A heritage of John Archibald Wheeler: the accurate measurement of dragging of inertial frames using the LAGEOS satellites and the forthcoming LARES satellite

- Dedication to John Archibald Wheeler and brief introduction on frame-dragging
- Main experimental efforts to measure frame-dragging
  brief description of Lunar Laser Ranging test
- Satellite laser ranging measurements of the Lense-Thirring effect using the LAGEOS satellites
- LARES (LAser RElativity Satellite)

Ignazio Ciufolini (Univ. Lecce): Virginia 7-7-2008

A heritage of John Archibald Wheeler: the accurate measurement of dragging of inertial frames using the LAGEOS satellites and the forthcoming LARES satellite

THIS TALK IS DEDICATED TO JOHN ARCHIBALD WHEELER ONE OF THE MASTERS OF PHYSICS OF XX CENTURY AND FATHER OF THE REINASSANCE OF GENERAL RELATIVITY

If today we are here we also owe this to "Johnny", he was the scientific father, grandfather and grand-grand father of many of us!

Ignazio Ciufolini (Univ. Salento): Virginia 7.7.2008

John’s office, Univ. Texas at Austin, nearly 20 years ago

A SCHOOL OF ASTROPHYSICAL RELATIVITY IS DEDICATED, SINCE 2006, TO JOHNNY

ETTORE MAJORANA FOUNDATION AND CENTRE FOR SCIENTIFIC CULTURE

TO PAY A PERMANENT TRIBUTE TO GALILEO GALILEI, FOUNDER OF MODERN SCIENCE AND TO ENRICO FERMI, "THE ITALIAN NAVIGATOR", FATHER OF THE WEAK FORCES

Frontiers in Numerical Gravitational Astrophysics

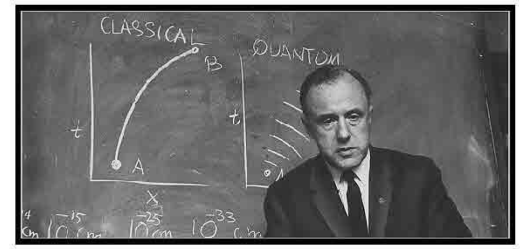
2nd Course of the International School on Astrophysical Relativity

«John Archibald Wheeler»

at EMFCSC (*Erice, Italy, June 27-July 5, 2008*)

sponsored also by: European Science Foundation, CINECA, Sapienza Università di Roma, National Science Foundation, The University of Texas at Austin

We are sad to inform that Prof.J.A.Wheeler passed away on sunday april 13 at his home in Hightstown, N.J.
He was 96.
Please read the obituary of New York Times



This Course is dedicated to the memory of this overwhelming scientist and human being.

Purpose of the Course

This Course has the scope to gather some of the best experts in computational simulations for the study of gravitating systems in astrophysics, to allow them to present their knowledge and expertise to deeply motivated students at the graduate level, as well as young researchers.

The main aim of the Course is that of clarifying whether and when the classic Newtonian approach fails and General Relativity is needed to approach reliably the complicated non-linear aspects of the physics involved. Consequently, the differences in the numerical and computational schemes approaches in the classic and relativistic cases will be illustrated and discussed. A small fraction of time is dedicated to the handling and visualization of the huge amount of data output.

Lectures will be on:

- stars: final evolutionary phases;
- compact interacting objects and gravitational waves;
- stellar systems: from small to large N-body problems;
- large and small scale structure in the Universe.
- supercomputing methods and tools.

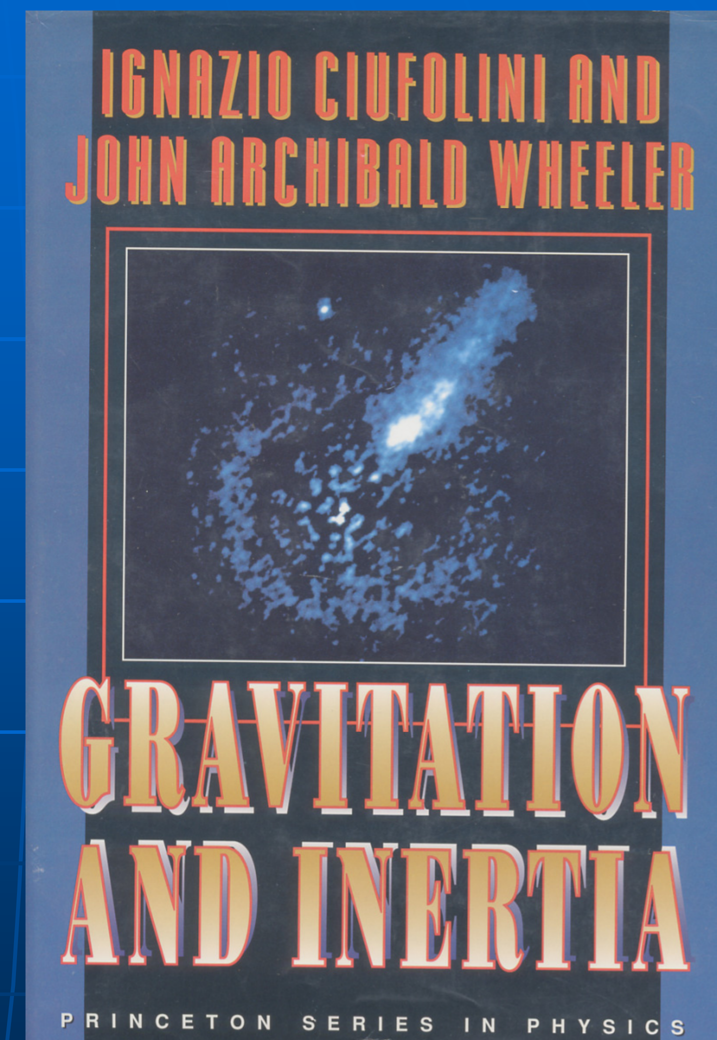
I.C. & E.Pavlis,
Letters to NATURE,
21 October, 2004.

I.C., E.Pavlis and R.Peron,, New Astronomy
2006.

I.C., *Dragging of Inertial Frames*, NATURE Review,
September, 2007.

I.C., *Lunar Laser Ranging, Frame Dragging and Gravitomagnentism by Spin*
(2008).

I.C., A. Paolozzi, E.Pavlis, R.Matzner, et al. in *"General Relativity and John Archibald Wheeler"* (Springer, 2008).

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THE INTERNATIONAL WEEKLY JOURNAL OF SCIENCE
nature
THE K/T IMPACT
Baptistina asteroids in the frame
BIOMETRICS
The questions you meant to ask
TSUNAMIS
Tracking risk off the Myanmar coast
THE RIDDLE OF INERTIA
How Earth's rotation reshapes space and time
NATUREJOBS
Hydrogen technology

DRAGGING OF INERTIAL FRAMES
(*FRAME-DRAGGING* as Einstein named it in 1913)

- **The local inertial frames are dragged by mass-energy currents: $\varepsilon\, u^{\alpha}$**

  gravitomagnetism:

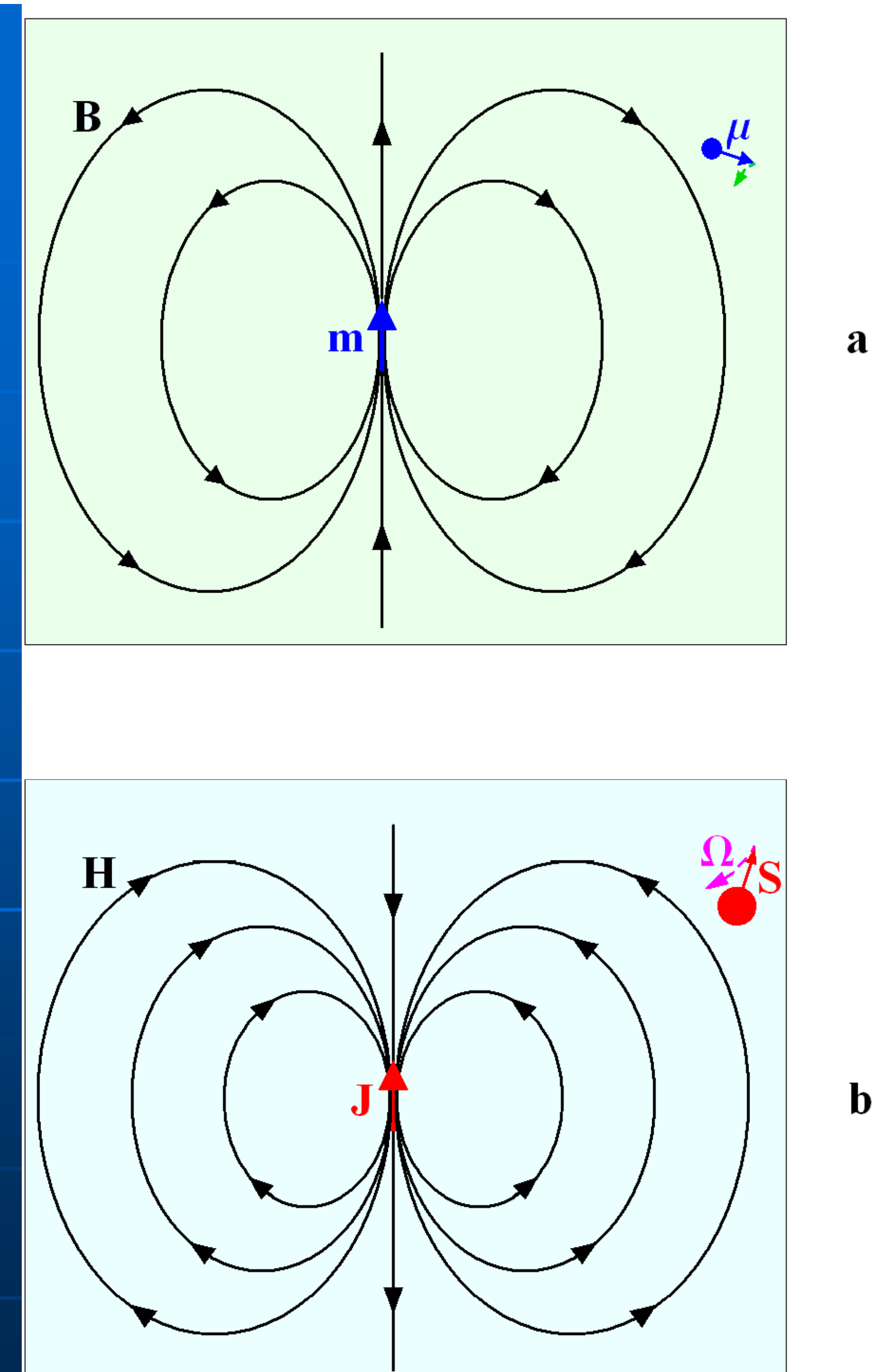
  $G^{\alpha\beta} = \chi\, T^{\alpha\beta} =$

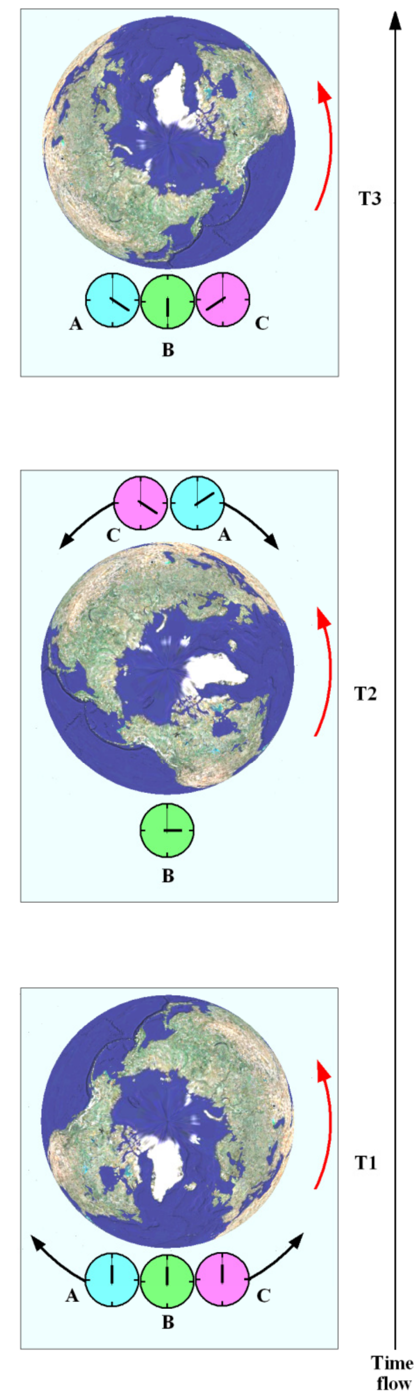
  $= \chi\, [(\varepsilon + p)\, u^{\alpha} u^{\beta} + p\, g^{\alpha\beta}]$
- It plays a key role in high energy astrophysics

  (Kerr metric)

Thirring 1918
Braginsky, Caves and Thorne 1977
Thorne 1986
Jantsen et al. 1992-97, 2001
I.C. 1994-2001

GRAVITOMAGNETISM





T3
A
B
C
C
A
T2
B
T1
A
B
C
Time flow

INVARIANT CHARACTERIZATION of GRAVITOMAGNETISM

Gravitomagnetism defined without approximations by the Riemann tensor in a local Fermi frame.

Matte-1953

By explicit spacetime invariants built with the Riemann tensor:

I.C. 1994

I.C. and Wheeler 1995:

for the Kerr metric:

$\frac{1}{2}\, \epsilon_{\alpha\beta\sigma\rho}\, R^{\sigma\rho}{}_{\mu\nu}\, R^{\alpha\beta\mu\nu} = 1536\, J\, M \cos\theta\, (\rho^5 \rho^{-6} - \rho^3 \rho^{-5} + 3/16\, \rho\, \rho^{-4})$

In weak-field and slow-motion:

$^*R \cdot R = 288\, (J\, M)/r^7 \cos\theta + \cdots$

J = aM = angular momentum

SOME EXPERIMENTAL ATTEMPTS TO MEASURE FRAME-DRAGGING AND GRAVITOMAGNETISM

- 1896: Benedict and Immanuel FRIEDLANDER
  (torsion balance near a heavy flying-wheel)
- 1904: August FOPPL (Earth-rotation effect on a gyroscope)
- 1916: DE SITTER (shift of perihelion of Mercury due to Sun rotation)
- 1918: LENSE AND THIRRING (perturbations of the Moons of solar system planets by the planet angular momentum)
- 1959: Yilmaz (satellites in polar orbit)
- 1976: Van Patten-Everitt
  (two non-passive counter-rotating satellites in polar orbit)
- 1960: Schiff-Fairbank-Everitt (Earth orbiting gyroscopes)
- 1986: I.C.: USE THE NODES OF TWO LAGEOS SATELLITES
  (two supplementary inclination, passive, laser ranged satellites)
- Lunar Laser Ranging measurements of geodetic precession
- 1988 : Nordtvedt (Astrophysical evidence from periastron
  rate of binary pulsar)
- 1995-2007: I.C. et al. measurements using LAGEOS and LAGEOS-II
- 1998: Some astrophysical evidence from accretion disks of black holes and neutron stars
- 2004 launch of Gravity Probe B
- Binary Pulsars/Gravitomagnetic deflection of radio waves due to the orbital motion of Jupiter (Kopeikin)
- LARES (2007-2009) by ASI

Lunar Laser Ranging has been and is a basic tool for testing fundamental physics, it has provided:

- **Very accurate test of the weak equivalence principle**
- **Accurate test of the strong equivalence principle (Nordtvedt effect)**
- **Accurate measurement of PPN (Post Newtonian Parametrized) parameters**
- **Limits on G-dot**
- **Very accurate measurement of the geodetic effect with accuracy of about 0.6 percent**

(see: Williams, J.G., Turyshev, S.G., and Boggs, D.H. Phys. Rev. Lett (2004).
Williams, J.G., Newhall, X.X., and Dickey, J.O. Phys. Rev(1996).

For a discussion on frame-dragging, Lense-Thirring effect and geodetic precession, see: O’Connell, R.F. A Note on Frame Dragging. Class. Quant. Grav. (2005) and Proc. of Course CLXVIII of the International School of Physics "Enrico Fermi", Varenna, Italy, 2007, ed. E. Arimondo, W. Ertmer and W. Schleich, (2008).

