Searching for Extra Dimensions
with
An Artificial Planetary System in Space
(APSIS)

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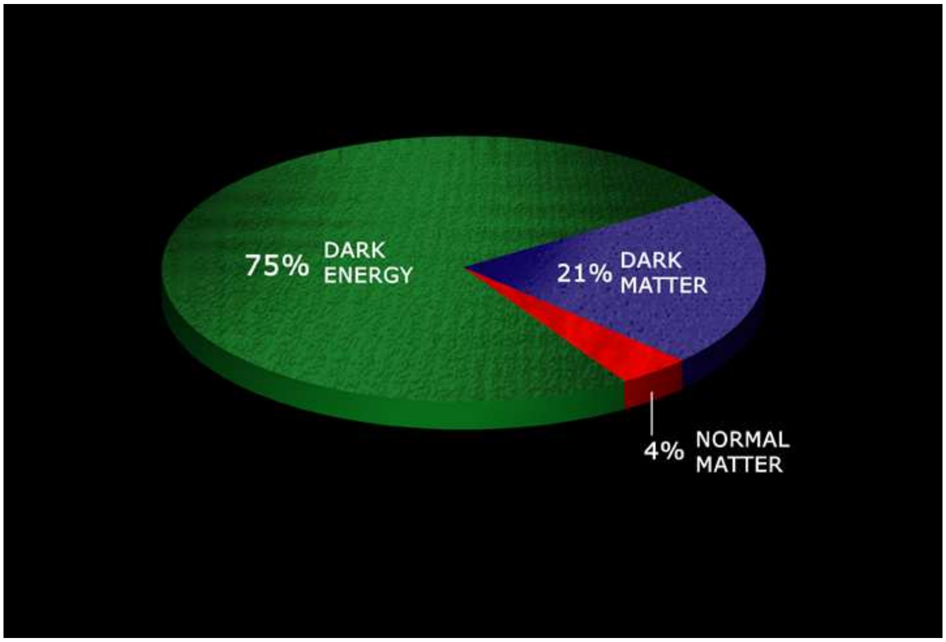
Based on: arXiv:gr-qc/0606063



It is quite remarkable that about 96% of the matter content of the Universe is of unknown origin !

We are accustomed to calling these unknowns Dark Matter and Dark Energy thereby implicitely implying a material basis for the discrepancy between what is observed (through light) and what is implied (through gravity).

But it could also be that the current situation may warrant a more fundamental revision of our understanding of the basic laws governing gravity.

One of the most EXCITING observational discoveries of the past decade is that the

Universe is Accelerating

The source responsible for Cosmic Acceleration is presently unknown and has been called

Dark Energy

If viewed as a fluid, Dark Energy has large negative pressure and could account for upto 70% of the total energy density in the Universe !!

Two ways of making the Universe ACCELERATE:

- modify the MATTER sector:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

This leads to Physical models of DE such as Quintessence, Chaplygin Gas, Phantom matter etc.

- modify the GRAVITY sector:

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

The cosmological constant introduced by Einstein in 1917 was the first model of this kind since

$$G_{\mu\nu} + \lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

This leads to modified gravity models which include: higher dimensional (Braneworld) Gravity, scalar-tensor gravity, string/M-theory inspired models, f(R) gravity, vacuum polarization, etc.

Models of dark matter and dark energy based on modified gravity could lead to new observational consequences including:

- Violation of the inverse-square law for gravity

- Modification of Newton's law of inertia

- Violation of the principle of equivalence

- New cosmological signatures:

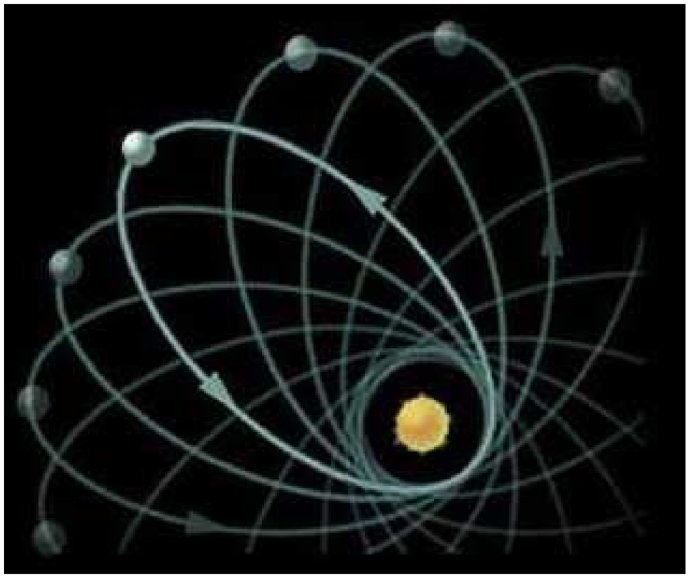
  (i) Change in the expansion law of the universe at early times (Randall-Sundrum braneworld)

  (ii) Change in the expansion law at late times (DGP braneworld)

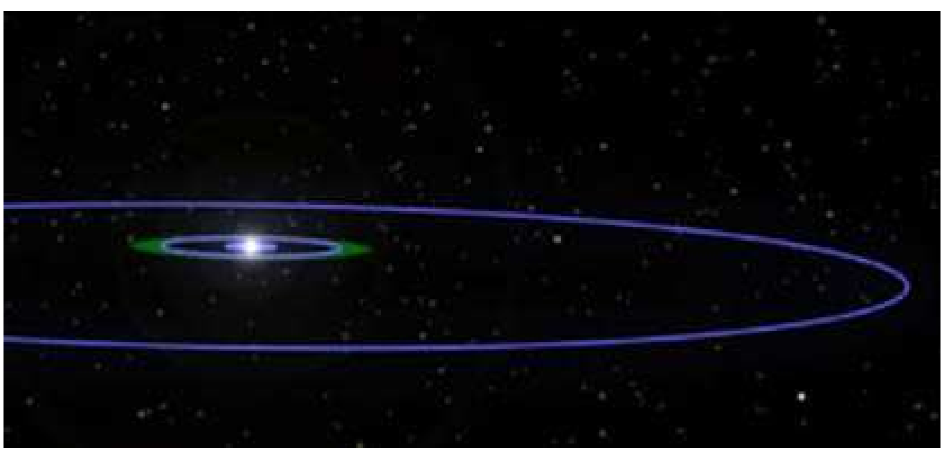
  (iii) Altered rate of growth for density perturbations, etc.

The Artificial Planetary System in Space (APSIS) will consist of two experiments.

The first will measure departures from the inverse square law and the possibility of extra dimensions using a binary system consisting of two 'planets' in an eccentric orbit. Key signature: shift in the periapsis.



The second will test extra-dimensions and MOND using 'planets' in circular orbits. Key signature: Modification of Kepler's $T \propto r^{3/2}$ law.

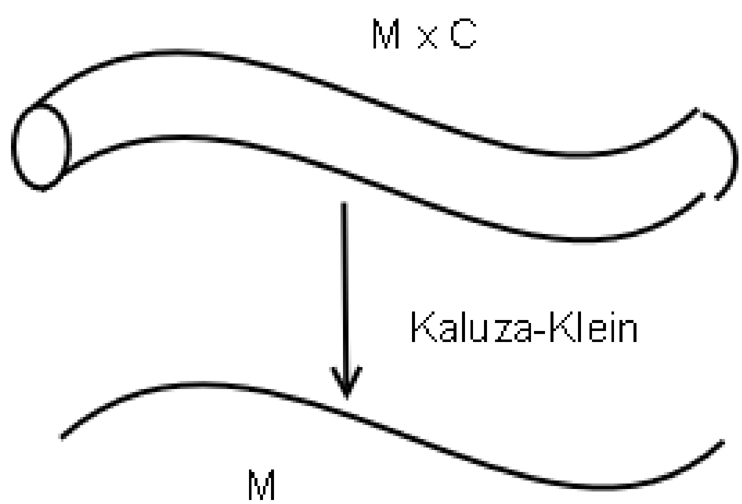


APSIS will be located at the L1 or L2 point: Far from massive bodies which could induce tidal forces on the spacecraft.

