Inflation after WMAP

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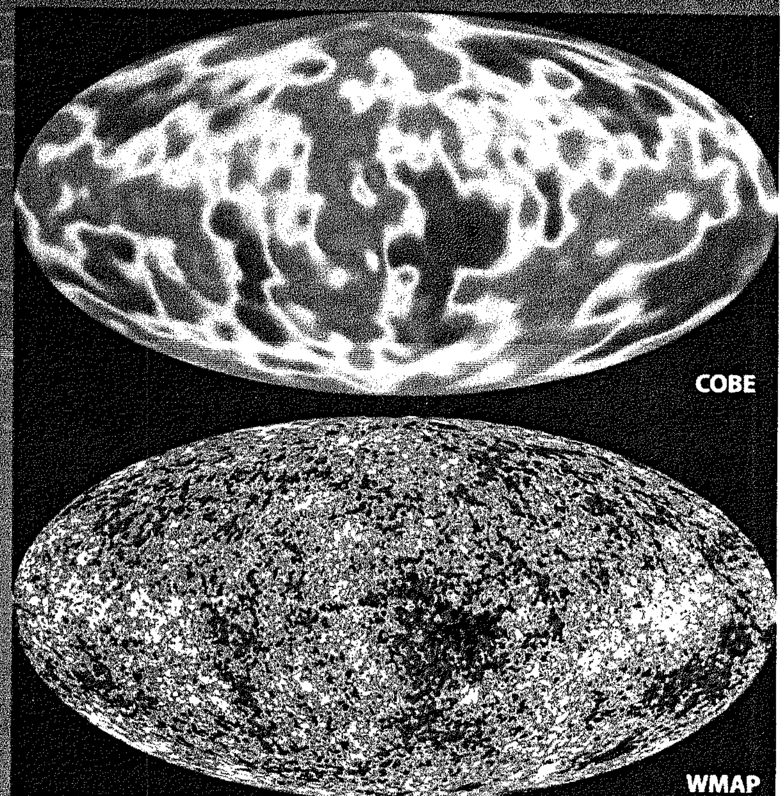


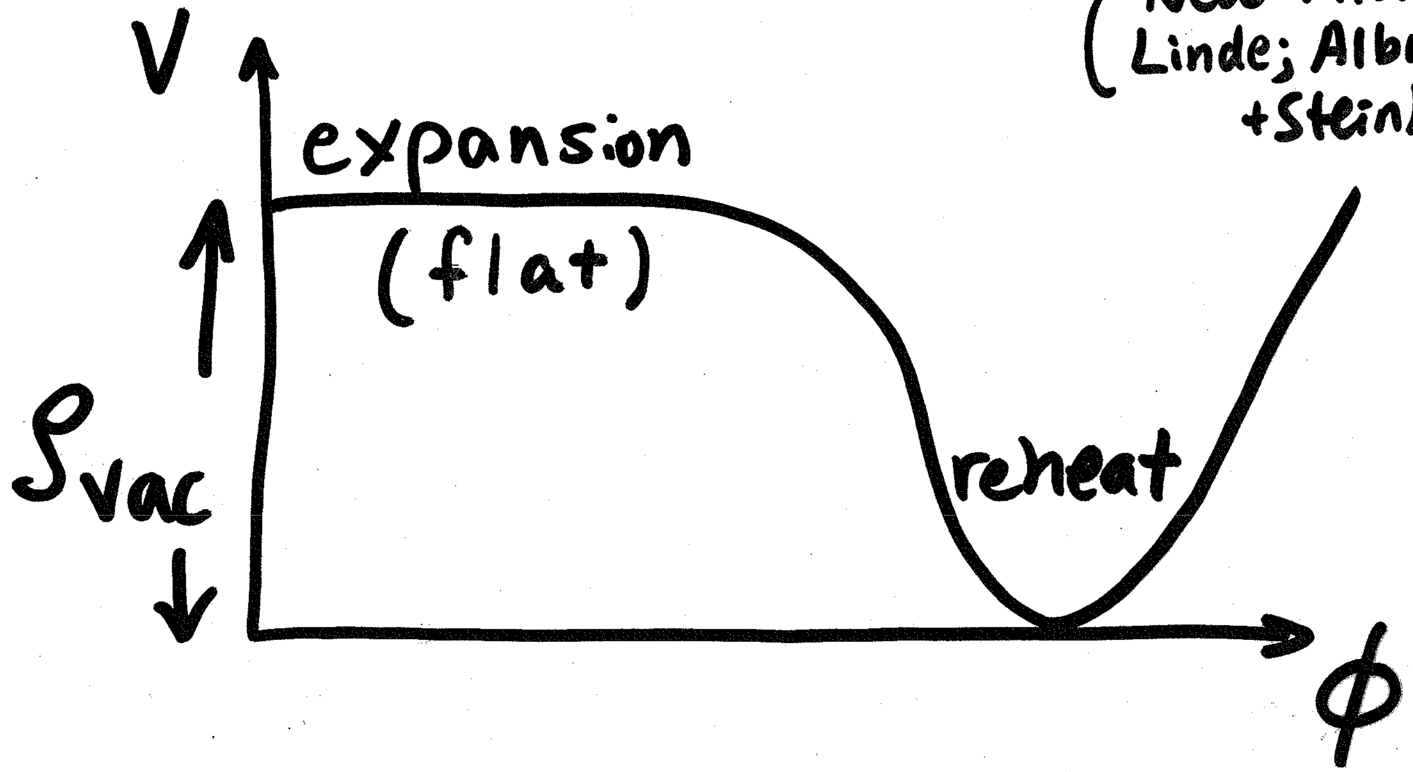
SUMMARY:

- **I. The predictions of inflation are right:**
  - (i) the universe has a critical density
  - (ii) Gaussian perturbations
  - (iii) density perturbation spectrum nearly scale invariant
  - iv) detection of polarization (from gravitational wave modes) in upcoming data may provide smoking gun for inflation
- **II. Polarization measurements will tell us which model is right.**
  - WMAP already selects between models.
  - Natural inflation (Freese, Frieman, Olinto) looks great

[redacted] 2)

Rolling Models of Inflation :

(New inflation: Linde; Albrecht + Steinhardt)



While ϕ rolls along flat part, V is almost constant and ρ_{vac} dominates energy density

$$\rightarrow R \approx R_i e^{Ht}$$

eqn. of motion for ϕ

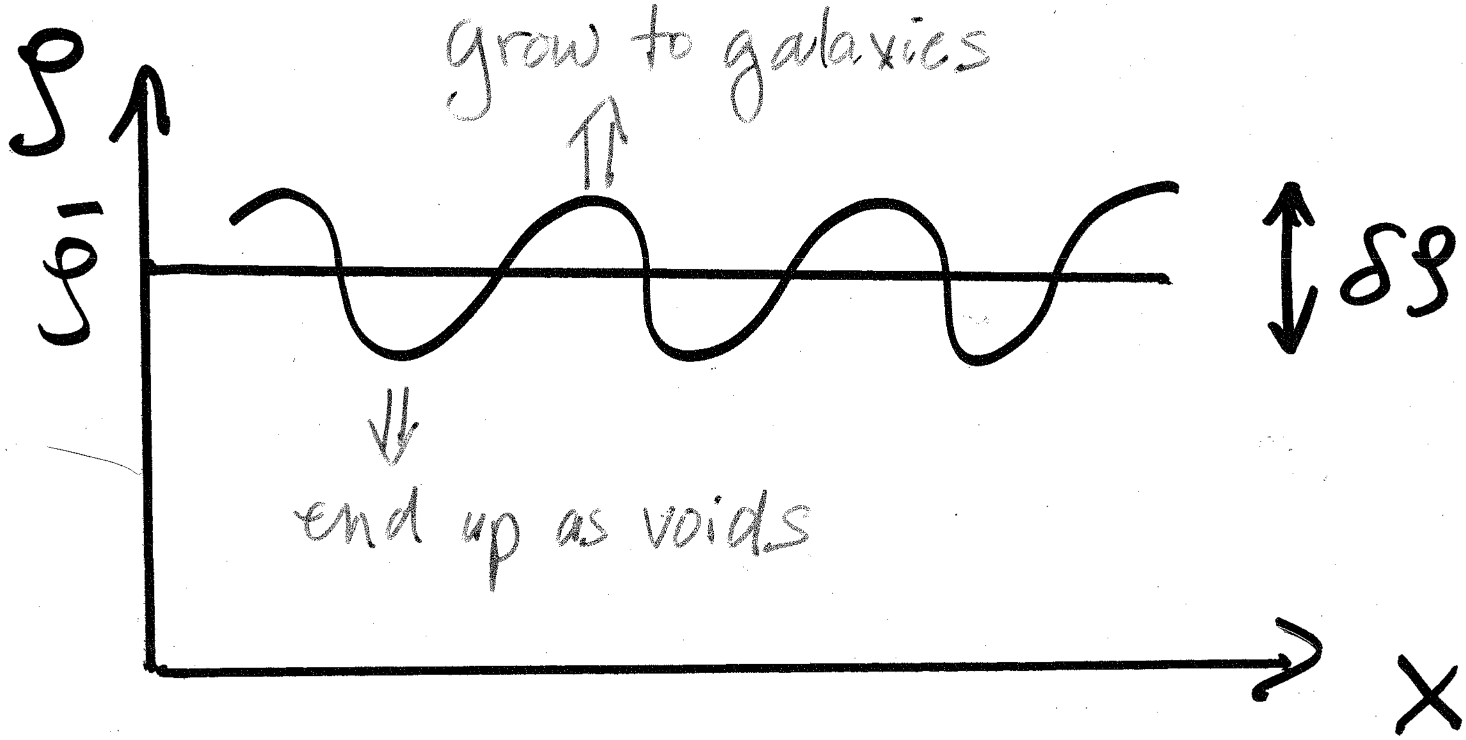
$$\ddot{\phi} + 3H\dot{\phi} + \Gamma_\phi \dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

decay of ϕ leads to particle production & reheating

Inflation Resolves Cosmological Problems

- Horizon Problem (homogeneity and isotropy): small causally connected region inflates to large region containing our universe
- Flatness Problem $k/a^2 \rightarrow \text{small} \quad \Omega \rightarrow 1$
- Monopole Problem: tightest bounds on GUT monopoles from neutron stars (Freese, Schramm, and Turner 1983); monopoles inflated away (outside our horizon)
- BONUS: Density Perturbations that give rise to large scale structure are generated by inflation

Density Fluctuations $\delta\rho/\rho$ are produced in rolling models of inflation



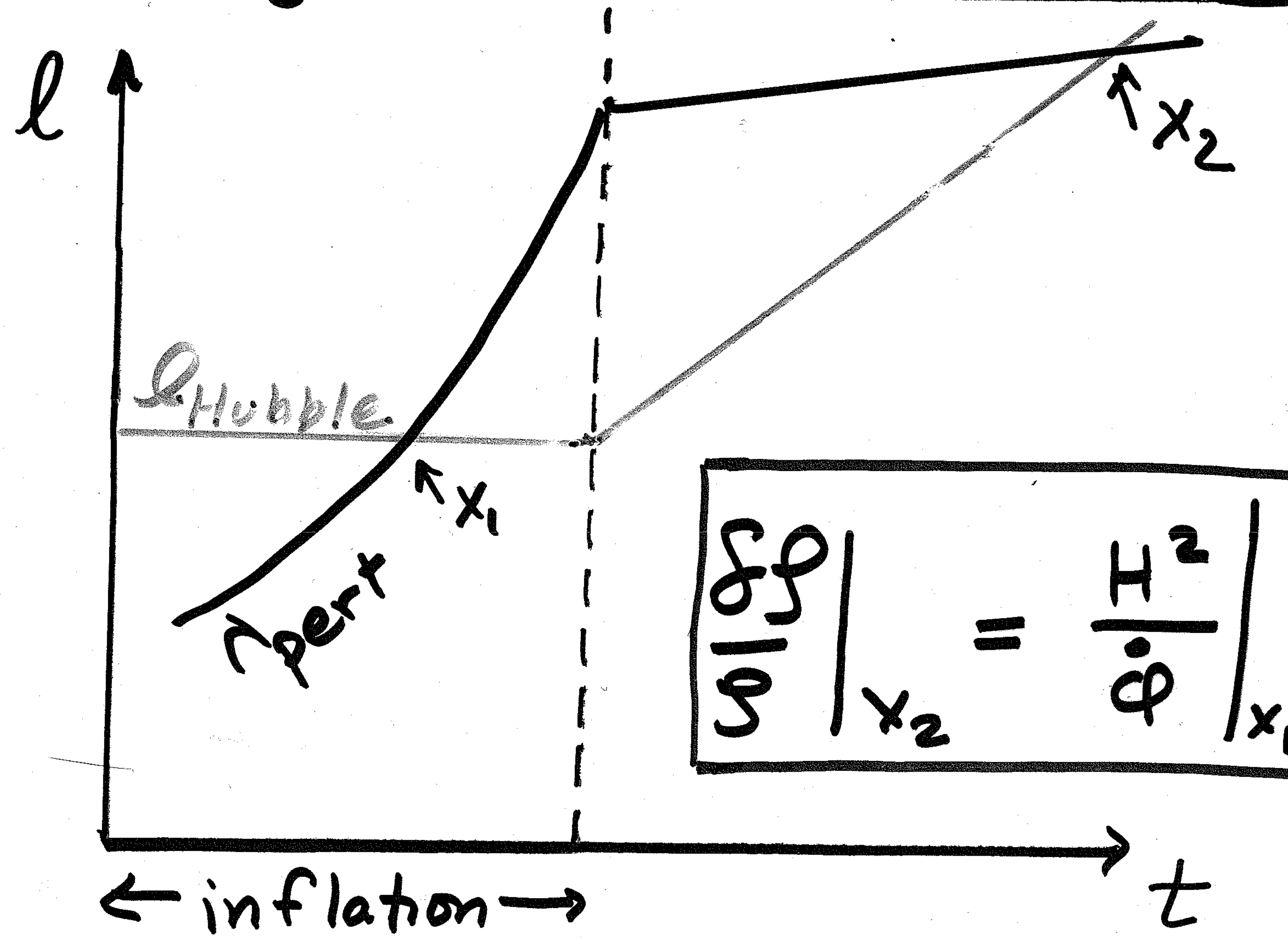
Origin of $\delta\rho/\rho$:

quantum fluctuations

$\langle \Delta\phi^2 \rangle \sim H/2\pi$

Different regions of the universe start at different values of ϕ, take different times to reach the bottom, end at different energy densities

Density Perturbations in Inflation



$$\left.\frac{\delta\rho}{\rho}\right|_{x_2} = \left.\frac{H^2}{\dot\phi}\right|_{x_1}$$

$$\ell_{Hubble} \sim \frac{1}{H} \sim \begin{cases} \text{const. during inflation} \\ t \quad \text{after inflation} \end{cases}$$

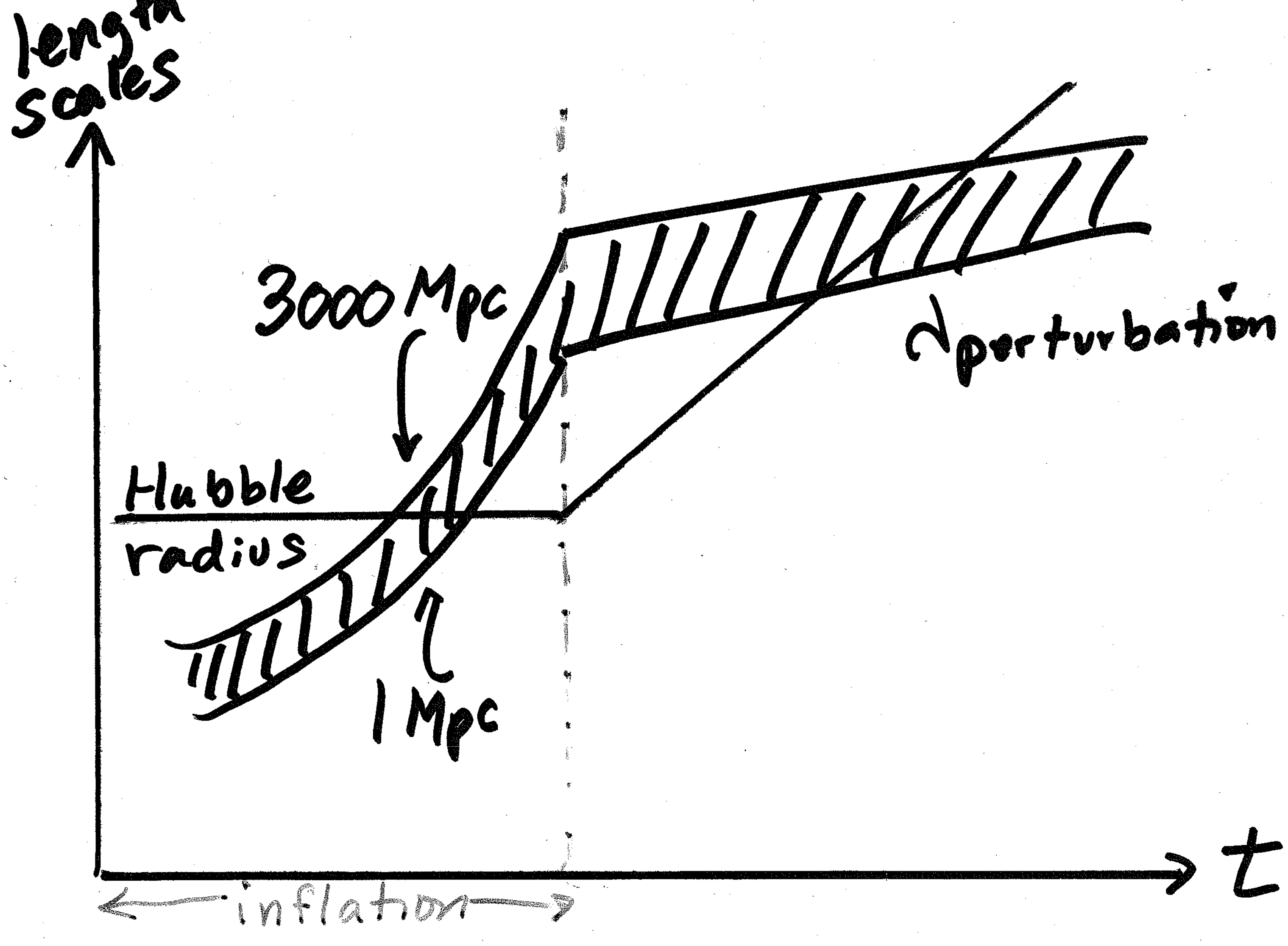
$$\lambda_{pert} \sim \begin{cases} \exp & \text{during inflation} \\ t^{2/3} & \text{after inflation} \end{cases}$$

Two horizon crossings x : causal microphysics before x_1 describes density perts. at x_2

Scales of structure in universe:

distance between galaxies ~1 Mpc

Horizon size (size of our observable universe) ~3000 Mpc



Density fluctuations
lead to test of inflation
theory:

Must match amplitude of
observations
$\delta\rho/\rho \sim \delta T/T \sim 10^{-5}$

and spectrum of
observations
(amplitude on all length
scales)

Spectrum of Perturbations in Inflation

$\delta_k = \text{F.T.}\,(\delta\rho/\rho)$

Power spectrum

$$P_k = |\delta_k|^2 \sim k^n$$

$n=1$: equal power on all length scales (when perturbations enter horizon)

$n<1$: extra power on large scales

During inflation, H and $\dot{\varphi}$ vary slowly

$\delta\rho/\rho = H^2/\dot{\varphi}$ when enter horizon

$\sim$ same $\forall$ scales

Predicts n near 1 : CORRECT

Precise predictions of n in different models leads to test of models e.g. total # of e-foldings