Has Lunar Laser Ranging measured the gravitomagnetic field by spin (Lense-Thirring effect) or not?.

Debate:

Murphy, T.W. Jr., Nordtvedt, K. $\&$ Turyshev, S.G. Gravitomagnetic Influence on Gyroscopes and on the Lunar Orbit. Phys. Rev. Lett. 98, 071102--1-4 (2007).

Kopeikin, S.M. Comment on "The gravitomagnetic influence on gyroscopes And on the lunar orbit". Phys. Rev. Lett. 98 229001 (2007).

Murphy, T.W. Jr., Nordtvedt, K. $\&$ Turyshev, S.G. A Reply to the Comment by Sergei M. Kopeikin. Phys. Rev. Lett. 98, 229002 (2007).

In:
I.C. Gravitomagnetism, Frame-Dragging and Lunar Laser Ranging. ArXiv:0704.3338v2 [gr-qc] 10 May 2007;);

I.C.and Pavlis, E. Proc. of 15th International Laser Ranging Workshop, Camberra, Australia, October 16-20, 2006 and

I.C. Lunar Laser Ranging, Frame-Dragging and Gravitomagnetism by Spin, to appear (2008)

It is proven, using two different methods, that LLR measures frame dragging, in the sense of geodetic precession, but NOT frame-dragging by spin, in the sense of Lense-Thirring effect. One method uses the spacetime invariant built out of the Riemann tensor that is substantially zero on the Moon orbit near the ecliptic plane.

GRAVITY PROBE B

R: A Work in Progress

eiser, and J. Turneaure

ntifying the Error

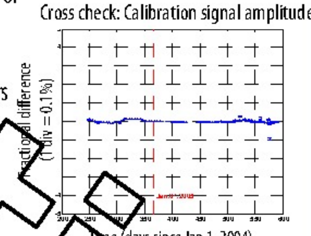
data for one parameter
nalysis with degraded data
lativity result to original
xperiment error for parameter

ency
confidence in result
results decreases error

build confidence
reference parameters

ganize effort

Cross check: Calibration signal amplitude



l relativity uncertainty associated with the modeling of
he systematic error of that effect.

pe, Roll Phase, Timing

Gyro Torques (I)

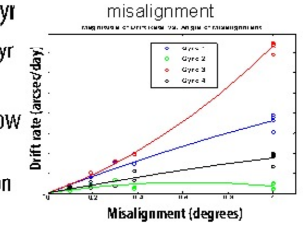
Misalignment torque: Torque proportional to angle subtended by SV roll axis and gyro spin axis

Cause: Interaction of patch fields on rotor and casing

Induced drift rate ~0.1 to 1.0 arcsec/yr

Require > 99 % removal for 10^{-3} arcsec/yr

Measured drift rate vs. misalignment



Misalignment torque: direction is know

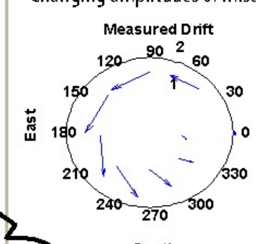
Mitigation: phase sensitive detection

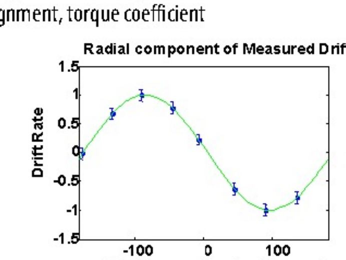
measure relativity in orthogonal direction

Error in resulting mitigated drift:

Uncertainty in torque direction: gives 4 marcsec/yr

Changing amplitudes of misalignment, torque coefficient

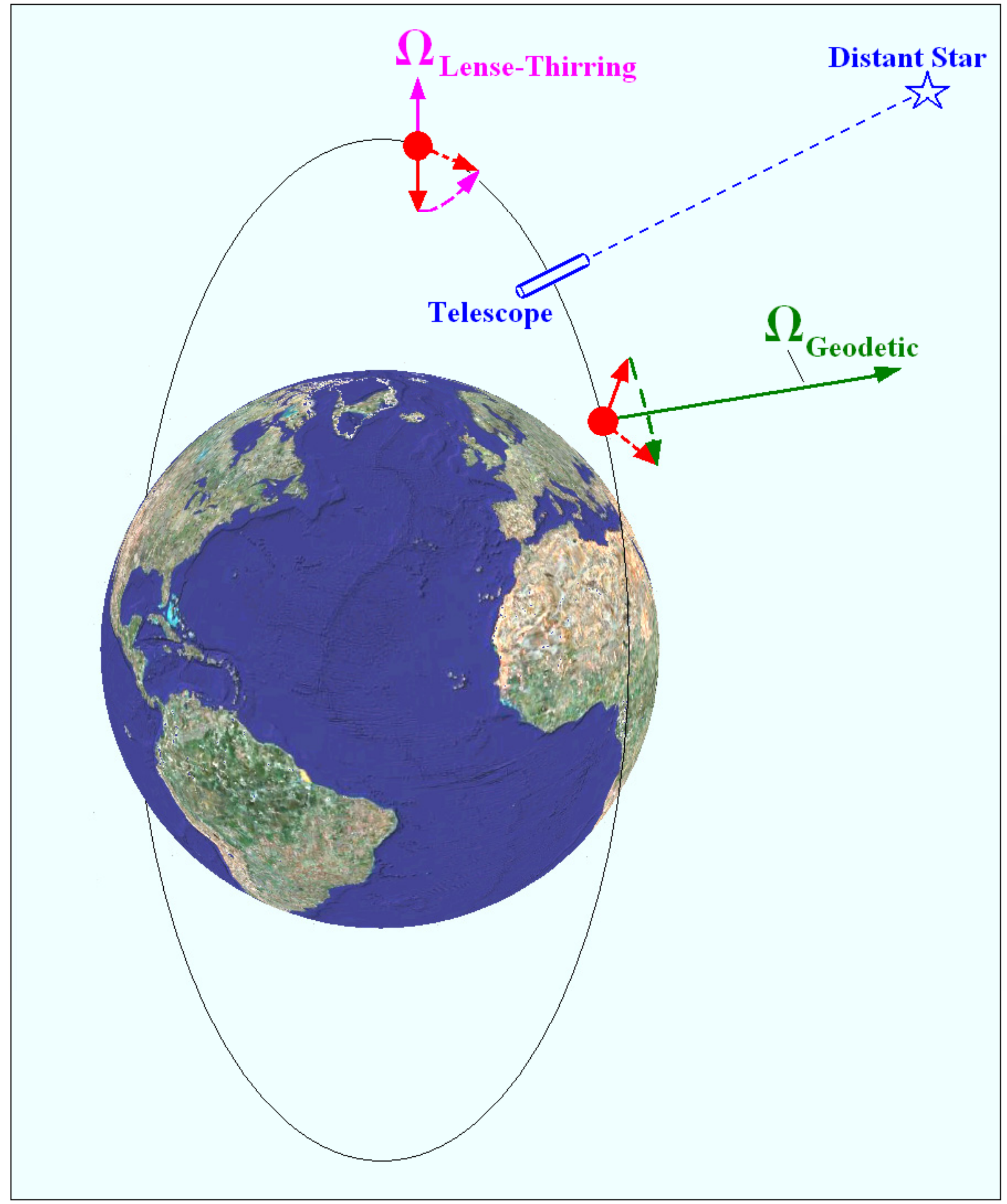


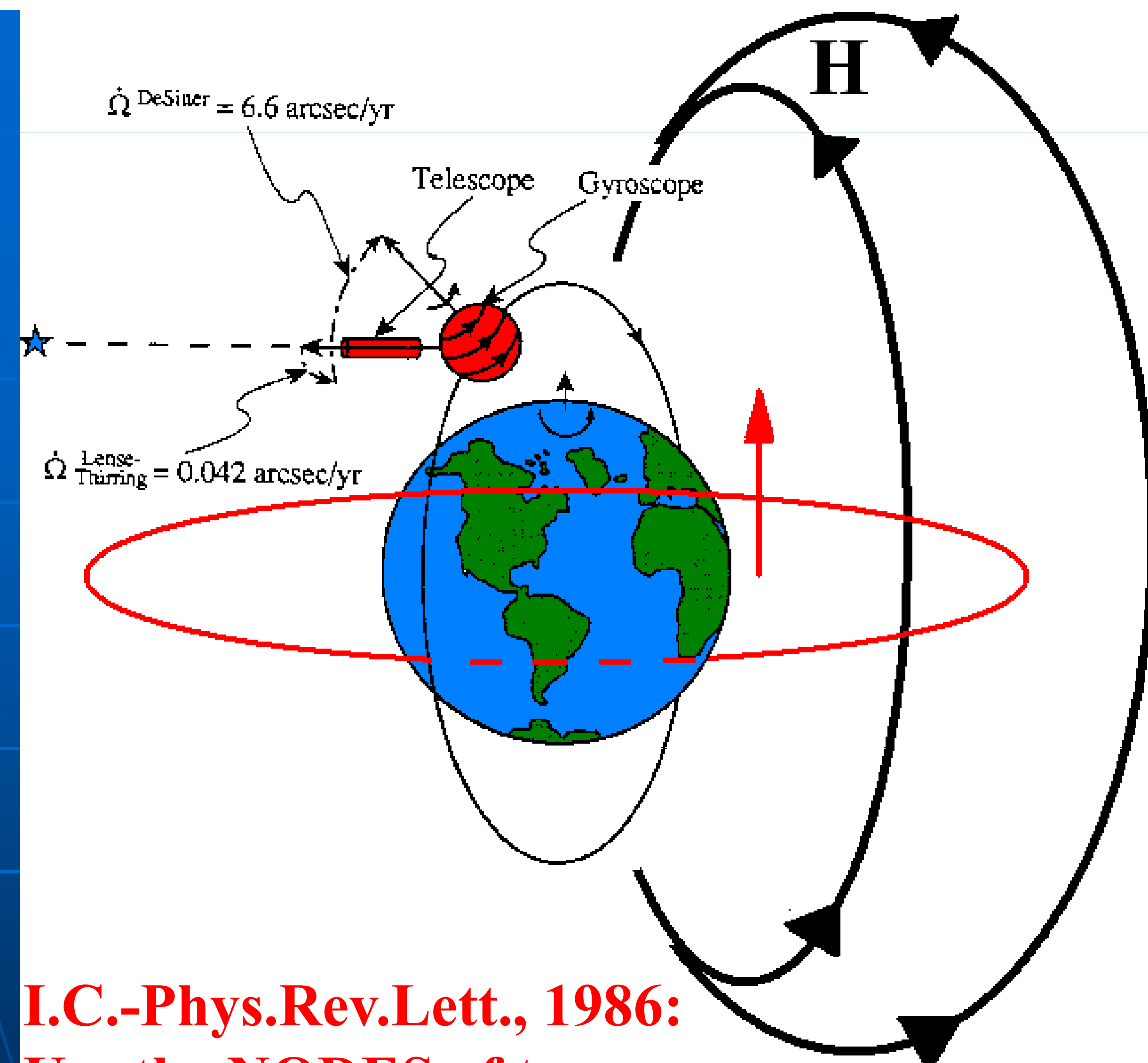


Next Steps

Current Limits on Experiment Error

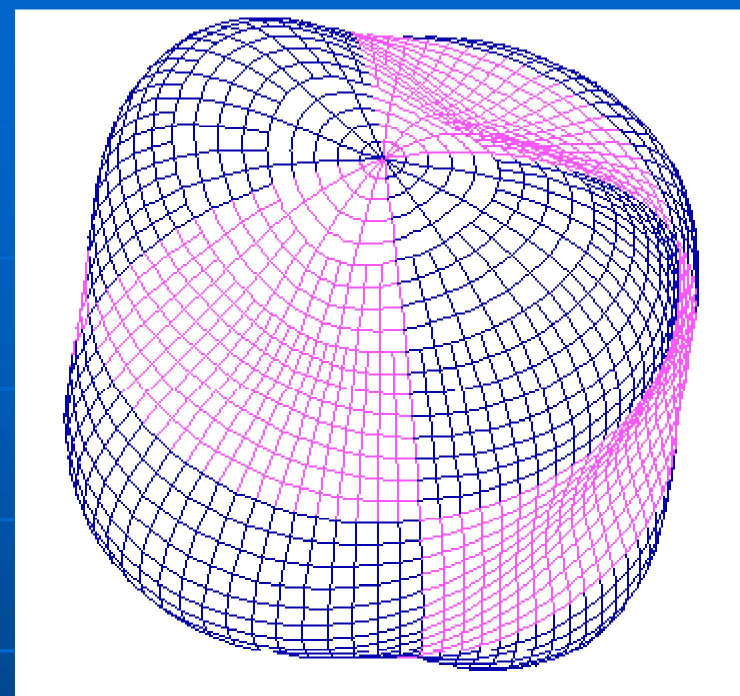
Error Source	Current Error (marcsec/yr)	Mitigation
Current error including resonance observation	100	Improved modeling
EMI effects	5	Data grading & noise removal
Misalignment patch-effect torque	4 *	Improved misalign-ment model
Polhode frequency error (Cg)	2.5	Trapped flux mapping.
Roll phase		None


$\Omega_{\text{Lense-Thirring}}$
Distant Star
Telescope
Ω_{Geodetic}



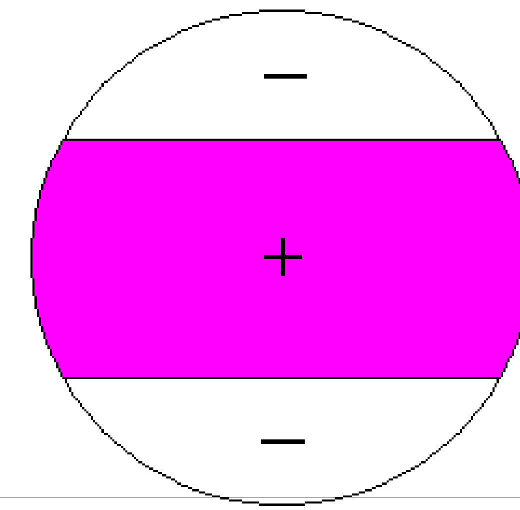
I.C.-Phys.Rev.Lett., 1986:
Use the NODES of two
LAGEOS satellites.

Satellite Laser Ranging

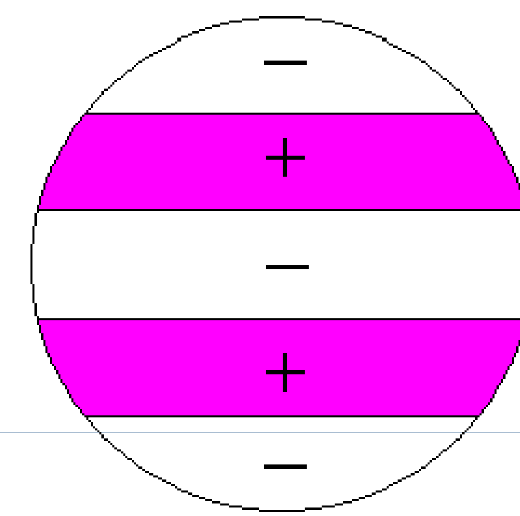


l=3, m=1

EVEN ZONAL HARMONICS



J_2



J_4

Volume 56, Number 4 PHYSICAL REVIEW LETTERS 27 January 1986

Measurement of the Lense-Thirring Drag on High-Altitude, Laser-Ranged Artificial Satellites

Ignazio Ciufolini

Center for Theoretical Physics, Center for Relativity, and Physics Department, University of Texas, Austin, Texas 78712

(Received 16 October 1984; revised manuscript received 19 April 1985)

We describe a new method of measuring the Lense-Thirring relativistic nodal drag using LAGEOS together with another similar high-altitude, laser-ranged satellite with appropriately chosen orbital parameters. We propose, for this purpose, that a future satellite such as LAGEOS II have an inclination supplementary to that of LAGEOS. The experiment proposed here would provide a method for experimental verification of the general relativistic formulation of Mach's principle and measurement of the gravitomagnetic field.