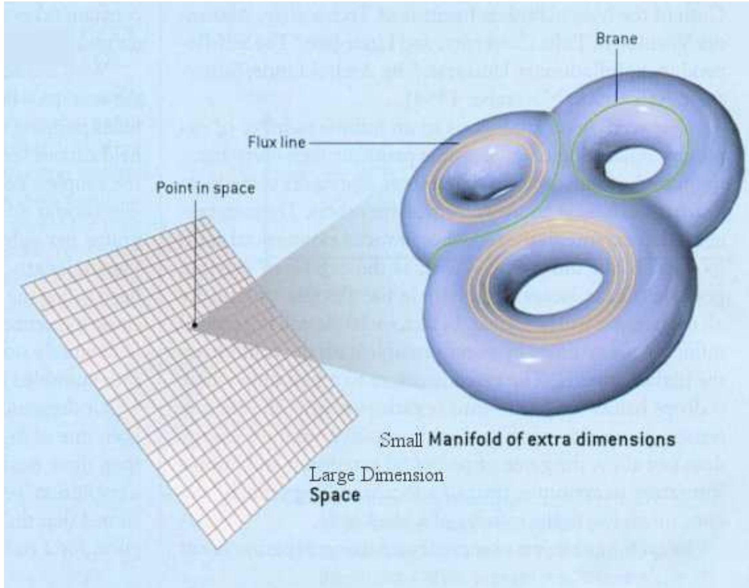
Extra Dimensions

The possibility that space could have more than three dimensions was originally suggested by Kaluza (1921) and Klein (1926), who demonstrated that a compact (circle-like) fifth dimension would unify gravity with the electromagnetic force.

However, the size of the extra dimensions in such models is close to the Planck scale, $\mathcal{R} \sim \ell_P \simeq 10^{-33}$ cm, which makes direct observational evidence of these dimensions virtually impossible.



A paradigm shift in our perception of a multi-dimensional universe occurred when it was suggested that extra dimensions, though compact, may be much larger than the Planck size and even macroscopic, $\mathcal{R} \lesssim 1$ mm. [Arkani-Hamed, Dimopoulos, and Dvali, 1998]



Consider two test masses m_1 and m_2 separated by a distance $r \ll \mathcal{R}$ in a $(4+n)$-dimensional universe and interacting via the gravitational potential

$$V(r) \sim \frac{m_1\, m_2}{M^{n+2}} \frac{1}{r^{n+1}}, \quad r \ll \mathcal{R}, \tag{1}$$

where M is the $(4+n)$-dimensional Planck mass. If the same two particles are placed much further apart

$$V(r) \sim \frac{m_1\, m_2}{M_P^2} \frac{1}{r}, \quad r \gg \mathcal{R}\,. \tag{2}$$

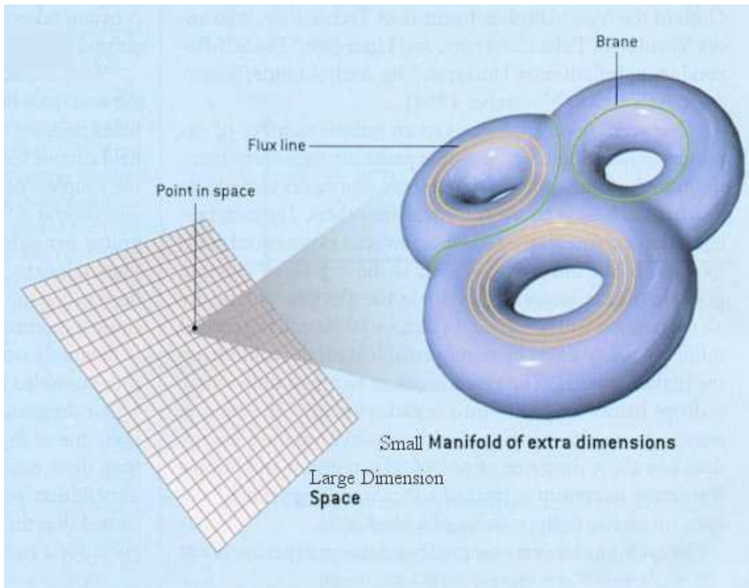
From (1) we find that gravity becomes higher-dimensional on length scales smaller than $\mathcal{R}$ – the radius of compactification.