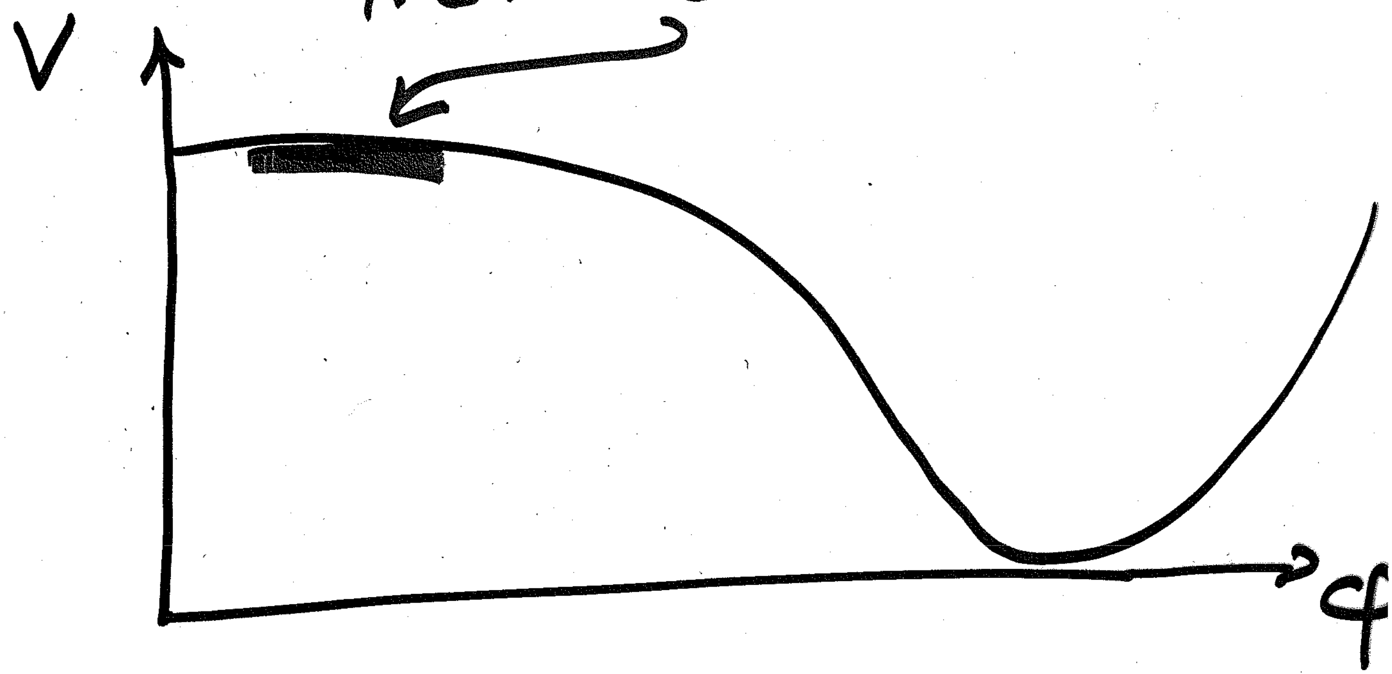
$$N_{tot} = 60$$

Then structure on observable scales is produced

3000 Mpc ← 60 - 50 e-foldings → 1 Mpc

before end of inflation

i.e. here

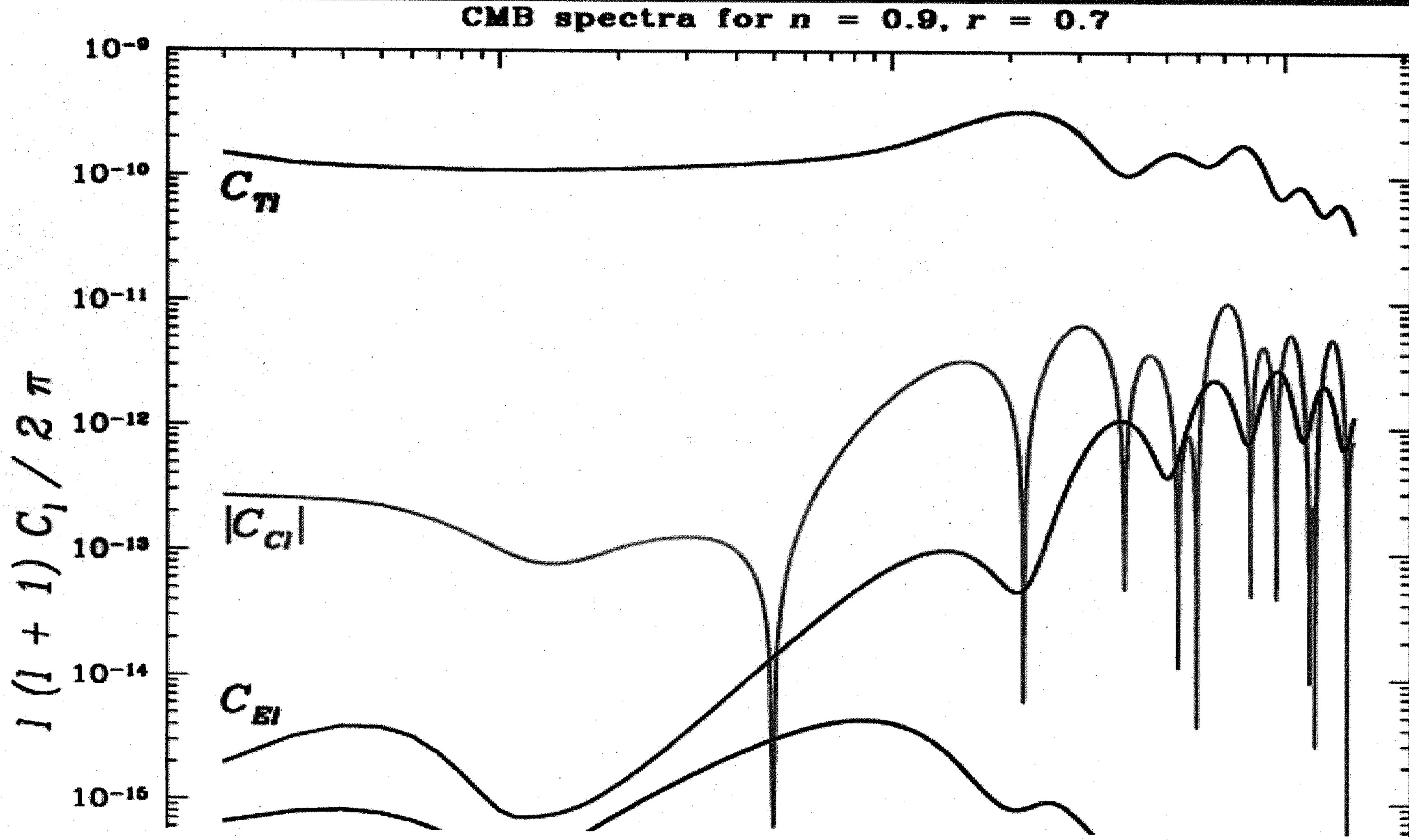


Tensor (gravitational wave) modes

- In addition to density fluctuations, inflation also predicts the generation of tensor fluctuations with amplitude $P_T^{1/2} = \frac{H}{2\pi}$.
- For comparison with observation, the tensor amplitude is conventionally expressed as:
- $r = \frac{P_T^{1/2}}{P_\zeta^{1/2}}$ (denominator: scalar modes)

In principle there are four parameters describing the scalar and tensor fluctuations: the amplitudes and spectra of both components. The amplitude of the scalar perturbations is normalized by the height of the potential (the energy density Λ^4).The tensor spectral index n_T is not an independent parameter since it is related to the tensor/scalar ratio by the inflationary consistency condition $r = -8n_T$. The remaining free parameters are the spectral index n of the scalar density fluctuations, and the tensor amplitude (given by r).

Gravity Modes are (at least) two orders of magnitude smaller than density fluctuations: hard to find!



Four parameters from inflationary perturbations:

I. Scalar perturbations:

amplitude $(\delta\rho/\rho)|_s$ spectral index n_s

II. Tensor (gravitational wave) modes:

amplitude $(\delta\rho/\rho)|_T$ spectral index n_T

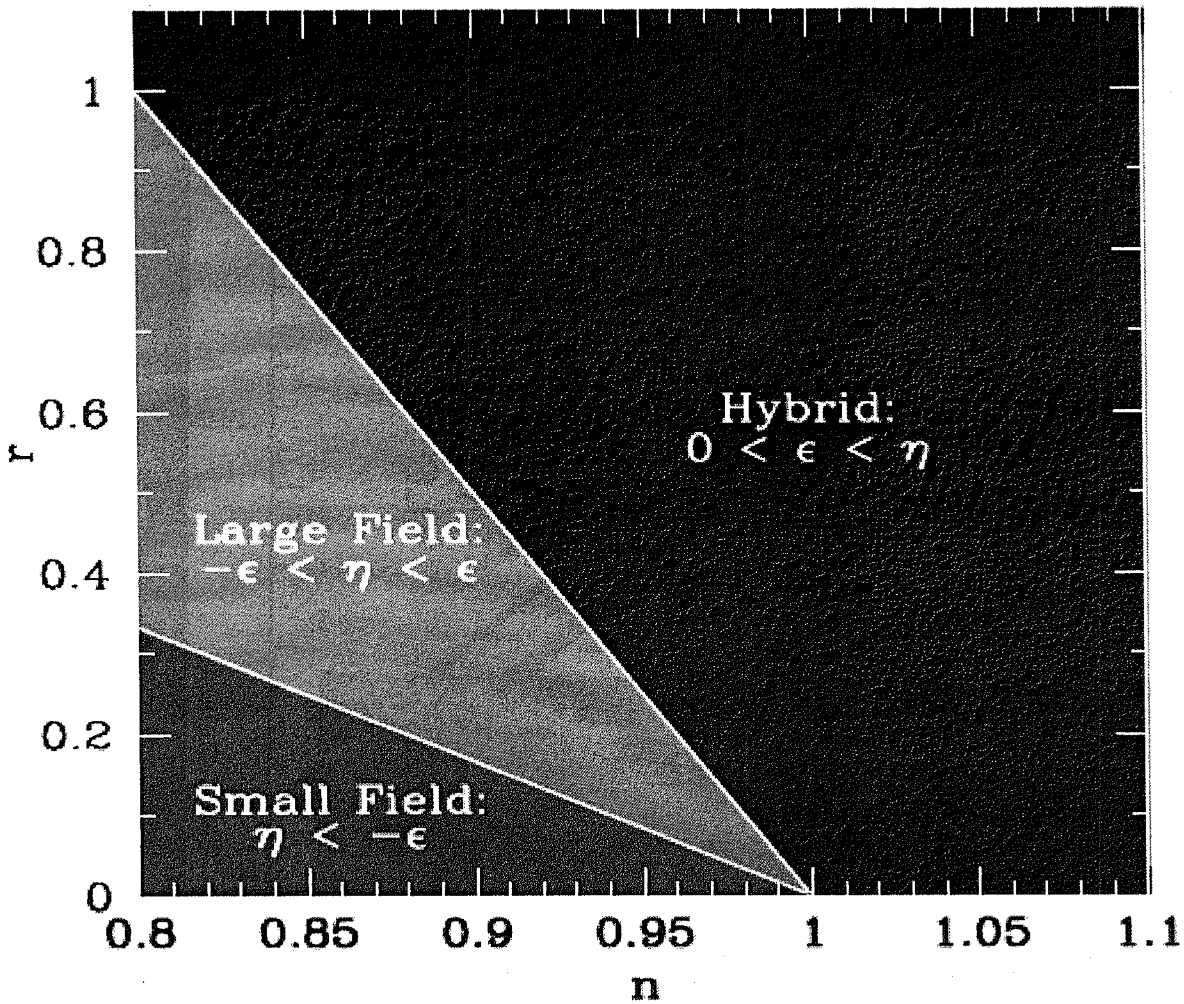
Expressed as

$$r \equiv \frac{P_T^{1/2}}{P_S^{1/2}}$$

Inflationary consistency condition: $r = -8n_T$

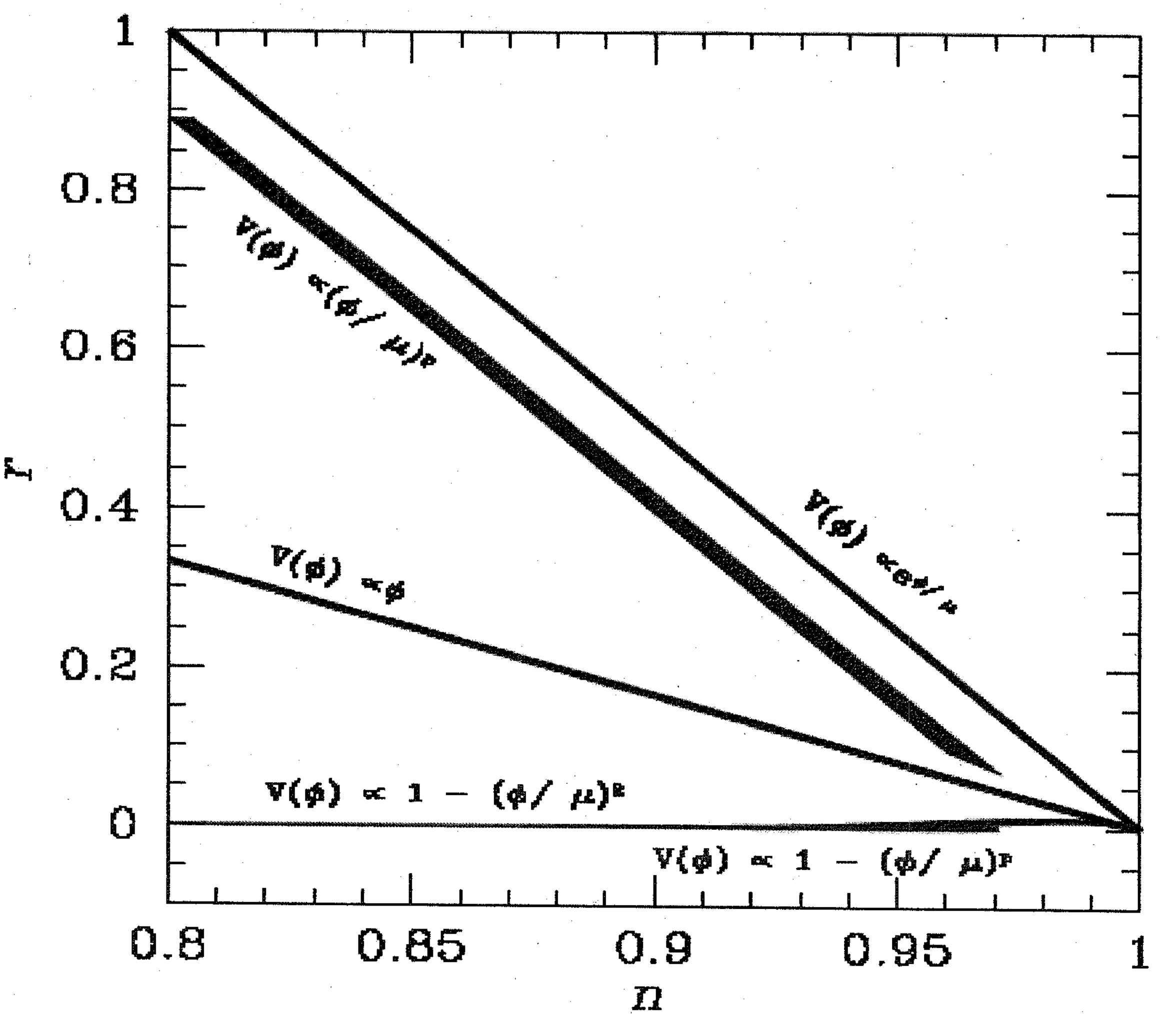
Plot in r-n plane

Different Types of Potentials in the r-n plane



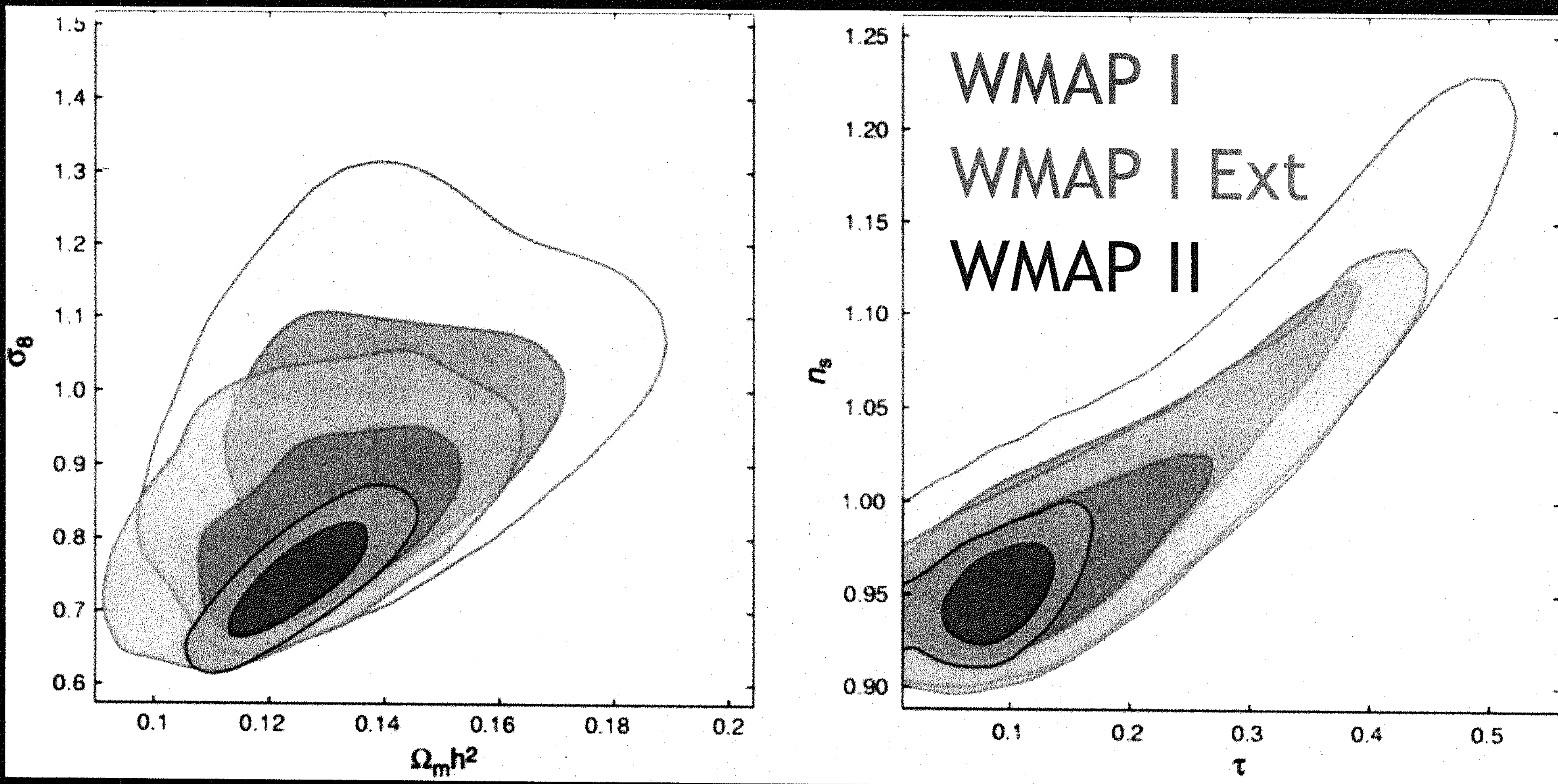
(KINNEY 2002)

Examples of Models



Effect of more data

LCDM model



Reducing the noise by 3 → degeneracies broken

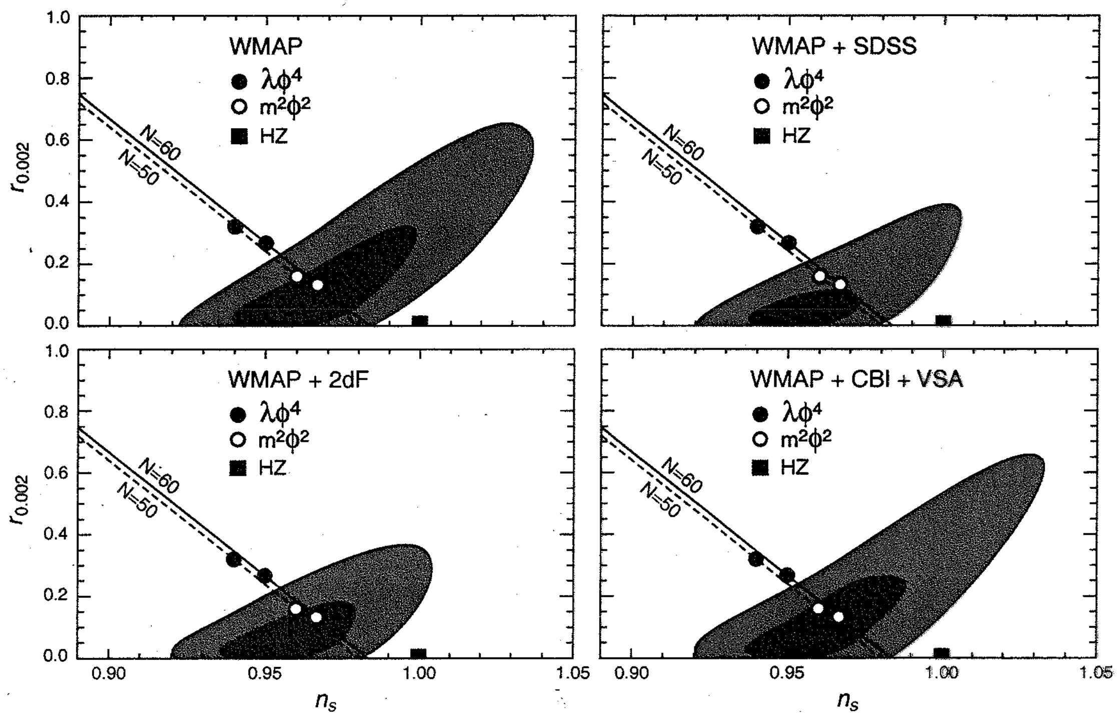
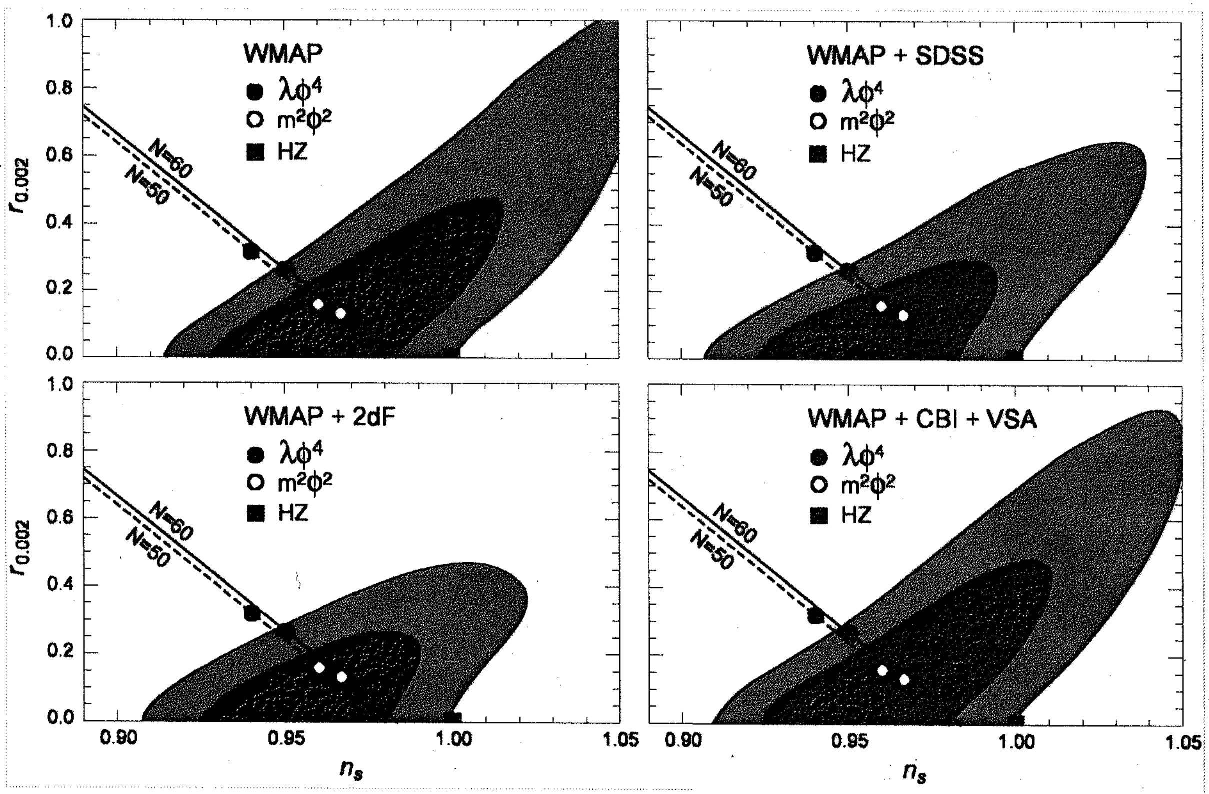


Fig. 14.— Joint two-dimensional marginalized contours (68% and 95% confidence levels) for inflationary parameters ($r_{0.002}$, n_s) predicted by monomial potential models, $V(\phi) \propto \phi^n$. We assume a power-law primordial power spectrum, $dn_s/d\ln k = 0$, as these models predict the negligible amount of running index, $dn_s/d\ln k \approx -10^{-3}$. (*Upper left*) WMAP only. (*Upper right*) WMAP+SDSS. (*Lower left*) WMAP+2dFGRS. (*Lower right*) WMAP+CBI+VSA. The dashed and solid lines show the range of values predicted for monomial inflaton models with 50 and 60 e-folds of inflation (equation (13), respectively. The open and filled circles show the predictions of $m^2\phi^2$ and $\lambda\phi^4$ models for 50 and 60 e-folds of inflation. The rectangle denotes the scale-invariant Harrison-Zel'dovich-Peebles (HZ) spectrum ($n_s = 1, r = 0$). Note that the current data prefers the $m^2\phi^2$ model over both the HZ spectrum and the $\lambda\phi^4$ model by likelihood ratios greater than 50.

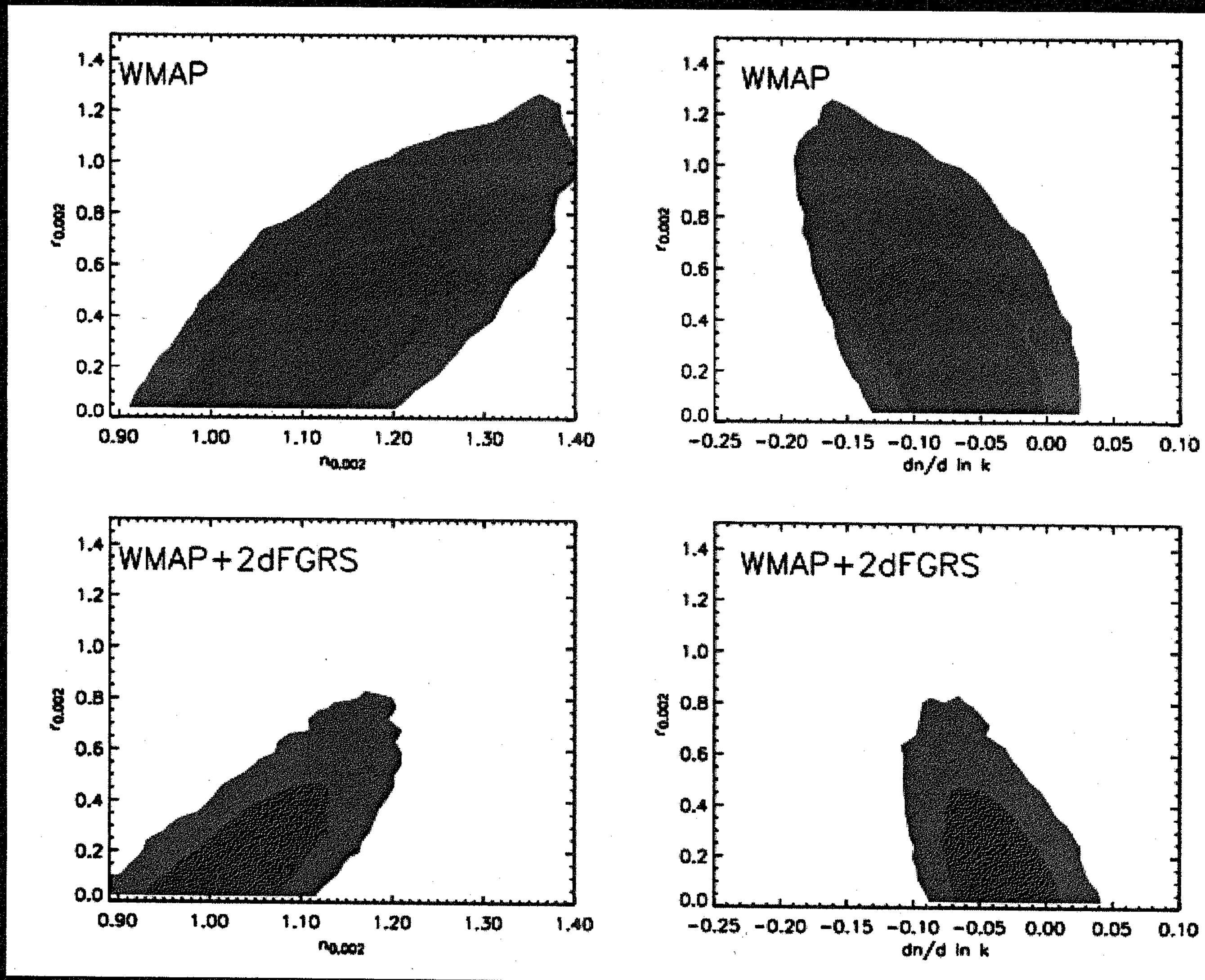
wrong

Testing Inflation with Tensors



Spectral index vs. tensors

The full treatment:

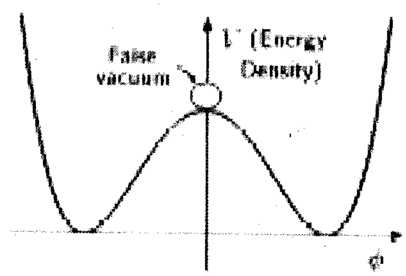


Natural Inflation after WMAP

Katherine Freese
Chris Savage

I. Fine Tuning in Rolling Models

- The potential must be very flat:



$$\frac{\Delta V}{(\Delta \Phi)^4} = \frac{\text{height}}{\text{width}^4} \leq 10^{-8},$$
$$e.g. V(\phi) = \lambda \Phi^4, \lambda \leq 10^{-12}$$

(Adams, Freese, and Guth 1990)

But particle physics typically gives this ratio = 1!