PACS numbers: 04.80.+z

In special and general relativity there are several precession phenomena associated with the angular momentum vector of a body. If a test particle is orbiting a rotating central body, the plane of the orbit of the particle is dragged by the intrinsic angular momentum J of the central body, in agreement with the general relativistic formulation of Mach's principle.[1]

In the weak-field and slow-motion limit the nodal lines are dragged in the sense of rotation, at a rate given by[2]

$$\dot{\Omega} = [2/a^3(1-e^2)^{3/2}]J, \tag{1}$$

where a is the semimajor axis of the orbit, e is the eccentricity of the orbit, and geometrized units are used, i.e., $G=c=1$. This phenomenon is the Lense-Thirring effect, from the names of its discoverers in 1918.[2]

In addition to this there are other precession phenomena associated with the intrinsic angular momentum or spin $\mathbf{S}$ of an orbiting particle. In the weak-field and slow-motion limit the vector $\mathbf{S}$ precesses at a rate given by[1] $d\mathbf{S}/d\tau = \dot{\mathbf{\Omega}} \times \mathbf{S}$ where

$$\dot{\mathbf{\Omega}} = -\tfrac{1}{2}\mathbf{v}\times\mathbf{a} + \tfrac{3}{2}\mathbf{v}\times\nabla U + \frac{1}{r^3}\left[-\mathbf{J} + \frac{3(\mathbf{J}\cdot\mathbf{r})\mathbf{r}}{r^2}\right], \tag{2}$$

where $\mathbf{v}$ is the particle velocity, $\mathbf{a} = d\mathbf{v}/d\tau - \nabla U$ is its nongravitational acceleration, $\mathbf{r}$ is its position vector, τ is its proper time, and U is the Newtonian potential.

The first term of this equation is the Thomas precession.[3] It is a special relativistic effect due to the noncommutativity of nonaligned Lorentz transformations. It may also be viewed as a coupling between the particle velocity $\mathbf{v}$ and the nongravitational forces acting on it.

The second (de Sitter[4]–Fokker[5]) term is general relativistic, arising even for a nonrotating source, from the parallel transport of a direction defined by $\mathbf{S}$; it may be viewed as spin precession due to the coupling between the particle velocity $\mathbf{v}$ and the static $-g_{\alpha\beta,0}=0$ and $g_{i0}=0$—part of the space-time geometry.

The third (Schiff[6]) term gives the general relativistic precession of the particle spin $\mathbf{S}$ caused by the intrinsic angular momentum $\mathbf{J}$ of the central body—$g_{i0}\neq 0$.

We also mention the precession of the periapsis of an orbiting test particle due to the angular momentum of the central body. This tiny shift of the perihelion of Mercury due to the rotation of the Sun was calculated by de Sitter in 1916.[7]

All these effects are quite small for an artificial satellite orbiting the Earth.

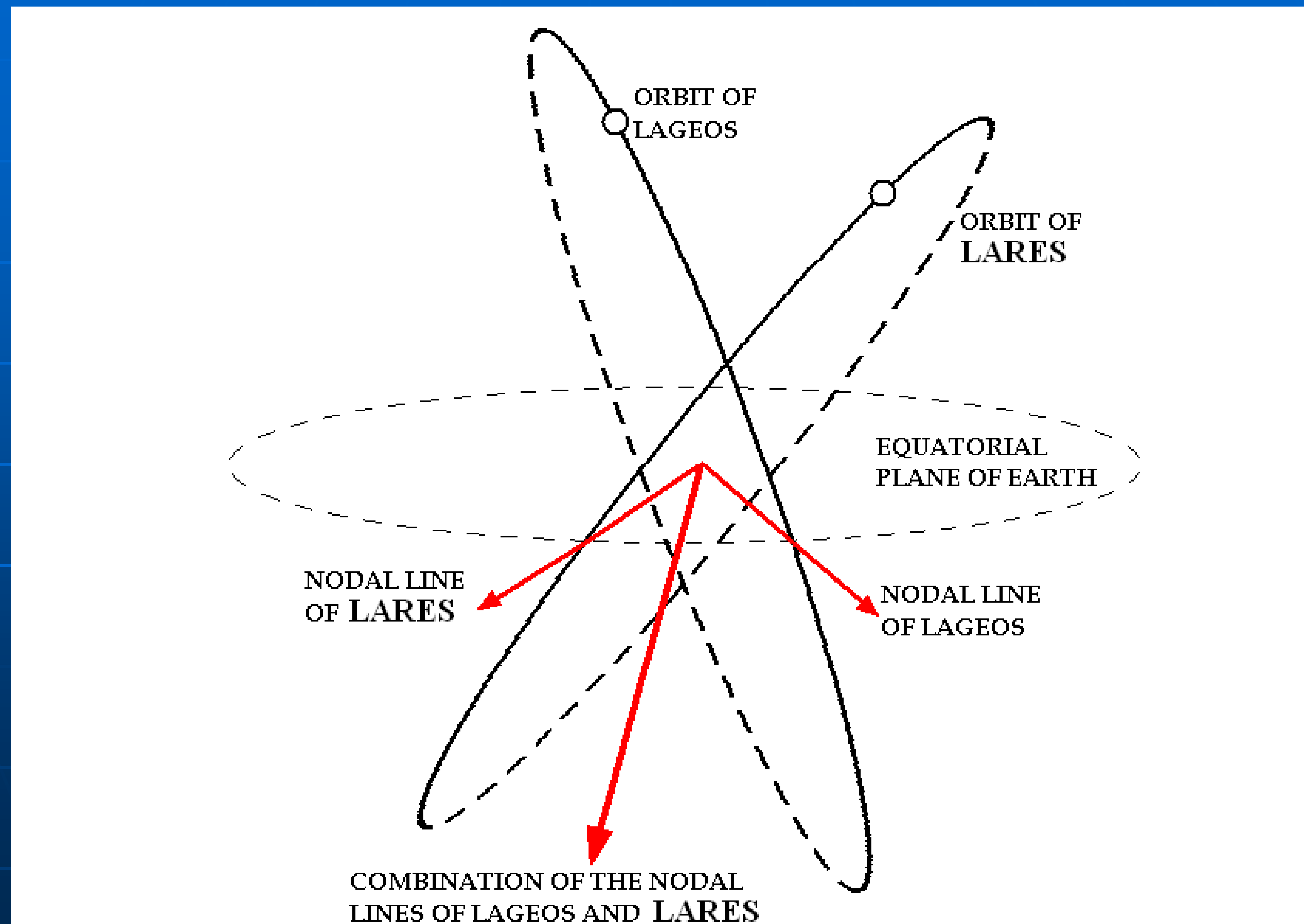
We propose here to measure the Lense-Thirring dragging by measuring the nodal precession of laser-ranged Earth satellites. We shall show that two satellites would be required; we propose that LAGEOS[8-10] together with a second satellite LAGEOS X with opposite inclination (i.e., with $I^X = 180° - I$, where $I \simeq 109.94°$ is the orbital inclination of LAGEOS) would provide the needed accuracy.

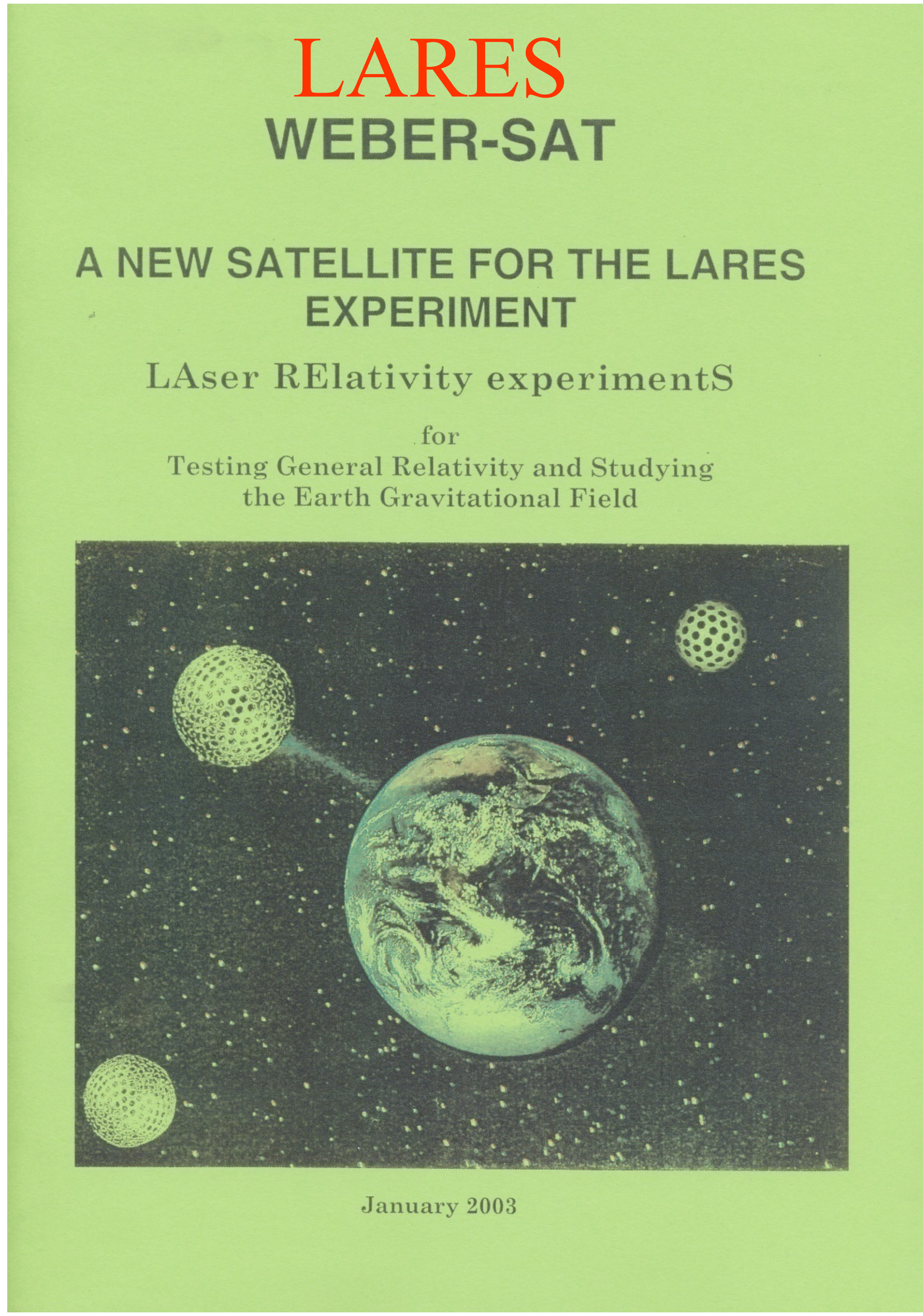
The major part of the nodal precession of an Earth satellite is a classical effect due to deviations from spherical symmetry of the Earth's gravity field—quadrupole and higher mass moments.[11] These deviations from sphericity are measured by the expansion of the potential $U(r)$ in spherical harmonics. From this expansion of $U(r)$ follows[11] the formula for the classical precession of the nodal lines of an Earth satellite:

$$\dot{\Omega}_{\text{class}} \simeq -\frac{3}{2}n\left(\frac{R_\oplus}{a}\right)^2 \frac{\cos I}{(1-e^2)^2}\left\{J_2 + J_4\left[\frac{5}{8}\left(\frac{R_\oplus}{a}\right)^2(7\sin^2 I - 4)\frac{1+\frac{3}{2}e^2}{(1-e^2)^2}\right] + \dots\right\}, \tag{3}$$

IC, PRL 1986:
Use of the nodes of two laser-ranged satellites to measure the Lense-Thirring effect

CONCEPT OF THE LAGEOS III / LARES EXPERIMENT




LARES
WEBER-SAT

A NEW SATELLITE FOR THE LARES EXPERIMENT

LAser RElativity experimentS

for
Testing General Relativity and Studying the Earth Gravitational Field

January 2003

MAIN COLLABORATION

I.C.

University of Lecce and INFN

A. Paolozzi

"Sapienza" University of Roma and INFN

LNF-INFN

E. Pavlis

University of Maryland

R. Matzner

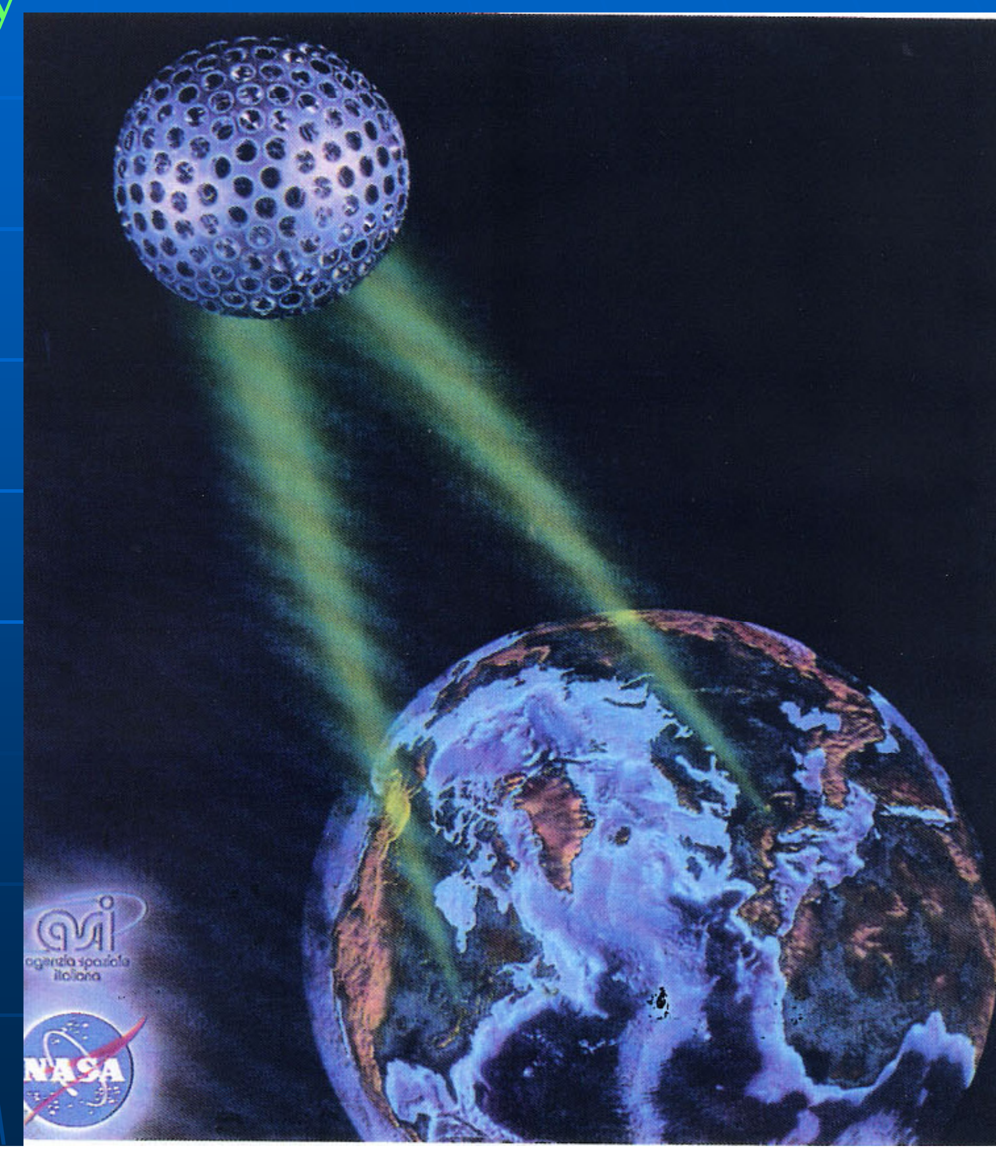
University of Texas at Austin

D. Rubincam

NASA-Goddard

Lageos II: 1992

However, NO LAGEOS satellite with supplementary inclination to LAGEOS has ever been launched. Nevertheless, LAGEOS II was launched in 1992.



