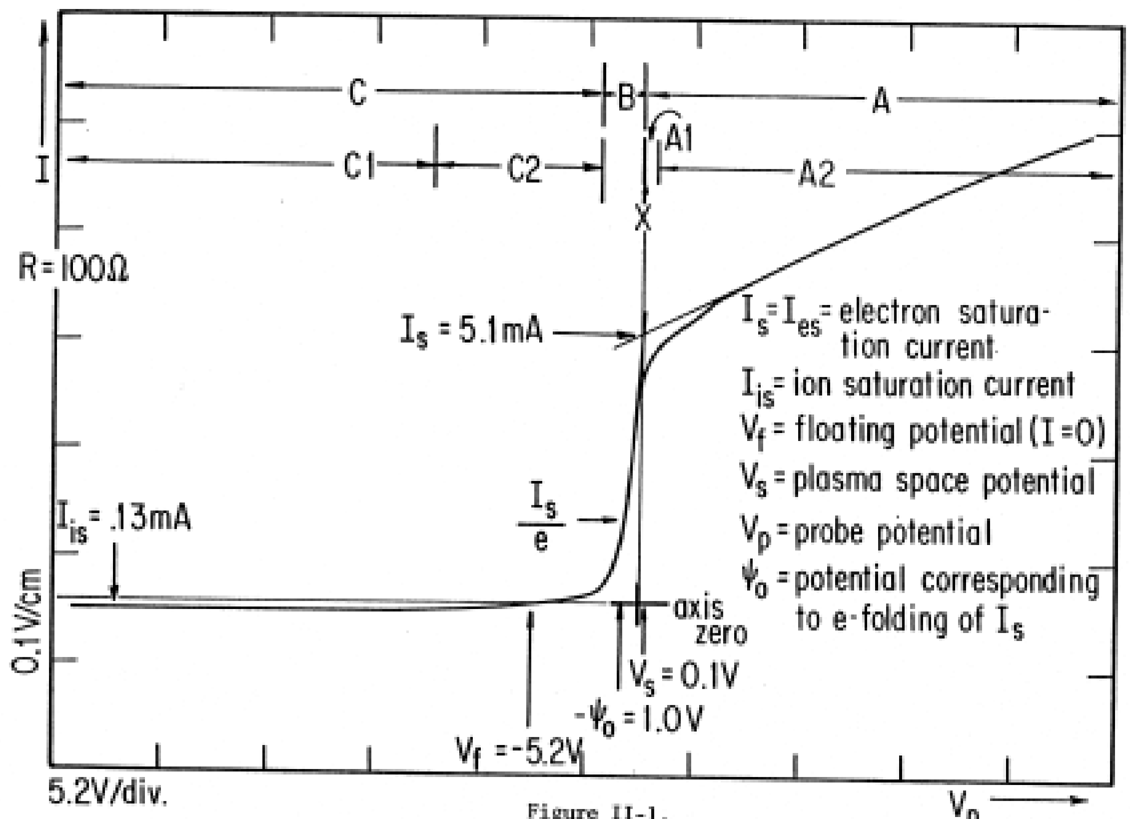

Figure II-1.

Sample Langmuir Probe Characteristic (Radial disc probe placed near center of single plasma): Region C1 - Ion saturation (electrons repelled); Region C2 - Ion saturation plus small primary electron current; Region B - Secondary electrons added to current of primaries and ions; X - Probe at space potential (zero electric probe field); Region A1 - Electron saturation with cooler ions being repelled; Region A2 - Electron saturation, no ion current. Note how ψ_o facilitates identification of secondary electron temperature, T_e (1.0 eV for this data).

a) The electron saturation current, I_{es}: In this region all electrons with v_x component toward probe are collected. We obtain the electron saturation current

$$I_{es} = -neA\int_0^\infty dv_x v_x \left(\frac{2\pi KT_e}{m_e}\right)^{-1/2} Exp\left(\frac{-\frac{1}{2}m_e v_x^{\,2}}{KT_e}\right) = -neA\left(\frac{KT_e}{2\pi m_e}\right)^{1/2}$$

(5)

Similarly, in region B and C where $V_p < V_s$ and electrons are repelled, the total current is

$$I(v) = I_{is} - neA\int_{v_{\min}}^\infty dv_x v_x \left(\frac{2\pi KT_e}{m_e}\right)^{-1/2} Exp\left(\frac{-\frac{1}{2}m_e v_x^{\,2}}{KT_e}\right).$$

(6)