Success of inflation models with rolling fields
$\Rightarrow$ constraints on $V(\varphi)$

1. enough inflation
Scale ~~factor~~ R must grow enough
e-folds of growth of R =

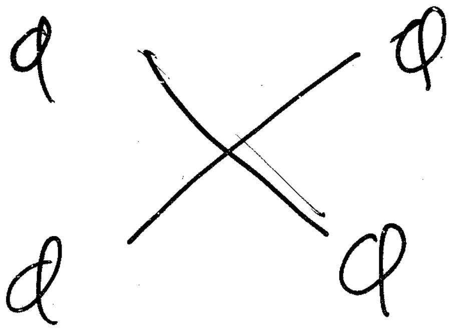
$$\ln(R_{end}/R_{begin}) = \int_{t_{begin}}^{t_{end}} H\,dt$$

$$= -8\pi G \int \frac{V(\phi)}{V'(\phi)}\,d\phi \gtrsim 60$$

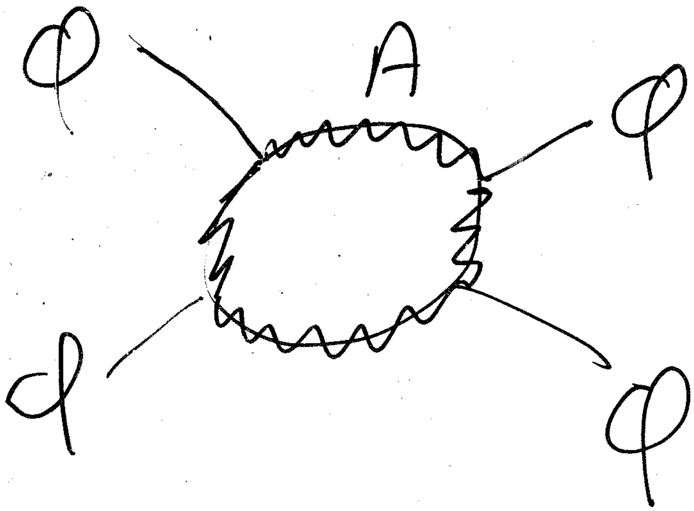
2. amplitude of density fluctuations not too big

$$\frac{\delta\rho}{\rho} \sim \left.\frac{H^2}{\dot{\varphi}}\right|_{\text{leave horizon}} \lesssim \frac{\delta T}{T} \sim 10^{-5}$$

Perturbation theory $\lambda\phi^4$

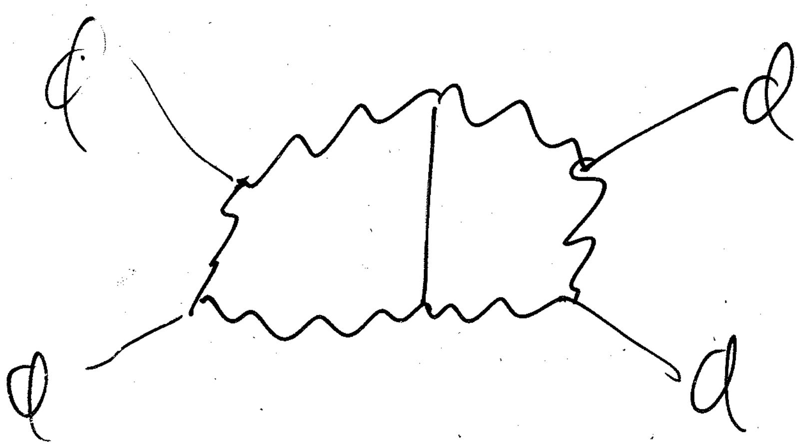


Tree level



coupling to gauge field A

to one-loop



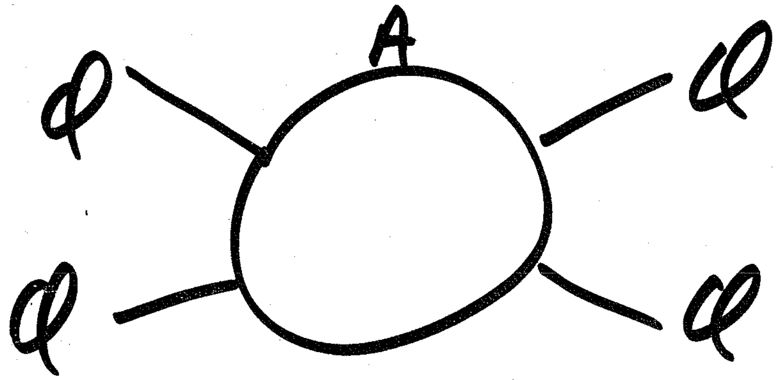
two loops

etc.

$V \sim \lambda_g \phi^4$ ← Higgs. Suppose, at tree level, put in small λ_g.

Higgs couples to gauge field A

radiative corrections ⇒



gauge loops

addit'l term: $V_{eff} \sim \phi^4 \alpha^2 \ln(\phi^2/\mu^2)$

↑ gauge coupling $O(\frac{1}{100})$

Gives correction to λ which is $O(\alpha^2) \sim 10^{-4}$, much bigger than constraints from inflation $\lambda < 10^{-8}$. To keep λ small, would have to balance against tree level term. Would have to do this to each order in pert. theory. UGLY!

Need small ratio of mass scales

$$\frac{\Delta V}{(\Delta \Phi)^4} = \frac{\text{height}}{\text{width}^4} \leq 10^{-8},$$

- Two attitudes:

1) We know there is a heirarchy problem, wait until it's explained

2) Two ways to get small masses in particles physics:

   (i) supersymmetry

   (ii) Goldstone bosons (shift symmetries)

Natural Inflation: Shift Symmetries

- Shift (axionic) symmetries protect flatness of inflaton potential
  $\Phi \rightarrow \Phi + \mathrm{constant}$ (inflaton is Goldstone boson)
- Additional explicit breaking allows field to roll.
- This mechanism, known as natural inflation, was first proposed in

Freese, Frieman, and Olinto 1990;
Adams, Bond, Freese, Frieman and Olinto 1993

Shift symmetries
→ "Natural Inflation"
(Freese, Frieman, Olinto '90)

We need $\chi = \frac{\Delta V}{(\Delta \varphi)^4} \leq 10^{-8}$

but most particle physics models require $\chi = O(1)$

BUT we know of a particle with a small ratio of scales:
the axion has $\chi_a \sim \left(\frac{\Lambda_{QCD}}{f_{PQ}}\right)^4 \sim 10^{-64}$

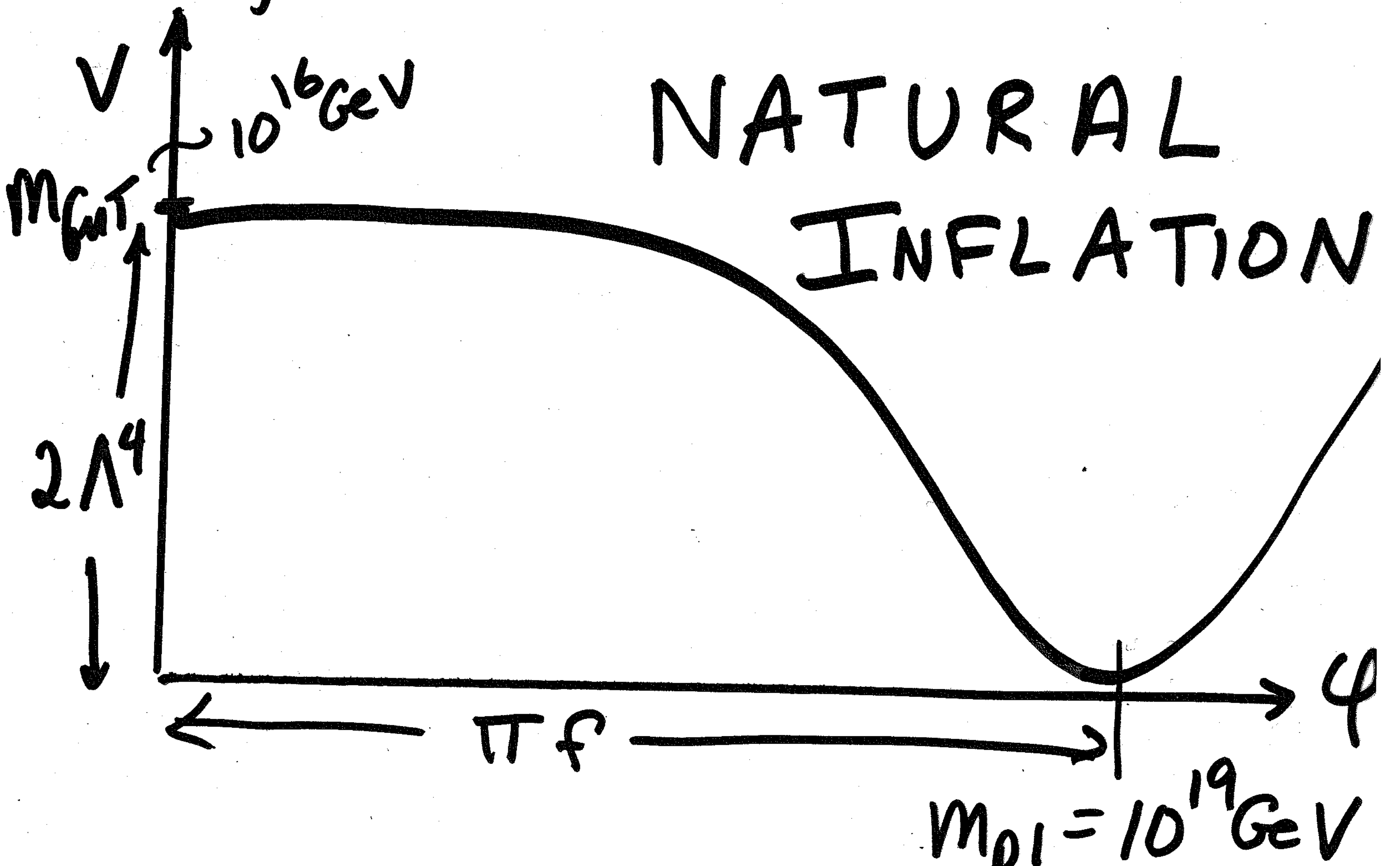
IDEA: Use a potential similar to that for axions in inflation ⇒ natural inflation (no fine-tuning).
n.b. here we do not use QCD axion
We use heavier particle with similar behavior

$$V(\varphi) = \Lambda^4 [1 + \cos \varphi/f]$$

Freese, Frieman, Olinto '90

Two different mass scales: height Λ and width f

Adams, Bond, Freese, Frieman, Olinto '94



f = scale of spontaneous symmetry breaking of some global symmetry

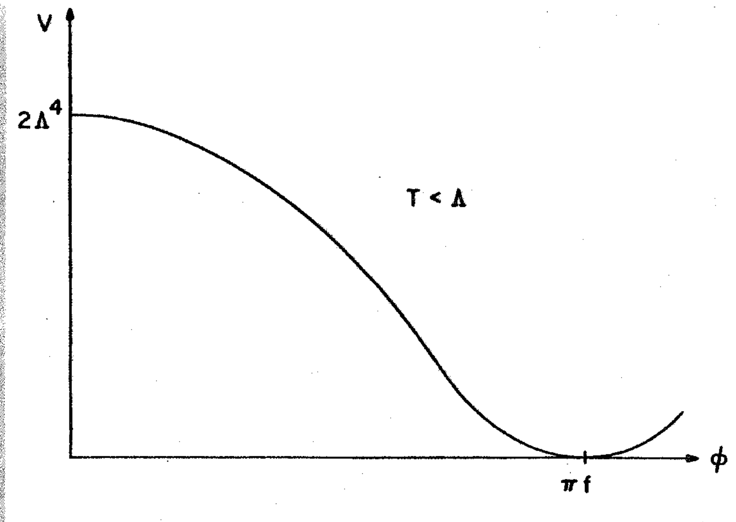
Λ = scale at which gauge group becomes strong