(For $\mathcal{R} \sim 1$ mm and $n = 2$, one gets $M \sim 1$ TeV.)

Since $M_P^2 \sim M^{n+2}\mathcal{R}^n$, $M \ll M_P$, which relaxes the hierarchy problem. (SM fields are localized to a 3+1 dimensional manifold.)

A hint that extra dimensions may be large and compact may be coming from observations of dark energy: $\rho_{\text{DE}} \sim 10^{-30} g/cm^3$.

Compact extra dimensions would distort the zero-point quantum fluctuations resulting in a Casimir-type origin for dark energy.



In this case, the fundamental length scale associated with dark energy

$$L_{\text{DE}} = \left(\frac{\hbar c}{\rho_{\text{DE}}}\right)^{1/4} \sim \mu m \ .$$

Therefore one may expect departures from Newton's law on scales of a few microns.

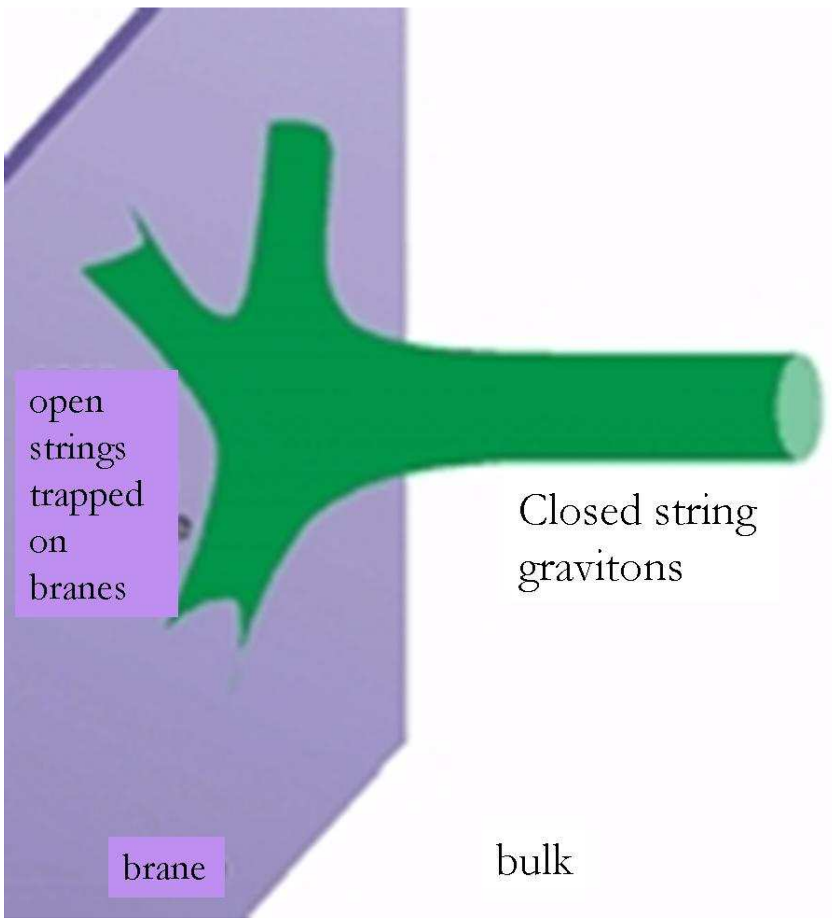
Proposals arguing for a change in Newtonian gravity on macroscopic scales include:

Sundrum JHEP07(1999)001, Dvali et al, PRD, 65, 024031, 2001; hep-ph/9910207, etc.