Substituting $\frac{1}{2}m_e v_{\min}^{\,2} = -eV$, (6) becomes

$$I(v) = I_{is} - neA\left(\frac{KT_e}{2\pi m_e}\right)^{1/2} Exp\left(\frac{eV}{KT_e}\right)$$

(7)

since $V < 0$ in region B and C. Equation (7) shows that the electron current increases exponentially until the probe voltage is the same as the plasma space potential ($V = V_p - V_s = 0$).

b) The ion saturation current, I_{is}: The ion saturation current is not simply given by an expression similar to (5). In order to repel all the electrons and observer I_{is}, V_p must be negative and have a magnitude near KT_e/e as shown in Figure II-3. The sheath criterion[2] requires that ions arriving at the periphery of the probe sheath be accelerated toward the probe with an energy $\sim KT_e$, which is much larger than their thermal energy KT_i. The ion saturation current is then approximately given as

$$I_{is} = neA\left(\frac{2KT_e}{m_i}\right)^{1/2}.$$

(8)

Even though this flux density is larger than the incident flux density at the periphery of the collecting sheath, the total particle flux is still conserved because the area at the probe is smaller than the outer collecting area at the sheath boundary.[2-5]

c) Floating potential V_f: Next we consider the floating potential. When $V = V_f$, the ion and electron currents are equal and the net probe current is zero. Combining equations (7) and (8), and letting $I = 0$, we get

$$V_f = -\frac{KT_e}{e}\ln\left(\frac{m_i}{4\pi m_e}\right)^{1/2}$$

(9)

d) The electron temperature, T_e : Measurement of the electron temperature can be obtained from equation (7). For $I_{is} <$ I we have

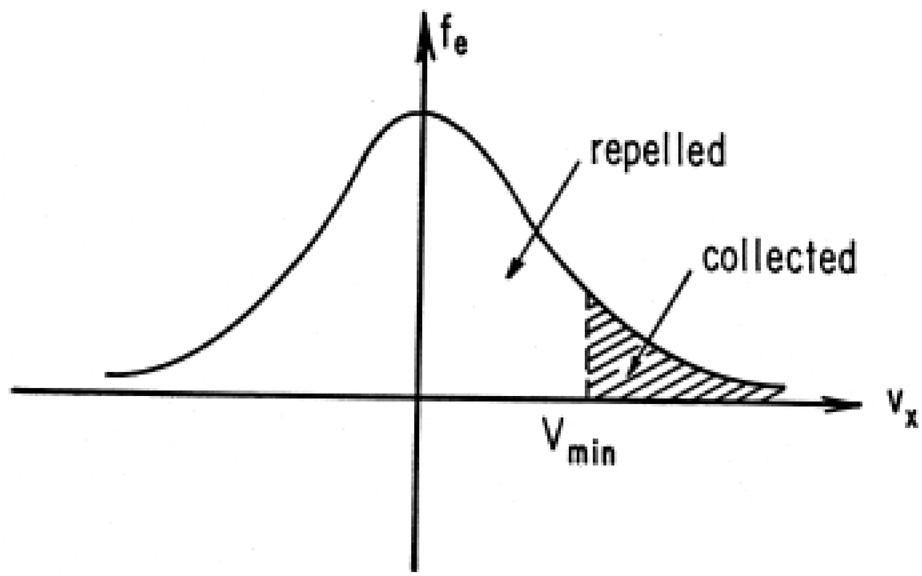

Figure II-2.

Electron Velocity Distribution
All electrons with energy $|eV|$ greater than $\frac{1}{2} m_e v_{min}^2$ are collected.

$$I(v) = -neA\left(\frac{KT_e}{2\pi m_e}\right)^{1/2} Exp\left(\frac{eV}{KT_e}\right) = I_{es}\ Exp\left(\frac{ev}{KT_e}\right)$$

(10)

$$\frac{d\ln|I|}{dV} = \frac{e}{KT_e}.$$

(11)

By differentiating the logarithm of the electron current with respect to the probe voltage V for V < 0, the electron temperature is obtained. We note that the slope of lnI vs. V is a straight line only if the distribution is a Maxwellian.

e) Measurement of the electron distribution function, $f_e(v_x)$: The electron current to a plane probe could be written in a more general expression as (again neglecting the ion current)

$$I = nqA\int_{v_{\min}}^{\infty} v_x f(v_x) dv_x = \frac{nqA}{m_e}\int_{qV}^{\infty} f(qV)d(qV)$$

(12)

$$\frac{dI}{d(qV)}\alpha\ f(qV)$$

where q = -e, the electron charge. This is a very simple way of obtaining the electron energy distribution function. If we measure $f(v_x)$ as a function of plasma position, we can obtain the phase space distribution $f(v_x,x)$. A further refinement is to observe the distribution at a given time t after a certain event using a sampling oscilloscope. This results in the complete description, f(v,x,t), of the electrons in a given system.

