# Predictive Power of the Strong Coupling in Large Distance Modification of Gravity

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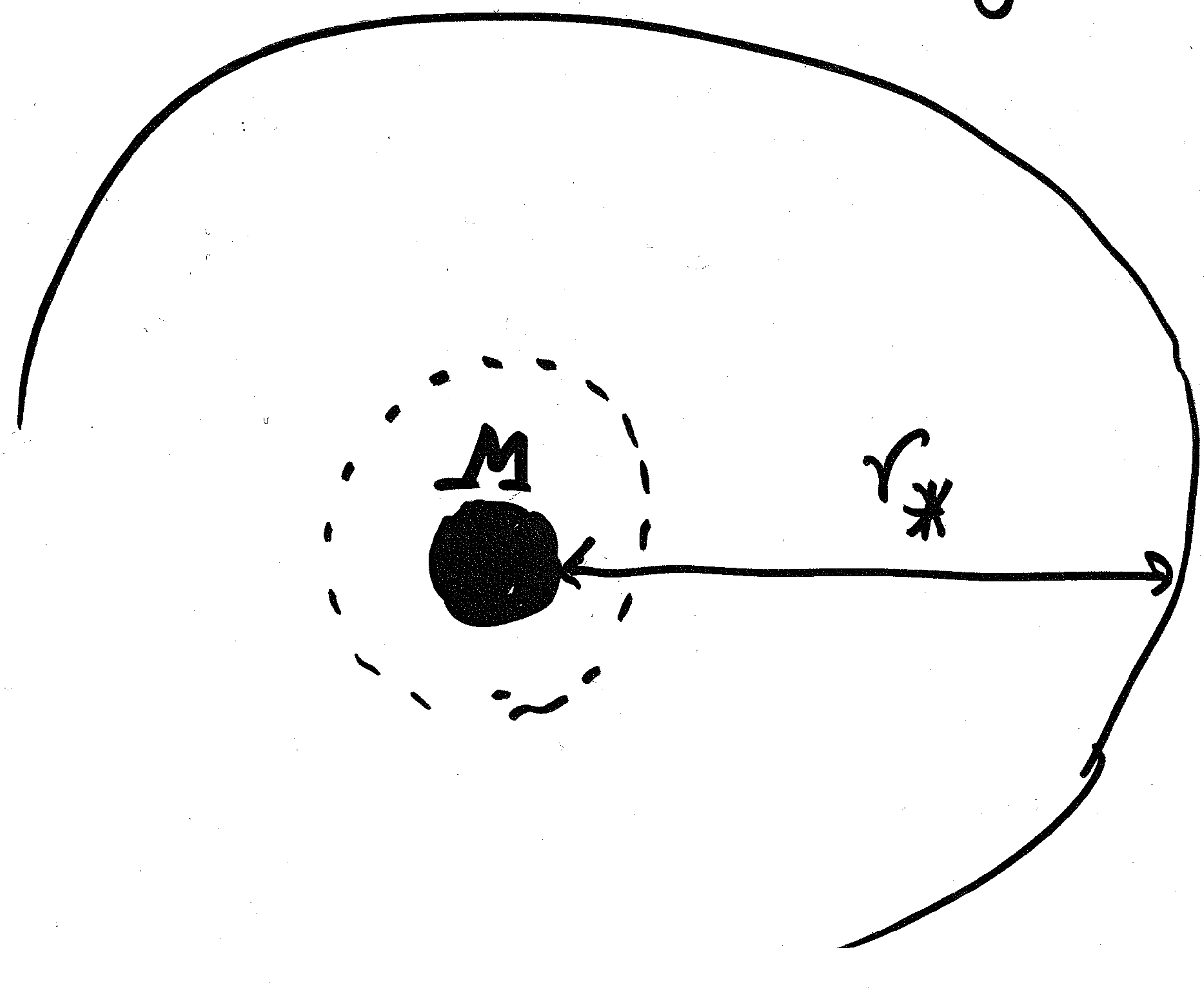
THE OBSERVED COSMIC ACCELERATION MAY SIGNAL THE MODIFICATION OF LAWS OF GRAVITY AT VERY LARGE DISTANCES

$$r_c \sim 10^{28}\ \text{cm}$$

I WILL DISCUSS FUNDAMENTAL THEORIES OF GRAVITY THAT PREDICT SUCH A MODIFICATION, AND THEIR EXPERIMENTAL CONSEQUENCES.

Any consistent theory of large distance modified gravity must exhibit a strong coupling phenomenon.

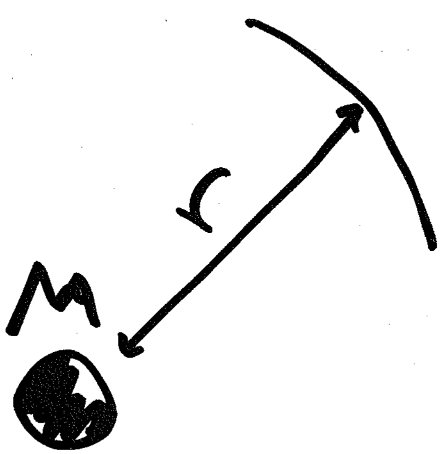
Due to this, gravitating sources have a second "horizon" $r_* \geq r_g \equiv 2G_N M$



Deviation from Einsteinian potential near gravitating objects ($r \ll r_*$)

$$\delta \approx \left(\frac{r}{10^{28}\,\mathrm{cm}}\right)^{2-2\alpha} \sqrt{\frac{r}{r_g}}$$

where $0 \le \alpha \le 1$




$$r_g \equiv 2 G_N M$$

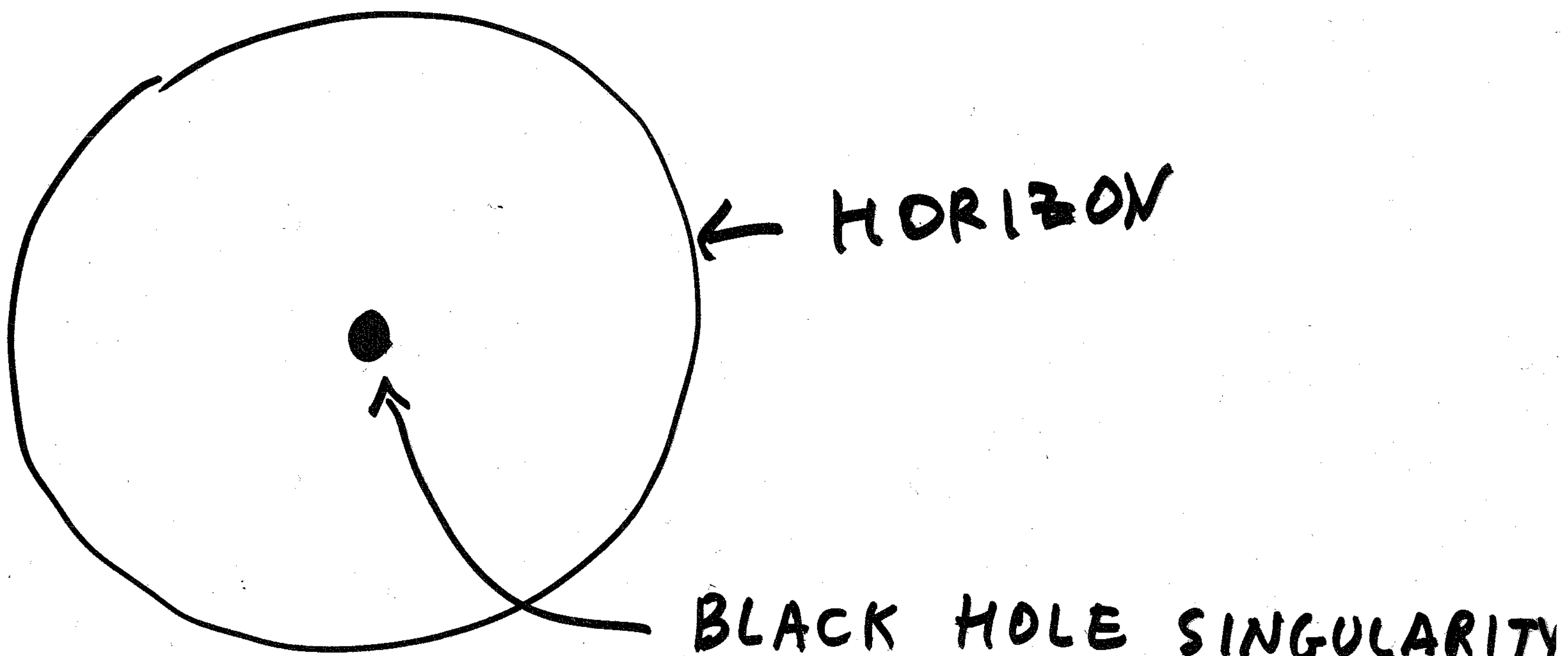
In contrast with the dark energy models, the modified gravity theories are <u>extremely</u> constrained, and imply new dynamics, which is testable by:

- ⊛ Precision cosmology;
- ⊛ Precision gravitational measurements at all distances.