Terrestrial experiments indicate that the inverse-square law holds down to about $10 - 50\mu m$: hep-ph/0611184, arXiv:0802.2350.

Changes in Newtonian gravity also occur in the braneworld scenario according to which our (3+1)-dimensional universe is a brane (from the word membrane) embedded in a higher-dimensional 'bulk' space time having large, even infinite, extra dimensions.

The fields of the Standard Model are confined to move along the brane and, therefore, do not 'feel' the presence of extra dimensions, whereas gravity can propagate in the bulk.



Considerable support for the multi-dimensional viewpoint comes from string and M-theory, in which extra dimensions play a crucial role in the unification of all forces at a fundamental level.

A seminal model of this kind was put forward by Randall and Sundrum (1999). It has one infinite extra dimension, and the space-time metric in this model has the form

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \ .$$

The gravitational potential between two point masses on the brane in this model is $V(r) = V_0(r)\,[1 + \Delta(r)]$, where

$$kr \ll 1 : \quad \Delta \simeq \frac{4}{3\pi kr} - \frac{1}{3} - \frac{1}{2\pi} kr \ln kr + ...,$$

$$kr \gg 1 : \quad \Delta \simeq \frac{2}{3(kr)^2} - \frac{4 \ln kr}{(kr)^4} + ...$$

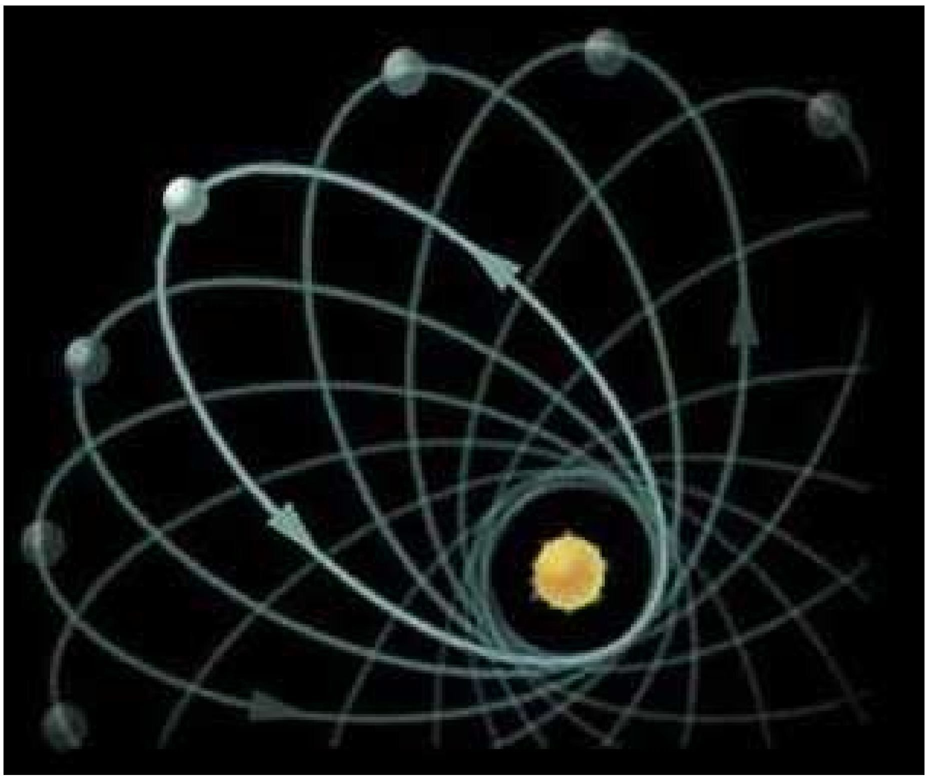
Thus, on length scales r smaller than the curvature radius k^{-1} of the fifth dimension, gravity becomes five-dimensional, and the gravitational potential changes from its familiar four-dimensional form $V_0(r) \propto 1/r$ to the five-dimensional $V(r) \propto 1/r^2$.