2) Double Probes[5-8]

A double probe consists of two electrodes of equal surface area, separated by a small distance and immersed in the plasma, Figure II-4. One probe draws current I_1 while the other is drawing current I_2. To find the electron temperature of the plasma, we consider quantitatively the current to the probe for various potential differences between the probes, Figure II-5. Since the probes are floating at V_f of the plasma, i.e., the double probe circuit has no plasma ground (anode) connection, the total current in the probe circuit must be zero. From (7) and (5), the current collected by probe #1 is

$$I_1 = I_1\ is - I_1\ es\ Exp\left(\frac{e(V_1 + V_f - V_s)}{KT_e}\right)$$

(13)

Using the definition of the floating potential with (7),

$$I_{es}\ Exp\left(\frac{e(V_f - V_s)}{KT_e}\right) = I_{is}$$

(14)

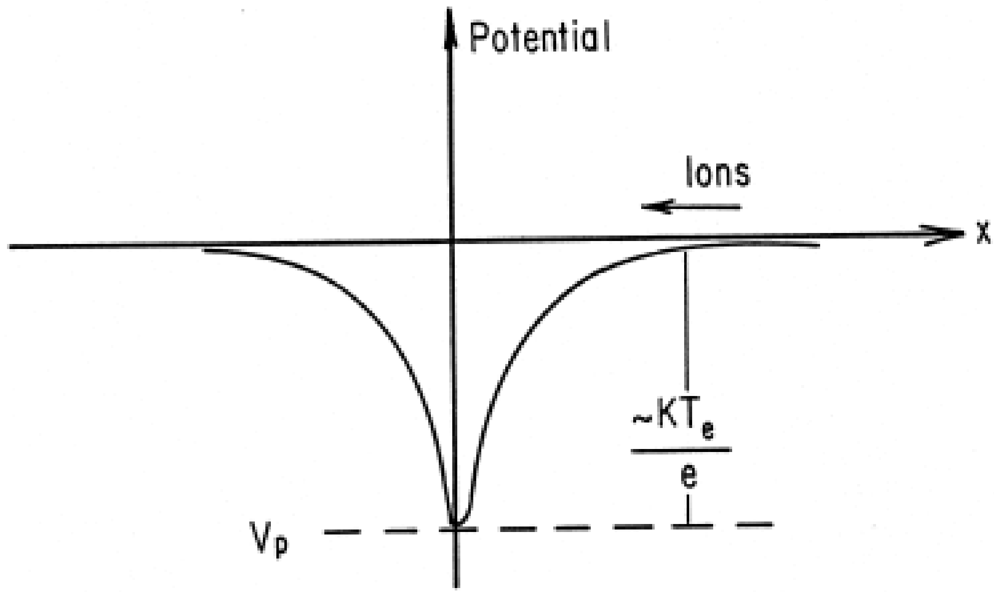

Figure II-3.

Sheath potential as function of distance x from infinite plane probe.

hence (13) becomes

$$I_1 = I_1\ is \left[1 - Exp\left(\frac{eV_1}{KT_e}\right)\right] \tag{15}$$

In the same manner we get

$$I_2 = I_2\ is \left[1 - Exp\left(\frac{eV_2}{KT_e}\right)\right] \tag{16}$$

If the probe areas are equal, then (8) implies

$$I_1\ is = I_2\ is = I_{is} \tag{17}$$

Zero net probe circuit current motivates the definition

$$I \equiv I_1 = -I_2$$

Combining this with equations (15), (16), and (17) yields

$$\frac{I - I_{is}}{-I - I_{is}} = Exp\left(\frac{e\psi}{KT_e}\right) \tag{18}$$

where the double probe potential is defined by $\psi \equiv V_1 - V_2$. Solving equation (18) for I

$$I = -I_{is}\ \tanh\left(\frac{e\psi}{2KT_e}\right) \tag{19}$$

Differentiating equation (19) with respect to ψ at ψ=0

$$\left.\frac{dI}{d\psi}\right|_{\psi=0} = -I_{is}\ \sec h^2 \left.\frac{e\psi}{2KT_e}\right|_{\psi} = 0\left(\frac{e}{2KT_e}\right) \tag{20}$$

i.e., electron temperature is related to the slope of the double probe characteristic by

$$\frac{dI}{d\psi} = -I_{is}\left(\frac{e}{2KT_e}\right) \tag{21}$$

The double probe can collect a maximum current equal to the ion saturation current and does not disturb the plasma as much as the single probe with its anode connection. However, the small amount of detected current (microampere range) does warrant a much more sensitive detection circuit, as in Figure II-4.