  E.g. Planetary motion

- ⊛ ~~Ex~~ In some cases by:
  1. sub-mm gravity measurements
  2. LHC

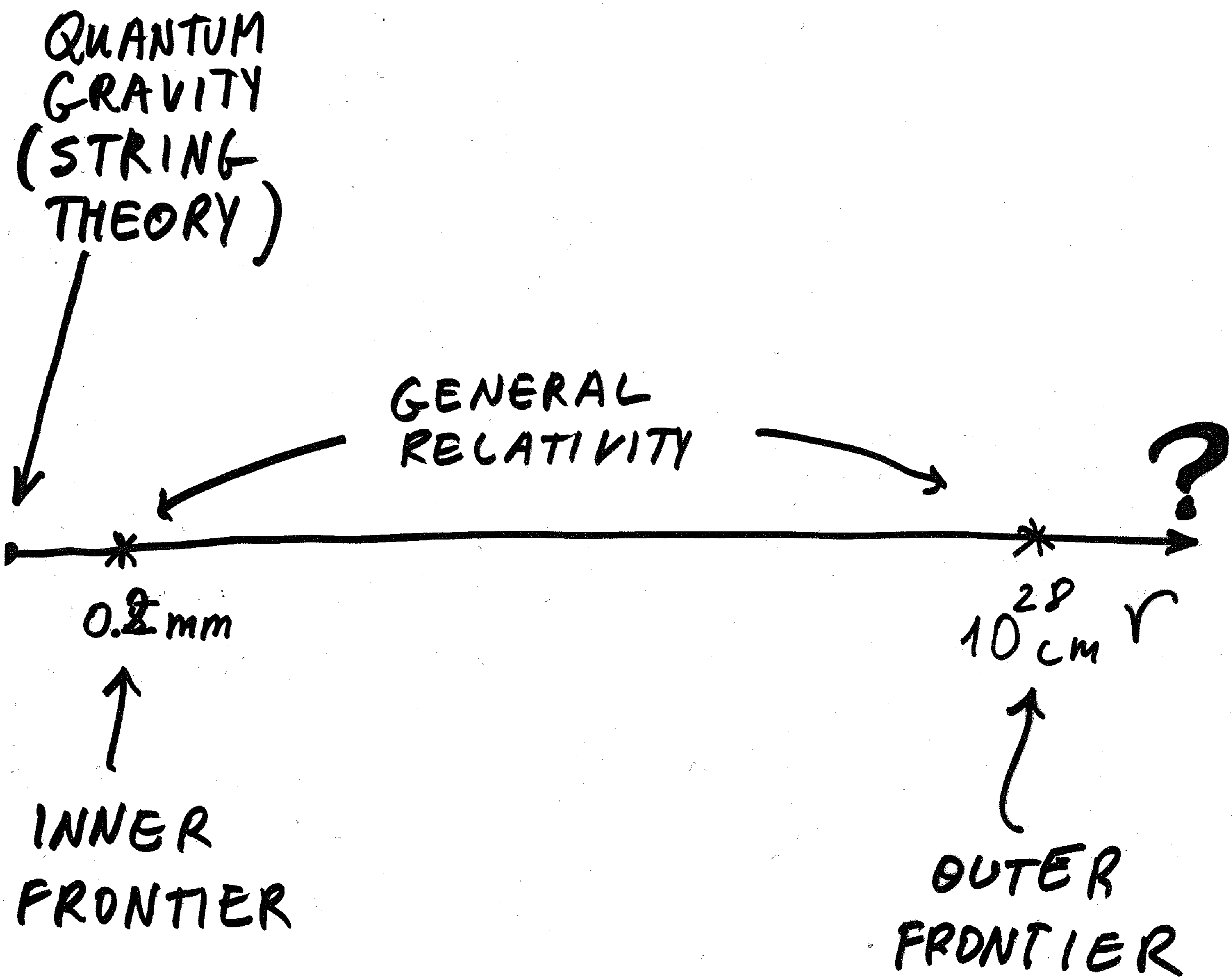
WE KNOW THAT GENERAL RELATIVITY MUST BE MODIFIED (EMBEDDED IN A BIGGER THEORY) AT THE SHORT DISTANCES:




ALSO,
BIG BANG SINGULARITY.

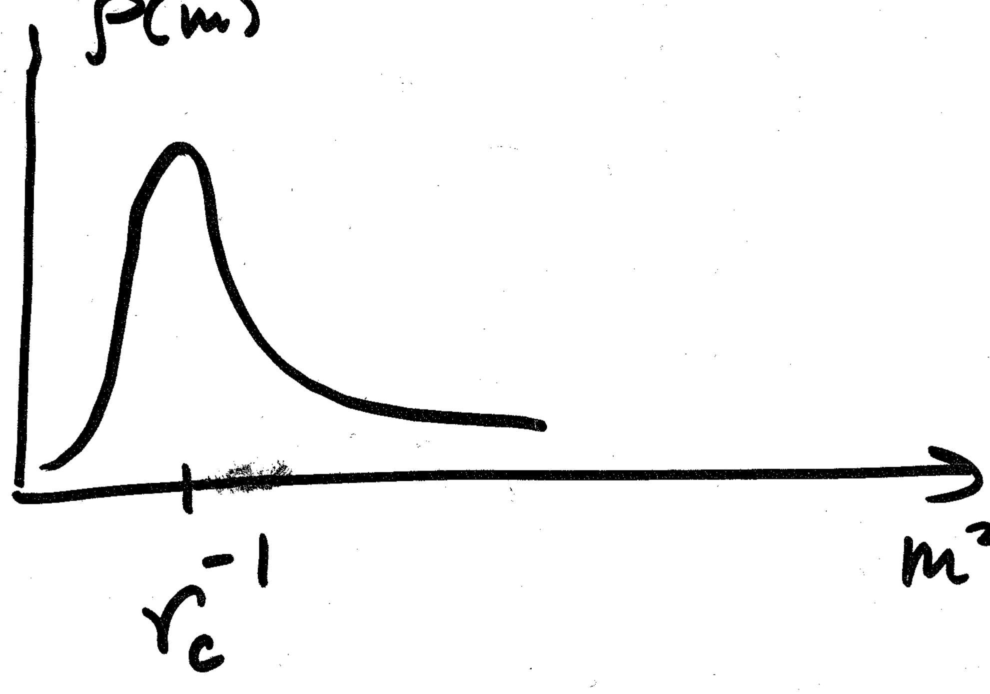
<u>WHAT ABOUT</u> <u>THE LARGE DISTANCES</u>?

# THE REGIONS OF KNOWN AND UNKNOWN:



Any consistent (general covariant, ghost-free) theory of infrared modified gravity must be a theory of infinite number (continuum) of massive spin-2 states:

$$h_{\mu\nu} = \int h^{(m)}_{\mu\nu}\, \rho(m)$$



The only ghost-free theory of linearized massive gravity
Pauli-Fierz mass

$$m_g^2 \left(h_{\mu\nu} h^{\mu\nu} - (h_\mu^\mu)^2\right)$$

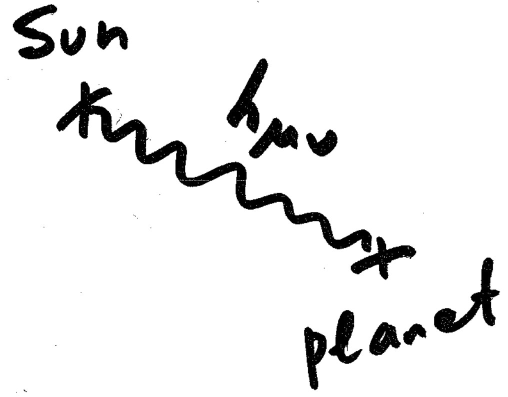
$h_{\mu\nu}$ contains 5 polarizations.

$$5 = \underset{\uparrow}{2} + 2 + 1$$

massless graviton contains 2.

