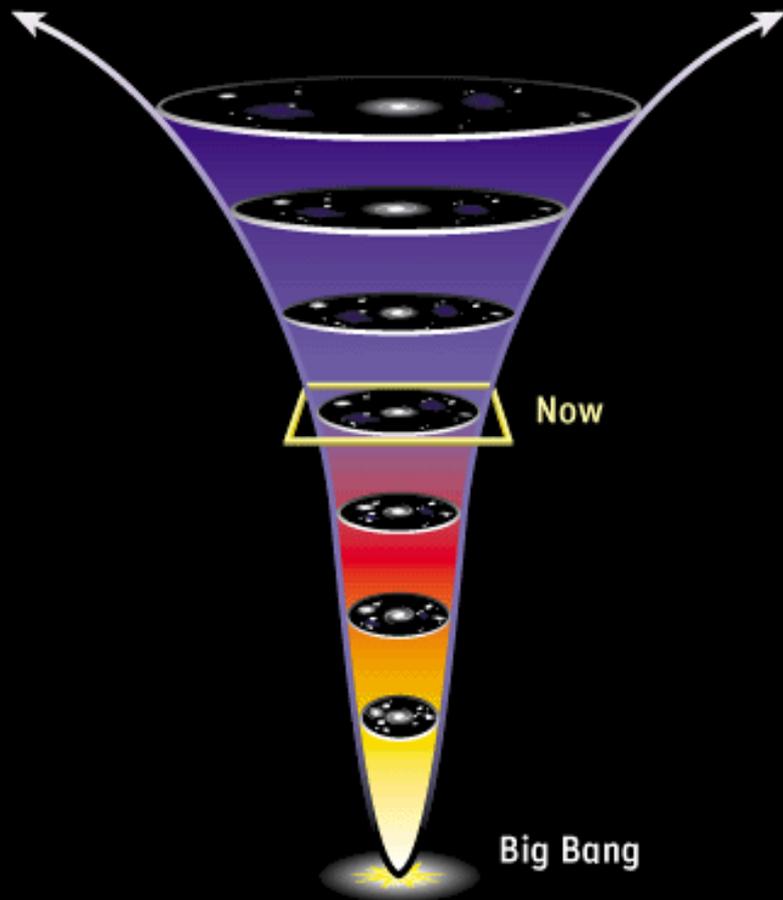


Modified Gravity and the Accelerating Universe

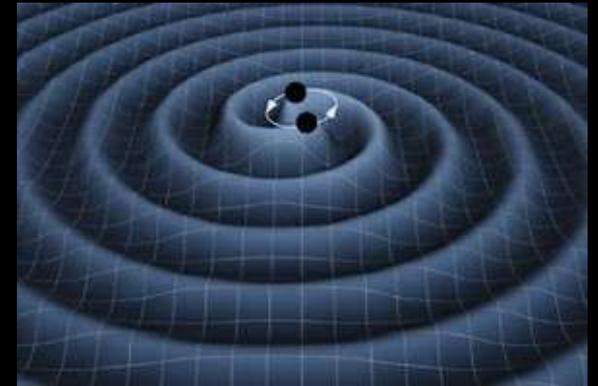
Sean Carroll



Something is making the universe accelerate. Could gravity be the culprit, and if so how will we know?