For QCD axion: $\Lambda_{QCD} \sim 100$ MeV, $f \sim 10^{12}$ GeV

For inflation, need: $\Lambda \sim m_{GUT}$, $f \sim m_{pl}$

Natural Inflation

(Freese, Frieman, and Olinto 1990;
Adams, Bond, Freese, Frieman and Olinto 1993)



$$V(\Phi) = \Lambda^4[1 + \cos(\Phi/f)]$$

- Two different mass scales:
- Width f is the scale of SSB of some global symmetry
- Height Λ is the scale at which some gauge group becomes strong

Two Mass Scales Provide required heirarchy

- For QCD axion,

$$\Lambda_{QCD} \sim 100\mathrm{MeV},\, f_{PQ} \sim 10^{12}\mathrm{GeV},\, \frac{\mathrm{height}}{\mathrm{width}} \sim 10^{-64}!!$$

- For inflation, need $\Lambda \sim m_{GUT},\, f \sim m_{pl}$

Enough inflation requires width = f = m_pl,
Amplitude of density fluctuations requires
height = $\Lambda \sim m_{GUT}$

Implementations of natural inflation's shift symmetry

- Natural chaotic inflation in SUGRA using shift symmetry in Kahler potential (Gaillard, Murayama, Olive 1995; Kawasaki, Yamaguchi, Yanagida 2000)
- In context of extra dimensions: Wilson line with $f \gg m_{pl}$ (Arkani-Hamed et al 2003) but Banks et al (2003) showed it fails in string theory.
- "Little" field models (Kaplan and Weiner 2004)
- In brane Inflation ideas (Firouzjahi and Tye 2004)
- Gaugino condensation in SU(N) $\times$ SU(M):

Adams, Bond, Freese, Frieman, Olinto 1993;

Blanco-Pillado et al 2004 (Racetrack inflation)

Legitimacy of large axion scale?

Natural Inflation needs $f > 0.6 m_{pl}$

Is such a high value compatible with an effective field theory description? Do quantum gravity effects break the global axion symmetry?

Kinney and Mahantappa 1995: symmetries suppress the mass term and $f \ll m_{pl}$ is OK.

Arkani-Hamed et al (2003):axion direction from Wilson line of U(1) field along compactified extra dimension provides $f \gg m_{pl}$

However, Banks et al (2003) showed it does not work in string theory.

A large effective axion scale

(Kim, Nilles, Peloso 2004)

- Two or more axions with low PQ scale can provide large $f_{eff} \sim m_{pl}$
- Two axions θ and ρ

$$V = \Lambda_1^4 [1 - \cos(\frac{\theta}{f} + \frac{\epsilon_1 \rho}{g})] + \Lambda_2^4 [1 - \cos(\frac{\theta}{f} + \frac{\epsilon_2 \rho}{g})]$$

- Mass eigenstates are linear combinations of θ and ρ
- Effective axion scale can be large,

$$f_\xi = \frac{\sqrt{\epsilon_1^2 f^2 + g^2}}{\epsilon_1 - \epsilon_2} >> f \text{ if } |\epsilon_1 - \epsilon_2| << 1$$

Density Fluctuations in Natural Inflation

- Power Spectrum:

$$|\delta_k|^2 \sim k^{n_s}, n_s = 1 - \frac{m_{pl}^2}{8\pi f^2}$$

- WMAP data:

$$|n_s - 1| < 0.1$$

implies

$$f \geq 0.6 m_{pl}$$

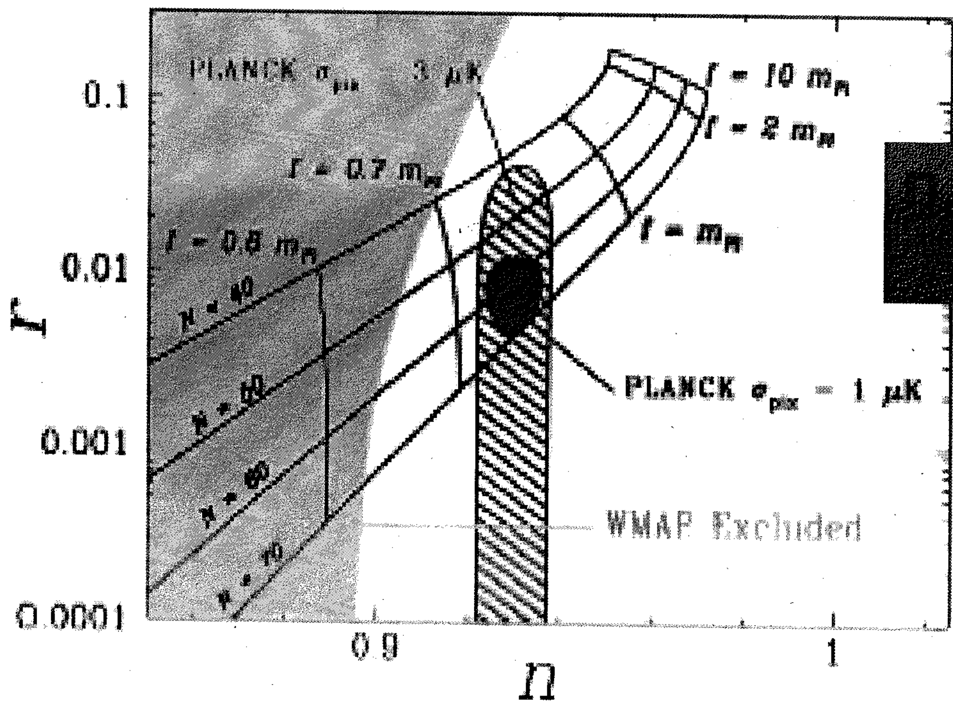
(Freese and Kinney 2004)

Tensor Modes in Natural Inflation

(Freese and Kinney 2004)

Two predictions, testable in next decade: Tensor modes, while smaller than in other models, should be found. Also, there is very little running of n in natural inflation.

$$r = \frac{P_T^{1/2}}{P_\zeta^{1/2}} = 16\epsilon_1$$



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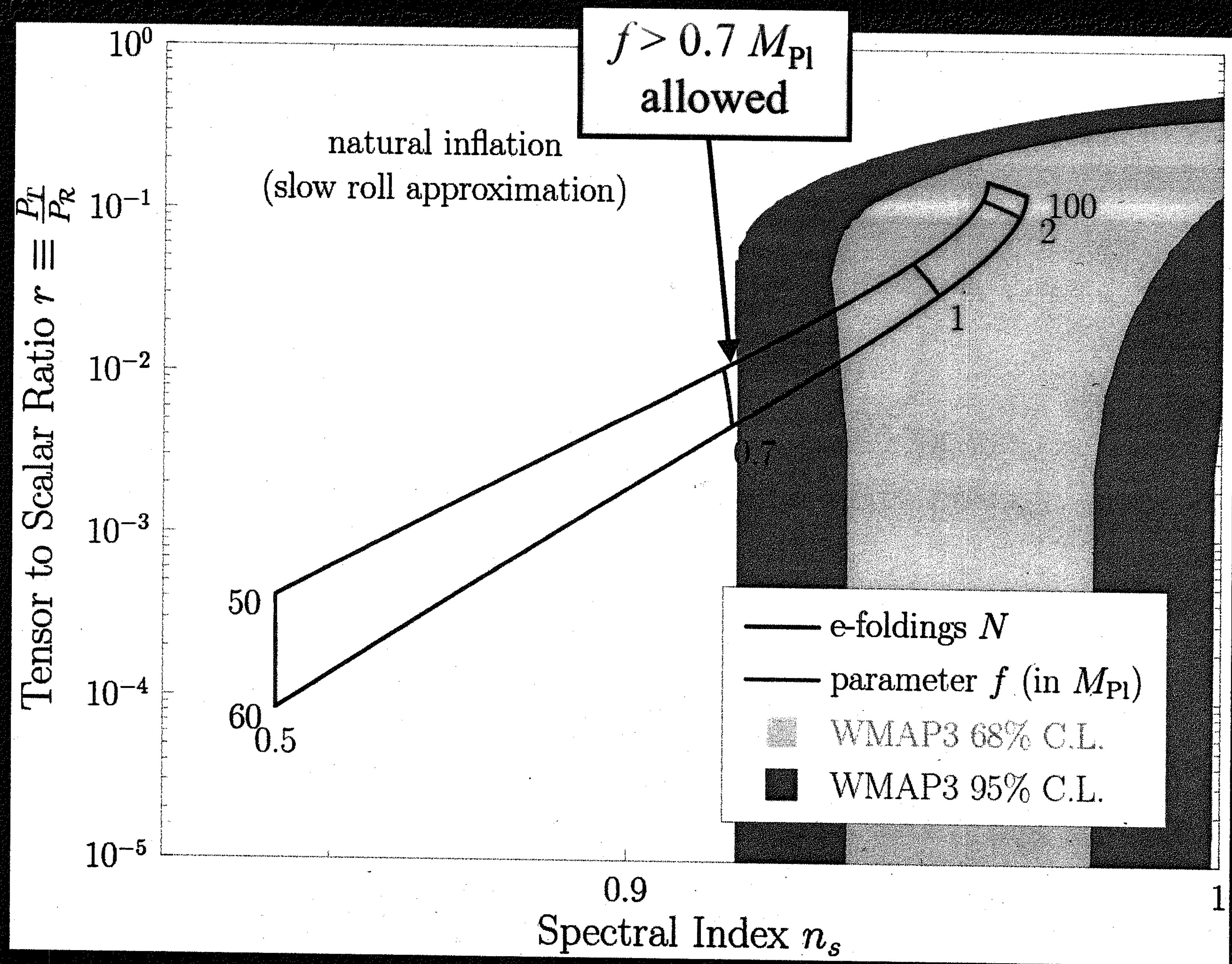
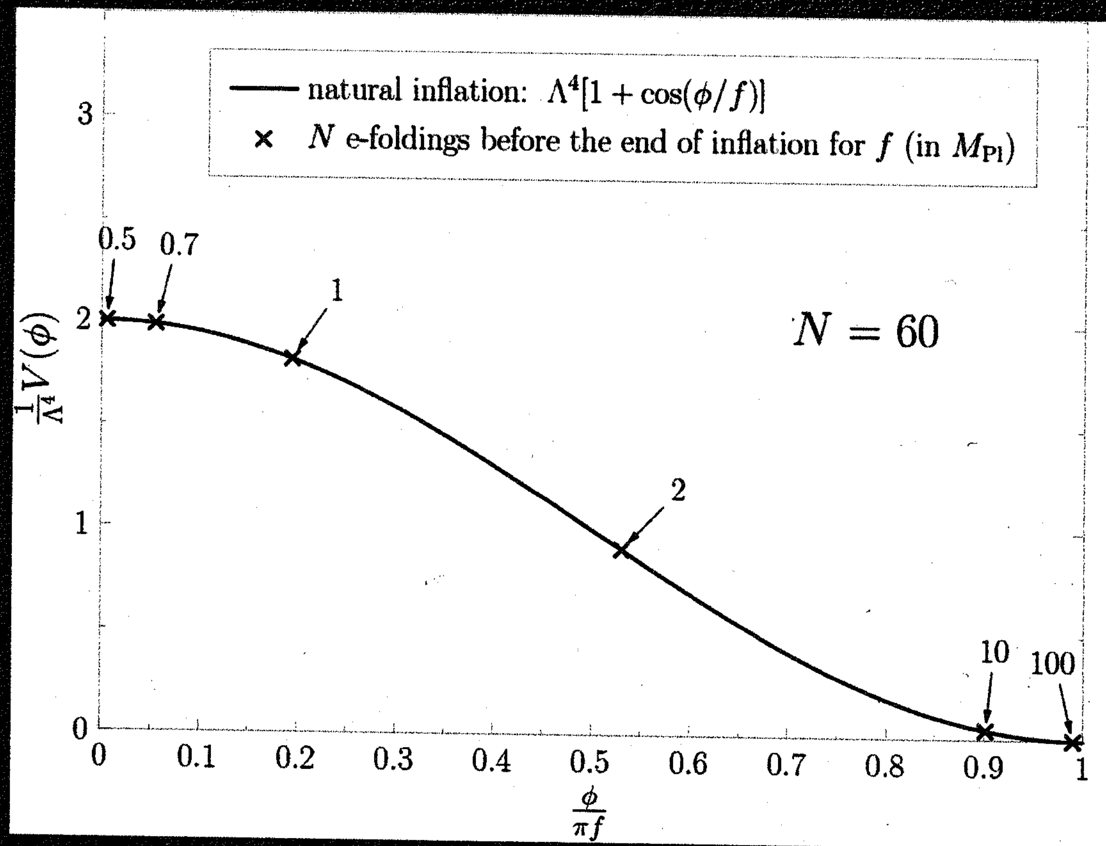
Sensitivity of PLANCK: error bars +/- 0.05 on r and 0.01 on n. Next generation expts (3 times more sensitive) must see it.