# vDVZ - discontinuity

Metric produced by a static gravitating source $T_{\mu\nu}$ in the two theories:




$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

massive $\longrightarrow$ $$h_{\mu\nu} = G_N \frac{T_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu} T^{\alpha}_{\alpha}}{r} e^{-rm}$$

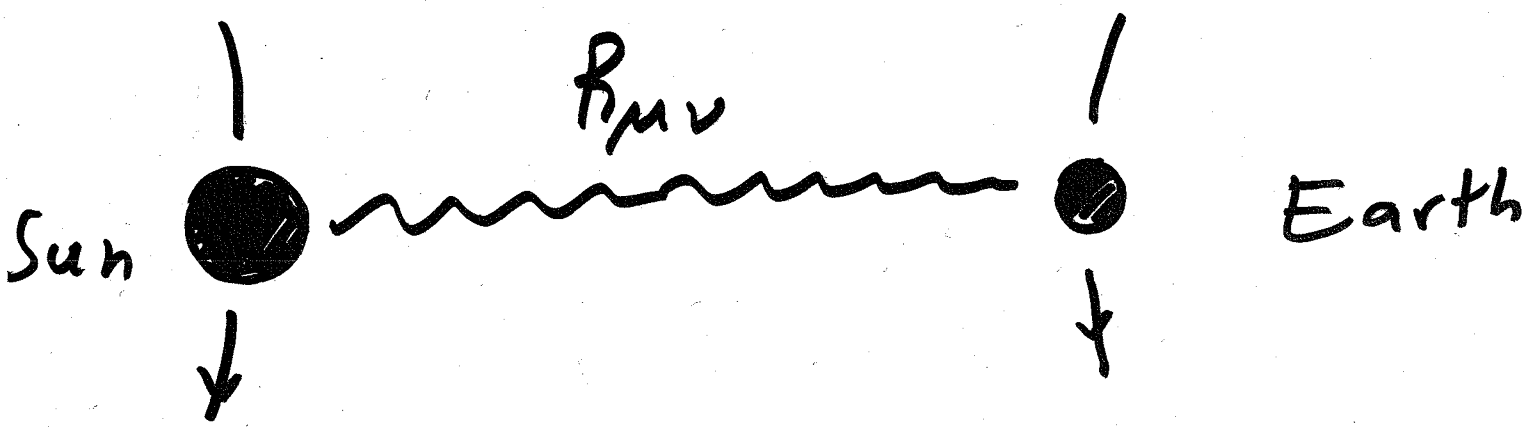
massless $\longrightarrow$ $$h_{\mu\nu} = G_N \frac{T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T^{\alpha}_{\alpha}}{r}$$

Thus, in any linear theory of large distance modified gravity, the metric produced by a static source $T_{\mu\nu}$ is

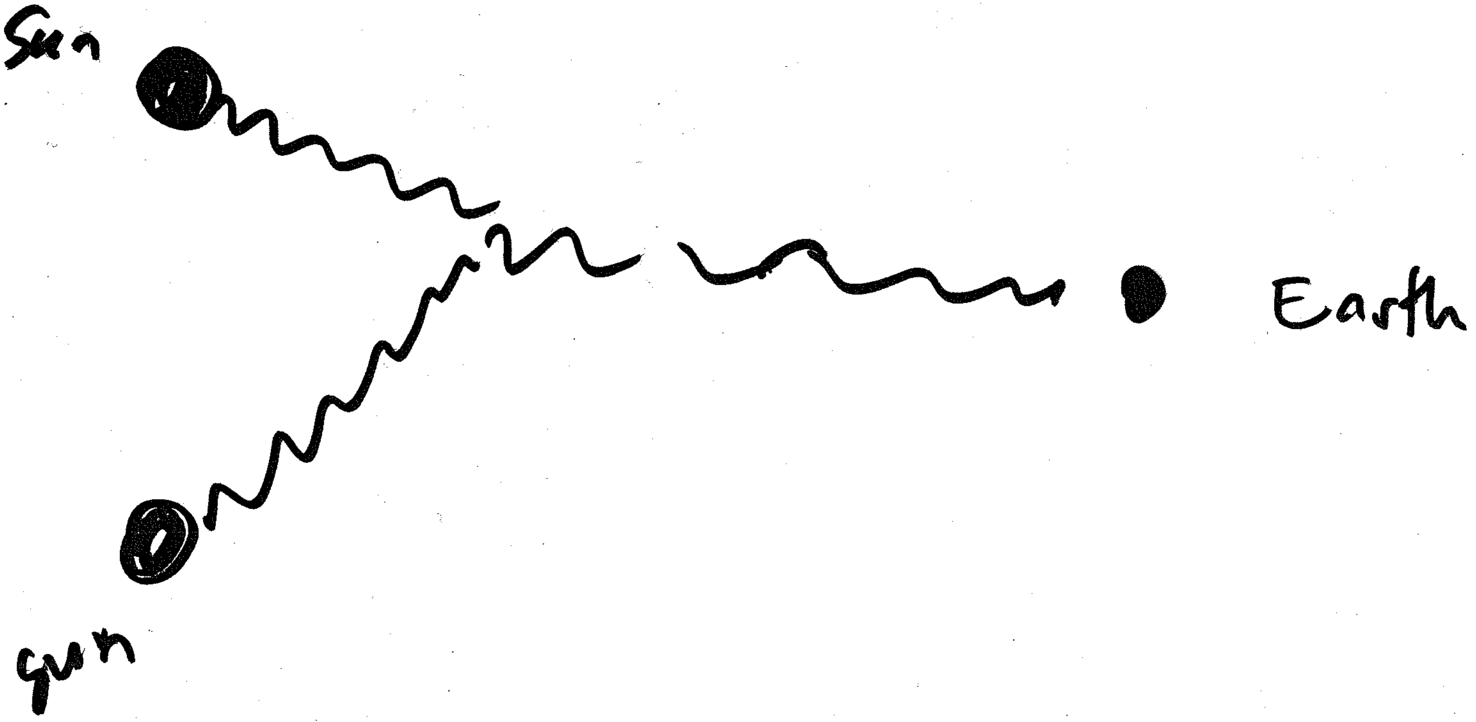
$$h_{\mu\nu} = G_N \left( T_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} T^{\alpha}_{\alpha} \right) \int_0^{\infty} dm \, \frac{e^{-rm}}{r} \rho(m)$$

So, is large distance modification of gravity ruled out?

Gravitational force between Earth and Sun is mediated by graviton $h_{\mu\nu}$



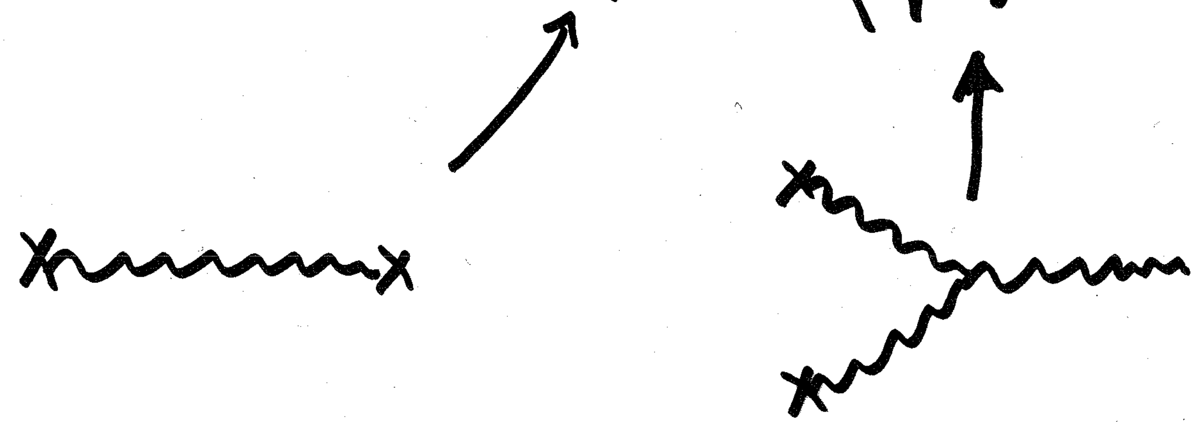

$$V(r) = G_N \frac{M_S m_E}{r}$$



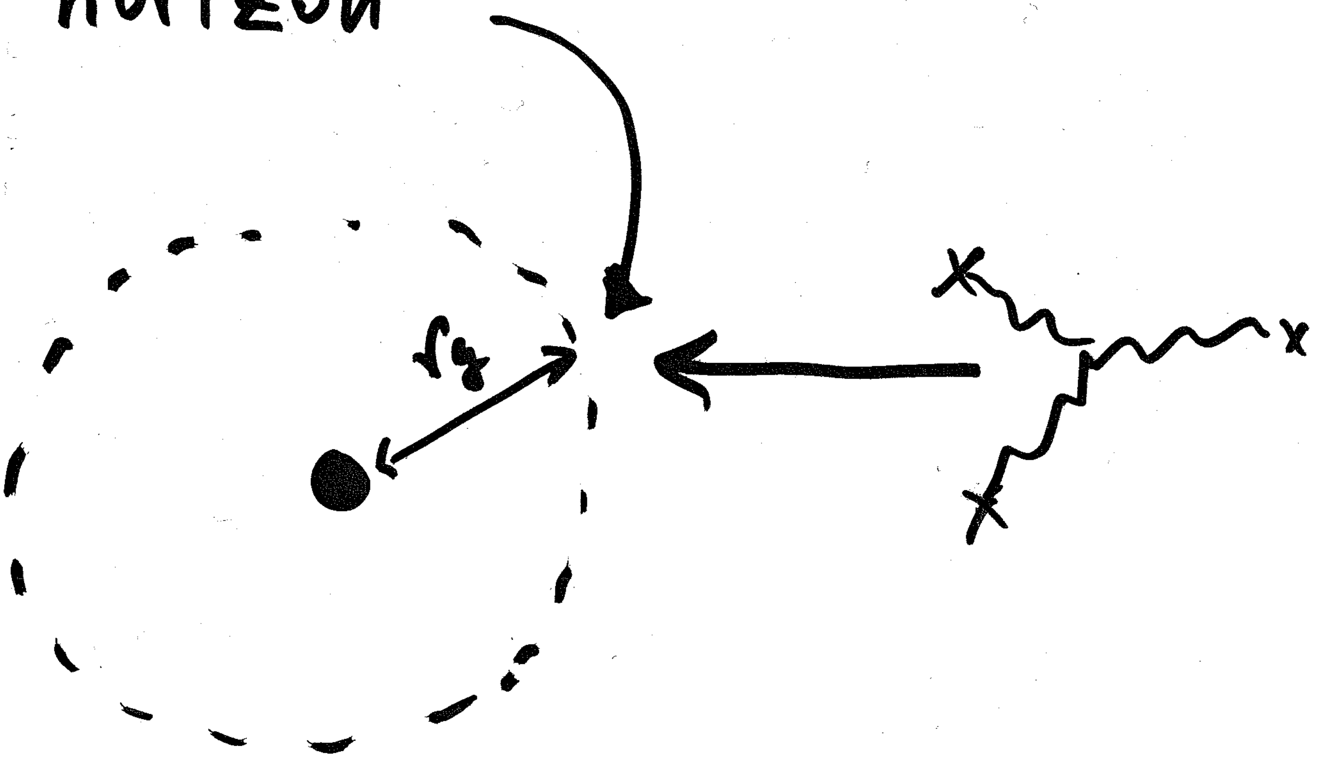
Metric produced by a heavy source in GR:

$$r_g \equiv 2G_N M$$

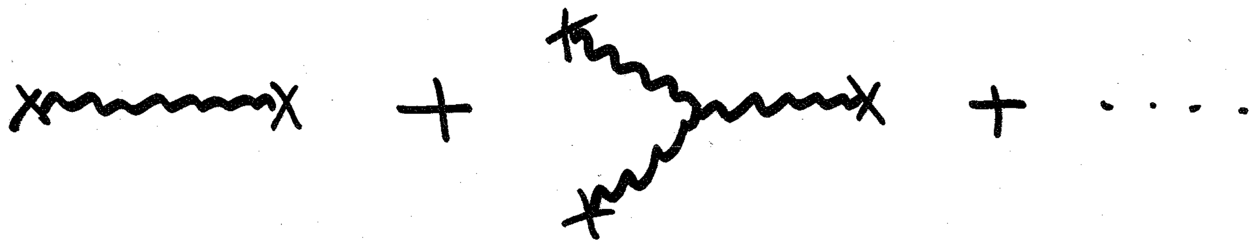
$$\text{Metric} = \text{Flat} + \frac{r_g}{r} + \left(\frac{r_g}{r}\right)^2 + \cdots$$



In Einstein's GR nonlinearities become important only at the horizon



So any modified gravity theory from the above class is ruled out unless the expansion in $G_N$ breaks down at the Solar system distances.

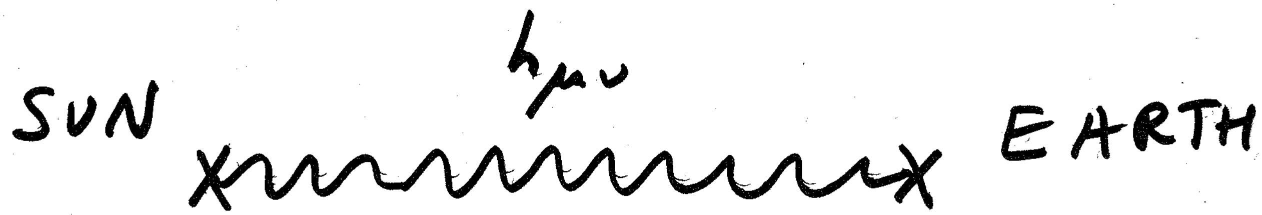


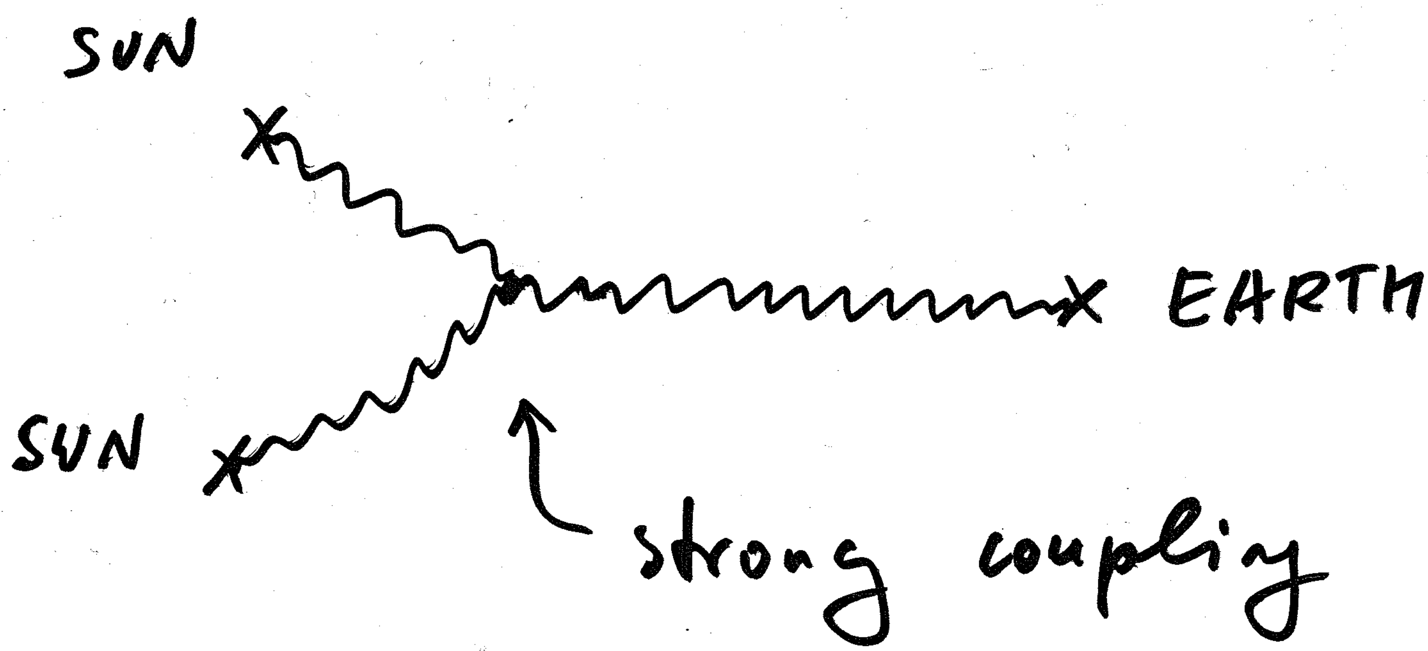
$$G_N \quad + \quad G_N^2 \quad + \cdots$$

This is exactly what happense in massive gravity and in any other consistent modified gravity theory!

# THE STRONG COUPLING EFFECT OF EXTRA POLARIZATIONS

Deffayet, GD, Gabadadze, Vainshtein (2001)



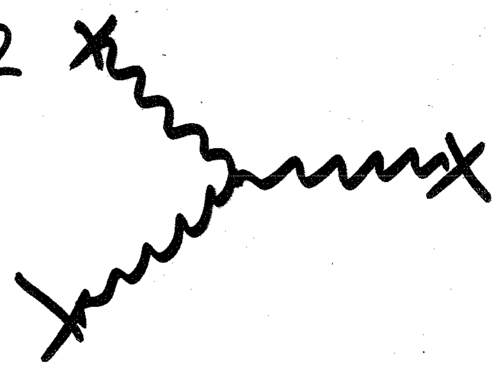



2-polarizations

$$h_{\mu\nu} = h^{\perp}_{\mu\nu} + \frac{1}{2}\eta_{\mu\nu} h^{\parallel} + \frac{\partial_\mu \partial_\nu}{m^2} h^{\parallel}$$