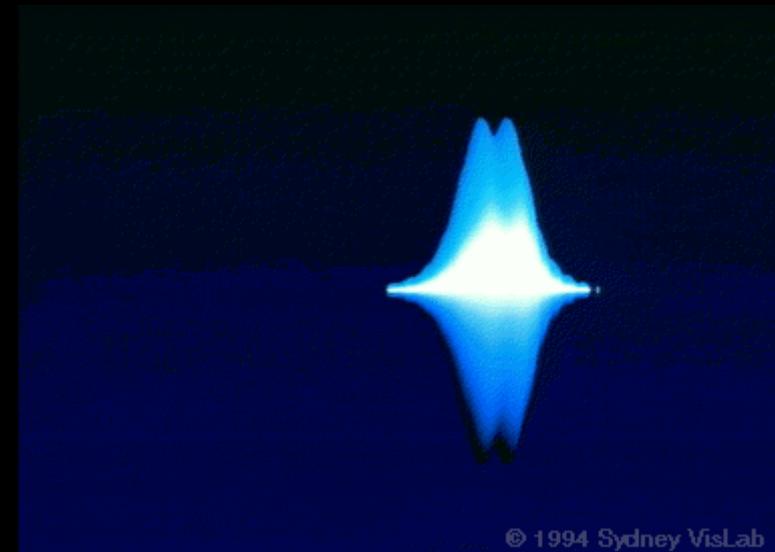
Degrees of Freedom

- We live in a low-energy world, in which it is useful to classify degrees of freedom into quantized weakly-coupled excitations: “particles.”
- In the weak-field limit, gravity is a theory of massless **spin-2 gravitons** coupled to the energy-momentum of matter (and each other).
- Such a theory is **essentially unique**, and gives rise to general relativity.
- Alternatives generically involve **new degrees of freedom**.



What sorts of new degrees of freedom?

- **Fermions** no good - don't pile up into classical fields.
- **Bosons:**
 - spin > 2 : no consistent theories.
 - spin-2: can't have more than one graviton.
 - spin-1: like charges repel.
not really gravity-like.
interesting, but complicated.
 - spin-0: attractive!



How do we hide the new degrees of freedom?

An action $S_\phi = \int \left[-\frac{\alpha}{2} (\nabla \phi)^2 - \frac{1}{2} m^2 \phi^2 + \lambda \phi \rho \right] d^4 x$

leads to an equation of motion

$$\alpha \nabla^2 \phi - m^2 \phi = \lambda Q, \quad Q = \int \rho d^3 x.$$

ϕ can be hidden by:

- **large mass m** , pinning ϕ to the potential minimum.
- **small coupling λ** , decoupling ϕ from other fields.
- **small kinetic coefficient α** , tying ϕ to the source Q .

“Spin-0” doesn't necessarily mean “scalar.”

A four-vector field:

$$A^\mu = (A^0, \vec{A})$$

spin-0
(one degree of freedom)

spin-1
(three degrees of freedom)

Spatial vectors can be further decomposed:

$$\vec{A} = \vec{\nabla} \times \vec{v} + \vec{\nabla} \lambda$$

curl gradient

massive spin-1
(three d.f.'s)

massless spin-1
(two d.f.'s)

spin-0
(one d.f.)

But we can't get both spin-0 and spin-1 from the same vector field; one would be a **ghost** (negative energy).

The same holds true (of course)
for the spacetime metric.

$$g_{\mu\nu} = \left(\begin{array}{c|c} g_{00} & g_{i0} \\ \hline g_{0j} & g_{ij} \end{array} \right)$$

Ordinary GR propagates a single massless spin-2 graviton:

$$S = \int (R - 2\Lambda) d^4x$$

But any other function of the curvature scalar R ,
Ricci tensor $R_{\mu\nu}$, and Riemann tensor $R_{\mu\nu\rho\sigma}$ will propagate
massive spin-0 and/or massive spin-2 particles.

$$S = \int f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma}) d^4x$$

[Stelle 1977]