International Journal of Modern Physics A, Vol. 4, No. 13 (1989) 3083–3145

A COMPREHENSIVE INTRODUCTION TO THE LAGEOS GRAVITOMAGNETIC EXPERIMENT: FROM THE IMPORTANCE OF THE GRAVITOMAGNETIC FIELD IN PHYSICS TO PRELIMINARY ERROR ANALYSIS AND ERROR BUDGET

IGNAZIO CIUFOLINI

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and

Center for Space Research, The University of Texas at Austin, Austin, Texas 78712, USA

Received 3 May 1988
Revised 7 October 1988

The existence of the gravitomagnetic field, generated by mass currents according to Einstein geometrodynamics, has never been proved. The author of this paper, after a discussion of the importance of the gravitomagnetic field in physics, describes the experiment that he proposed in 1984 to measure this field using LAGEOS (Laser geodynamics satellite) together with another non-polar, laser-ranged satellite with the same orbital parameters as LAGEOS but a supplementary inclination.

The author then studies the main perturbations and measurement uncertainties that may affect the measurement of the Lense-Thirring drag. He concludes that, over the period of the node of ~ 3 years, the maximum error, using two nonpolar laser ranged satellites with supplementary inclinations, should not be larger than ~ 10% of the gravitomagnetic effect to be measured.

Table of Contents

IC IJMPA 1989:
Analysis of the orbital perturbations affecting the nodes of LAGEOS-type satellites

(1) Use two LAGEOS satellites with supplementary inclinations

OR:

Use n satellites of LAGEOS-type to measure the first n-1 even zonal harmonics: J_2, J_4, … and the Lense-Thirring effect

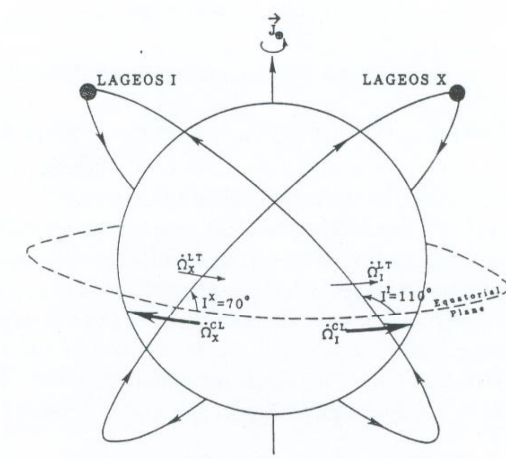


Fig. 5. The LAGEOS and LAGEOS X orbits and their classical and gravitomagnetic nodal precessions. A new[17] configuration to measure the Lense-Thirring effect.

For J_2, this corresponds, from formula (3.2), to an uncertainty in the nodal precession of 450 milliarcsec/year, and similarly for higher J_{2n} coefficients. Therefore, the uncertainty in $\dot{\Omega}^{\text{Class}}_{\text{Lageos}}$ is more than ten times larger than the Lense-Thirring precession.

A solution would be to orbit several high-altitude, laser-ranged satellites, similar to LAGEOS, to measure J_2, J_4, J_6, etc., and one satellite to measure $\dot{\Omega}^{\text{Lense-Thirring}}$.

Another solution would be to orbit polar satellites; in fact, from formula (3.2), for polar satellites, since $I = 90°$, $\dot{\Omega}^{\text{Class}}$ is equal to zero. As mentioned before, Yilmaz proposed the use of polar satellites in 1959.[40,41] In 1976, Van Patten and Everitt[46,47] proposed an experiment with two drag-free, guided, counter-rotating, polar satellites to avoid inclination measurement errors.

A new solution[15,16,17,21,22,23] would be to orbit a second satellite, of LAGEOS type, with the same semimajor axis, the same eccentricity, but the inclination supplementary to that of LAGEOS (see Fig. 5). Therefore, "LAGEOS X" should have the following orbital parameters:

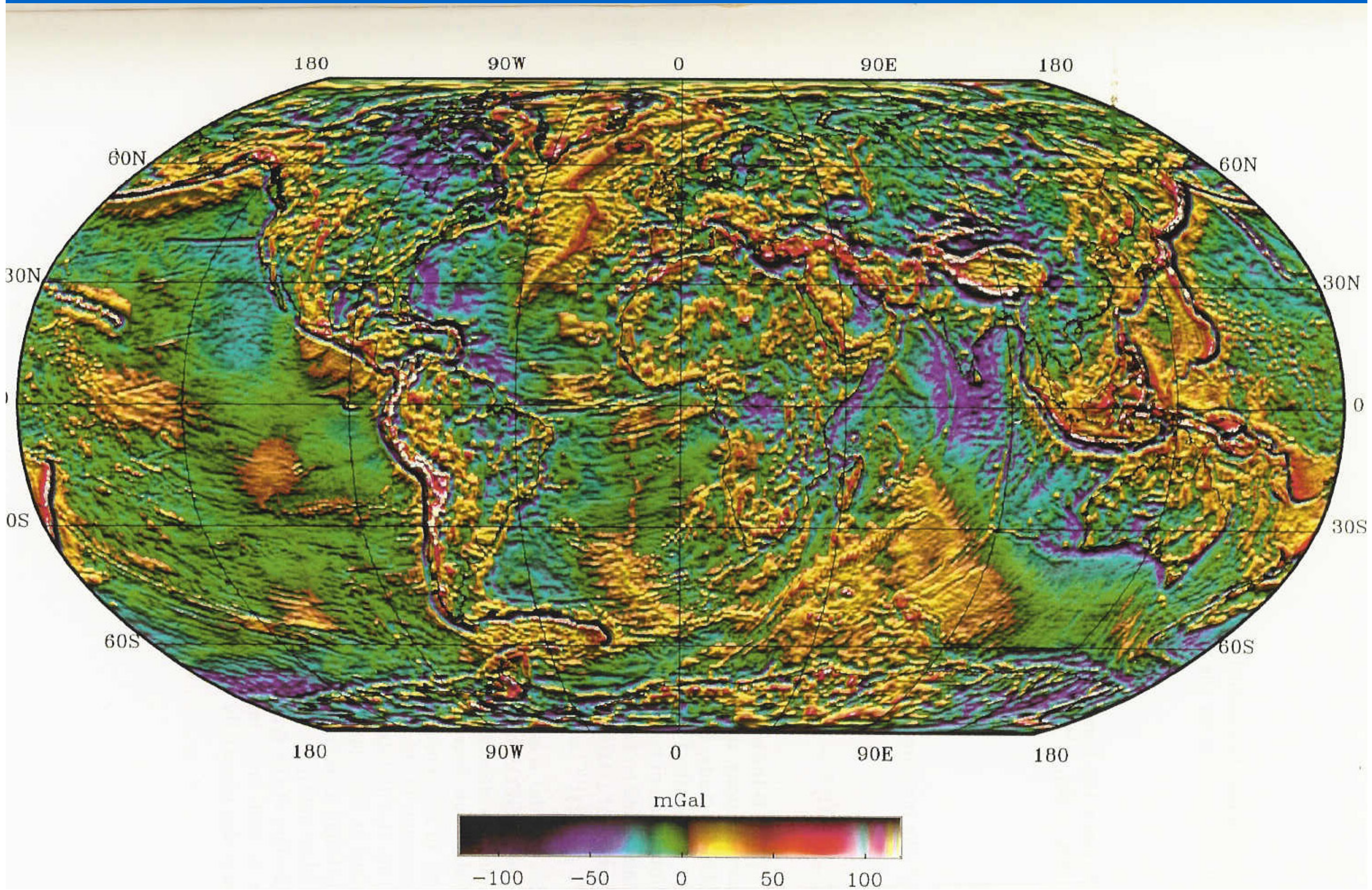
$$I^X \cong \pi - I^I \cong 70°, \qquad a^X \cong a^I, \qquad e^X \cong e^I. \tag{3.3}$$

With this choice, since the classical precession $\dot{\Omega}^{\text{Class}}$ is linearly proportional to $\cos I$, $\dot{\Omega}^{\text{Class}}$ would be equal and opposite for the two satellites:

$$\dot{\Omega}^{\text{Class}}_X = -\dot{\Omega}^{\text{Class}}_I. \tag{3.4}$$

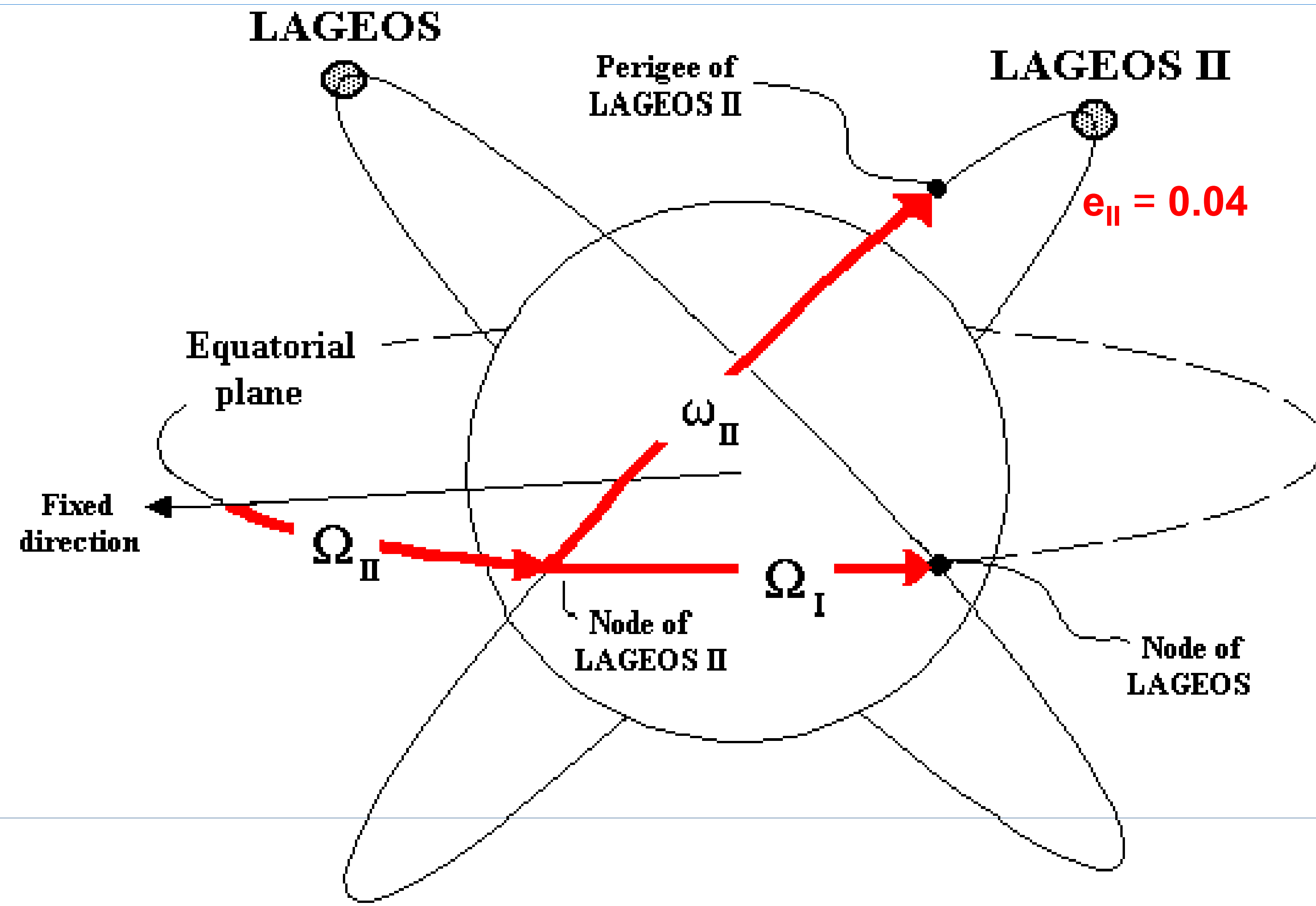
By contrast, since the Lense-Thirring precession $\dot{\Omega}^{\text{Lense-Thirring}}$ is independent of the inclination (Eq. (3.1)), $\dot{\Omega}^{\text{Lense-Thirring}}$ will be the same in magnitude and sign for both satellites:

EGM-96 GRAVITY MODEL



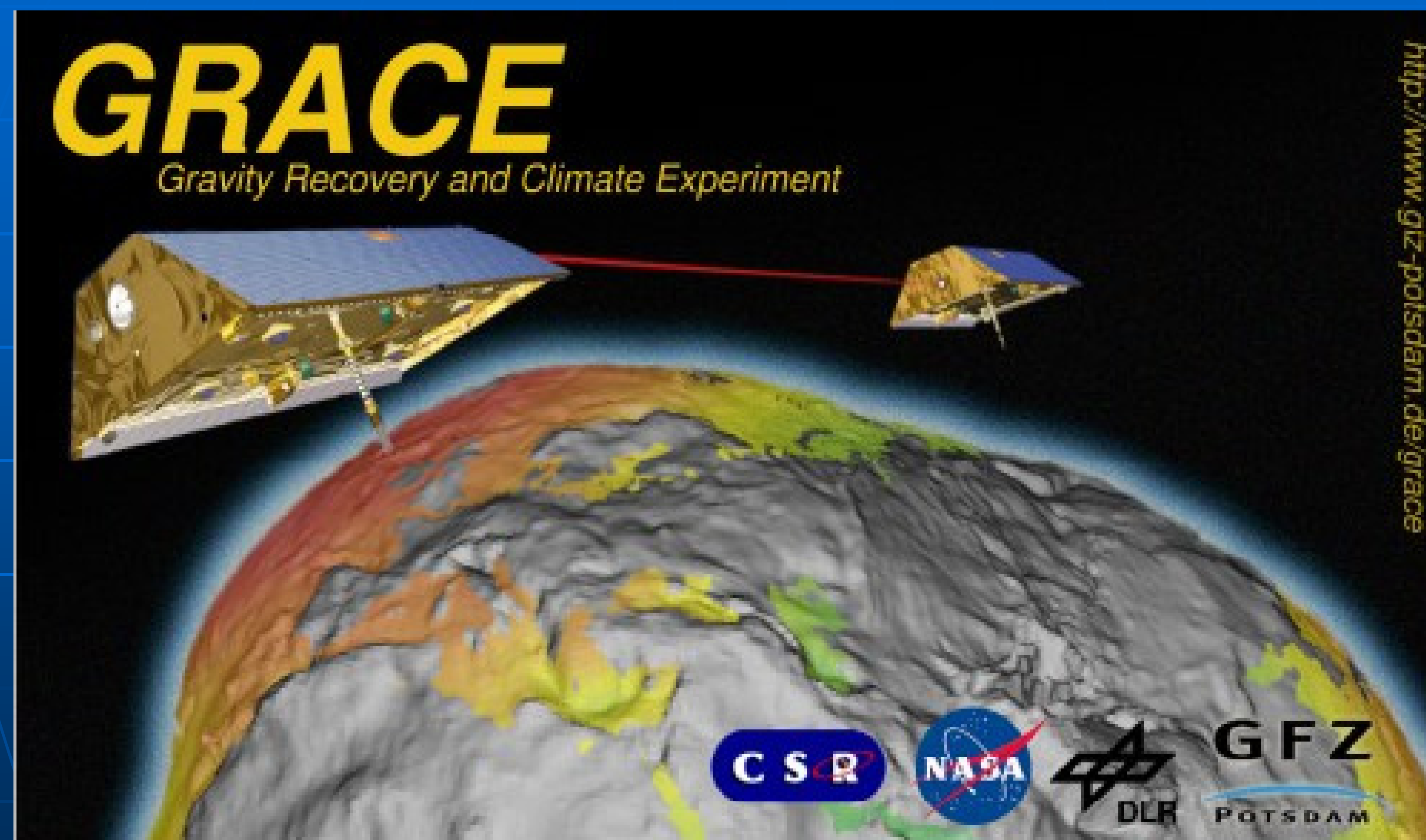
EGM96 Model and its uncertainties

Even zonals **l m**	value	Uncer -tainty in value	Uncer-tainty on node I	Uncer-tainty on node II	Uncer-tainty on Perigee II
20	**-0.484165 37 x 10^{-03}**	**0.36x10^{-10}**	**1 Ω_{LT}**	**2 Ω_{LT}**	**0.8 ω_{LT}**
40	**0.53987386 x 10^{-06}**	**0.1 x 10^{-09}**	**1.5 Ω_{LT}**	**0.5 Ω_{LT}**	**2.1 ω_{LT}**
60	**-0.14995**	**0.15x10^{-09}**	**0.6 Ω_{LT}**	**0.9 Ω_{LT}**	**0.31 ω_{LT}**