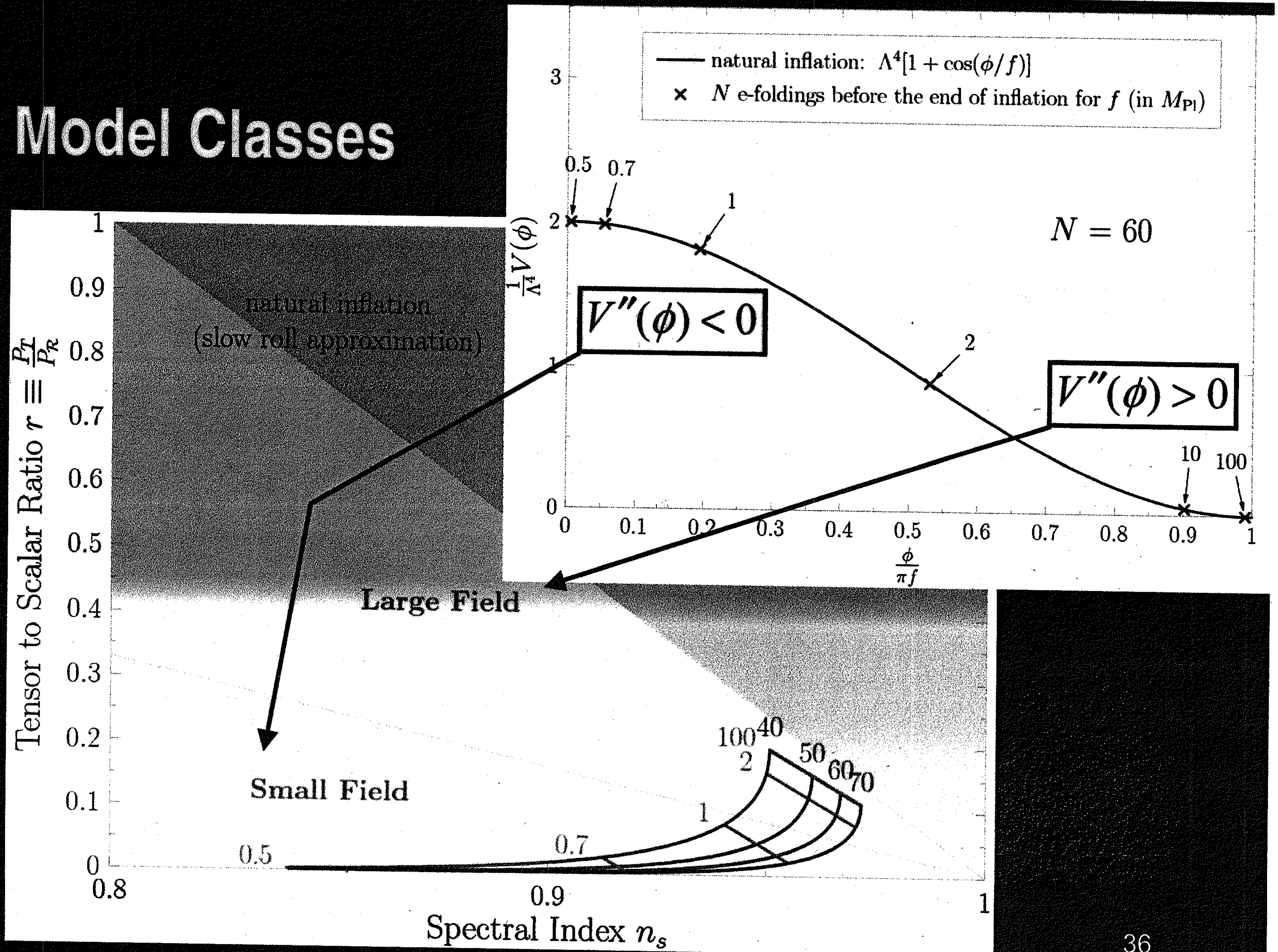
$f > 0.7\ M_{Pl}$ allowed
natural inflation
(slow roll approximation)
Tensor to Scalar Ratio $r \equiv \frac{P_T}{P_R}$
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
100
2
1
0.7
50
60
0.5
0.9
1
Spectral Index n_s
e-foldings N
parameter f (in M_{Pl})
WMAP3 68% C.L.
WMAP3 95% C.L.

Potential

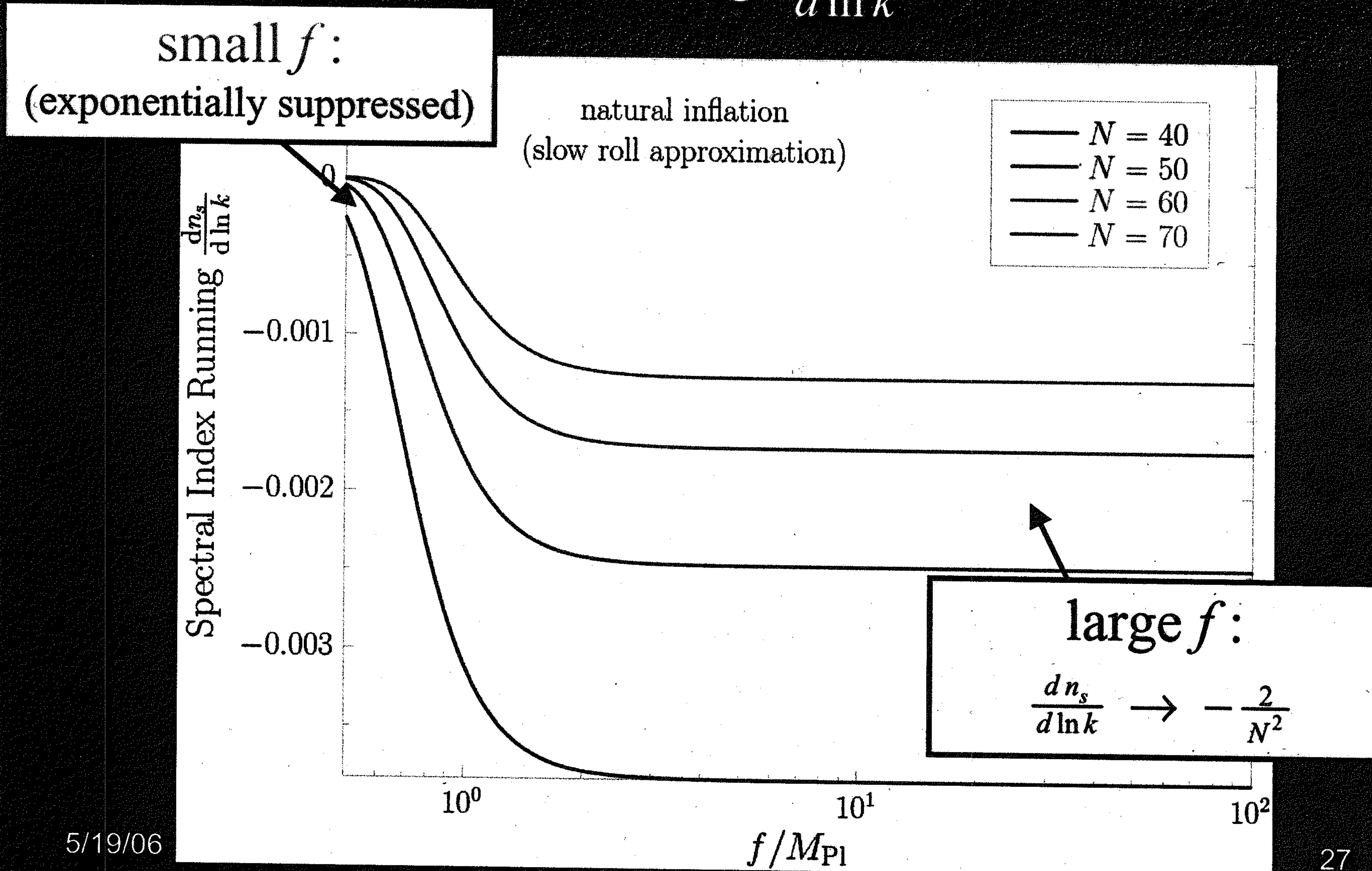


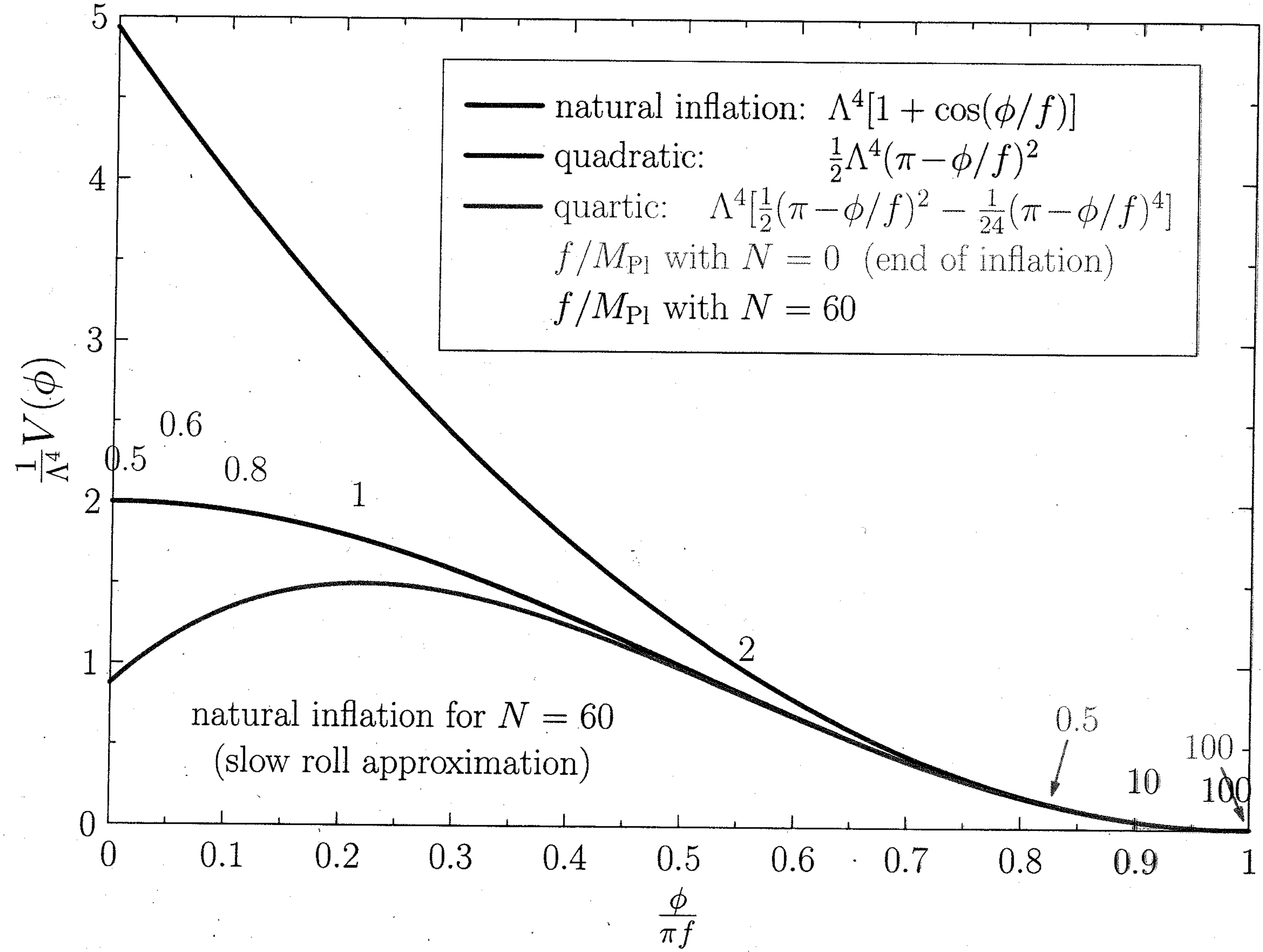
60 e-foldings before the end of inflation
~ present day horizon