The student is required to compare the electron temperatures and plasma densities obtained with the single and the double probes.

3) Microwave Interferometer[9,10]

The basic idea behind this diagnostic scheme is as follows. The plasma acts like a dielectric medium to electromagnetic radiation, and a wave propagating through the plasma will suffer a change in phase

$$\Delta\phi = \int_0^L \left(k_{vacuum} - k_{plasma}\right) dx \tag{22}$$

where L is the path length of the plasma, k_{vacuum} = w/c is the free space wave number of the electromagnetic waves, and k_{plasma} is the wave number of the wave propagating in the plasma, which is given by the dispersion relation

$$k_{plasma} = \frac{\left(\omega^2 - \omega_{pe}{}^2\right)^{1/2}}{c} \tag{23}$$

Here w is the wave frequency, $\omega_{pe} = \left(\frac{4\pi ne^2}{m}\right)^{1/2}$, the electron plasma frequency and c is the speed of light. If the plasma density is uniform over the distance L, we obtain from equation (23) for the phase shift

$$\Delta\phi = \frac{\omega}{c}\left(1 - \left(1 - \frac{\omega_{pe}{}^2}{\omega^2}\right)^{1/2}\right)L. \tag{24}$$

a) Density measurement by phase shift: When w_{pe} << w (note this restriction) we obtain a relation between the phase shift and plasma density

$$\Delta\phi = \frac{\omega}{c}\left(\frac{1}{2}\frac{\omega_{pe}{}^2}{\omega^2}\right)L. \tag{25}$$

Defining the critical density by $\frac{4\pi n_c e^2}{m_e} \equiv \omega^2$ we can express equation (25) alternatively by

$$\frac{n}{n_c} = \frac{2\Delta\phi}{k_{vac}L}, \; where \; k_{vac} \equiv \frac{w}{c} \tag{26}$$

Since all laboratory plasmas have a certain degree of inhomogeneity, i.e., some density gradient, the phase shift Df is an integrated quantity as represented by equation (23). However, the density profile can be obtained by relative density measurement using movable probes. If we write $n(x)=n_0f(x)$, where f(x) contains the spatial variation in the plasma density, a relation similar to equation (26) can be achieved using

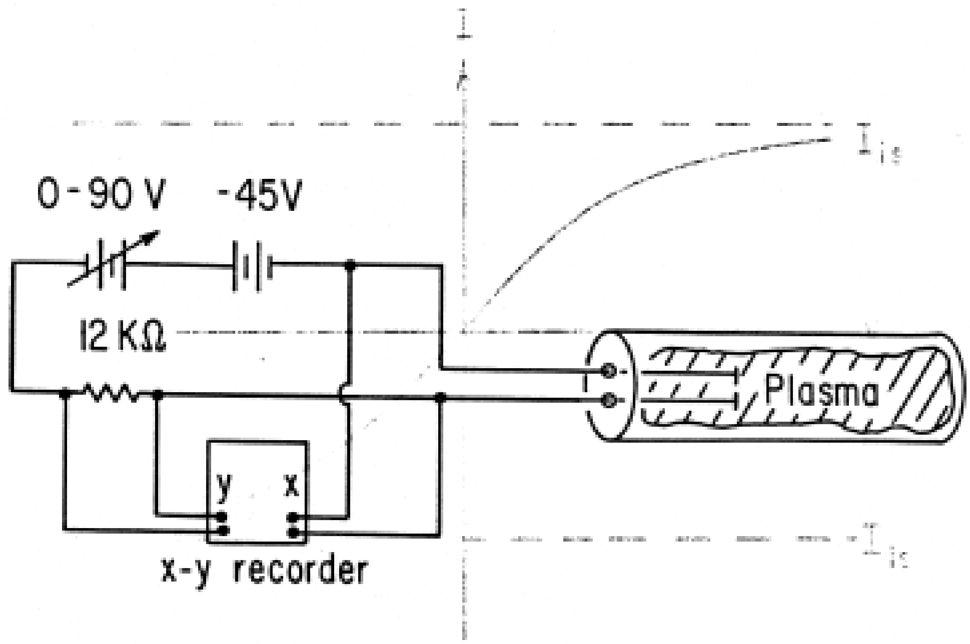

Figure II-4.

Typical Double Probe Manual Sweep Circuit. Note that there is no connection made to plasma ground in this diagnostic; it is independent of plasma potential.