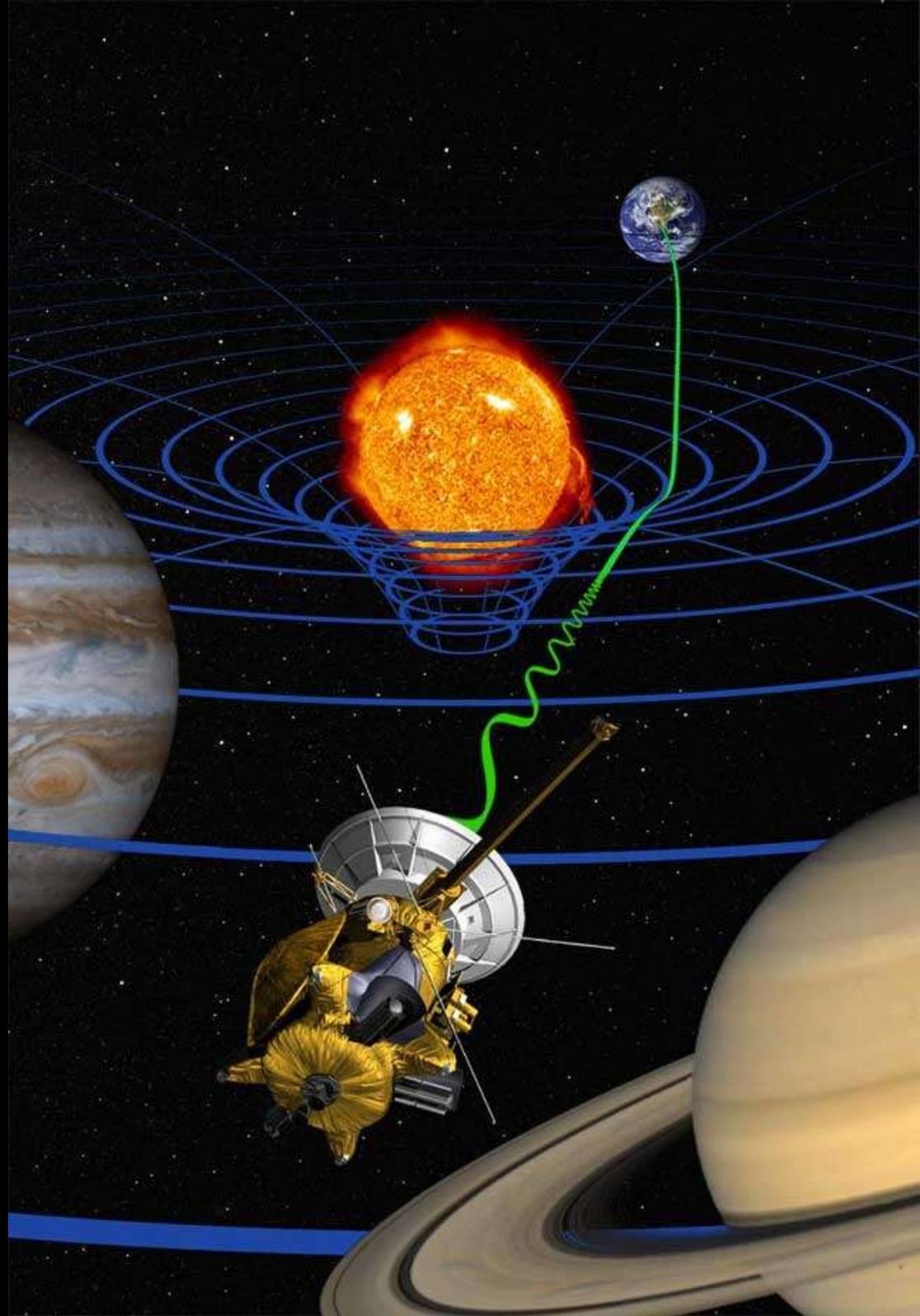
But **the massive spin-2 particle is a ghost**, so sensible
theories have the massless graviton and a massive scalar.

The simplest model is

$$S = \int \left(R - \frac{1}{R} \right) d^4 x.$$

It has a tachyonic scalar as well as the graviton. It can make the universe accelerate, but the scalar is extremely light and distorts the metric away from Schwarzschild, leading to conflict with Solar-System tests. Have to think harder.

[Capozziello, Carloni & Troisi 2003;
Carroll, Duvvuri, Trodden & Turner 2003;
Chiba 2003]



It turns out that ghost-free actions are of the form

$$S = \int f(R, R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu}) d^4 x$$

The scalar degree of freedom has a typical mass $m \sim H_0 \sim 10^{-33}$ eV. But by playing $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu}$ against R^2 , we can make the mass depend significantly on the spacetime curvature - and thus on the density.

Claim: You can get a large mass inside astrophysical sources, freezing ϕ at some fixed value, effectively decoupling its dynamics. It may be possible to be consistent with Solar-System tests, while introducing interesting possibilities in the galaxy and at mm scales.

An alternative: Modified-Source Gravity

Kill off the scalar mode by setting kinetic term to zero.

$$S = \int \left[\frac{M_P^2}{2} e^{2\psi} R + 3 M_P^2 e^{2\psi} (\nabla \psi)^2 - U(\psi) + L_m(g_{\mu\nu}, \chi^i) \right] d^4 x$$

Looks like a dynamical scalar ψ , but really isn't.

$$G_{\mu\nu} = 8\pi G e^{-2\psi} \left(T_{\mu\nu}^{(matter)} + T_{\mu\nu}^{(\psi)} \right)$$

Similar to a scalar-tensor theory, but now the scalar is **determined algebraically** by the ordinary matter fields, once we specify the potential $U(\psi)$.