We therefore find that in higher dimensional models the inverse-square law may change on small scales !

This could lead to important observational consequences !!

Corrections to Newton's inverse-square law on small scales will lead to a violation of Kepler's famous laws of planetary motion.

Kepler's first law which states that the motion of planets is elliptical will no longer be true: Planets will now have open orbits with a constantly changing major/minor axis (apsis).



Apsis – point of greatest or least distance of an elliptical orbit to its centre of attraction.

So our Artificial Planetary System in Space (APSIS) will look for changes in the apsis of two 'planets' less than a meter apart and moving in a bound orbit.

Consider, for instance, the following potential usually associated with higher-dimensional cosmological models:

$$U(r) = -\frac{\alpha}{r}\left[1 + \left(\frac{r_0}{r}\right)^n\right] , \quad \alpha = Gm_1m_2 \ .$$

r_0 is the scale below which gravity becomes non-Newtonian, and n is model dependent; for instance, $n = 2$ in the five-dimensional Randall–Sundrum model.

Advantages of miniaturisation

- Apsis shift for a quasi-circular orbit of radius r

  $$\delta\phi = 2\pi(2n-1)\left(\frac{r_0}{r}\right)^n ,$$

  smaller r will lead to larger $\delta\phi$!

- The orbital period is $\propto r^{3/2}$, a planet in a smaller orbit will execute more revolutions per year leading to a larger cumulative shift $\delta\phi$! A planet in an orbit of radius 10 cm around a 1 kg central mass will execute several orbits per day !

  Since the yearly shift in the apsis is

  $$\delta\phi_{\text{one year}} \ \propto \ \left(\frac{M}{r^{3+2n}}\right)^{1/2} ,$$

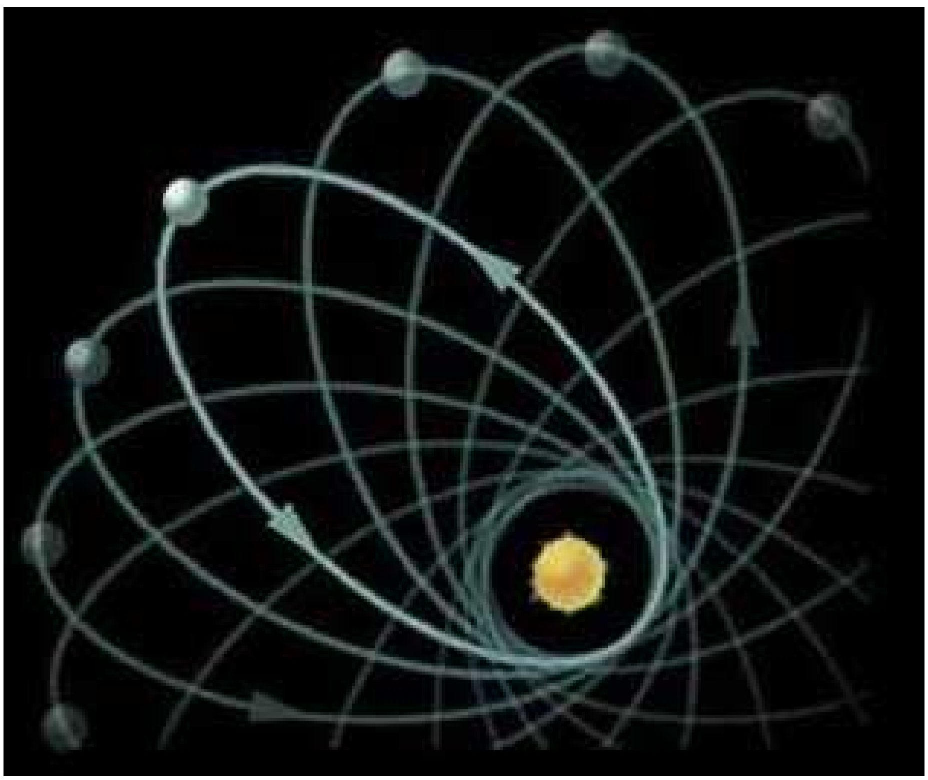
  its advantageous to have smaller orbital radii for the planets.