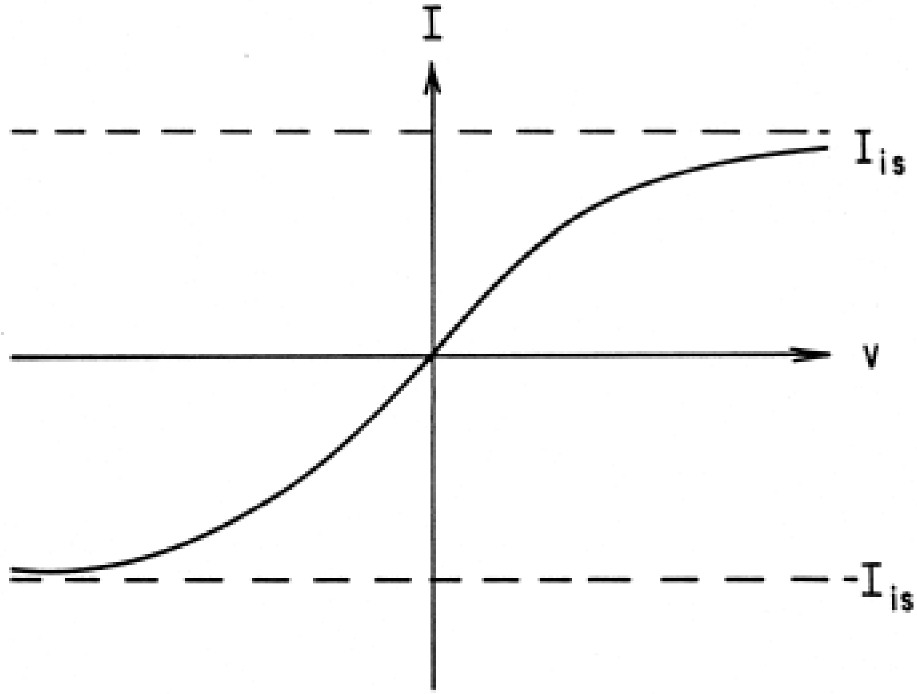

Figure II-5.

Current vs. voltage characteristic of a double probe.

$$n_0 = \frac{2 n_c \Delta\phi}{k_{vac}\int_0^L f(x)dx}$$

(27)

Thus with the help of the radial probe measurement on the relative density profile, the microwave interferometer technique could be used to obtain the absolute density at any radial position.

b) Observation of cut-off: For $w = w_{pe}$ the relation given by equation (26) or (27) is no longer valid (why?). One must use equation (23) or (24) directly and the relation between Dy and n becomes quite complicated. However, when $w = w_{pe}$, the wave number becomes purely imaginary and no propagation is possible. By observing the cut-off, we can calculate the maximum density in the plasma, $n_{max} = n_c$, where n_c is the critical density defined above. The student is required to compare this microwave method with the Langmuir probe result.

4) Experimental Procedure

a) Follow the procedure as described in Chapter I to obtain a D.C. discharge. Clean up probes and set up a sweeper circuit as described in Appendix A. Observe a Langmuir characteristic curve by using the oscilloscope. For recording, the single sweep mode must be used with sweep rate of 1 to 2 seconds per cm, such that the mechanical movement of the recorder pin can follow the changes of the signal. The manual sweeping circuit of Figure II-7a is a possible substitute for oscilloscope and automatic sweeper when recording the Langmuir curve graphically.

b) Details of the probe characteristic: Take several single probe traces using the probe sweeper circuit and the x-y recorder. Replot each trace on semi-log paper and subtract out the ion saturation current to obtain the current contributed by the secondary electrons. At low neutral pressures the primary electron current will appear as a long, high temperature (gently sloping) tail with negative current in the ion saturation region of probe bias. In this case, subtract the primary electron current (as well as the ion saturation current) from the total probe current to obtain actual secondary electron current. (Primary electron current collected for a given probe bias can be estimated by extrapolating the straight-line primary tail.) Obtain the electron temperature from the slope of the curve and the density from both the ion and electron saturation currents. Experimentally determine how the ratio I_{es}/I_{is} depends on the mass ratio.