Model Classes



Spectral Index Running $\frac{dn_s}{d\ln k}$

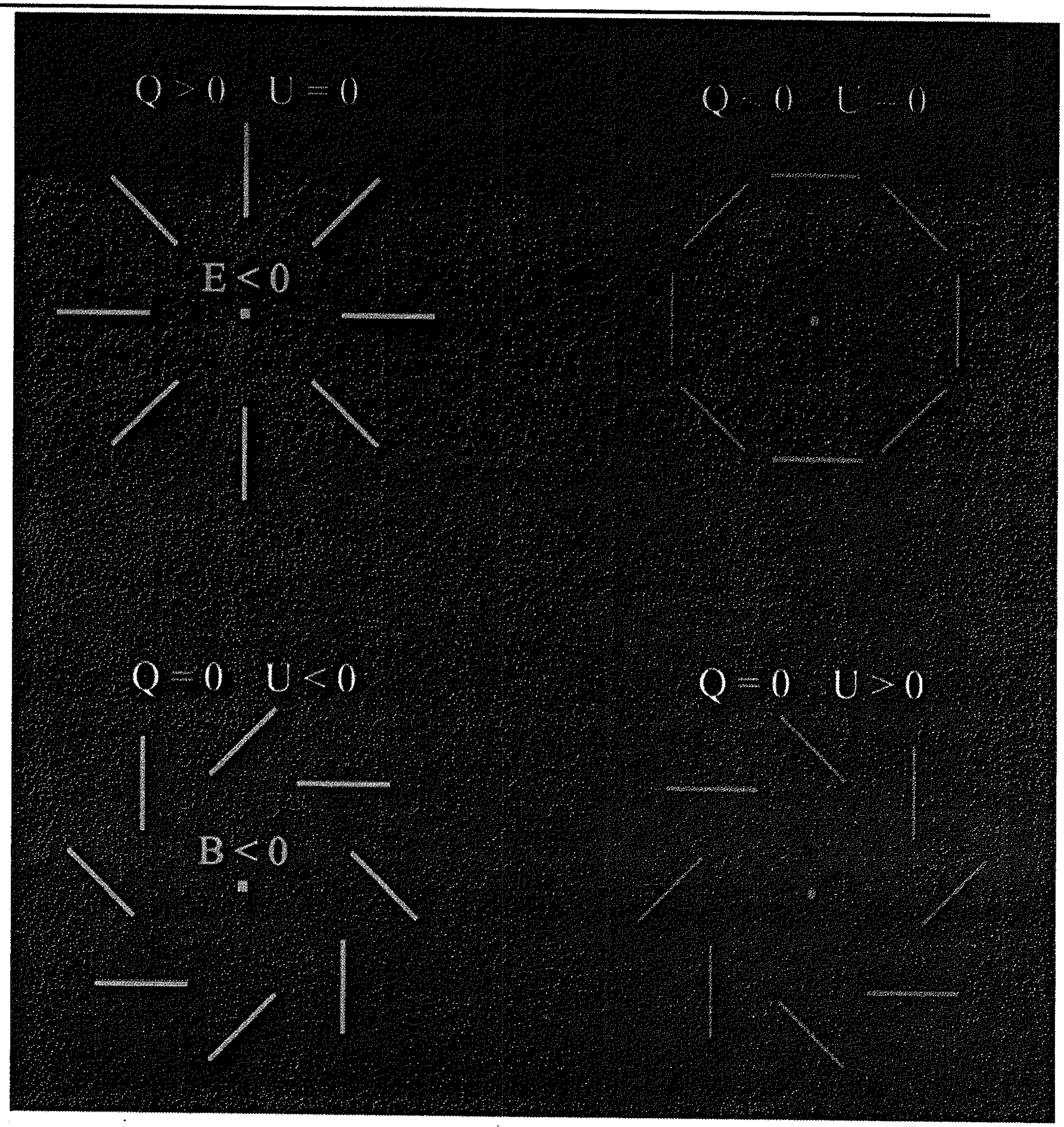




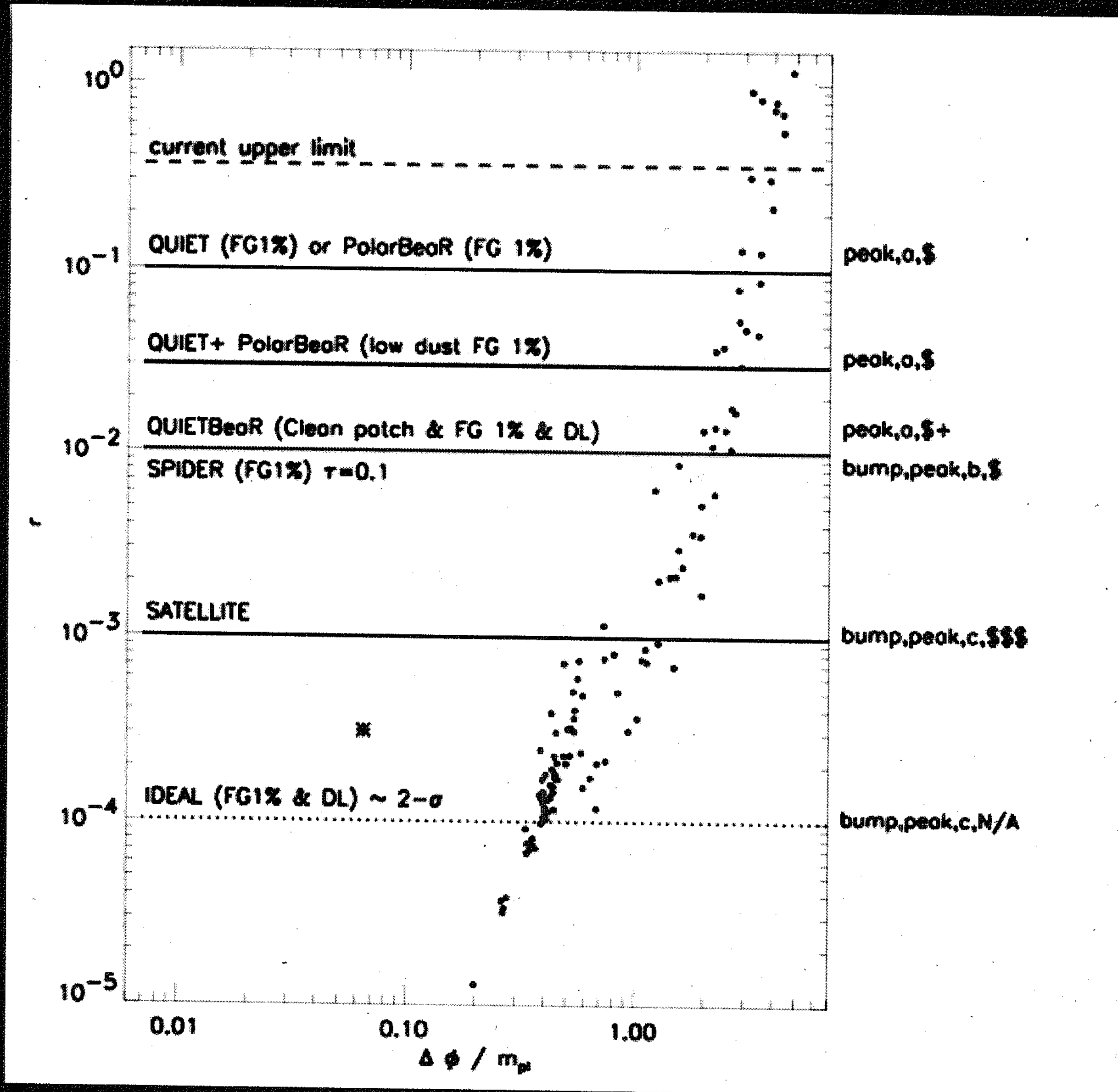
natural inflation: $\Lambda^4[1+\cos(\phi/f)]$
quadratic: $\frac{1}{2}\Lambda^4(\pi-\phi/f)^2$
quartic: $\Lambda^4[\frac{1}{2}(\pi-\phi/f)^2-\frac{1}{24}(\pi-\phi/f)^4]$
$f/M_{\rm Pl}$ with $N=0$ (end of inflation)
$f/M_{\rm Pl}$ with $N=60$
natural inflation for $N=60$
(slow roll approximation)
$\frac{1}{\Lambda^4}V(\phi)$
$\frac{\phi}{\pi f}$
0.5
0.6
0.8
1
2
0.5
10
100
100
0
1
2
3
4
5
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

The E's and B's of Polarization Spectra

- Polarization decomposable into E mode (gradient) and B mode (curl) components.
- Tensor fluctuations produce both E and B mode components.
- Scalar fluctuations produce only E mode component (except for transformation by gravitational lensing).
- B modes directly probe gravity waves.



Prospects for finding B modes in the CMB



TeV

3.2×10^{13}

1.7×10^{13}

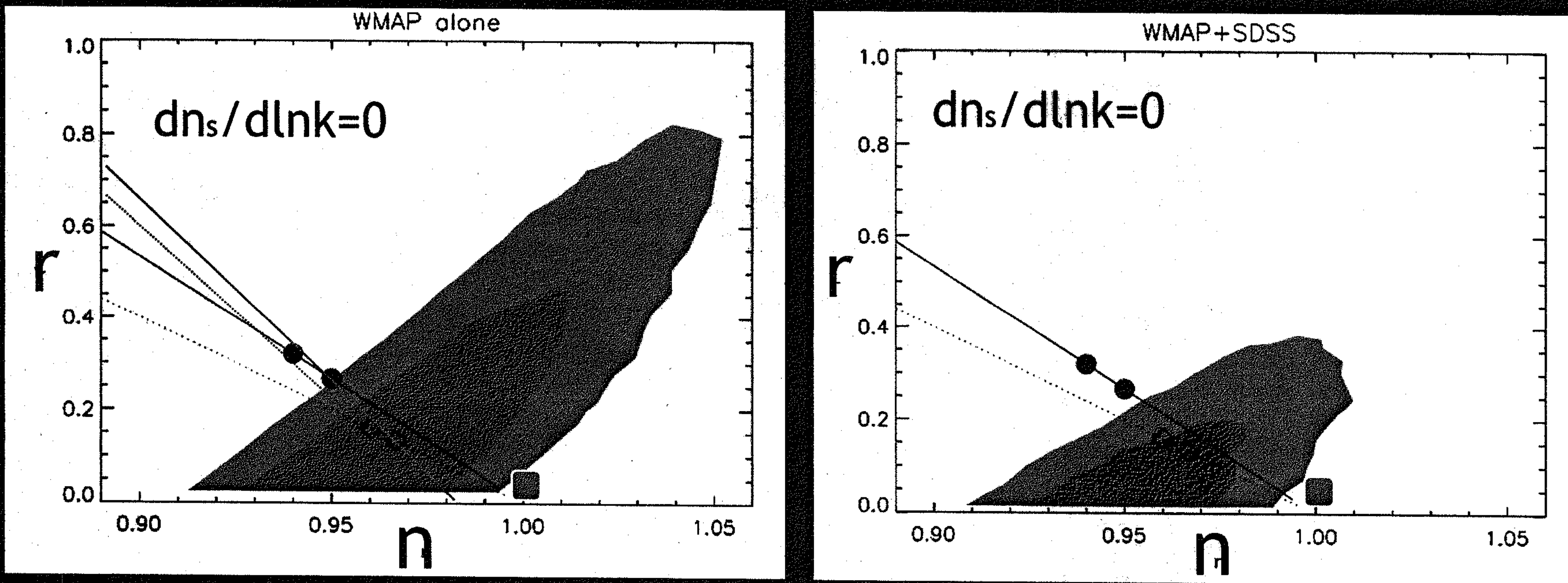
9.7×10^{12}

5.5×10^{12}

3.0×10^{12}

Verde, Peiris & Jimenez (2005)

Specific models critically tested



Models like $V(\phi) \sim \phi^p$

○ p=4 ◆ p=2 For 50 and 60 e-foldings

HZ

p fix, Ne varies

p varies, Ne fix

SUMMARY:

- **I. The predictions of inflation are right:**

  (i) the universe has a critical density

  (ii) Gaussian perturbations

  (iii) density perturbation spectrum nearly scale invariant

  iv) detection of polarization (from gravitational wave modes) in upcoming data may provide smoking gun for inflation

- **II. Polarization measurements will tell us which model is right.**

  WMAP already selects between models.

  Natural inflation (Freese, Frieman, Olinto) looks great