The advance in the periapsis increases for eccentric orbits roughly as $(1-\epsilon^2)^{-2}$, where ϵ is the eccentricity.

Two tiny quartz planets in an elliptical orbit with eccentricity $\epsilon = 0.7$ and semimajor axis = 1 cm, will show a periapsis shift of

$$\delta\phi_{\text{one year}} \simeq 80 \text{ arcseconds} ,$$

if the gravitational potential becomes five dimensional at $r \leq 1$ micron.



• Gravity Probe B measured a 40×10^{-3} arcseconds drift in frame dragging with an accuracy better than 0.5 milliarcseconds !

• The Hipparcus satellite measured the astrometric parameters of over 100,000 stars to a precision of a few milliarcseconds !!

Extra-dimensional corrections to the Newtonian gravitational potential

$$U(r) = -\frac{\alpha}{r}\left[1 + \left(\frac{r_0}{r}\right)^n\right] , \quad \alpha = G m_1 m_2 .$$

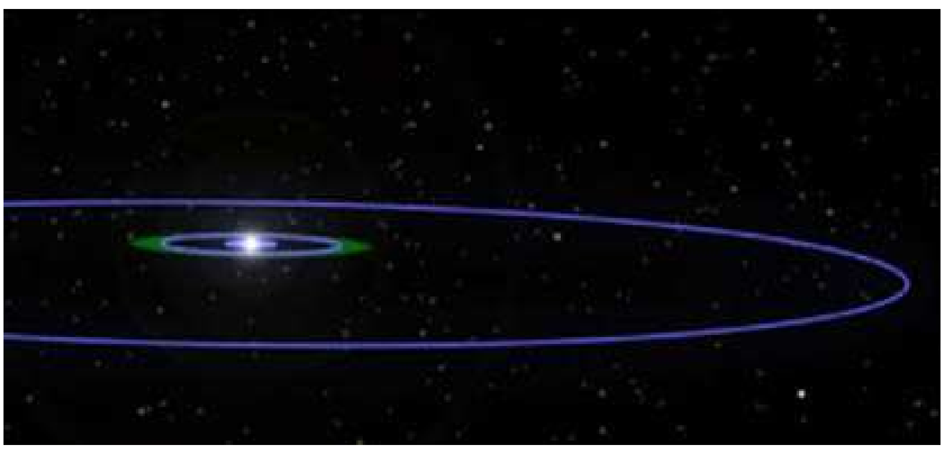
modify Kepler's third law ($T_{\mathsf{k}} \propto r^{3/2}$) to

$$T = \frac{T_{\mathsf{k}}}{\sqrt{1 + (n+1)\left(\frac{r_0}{r}\right)^n}} , \quad r \gg r_0 . \qquad (3)$$