You will find that the primary electron current usually overshadows the ion current. However, the ratio of primary electron to ion current can be reduced significantly and the ion saturation current can be observed by simply raising the neutral pressure to about 10^{-3} torr (explain why).

c) Double probe: Set up the double probe manual sweeper circuit with the x-y recorder and obtain a double probe trace. Compute the density and electron temperature and compare the result with n and T_e obtained from a single probe characteristic at the same time in the same region of the plasma.

d) Microwave interferometer: The X-band microwave interferometer set-up is shown in Figure II-6. Microwave signal generated by the oscillator is split into two paths, one propagating through the plasma, V_1 cos (wt + o_1), the other through a variable phase shifter to provide a reference signal, V_2 cos(wt + o_2). The signal propagating through the plasma is received by a pickup horn on the other side of the vacuum system, and then added to the reference signal by the magic tee: $V_{sum} = V_1 \cos (wt + 0_1) + V_2 \cos (wt + 0_2)$. This signal is then fed into a crystal detector, which produces a current signal I V^2_{sum}. By taking a time average of the current I from the crystal,

$$\langle I \rangle \propto \langle V_{sum}{}^2 \rangle = \tfrac{1}{2} V_1^2 + \tfrac{1}{2} V_2^2 + V_1 V_2 \cos(\phi_1 - \phi_2) \tag{28}$$

For a given plasma density, f_1 is fixed. Vary V_2 with the variable attenuator and f_2 with the phase shifter to obtain a null (f_1 f_2 = p). Be careful not to overattenuate the reference signal V_2 as this will result in a phase independent signal [V_2 = 0 in (28)].

The crystal diode output signal is so small that a high gain amplifier must be used. Furthermore, the microwave signal should be gated on and off (i.e., square wave modulated output) to avoid confusion over D.C. shifting of the output signal in the high gain amplifier.

Turn off the plasma (by turning off the discharge), find the null again and record Df. In finding the null, large errors can develop. To minimize these, plot $<V_{sum}{}^2>$ versus f_z for the case where the plasma is present and the case where it is turned off to obtain two cosine curves offset by phase Df. Although this procedure is tedious, it is recommended for improved accuracy.

Calculate the plasma density using equation (26) and compare the results with those obtained from the probe measurements. If the density is non-uniform, try to correct the result by measuring relative density profile using a movable Langmuir probe. Difficulties arise whenever the dimensions of the plasma container are comparable to the wavelength of the microwaves. In this case, the waveguide horn is not large enough to sharply define the microwave beam and unwanted cavity modes are excited in the vacuum chamber as a result. These modes have multiple paths through the plasma and can drastically alter the measurement of Df. Hence, steel wool has been placed near both the sending and receiving horns to attenuate these undesirable multiple path signals.

e) Density Measurement via Plasma Resonance: The most accurate local density measurement in an inhomogeneous plasma is achieved by exciting the local plasma resonance $_p$(x). An oscillating electric field of frequency w_0 is externally excited in the plasma by a capacitor plate oriented to give an electric field along the density gradient as shown in Figure II-7. Where ever the