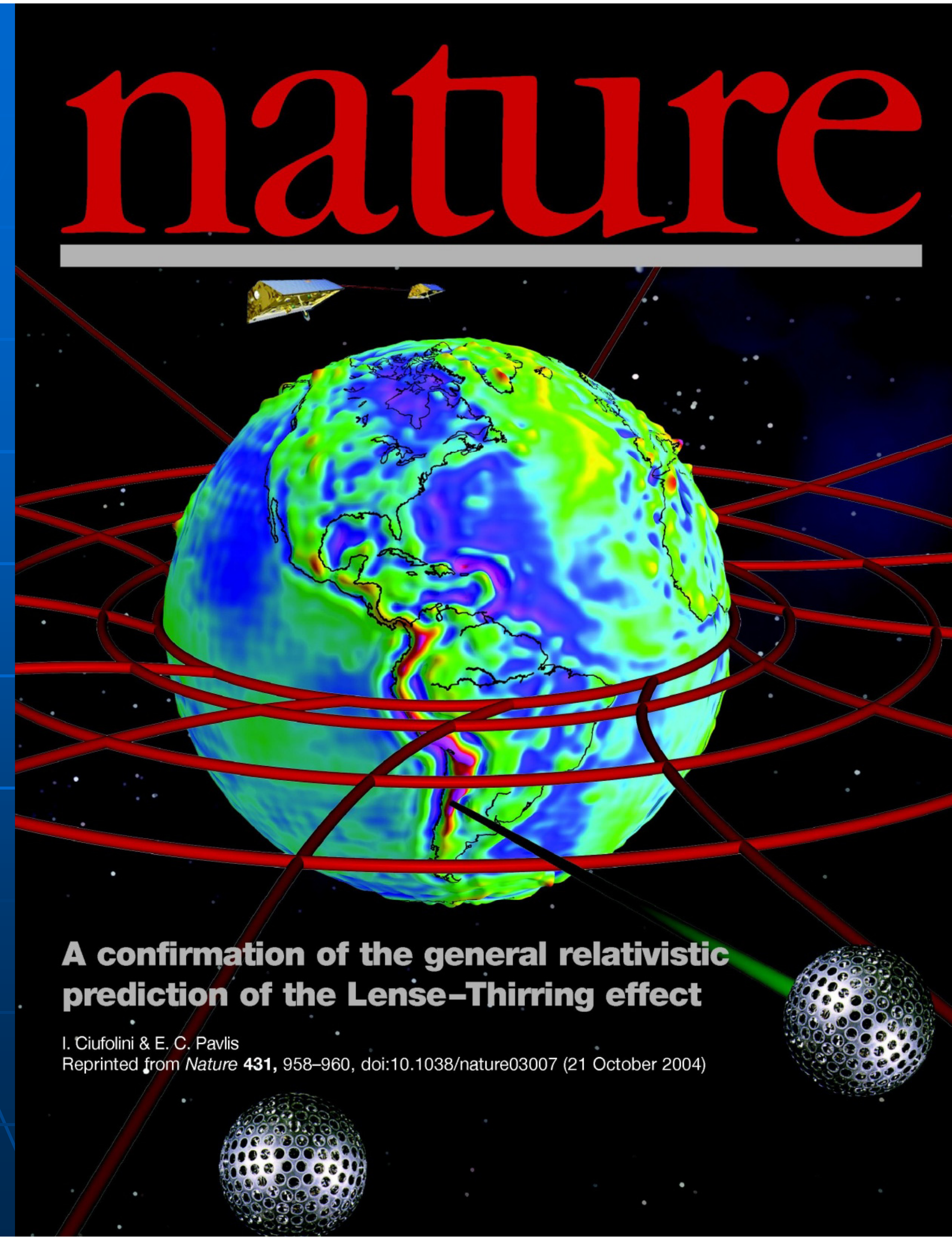

LAGEOS
Perigee of
LAGEOS II
LAGEOS II
e_{II} = 0.04
Equatorial
plane
ω_{II}
Fixed
direction
Ω_{II}
Ω_{I}
Node of
LAGEOS II
Node of
LAGEOS

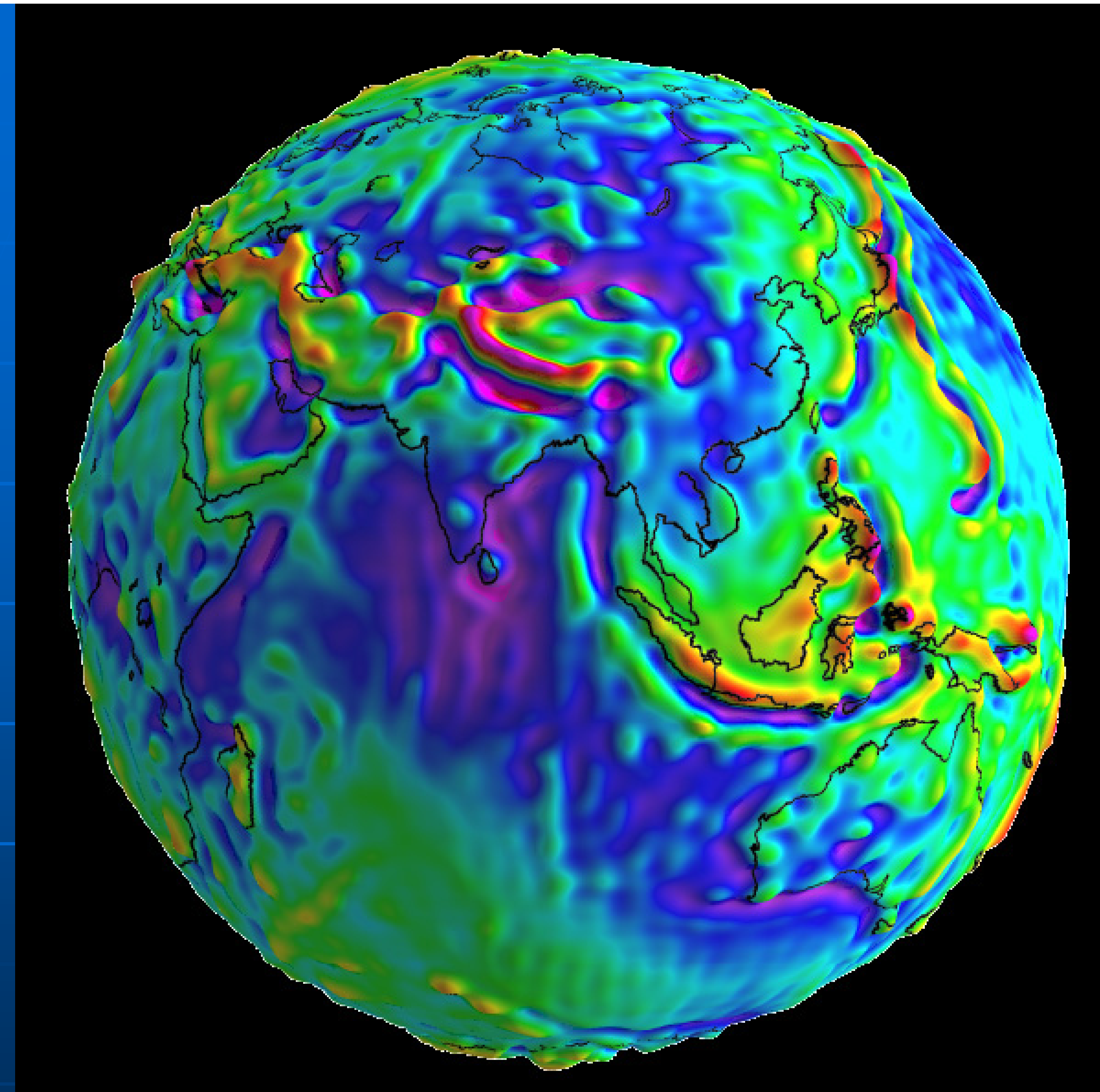
I.C., NC A, 1996

2002



Use of GRACE to test Lense-Thirring at a few percent level:
J. Ries et al. 2003 (1999),E. Pavlis 2002 (2000)


nature
A confirmation of the general relativistic prediction of the Lense–Thirring effect
I. Ciufolini & E. C. Pavlis
Reprinted from Nature 431, 958–960, doi:10.1038/nature03007 (21 October 2004)



EIGEN-GRACE-S (GFZ 2004)

EIGEN-GRACE02S Model and Uncertainties

Even zonals lm	Value $\cdot 10^{-6}$	Uncertainty	Uncertainty on node I	Uncertainty on node II	Uncertainty on perigee II
20	-484.16519788	$0.53 \cdot 10^{-10}$	$1.59\ \Omega_{LT}$	$2.86\ \Omega_{LT}$	$1.17\ \omega_{LT}$
40	0.53999294	$0.39 \cdot 10^{-11}$	$0.058\ \Omega_{LT}$	$0.02\ \Omega_{LT}$	$0.082\ \omega_{LT}$
60	-.14993038	$0.20 \cdot 10^{-11}$	$0.0076\ \Omega_{LT}$	$0.012\ \Omega_{LT}$	$0.0041\ \omega_{LT}$
80	0.04948789	$0.15 \cdot 10^{-11}$	$0.00045\ \Omega_{LT}$	$0.0021\ \Omega_{LT}$	$0.0051\ \omega_{LT}$
10,0	0.05332122	$0.21 \cdot 10^{-11}$	$0.00042\ \Omega_{LT}$	$0.00074\ \Omega_{LT}$	$0.0023\ \omega_{LT}$

Using EIGEN-GRACE02S:
2 main unknowns: δC_{20} and LT
Needed 2 observables:

$\delta\Omega_I, \delta\Omega_{II}$ (orbital angular momentum vector)

- $\delta\Omega_I$ = K_2 x δC_{20} + K_{2n} x $\delta C_{2n,0}$ + μ (31 mas/yr)
- $\delta\Omega_{II}$ = K'_2 x δC_{20} + K'_{2n} x $\delta C_{2n,0}$ + μ (31.5 mas/yr)

$$\mu = \delta\Omega_I + K * \delta\Omega_{II}:$$

not dependent on δC_{20}
free from non-gravitational errors on the perigee

TOTAL ERROR FROM EVEN ZONALS ρ C40 =
= 3% to 4 % Lense-Thirring

I.C. PRL 1986; I.C. IJMP A 1989;
I.C. NC A, 1996; I.C. Proc. I SIGRAV School, Frascati 2002, IOP.

IC
Nuovo Cimento A
1996

for LAGEOS II: $\dot{\omega}_{\text{LAGEOS II}} \cong 160°/\text{year}$, and the classical perigee precession is:

$$\dot{\omega}^{\text{Class}} = -\frac{3}{4}\, n \left(\frac{R_\oplus}{a}\right)^2 \frac{1-5\cos^2 I}{(1-e^2)^2}\, J_2 - \tag{11}$$
$$-\big[\big[15 n R_\oplus^4 (108 + 135 e^2 + 208\cos(2I) + 252 e^2 \cos(2I) + 196\cos(4I) + $$
$$+ 189 e^2 \cos(4I))\big]/(1024 a^4 (1-e^2)^4)\big]\, J_4 + \Sigma P_{2n} \times J_{2n}\,,$$

where the P_{2n} are the coefficients (in the equation for the perigee rate) of the nonnormalized even zonal harmonics $J_{2n} \equiv -\sqrt{4n+1}\, C_{2n0}$. Thus, for the perigee of LAGEOS II, one has (in units of $\dot{\omega}_{\text{II}}^{\text{Lense-Thirring}}$):

	$\delta\dot{\omega}_{\text{II}}/\dot{\omega}_{\text{II}}^{\text{L-T}}$ due to JGM3 estimated errors	$\delta\dot{\omega}_{\text{II}}/\dot{\omega}_{\text{II}}^{\text{L-T}}$ due to difference (JGM3 − GEMT3)
δC_{20}	~ 1.1	~ 5.9
δC_{40}	~ 2.1	~ 5.3
δC_{60}	~ 0.41	~ 0.32
δC_{80}	~ 0.68	~ 0.8
$\delta C_{10,0}$	~ 0.22	~ 0.07

From these uncertainties in the perigee rate of LAGEOS II, similarly to what inferred for the nodal rates, it is manifest that the dominating error sources are due to the uncertainties in C_{20} and C_{40}.

Thus, summarizing, we have now the three unknowns δC_{20}, δC_{40} and Lense-Thirring effect, and the three observable quantities $\dot{\Omega}_{\text{LAGEOS}}$, $\dot{\Omega}_{\text{LAGEOS II}}$, and $\dot{\omega}_{\text{LAGEOS II}}$.

The main unmodeled part of the LAGEOS I nodal rate, due to the uncertainties in the even zonal harmonics, to the errors in the value of the orbital parameters (mainly the inclination), and including the Lense-Thirring effect (to be determined), is:

$$\delta\dot{\Omega}_I = (-9.3\cdot 10^{11}) \times \delta C_{20} - (4.62\cdot 10^{11}) \times \delta C_{40} + \Sigma N_{2n} \times \delta C_{2n0} + 6 \times \delta I_I + 31\mu\,, \tag{12}$$

where $\delta\dot{\Omega}$ is in units of milliarcsec/year, and δI in milliarcsec. This formula shows the main error sources in the calculated nodal rate (apart from the errors due to tides and to nongravitational perturbations; see below). In this formula the first two contributions are due to the uncertainties δC_{20} and δC_{40}, we then have the error due to the uncertainties in the higher even zonal harmonics δC_{2n0} (with $2n \geqslant 6$), and the error due to the uncertainties in the determination of the inclination δI_I. In this formula we have also included the Lense-Thirring [2] parameter μ, by definition 1 in general relativity: $\mu^{\text{GR}} \equiv 1$, that, if not incorporated in the modeling of the orbital perturbations, will affect the orbital residuals. One can write a similar expression for the node of LAGEOS II:

$$\delta\dot{\Omega}_{\text{II}} = (17.17\cdot 10^{11}) \times \delta C_{20} + \tag{13}$$
$$+ (1.68\cdot 10^{11}) \times \delta C_{40} + \Sigma N''_{2n} \times \delta C_{2n0} + 5.3 \times \delta I_{\text{II}} + 31.5\mu$$