$$\frac{p^5}{m_g^4 M_p}$$

Because of the strong coupling in the solar system

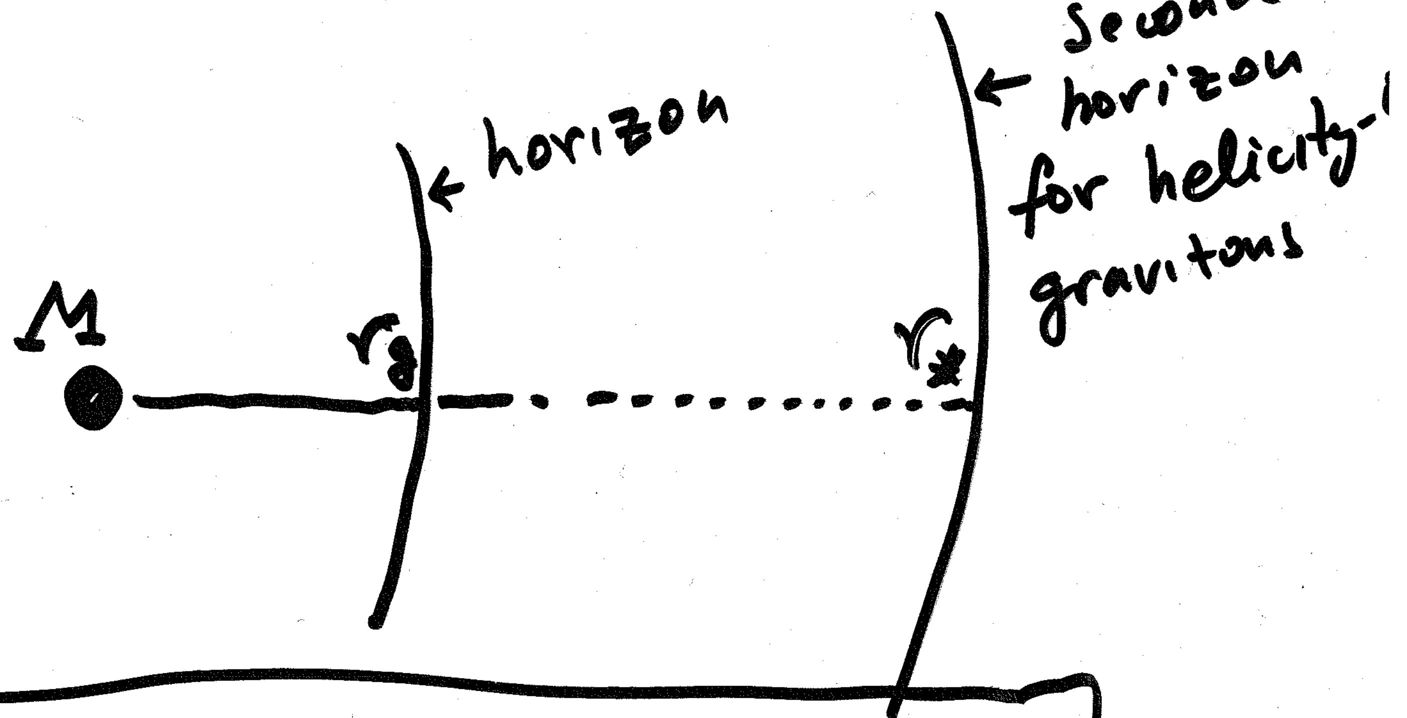
 $= 10^{-32}$  !

$G_N$ is not a good expansion parameter.

So is modified gravity compatible with observations?

The concept of $r_*$

For a source of gravitational radius $r_g \equiv 2 G_N M$




$$\boxed{r_* \equiv \left( r_c^{4-4\alpha}\, r_g \right)^{\frac{1}{5-4\alpha}}}$$

For $r \ll r_*$

$$h^{(0)} \simeq \frac{r_g}{r_*}\left(\frac{r}{r_*}\right)^{\frac{3}{2}-2\alpha}$$

which predicts the following relative correction to Einstein's gravitational potential:

$$\delta \simeq \left(\frac{r}{r_c}\right)^{2-2\alpha}\sqrt{\frac{r}{r_g}}$$

$$r_c \simeq 10^{28}\,\mathrm{cm}$$

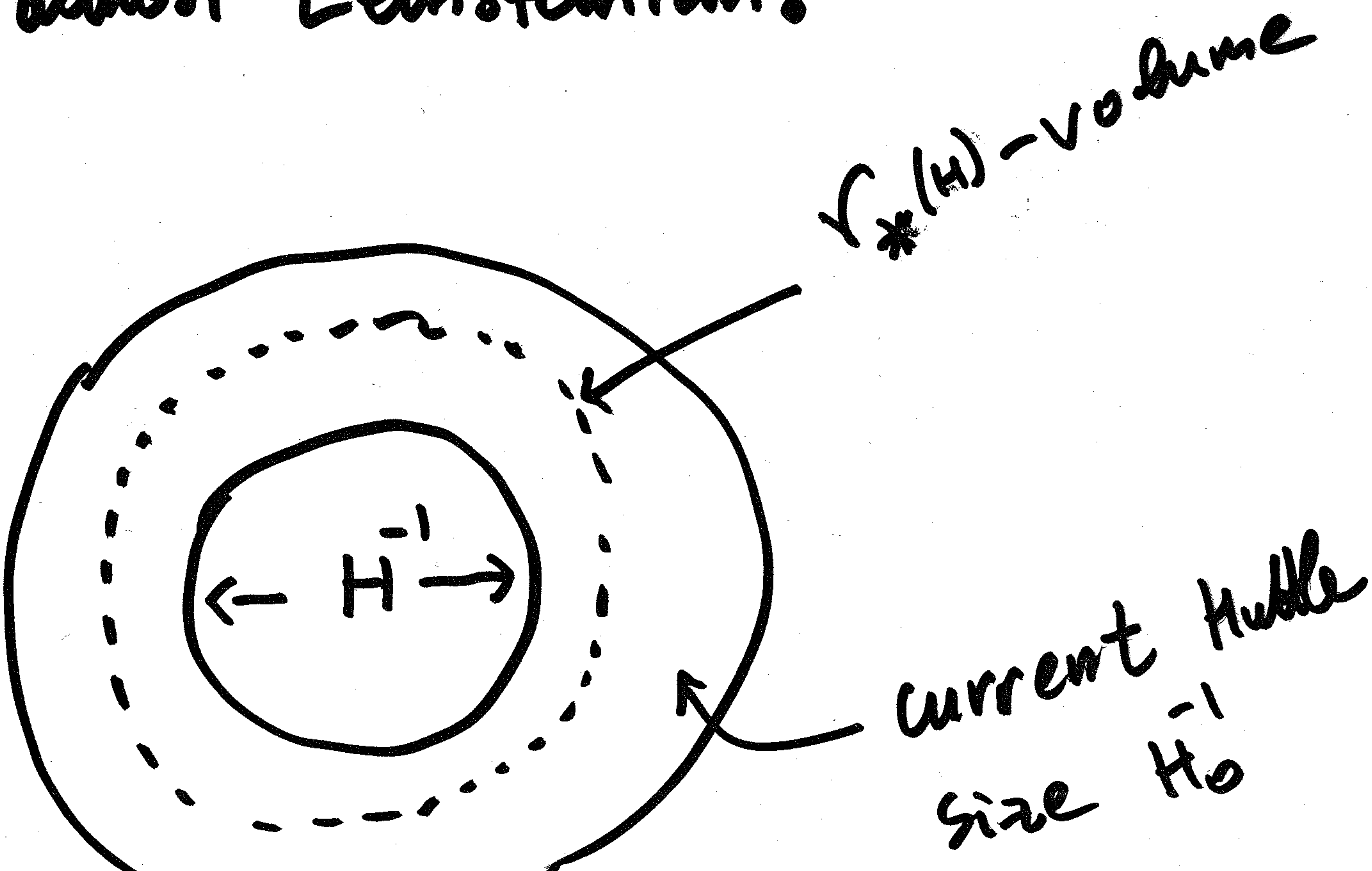
For example, take the Universe.

Because, $r_c = 10^{28}\,\text{cm} = \text{current Hubble size} \equiv H_0^{-1}$

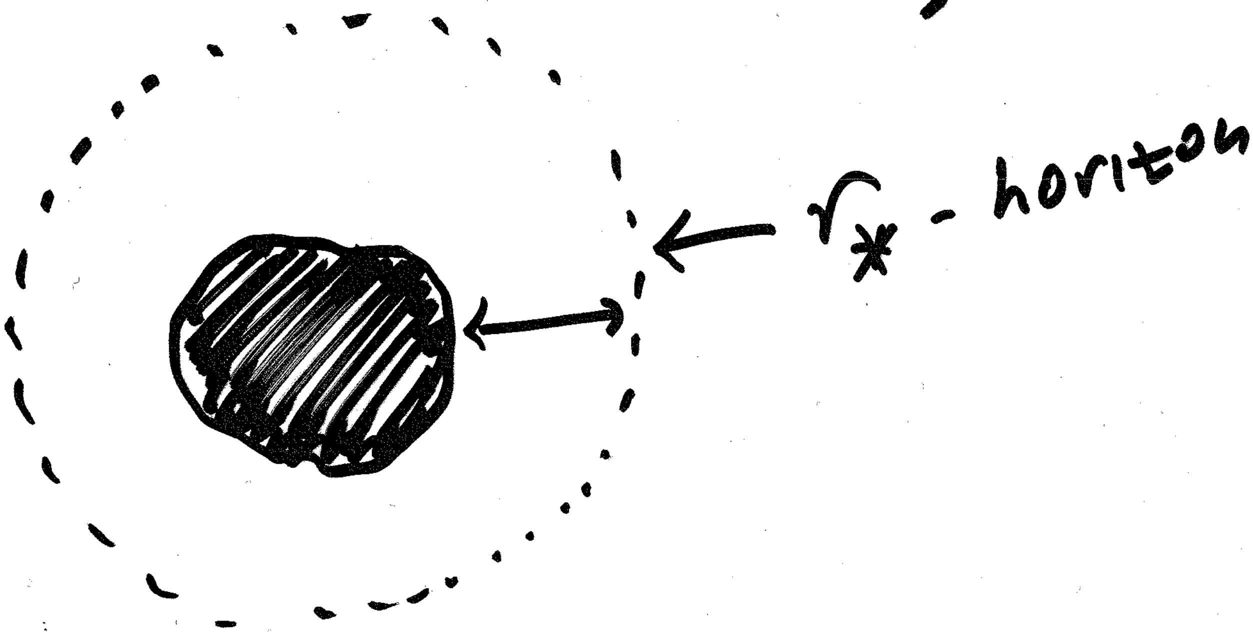
in any epoch in which

$$H > H_0$$

the Universe is within its own $r_*(H)$, and cosmology is almost Einsteinian!