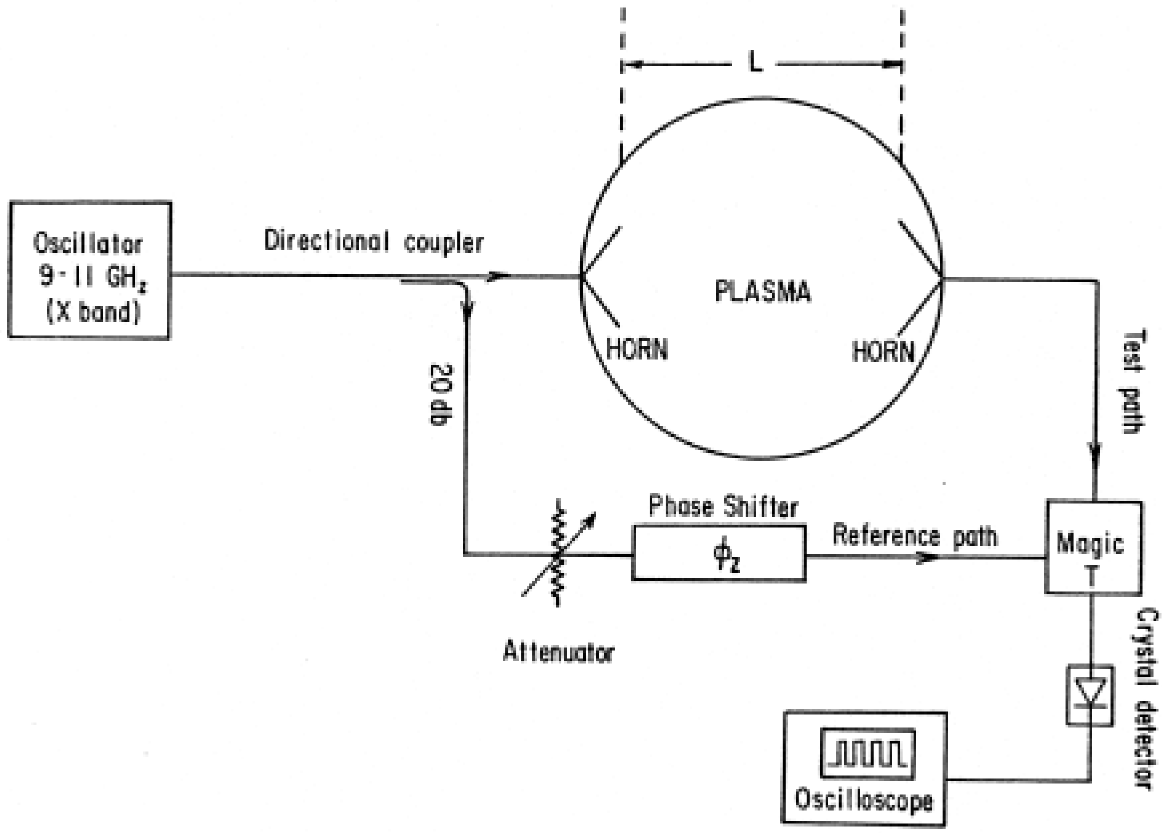

Figure II-6.

Interferometer Setup

external frequency matches the local plasma frequency ${}_0 = {}_p(x_0)$, the amplitude of the external electric field is found to be enhanced by an order of magnitude or more. This resonance is best detected by noting the deflection of an electron beam traversing the resonant location (Figure II-7) in a direction perpendicular to the density gradient. (A detailed description of this electron beam diagnostic is to be presented in Chapter V.) Since the external oscillating field frequency ${}_0$ can be precisely measured, the plasma density can be determined to better than 1%. A Langmuir probe can be calibrated using this technique.

Questions

1. Why does the ion saturation current depend on kT_e?
2 If there is an excess of primary electrons in the plasma, what kind of effect can one see by using (1) single probe, (2) double probe, and (3) microwave interferometer? Can you measure the density of the primary electrons?
3. For a plasma consisting of positive and negative ions of equal mass, draw the probe characteristics, carefully labeling the quantities I_{s+}, I_{s-}, V_f and V_s. How do you deduce T_+ and T_-?
4. What processes determine potential difference between the plasma and the anode?
5. When a fine conducting grid (called "plasma demon") is biased to some positive potential, it was found that the electron temperature increases by a factor up to 2-3. Can you explain this effect?

5) Appendix A: Description of Probe and Circuitry

a) Langmuir probe: A typical disc Langmuir probe in our laboratory experiment is spot welded onto a 0.010" diameter tungsten wire extending from the end of a glass insulating tube which is surrounded by a ceramic support sleeve, [c.f. Chapter I, Figure I-14]. A circuit diagram which is useful for plotting the current-voltage characteristics on an x-y recorder is given in Figure II- 4.

The probe voltage is obtained from a stabilized power supply (0 to 100 V), or better still a large battery, and can be varied in polarity and magnitude with a potentiometer. The probe voltage is also monitored on the x-axis of an x-y recorder which is easily calibrated with a voltmeter. As in Chapter I, the probe current is found from the voltage drop across a small resistor. The y axis is most conveniently calibrated by replacing the probe with a known resistor. More sophisticated circuits which electronically generate a voltage ramp may also be used to obtain probe I-V characteristics. One of these is shown in Figure II-8b.

The Langmuir characteristic is obtained by sweeping the probe voltage slowly between the negative and positive limits so as to include all the important regions of the curve.

c) Double probe: The double probe used in these experiments consists of two planar discs separated by a small distance. An alternative circuit to that of Figure II-4, suitable for measuring the double probe characteristic is the same as shown in Figure II-7, except that the ground (anode) is replaced by the second probe. No part of this circuit should be grounded. Each component probe of the double probe should be cleaned in the same manner as the Langmuir probe.