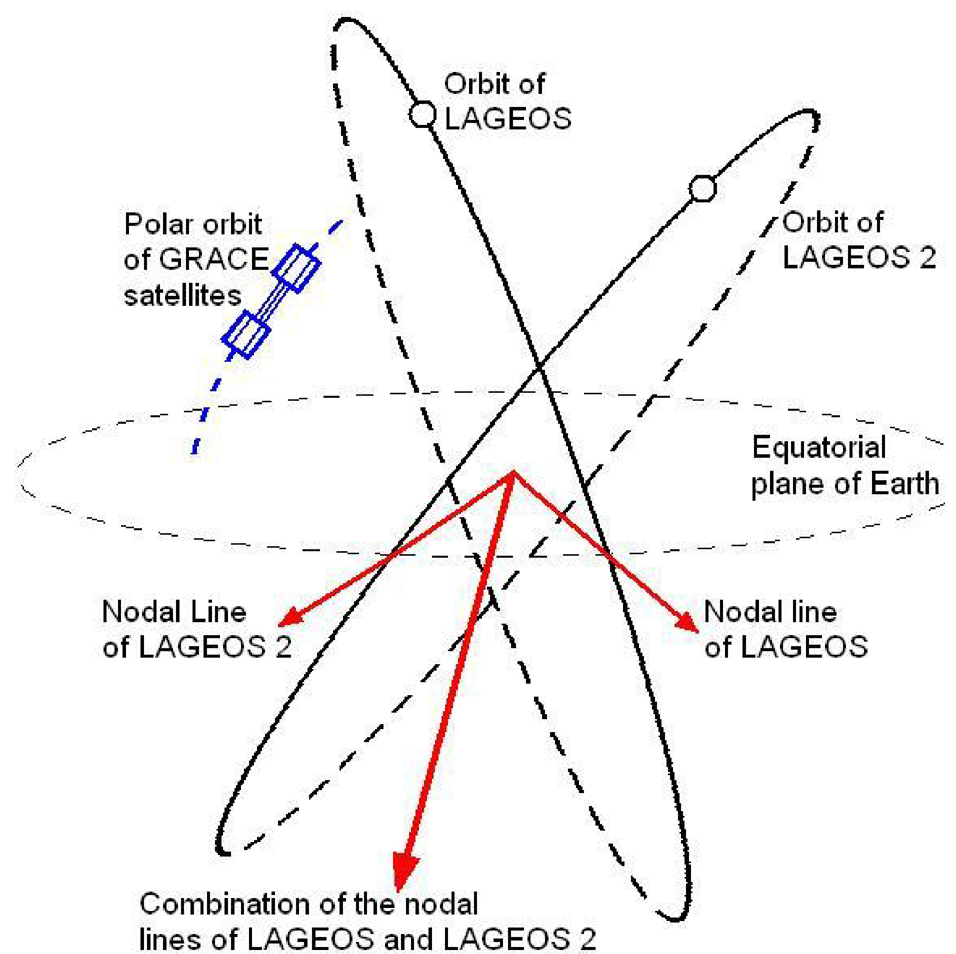
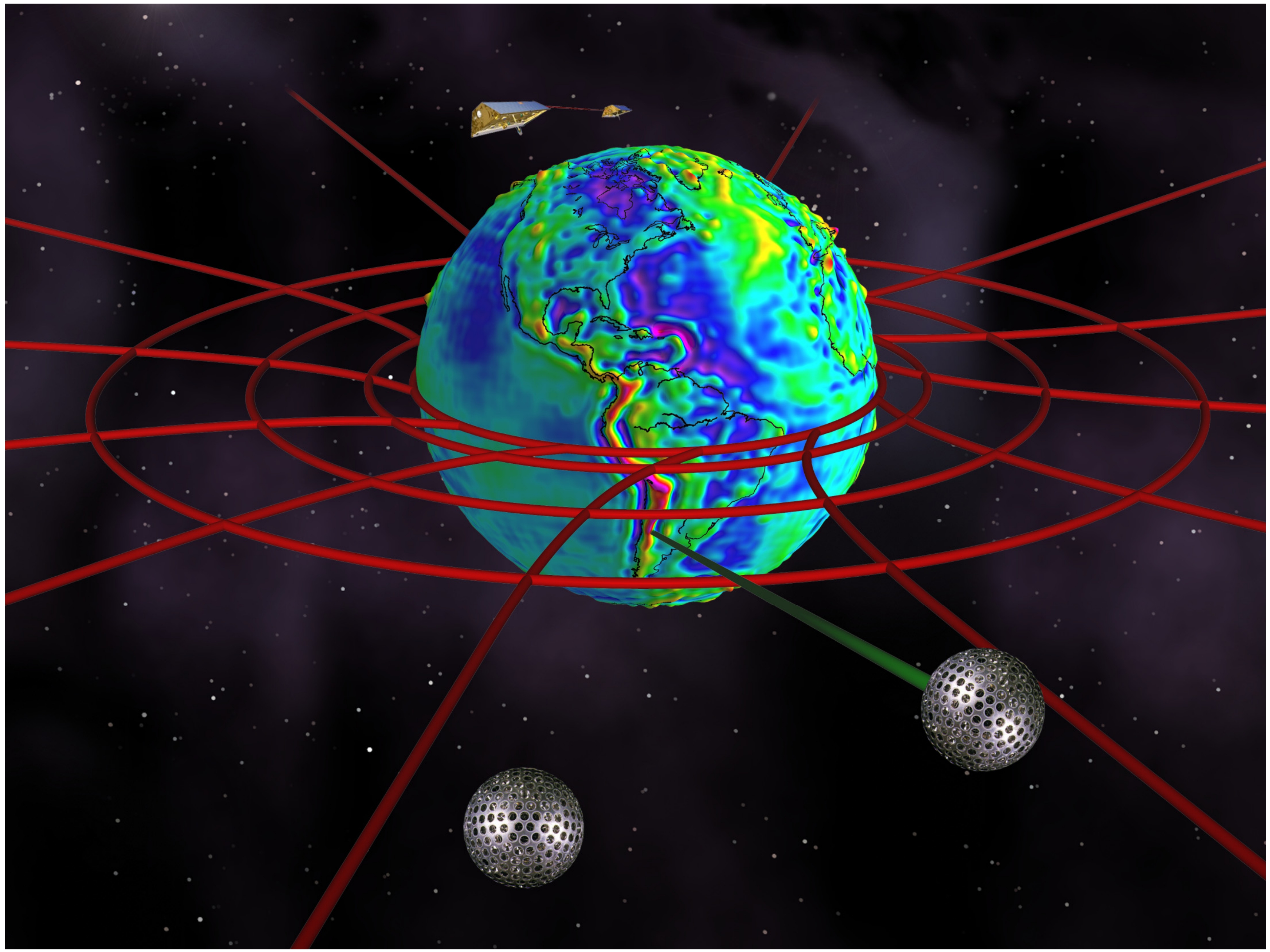

Orbit of
LAGEOS
Orbit of
LAGEOS 2
Polar orbit
of GRACE
satellites
Equatorial
plane of Earth
Nodal Line
of LAGEOS 2
Nodal line
of LAGEOS
Combination of the nodal
lines of LAGEOS and LAGEOS 2

Figure 1



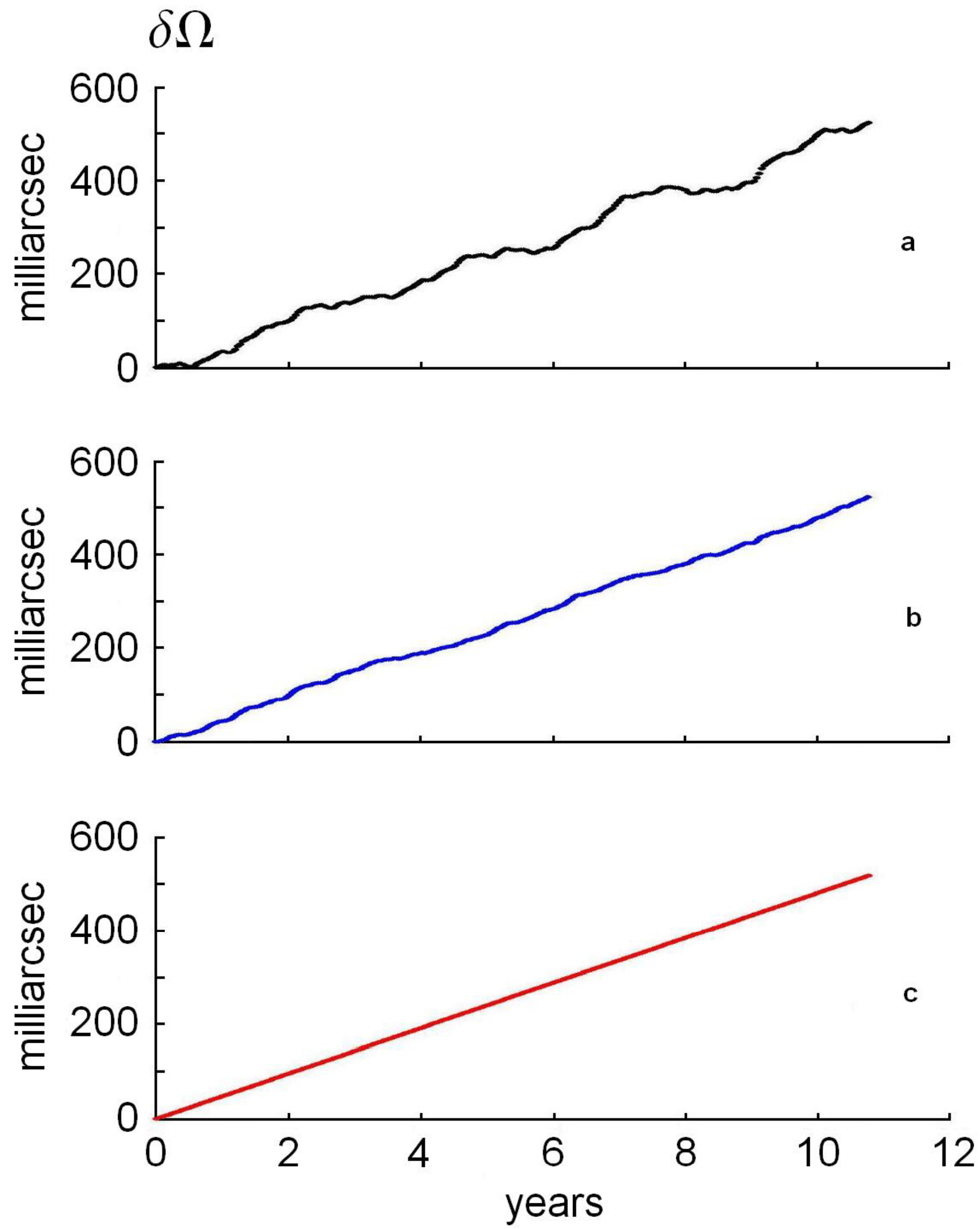


Figure 2

Observed value of
Lense-Thirring effect using
The combination of the
LAGEOS nodes.

Observed value of
Lense-Thirring effect = 99%
of the general relativistic
prediction. Fit of linear trend
plus 6 known frequencies

General relativistic
Prediction = 48.2 mas/yr

I.C. & E.Pavlis,
Letters to NATURE,
431, 958, 2004.

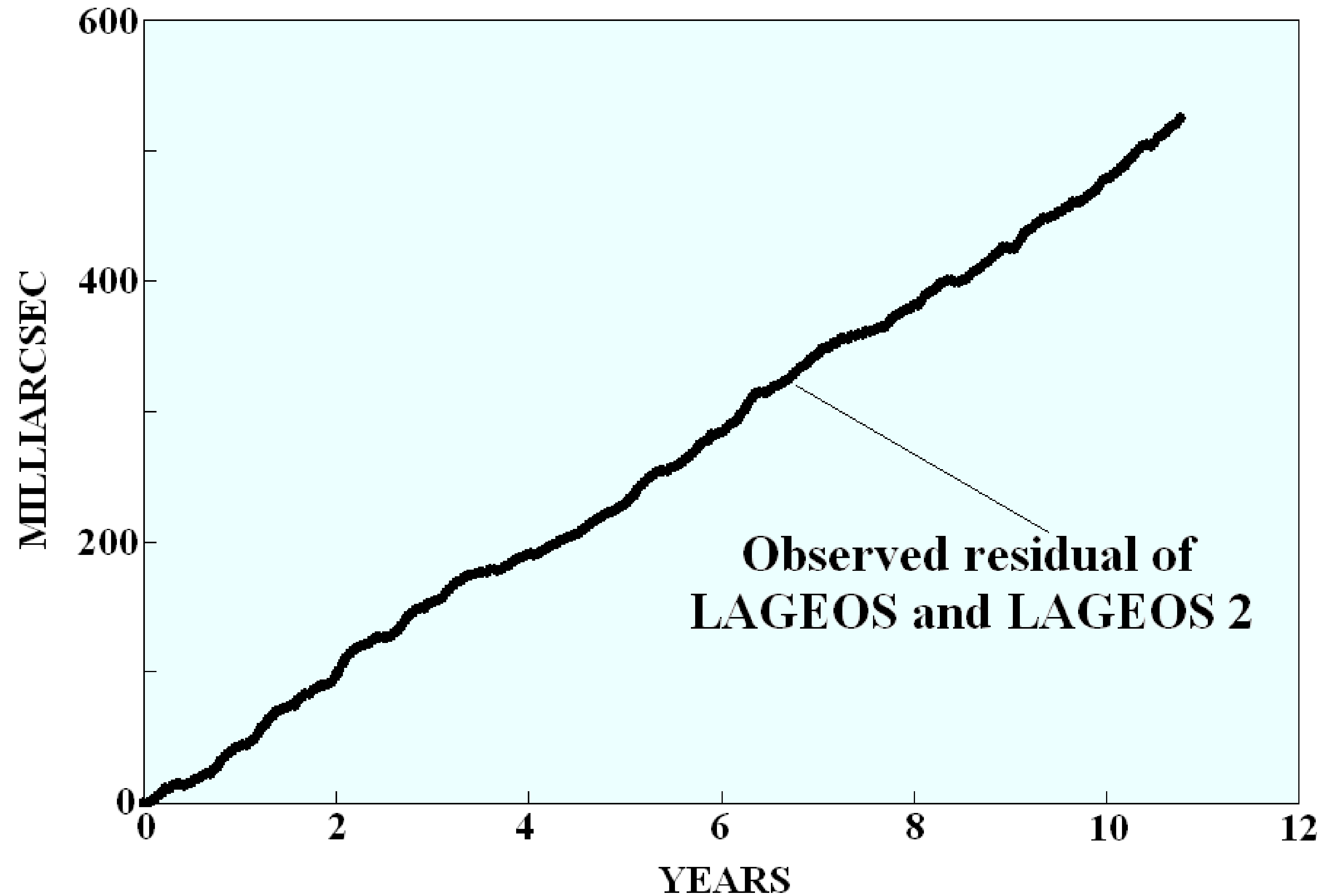
Error budget

- **Static gravitational field (using the EIGEN-GRACE02S uncertainties):**
  3 % to 4 % (the EIGEN-GRACE02S uncertainties include systematic errors) or 6 % to 8 % doubling the uncertainty published with EIGEN-GRACE02S.
  Time dependent gravitational field error:
  2 %
- **Non-Gravitational perturbations:**
  2 % to 3% [most of the modeling errors due to the non-gravitational perturbations are on the perigee, in particular due the Yarkowski effect on the perigee, but with in this combination we only used the nodes]
- 2% error due to random and stochastic errors and other errors

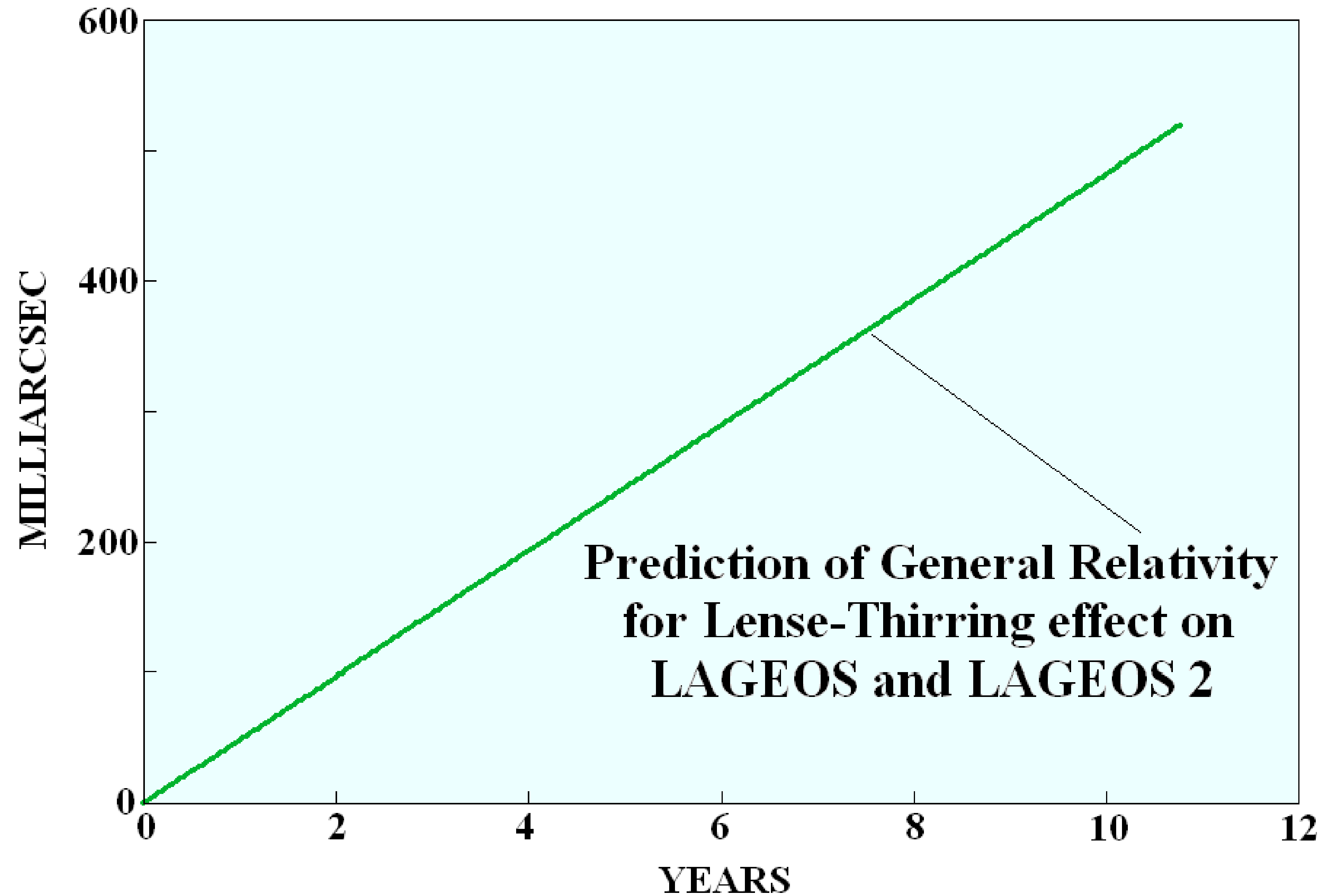
TOTAL: about 10 % (RSS)

I.C., E. Pavlis and R. Peron, New Astronomy (2006).
I.C. and E. Pavlis, New Astronomy (2005).