$$\frac{dU}{d\psi} - 4U(\psi) = -T = \rho - 3p$$

Cosmology in modified-source gravity

The effective Friedmann equation is

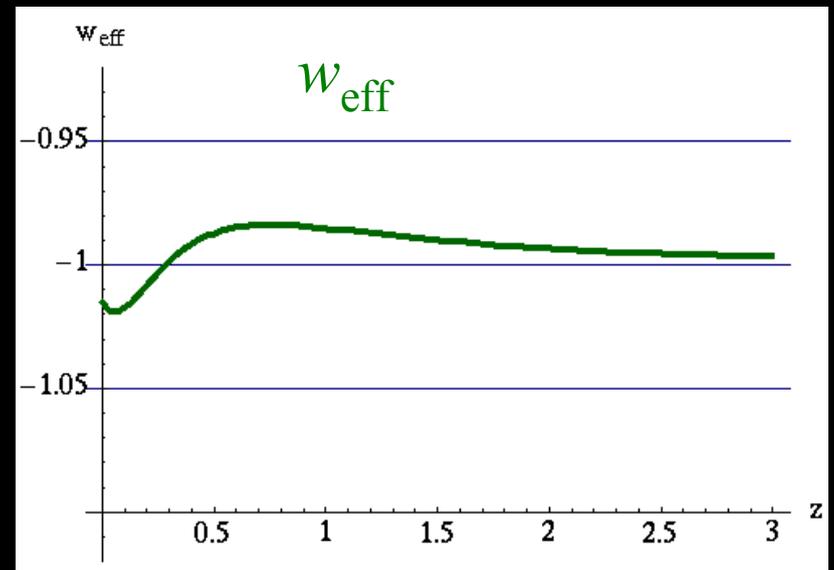
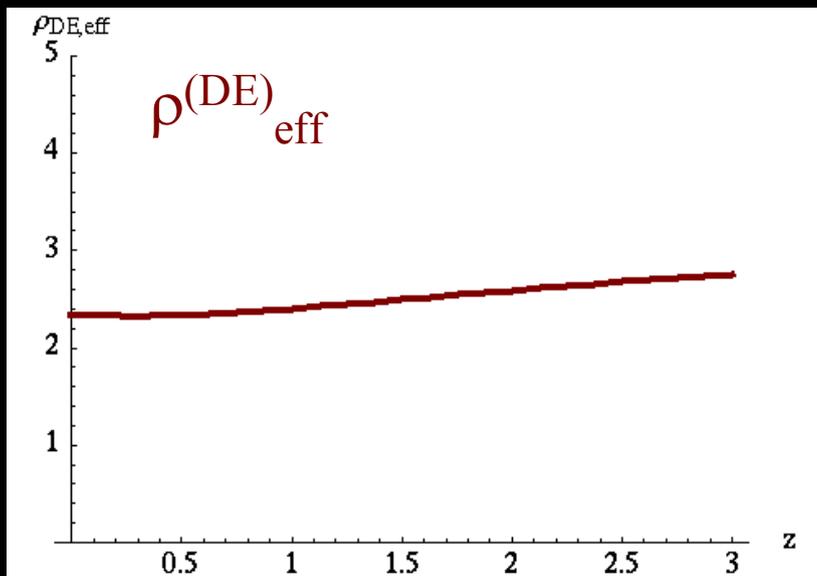
$$H^2 = \frac{8\pi G}{3} e^{-2\psi} \left[1 - 3\rho \left(\frac{d\psi}{d\rho} \right) \right]^{-2} [\rho + U(\psi)]$$