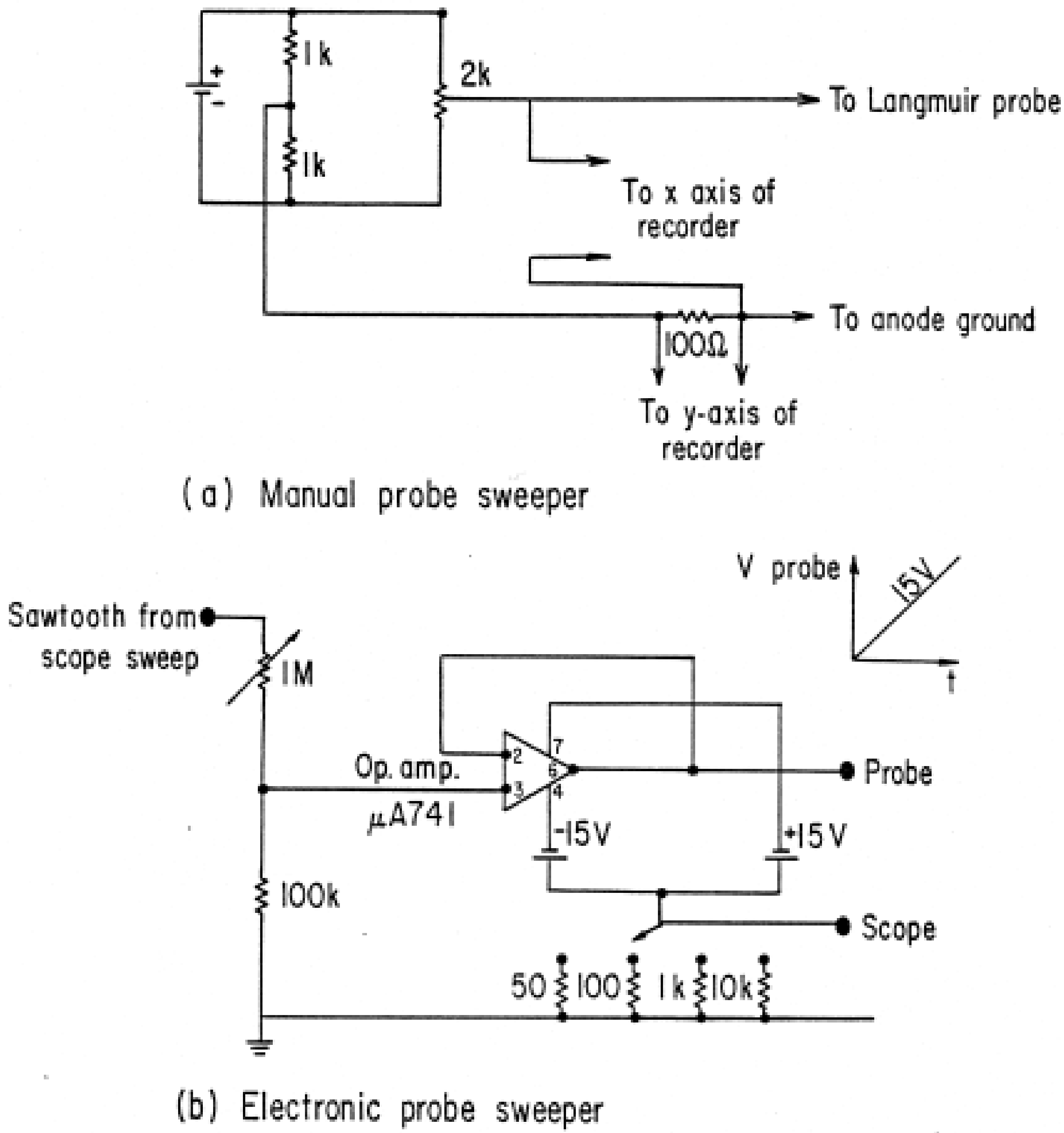

Figure II-8.

Probe Sweeper Circuits. For the electronic sweeper, a constant, positive bias is often applied in the form of a battery (0 - 9 volts) placed in series with the probe; this shifts the limits of the voltage ramp to sweep the desired features of the trace.

References

General Langmuir Probe

1. 0. Mawardi, Amer. J. Phys. 34, 112 (1966).
2. F. F. Chen, "Electric Probes," Plasma Diagnostic Techniques, Ed. by Huddlestone and Leonard (Academic Press, 1965).
3. F. F. Chen, C. Etievant and D. Mosher, Phys. Fluids 2, 811 (1968).
4. J. G. Laframboise, "Theory of Spherical and Cylindrical Langmuir Probes in a Collisionless, Maxwellian Plasma at Rest," UTIAS Report #100, Univ. Toronto (1966).
5. L. Schott, "Chapter 11," Plasma Diagnostics, Ed. by W. Lochte -Holtgreven (North Holland, 1968).

Double Probe

6. F. Johnson and L. Malter, Phys. Rev. 80, 59 (1950).
7. S. Ariga, J. Phys. Soc. Japan 31, 4, 1221 (1971).
8. P. Demchenko, L. Krupnik and N. Shulika, Sov. Phys. -Tec. Phys. 18, 8, 1054 (1974).

Microwave in Plasmas

9. C. Wharton, "Microwave Techniques," Plasma Diagnostic Techniques.
10. M. Heald and C. Wharton, Plasma Diagnostics with Microwaves (Wiley, 1965).