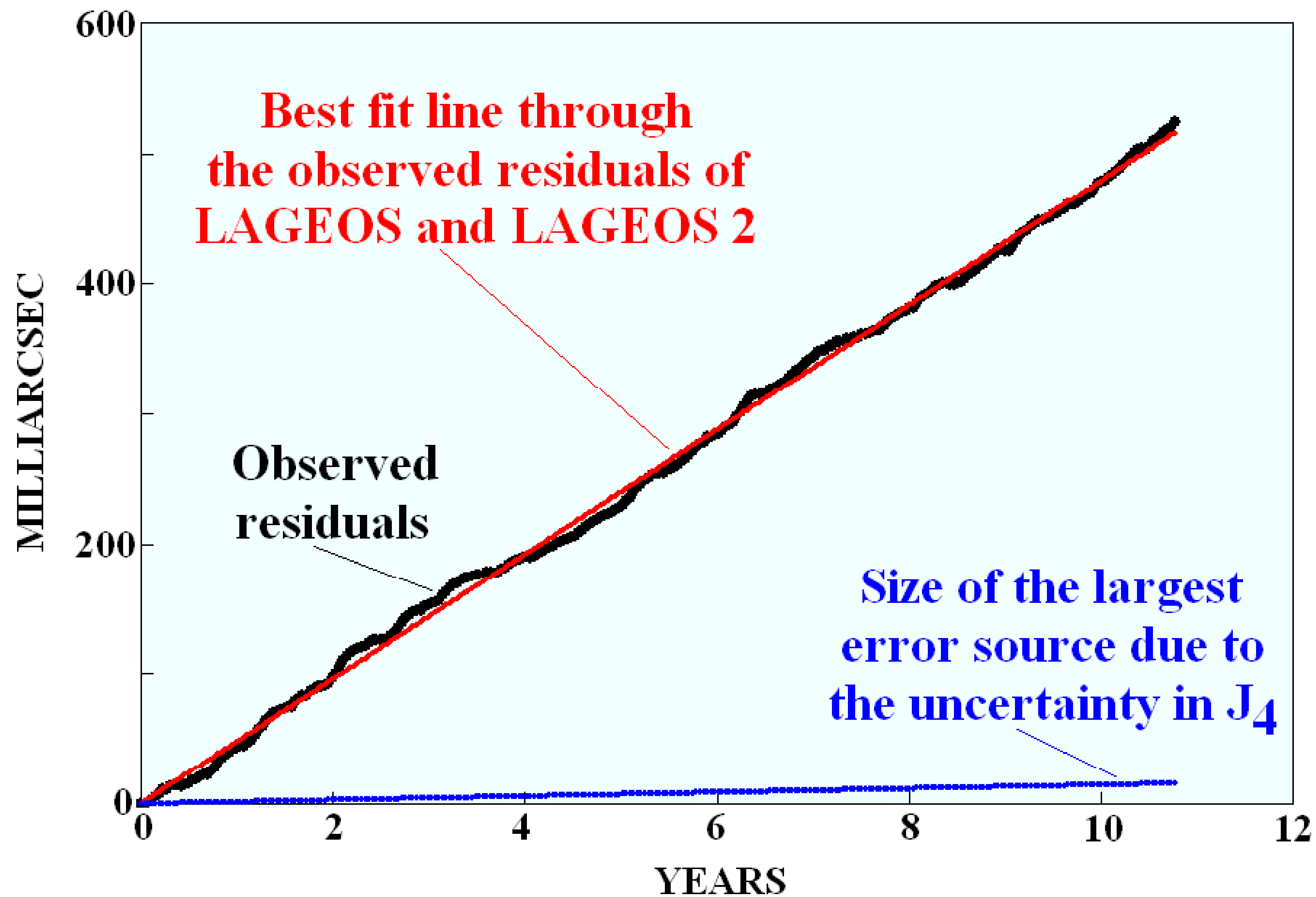
a


600
400
200
0
MILLIARCSEC
0
2
4
6
8
10
12
YEARS
Observed residual of
LAGEOS and LAGEOS 2

b



600
400
200
0
MILLIARCSEC
0
2
4
6
8
10
12
YEARS
Prediction of General Relativity
for Lense-Thirring effect on
LAGEOS and LAGEOS 2



600
400
200
0
MILLIARCSEC
0
2
4
6
8
10
12
YEARS
Best fit line through
the observed residuals of
LAGEOS and LAGEOS 2
Observed
residuals
Size of the largest
error source due to
the uncertainty in J_4

The 2004 analysis with EIGENGRACE02S:

- Does not use the perigee (i.e., no problems to assess the non-gravitational errors)
- In the error analysis we have summed up the absolute values of the errors due to each individual even zonal harmonic uncertainty: thus we did not use the correlation (anyhow small) among the even zonal harmonic coefficients
- The EIGENGRACE02S model was obtained with the use of GRACE data only and did NOT use any LAGEOS data
- The even zonal harmonics obtained from GRACE are independent of the Lense-Thirring effect (the acceleration of a polar, circular orbit satellite generated by the even zonals is orthogonal to the acceleration generated by the Lense-Thirring effect).

Potentially weak points of the 2004 analysis:

•The analysis was performed with the NASA orbital Estimator GEODYN, but what would happen by Performing it with a different orbital estimator ?

•The 2004 analysis was perfomed with EIGENGRACE02S but what happens if we change the gravity field model (and the corresponding value of the even zonal harmonics) ?

Answer:

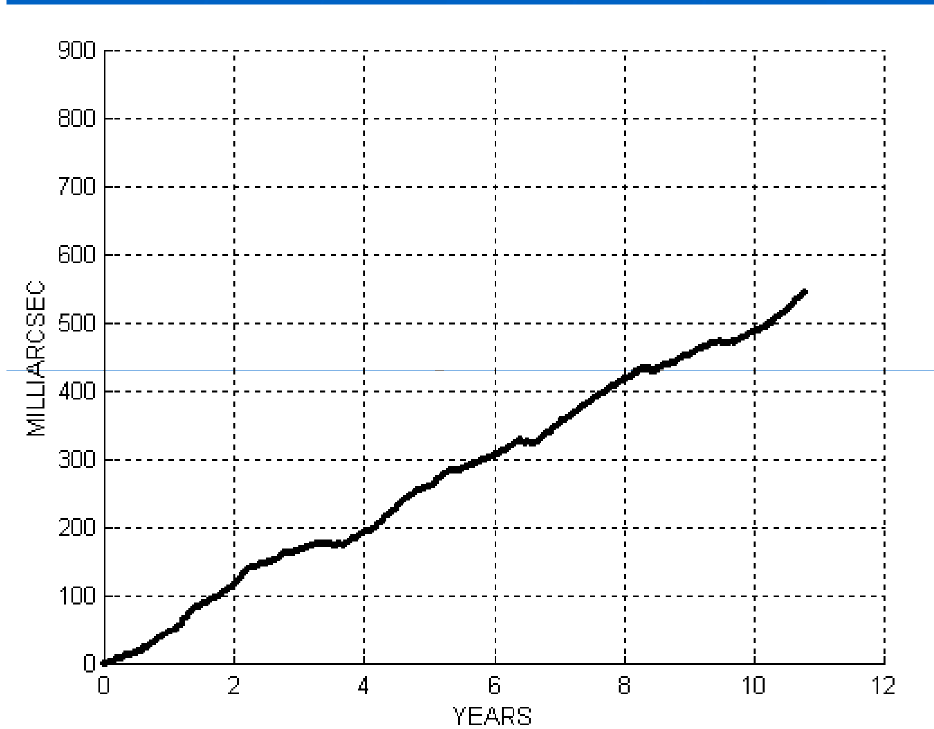
•Let us use the GFZ German orbital estimator EPOS (independent of GEODYN)

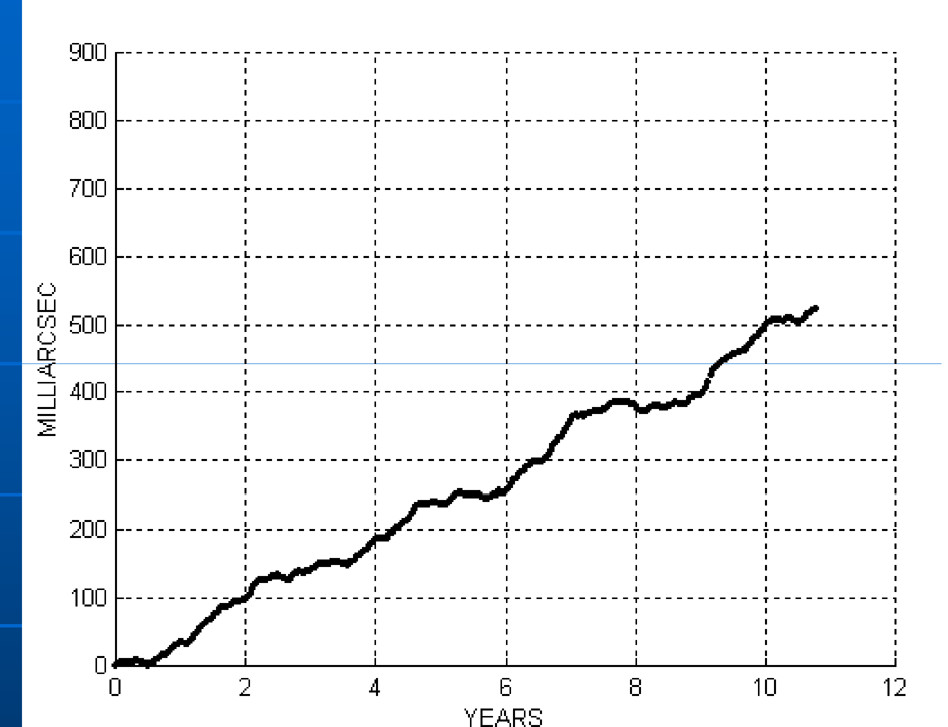
•Let us use different gravity field models obtained using GRACE

IC (Univ. Lecce), E. Pavlis (Univ Maryland Baltimore County),
R. Koenig and Neumayer (GFZ Munich/Potsdam),
G. Sindoni and A. Paolozzi (Univ. Roma I),
R. Matzner (Univ. Texas, Austin)

Using GEODYN (NASA) and EPOS (GFZ)