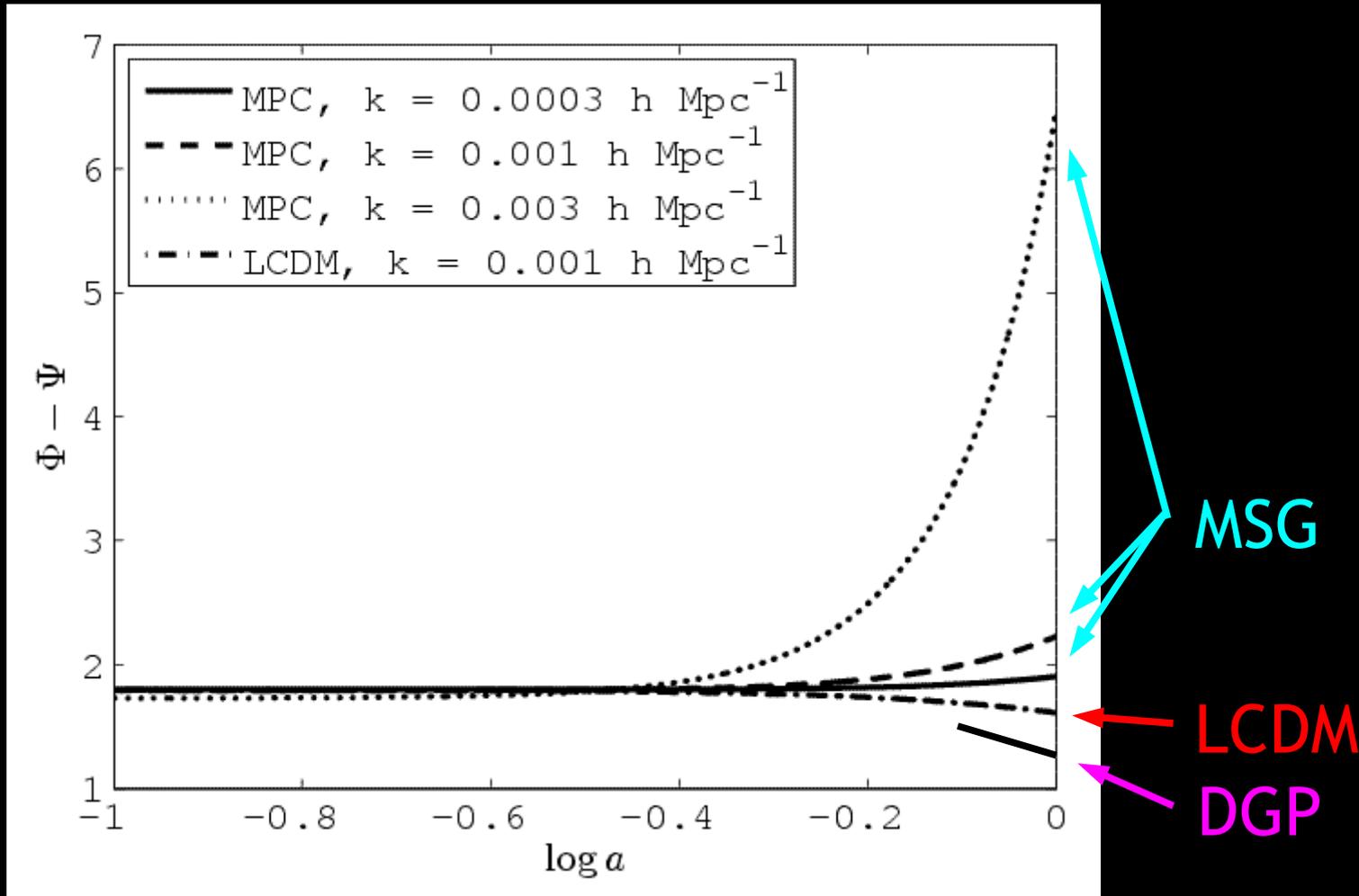
density-dependent
correction to
Newton's constant

ordinary
matter
energy
density

density-
dependent
vacuum
energy



Growth of perturbations: MSG gives a scale-dependent boost to perturbations at late times.



Linear perturbation theory breaks down; further investigation needed.

This suggests a way to **test GR on cosmological scales**: compare kinematic probes of DE to dynamical ones, look for consistency. (Relevant to DGP, MSG, ...)

Kinematic probes [only sensitive to $a(t)$]:

- Standard candles (luminosity distance vs. redshift)
- Baryon oscillations (angular diameter distance)

Dynamical probes [sensitive to $a(t)$ and growth factor]:

- Weak lensing
- Cluster counts (SZ effect)

Outlook

- The universe is accelerating. Time to get serious.
- Plausible modifications of GR are hard to come by. Theoretical requirements and experimental bounds are highly constraining.
- Phenomenological actions in 4 dimensions provide a playground in which to explore manifestations of modified gravity. Nothing natural jumps out.
- We can observationally distinguish between dark energy and modified gravity by comparing the scale factor to perturbation growth.
- Still need to connect with fundamental physics, come up with models that are not only allowed, but actually compelling.