The rule:
For a source that is localized within its own $r_*$, gravity is almost Einsteinian,




with small corrections given by:

$$\delta \simeq \left(\frac{r}{10^{28}\,cm}\right)^{2-2\alpha} \sqrt{\frac{r}{r_g}}$$

## Very important:

We have a good controle of the corrections both for $r > r_*$ and for $r < r_*$.

But, at $r \sim r_*$ both expansions, in $\frac{r}{r_c}$ and in $\frac{r_g}{r}$, break down.

So, there are no known perturbative methods for finding the spectrum of Longitudinal gravitons, for sources $r \sim r_*$!

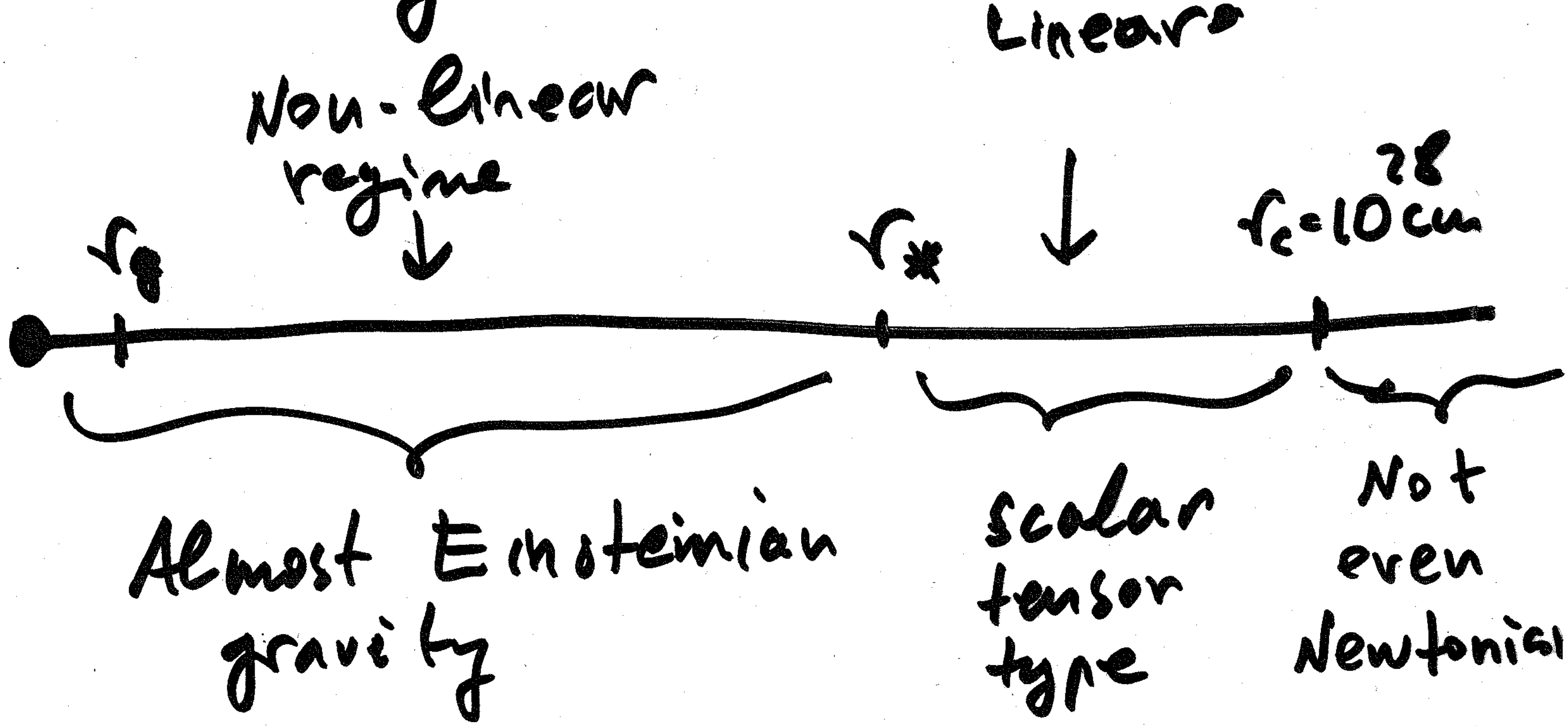
For example, in $\alpha = \frac{1}{2}$ model (DGP) on self-accelerating cosmological solution (see Deffayet) Universe today is at its $r_*$, and

$$\Box h^{(0)} \sim \Lambda^3.$$

So analysis of instabilities in $h^{(0)}$ cannot be trusted unless one knows how to resum the full series

$$\left[1 - \frac{\Box h^{(0)}}{\Lambda^3} + \text{higher terms}\right] \partial_\mu h^{(0)} \partial^\mu h^{(0)}$$

Now we are ready to formulate general properties of any large distance modified gravity theory:




For $r < r_*$ corrections to the gravitational potential are:

$$\delta \simeq \left(\frac{r}{10^{28}\,\mathrm{cm}}\right)^{2-2\alpha} \sqrt{\frac{r}{r_g}}$$

Of course, for some $\alpha$ the set of consistent theories may be empty.

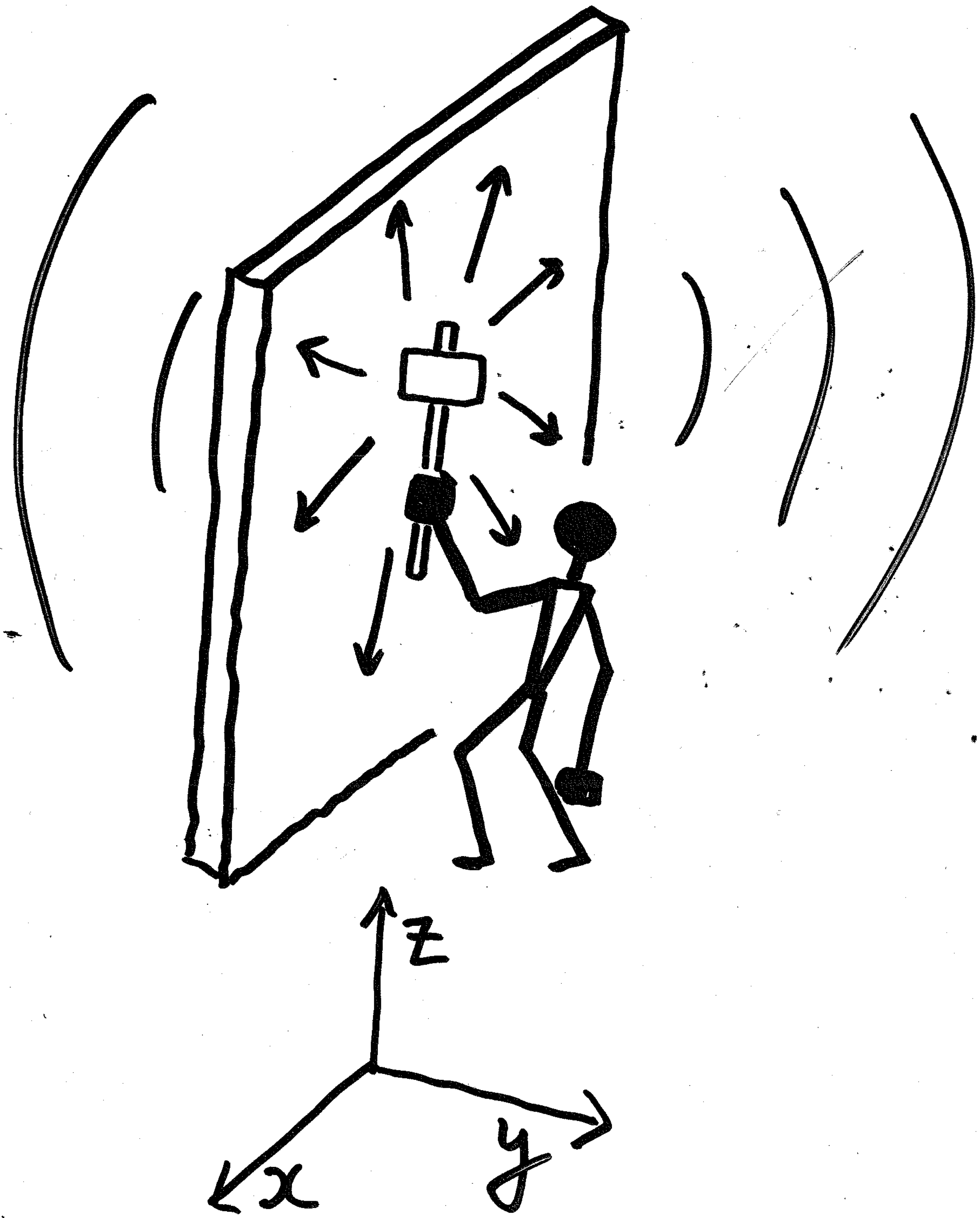
In fact, currently the only known consistent theories are $\alpha = \frac{1}{2}$ (and $\alpha = 0$?) in which gravity becomes high-D at $r > r_c$.