NEW 2006-2007 ANALYSIS OF THE LAGEOS ORBITS USING THE GFZ ORBITAL ESTIMATOR **EPOS**

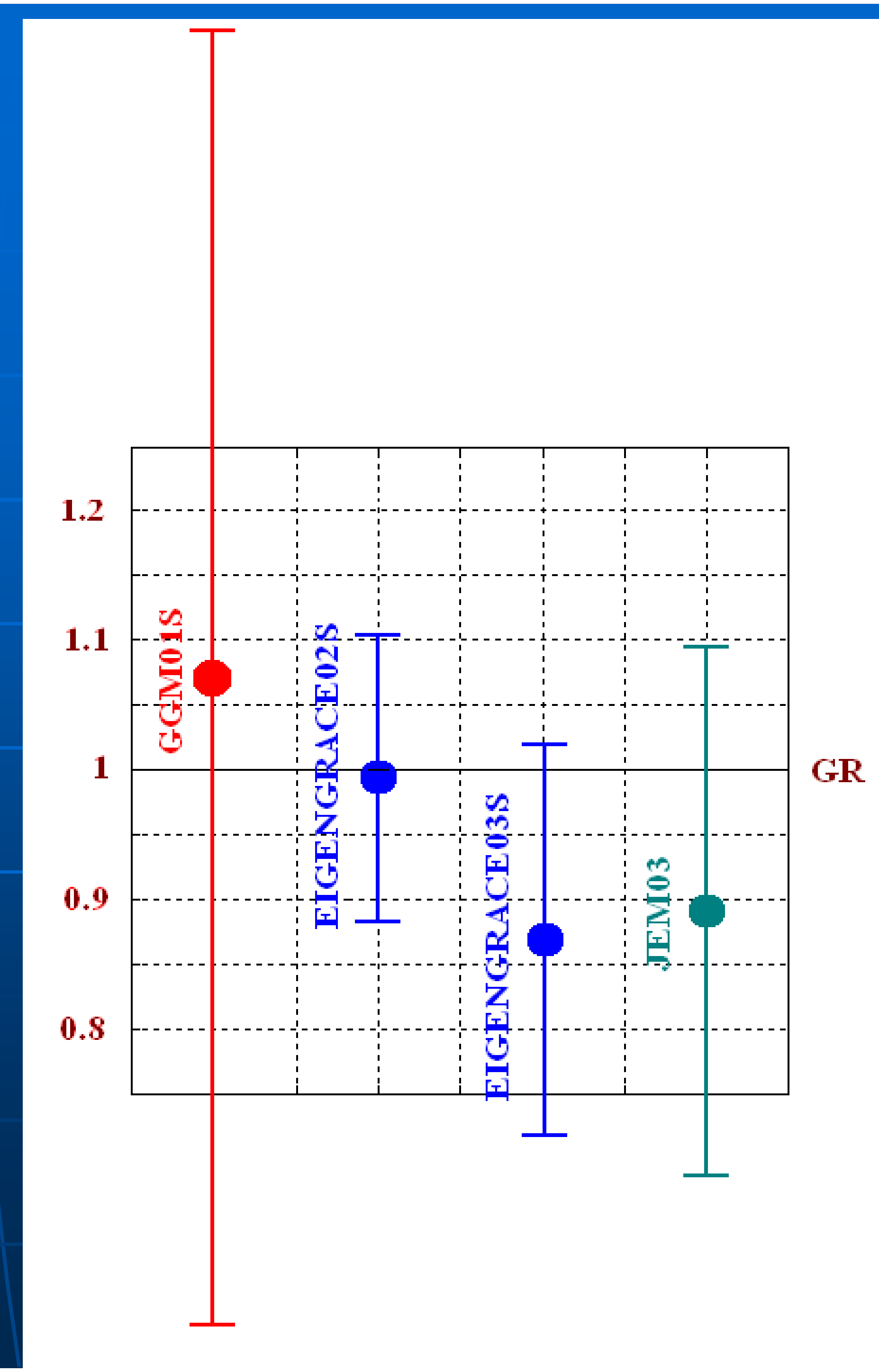


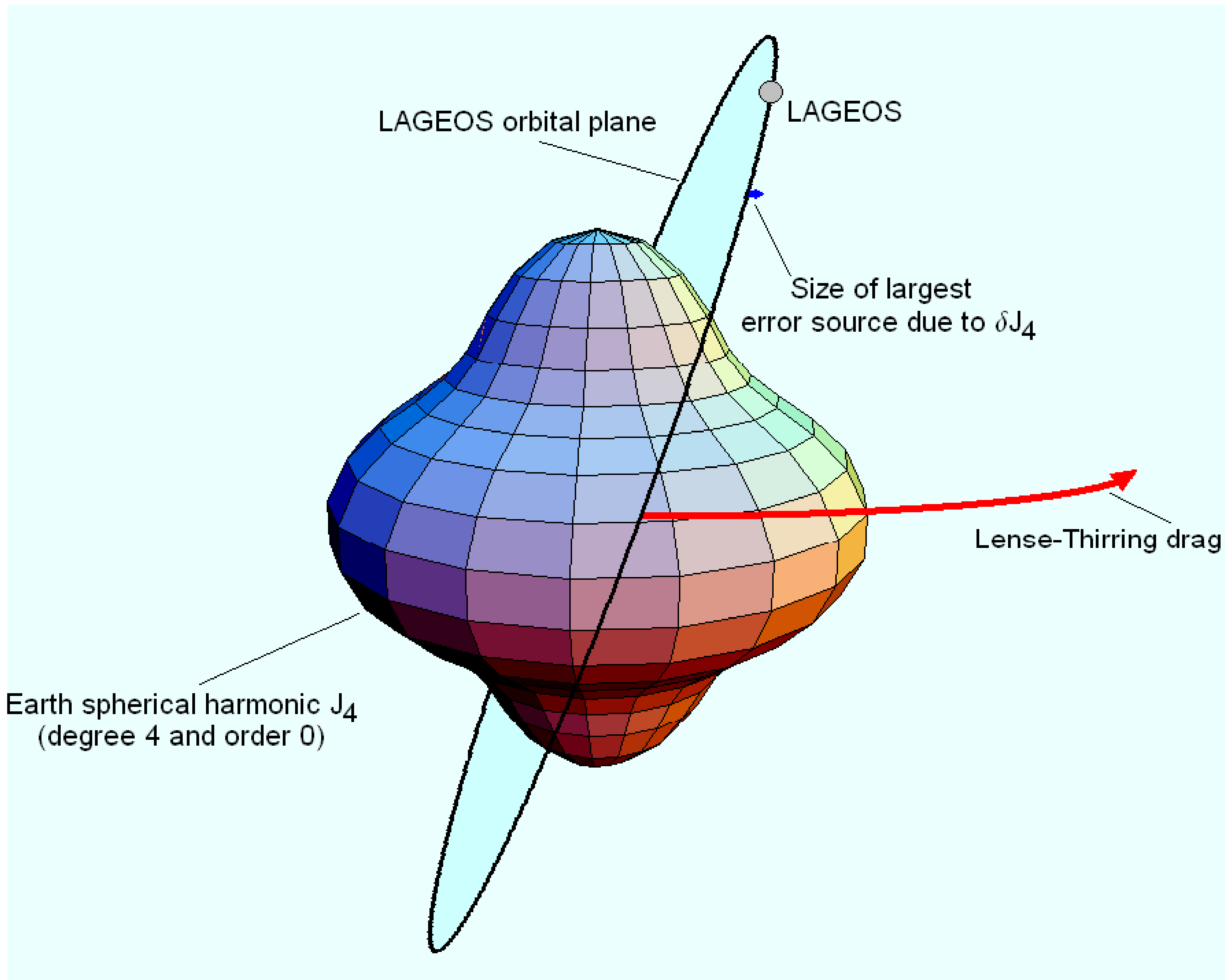


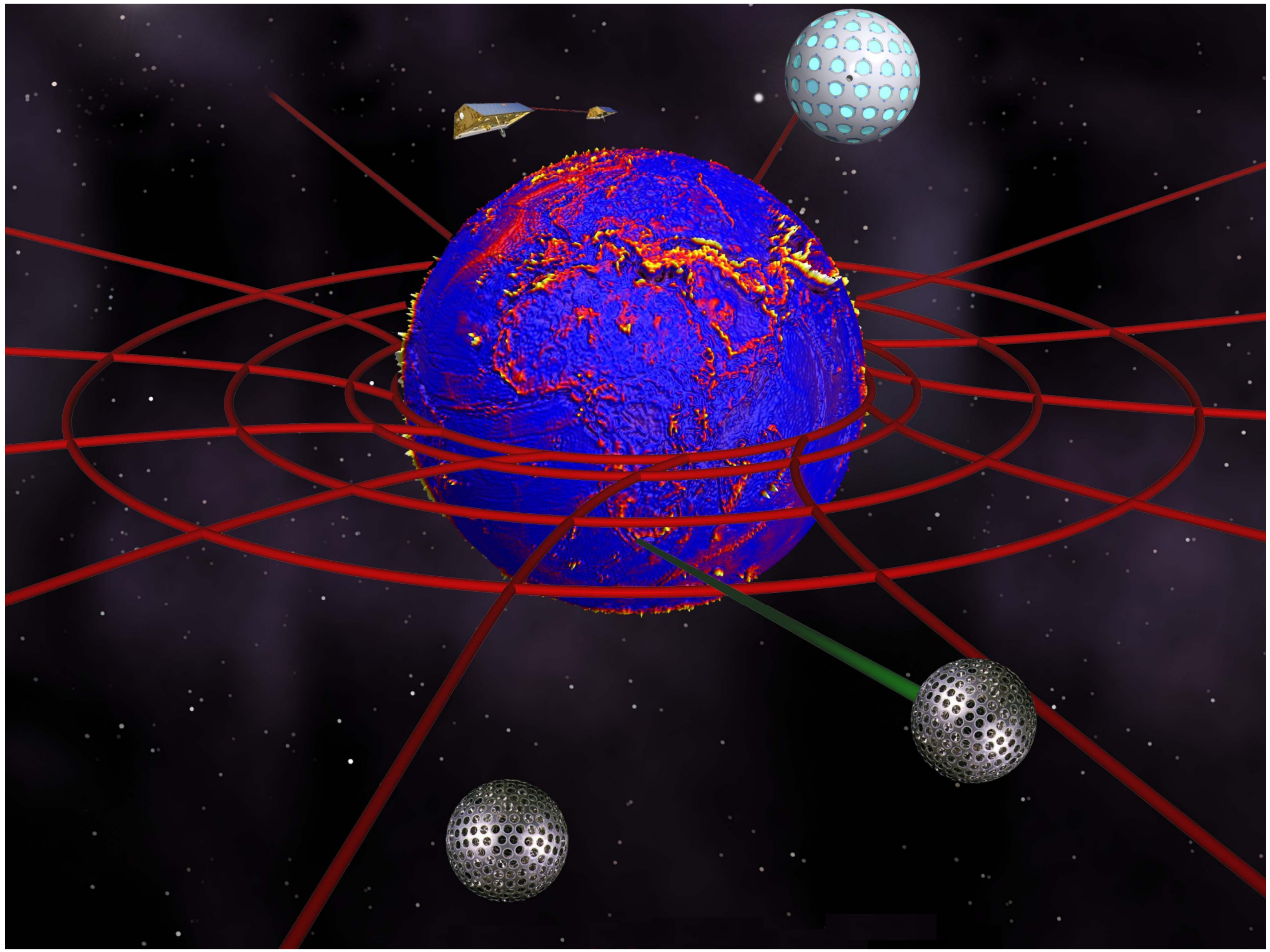
*by adding the geodetic precession of the orbital plane of an Earth satellite in the EPOS orbital estimator.

OLD 2004 ANALYSIS OF THE LAGEOS ORBITS USING THE NASA ORBITAL ESTIMATOR **GEODYN**

Comparison of Lense-Thirring effect measured using different Earth gravity field models




LAGEOS orbital plane
LAGEOS
Size of largest
error source due to δJ_4
Lense-Thirring drag
Earth spherical harmonic J_4
(degree 4 and order 0)



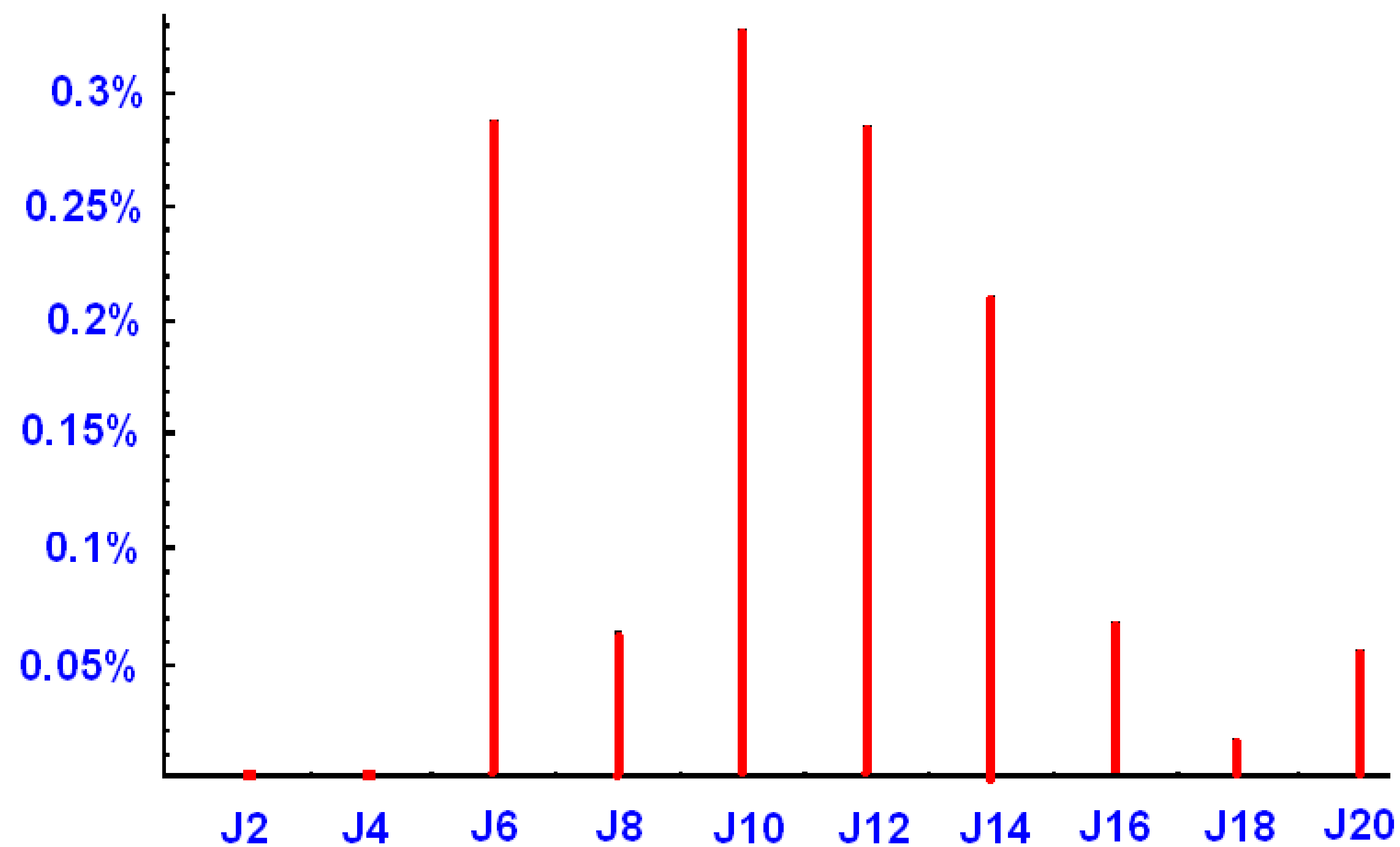
LARES

- Weight about 400 kg
- Radius about 18 cm
- Material Solid sphere of Tungsten alloy
- Semimajor Axis about 7850 km
- Eccentricity nearly zero
- Inclination about 70 degrees
- Combined with LAGEOS and LAGEOS 2 data it will provide a measurement of frame-dragging with accuracy of the order of 1 %

LARES

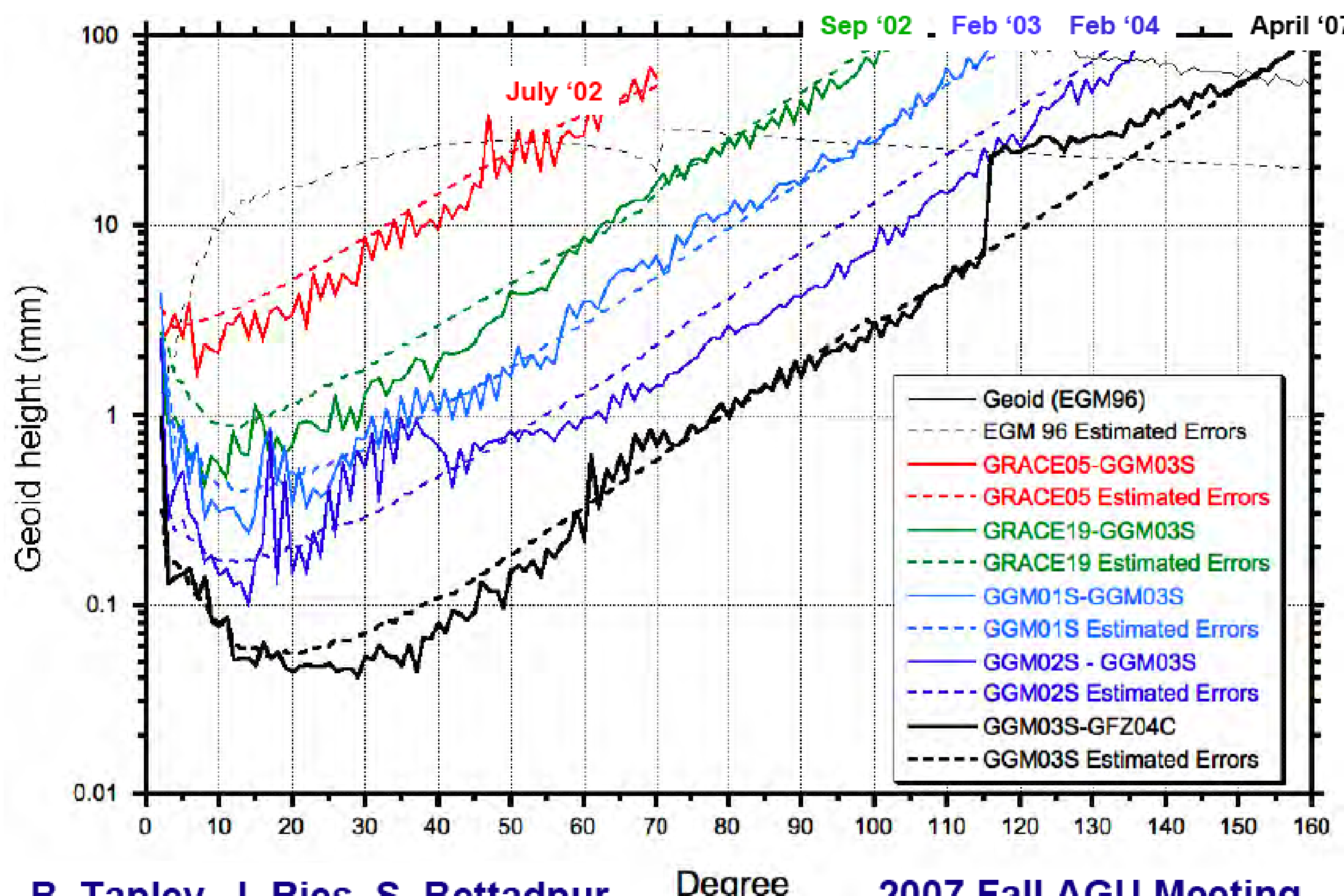
- Funded by the Italian Space Agency (ASI) in a collaboration with “Sapienza” Università di Roma, University of Salento and INFN.
- Main contractor Carlo Gavazzi Space.
- Launched in 2009 using the new European Space Agency launcher VEGA (by ASI, ELV, AVIO)
- Designed by Sapienza Università di Roma,

  School of Aerospace Engineering (Prof. Antonio Paolozzi)



Percent error (relative to the Lense-Thirring effect) due to each even zonal harmonic using LARES, LAGEOS and LAGEOS 2

Progress in Mean Gravity Models



B. Tapley, J. Ries, S. Bettadpur, D. Chambers, M. Cheng, F. Condi, S. Poole

2007 Fall AGU Meeting
San Francisco
December 10-14, 2007