(*)

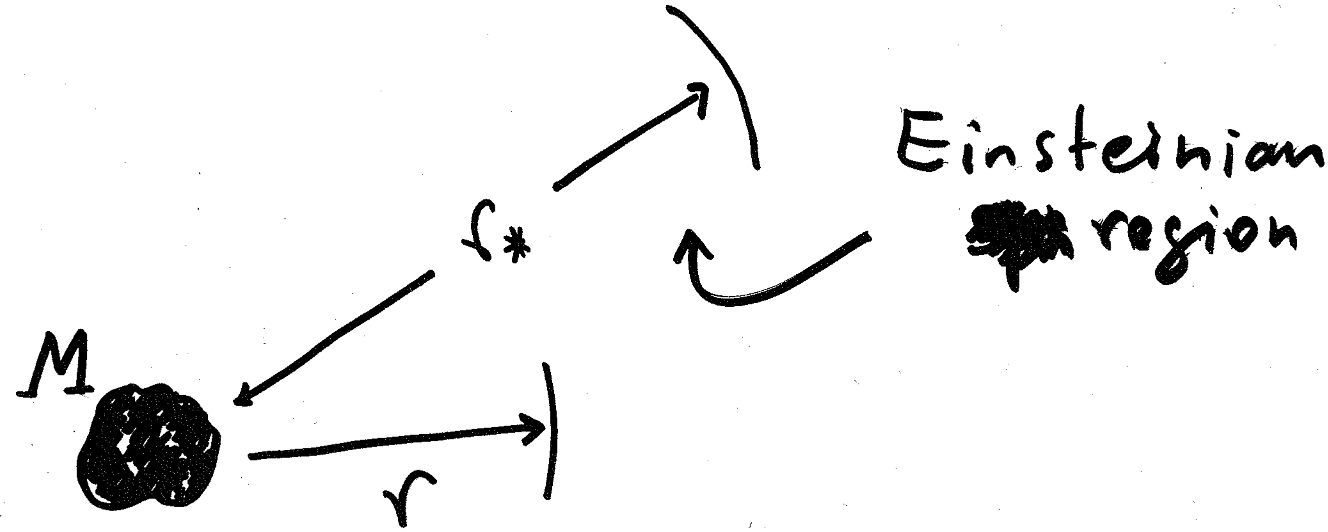
$\alpha = \frac{1}{2}$ - theory (see talk by Deffayet) can be tested both by cosmology and by precision gravity experiments.

# SOUND WAVE

DGP '00




$$\left\{ \delta(y)\, \Box_{2+1} + \frac{1}{r_c} \Box_{3+1} \right\} \psi = \delta^4(x)$$




$$r_g = 2 G_N M$$

Change of gravitational potential relative to GR

$$\frac{\delta\varphi}{\varphi} \sim \frac{r}{10^{28}\,\mathrm{cm}} \sqrt{\frac{r}{r_g}}$$

Predicted anomalous perihelion precession:

C.D., Gruzinov, Zaldarriaga; Lue, Starkman

$$\delta\phi = \left(\frac{3\pi}{4}\right)\left(\frac{r}{10^{28}\,\mathrm{cm}}\right)\sqrt{\frac{r}{r_g}}$$

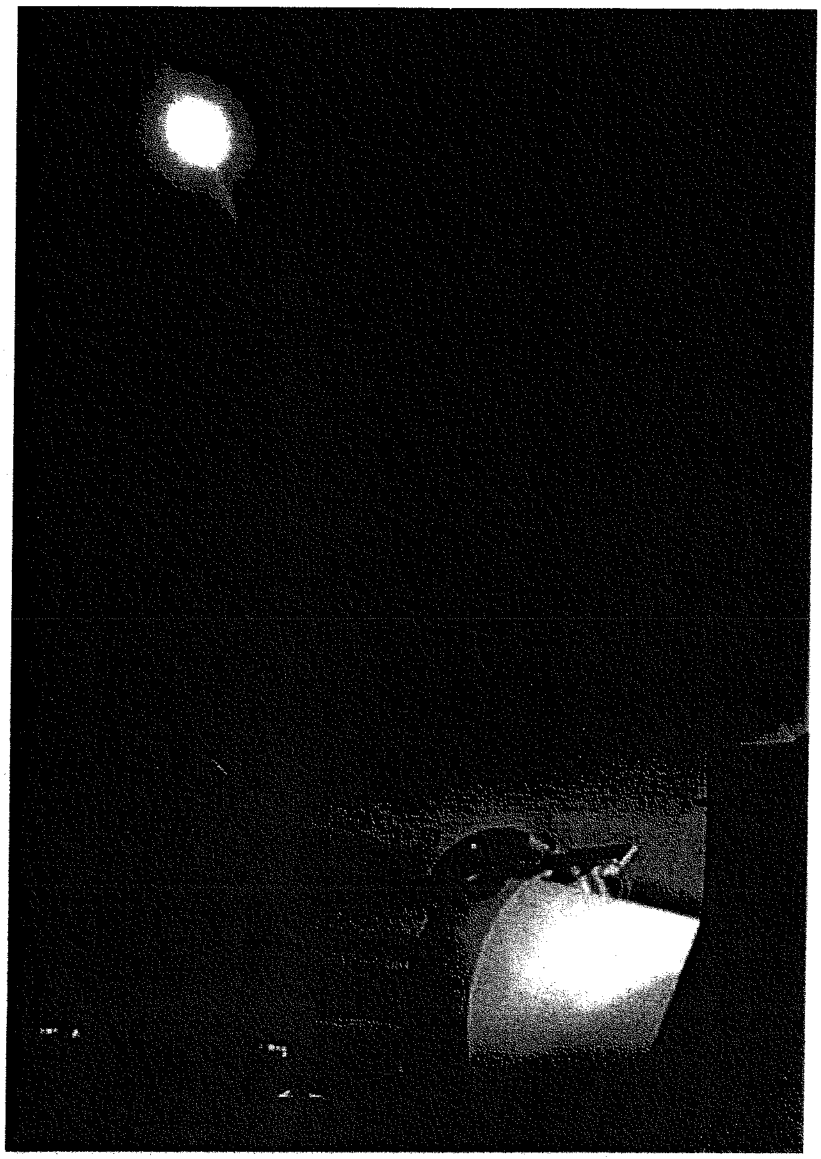
For Earth-Moon System:

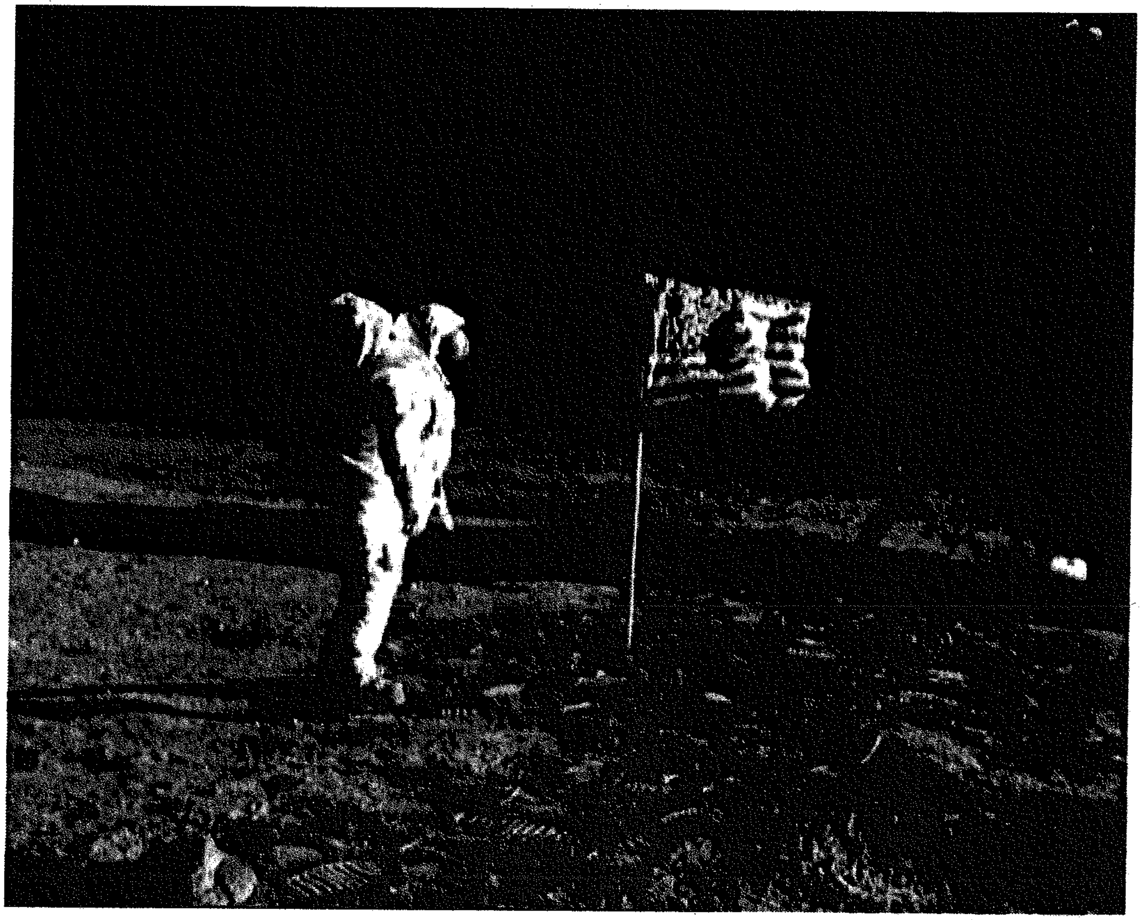
$$\delta\phi = 1.4 \times 10^{-12}$$

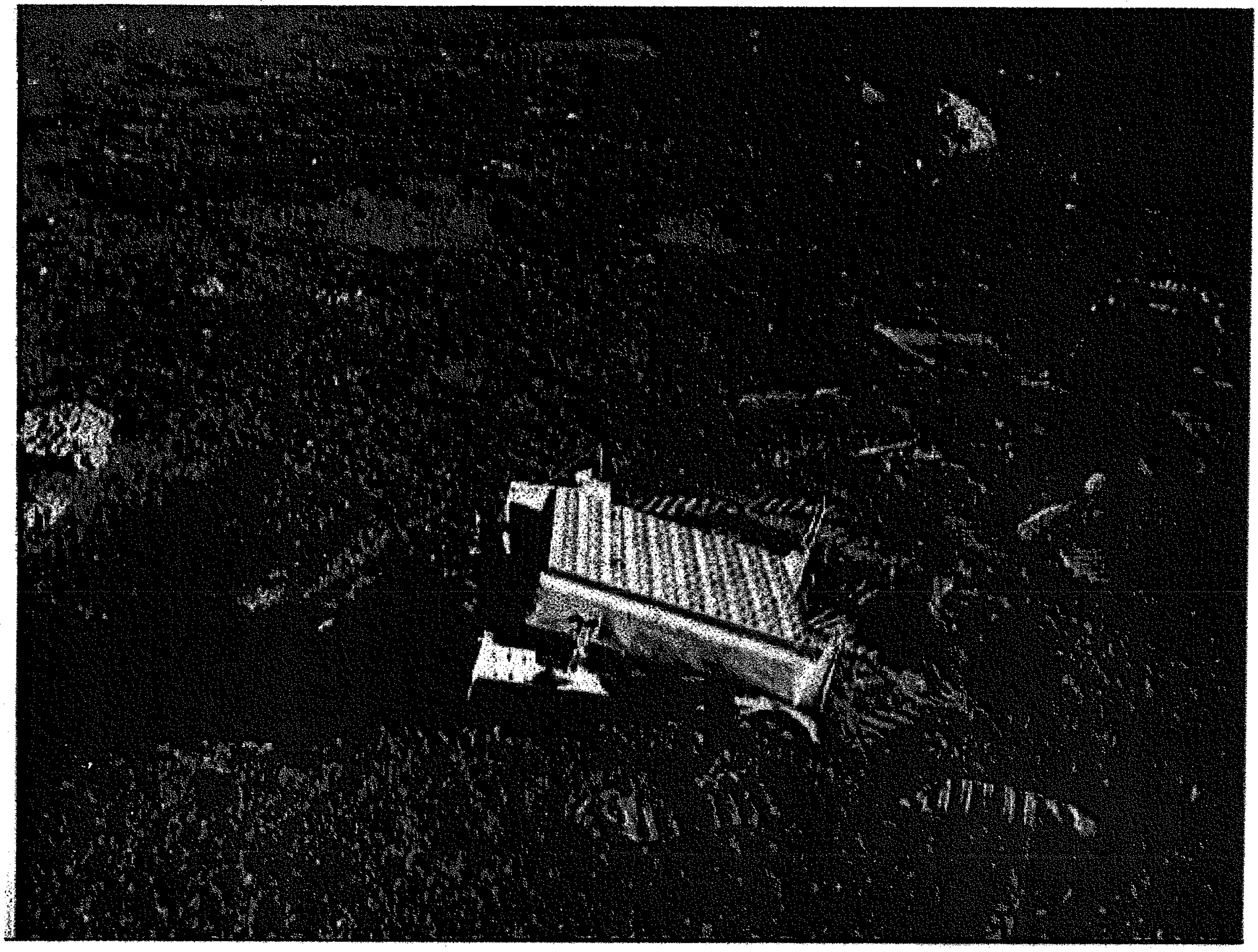
Today's accuracy: $\sigma_\phi = 2.4 \times 10^{-11}$

Can be tested by improved Lunar Laser Ranging

Adelberger, Stubbs, Murphy ....







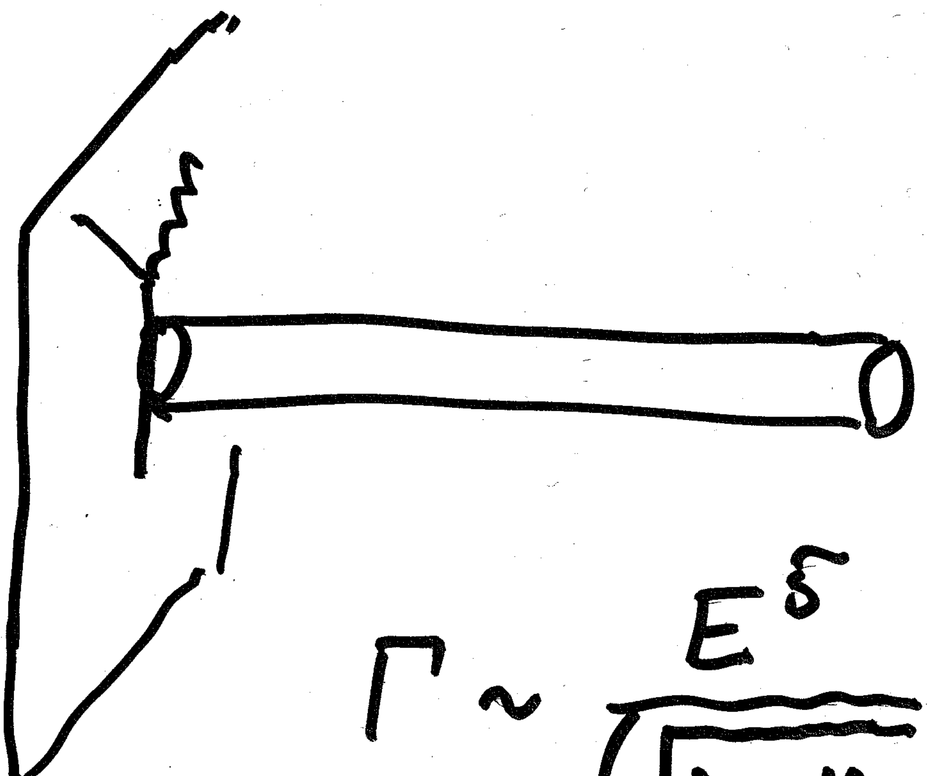
Codimension $N > 1$

$$r_c = \frac{M_p}{M_*^2} \longrightarrow M_* \sim 10^{-3}\,\text{eV}$$

$$\downarrow$$

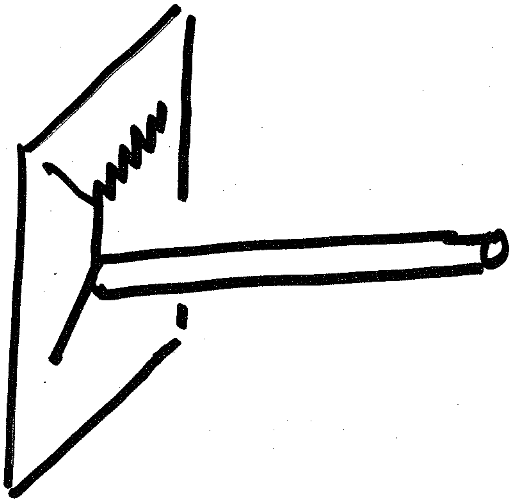
$$r_c \sim H_0^{-1} = 10^{28}\,\text{cm}$$

Hagedorn transition at LHC



$$\Gamma \sim \frac{E^5}{\left(\sqrt{M_p M_*}\right)^4}$$

Provided bulk spectrum looks something like closed strings,



HAGEDORN "CATASTROPHE" ABOVE THE SCALE $\sqrt{M_{pe} M_*} \sim \text{TeV}$

$$\Gamma \sim \sum_{n=2}^{\infty} E \left( \frac{E^2}{M_{pe} M_*} \right)^{2n-2} \sim \sum E \left( \frac{E}{\text{Tev}} \right)^{4n-4}$$

$$\Gamma_{E<\text{Tev}} \sim \frac{E^5}{\text{TeV}^4}$$