For $r_0 = 10^{-4}$ cm, $r = 10$ cm and $n = 2$

$$\left|\frac{\Delta T}{T}\right| \simeq 1.5 \times 10^{-10} ,$$

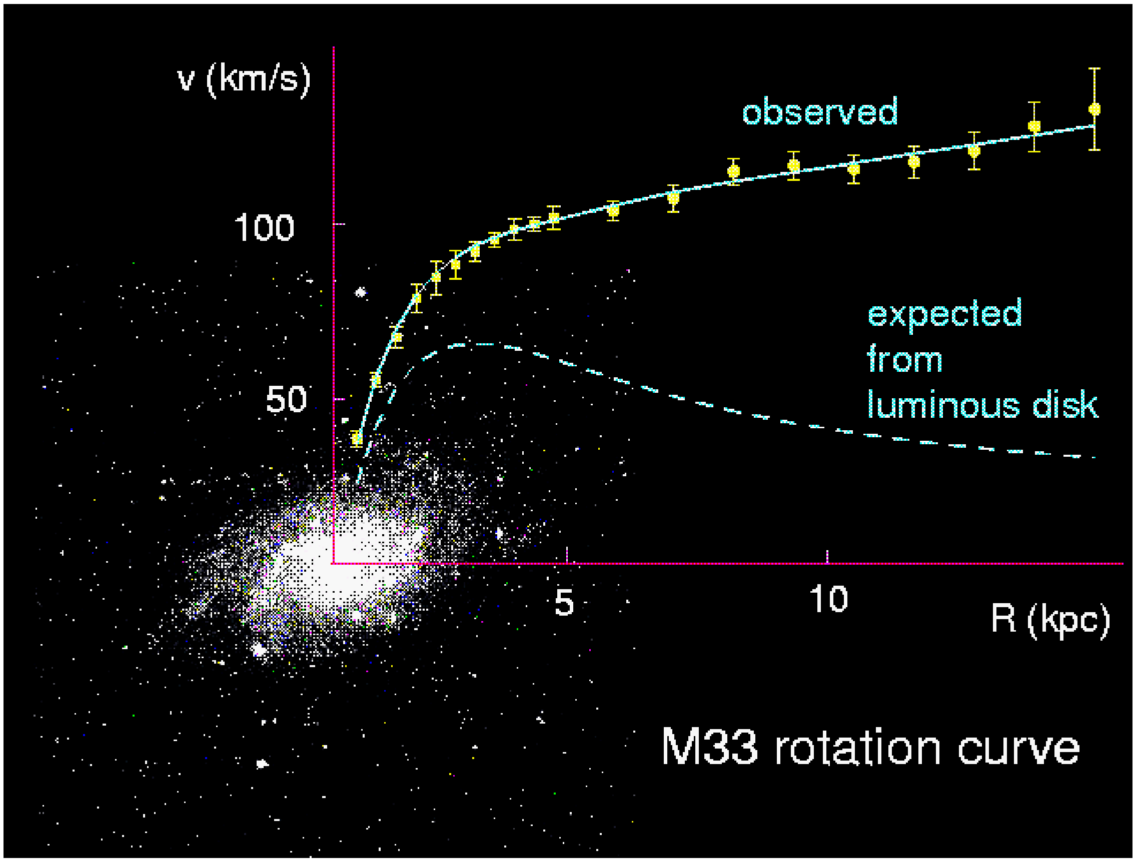
where $\Delta T = T - T_{\mathsf{kepler}}$. If our APSIS planet makes a revolution a day then the increment in the time period $\Delta T \simeq 10^{-5}$ seconds.



Microsecond accuracy in timing will allow us to probe gravity on submicron scales.

The period can be measured quite accurately since the planets will execute several hundred orbits per year !

Are the excess velocities observed in galaxies due to Dark Matter or MOND ?



If all of matter is associated with the luminous central region of a galaxy then, equating the centripetal acceleration v^2/r with the gravitational force GM/r^2, one expects a steady decline in the rotational velocity far from the galactic center

$$v \propto r^{-1/2} \ .$$

Instead one finds $v \simeq constant$, which is interpreted as evidence for the existence of a halo of *dark matter* surrounding the galaxy with $M(r) \propto r$.

A different explanation for the observed *flat rotation curves* of galaxies is provided by Modified Newtonian Dynamics (MOND).

MOND assumes that, at sufficiently low accelerations $a < a_0 = 1.2 \times 10^{-8}$ cm/s^2, Newton's law of inertia ($F = ma$) is modified to $F = ma(a/a_0)$.

Consequently a body of mass m orbiting a larger mass M will experience an acceleration

$$ma\left(\frac{a}{a_0}\right) = \frac{GMm}{r^2}$$

or

$$a = \frac{\sqrt{GMa_0}}{r} \; .$$

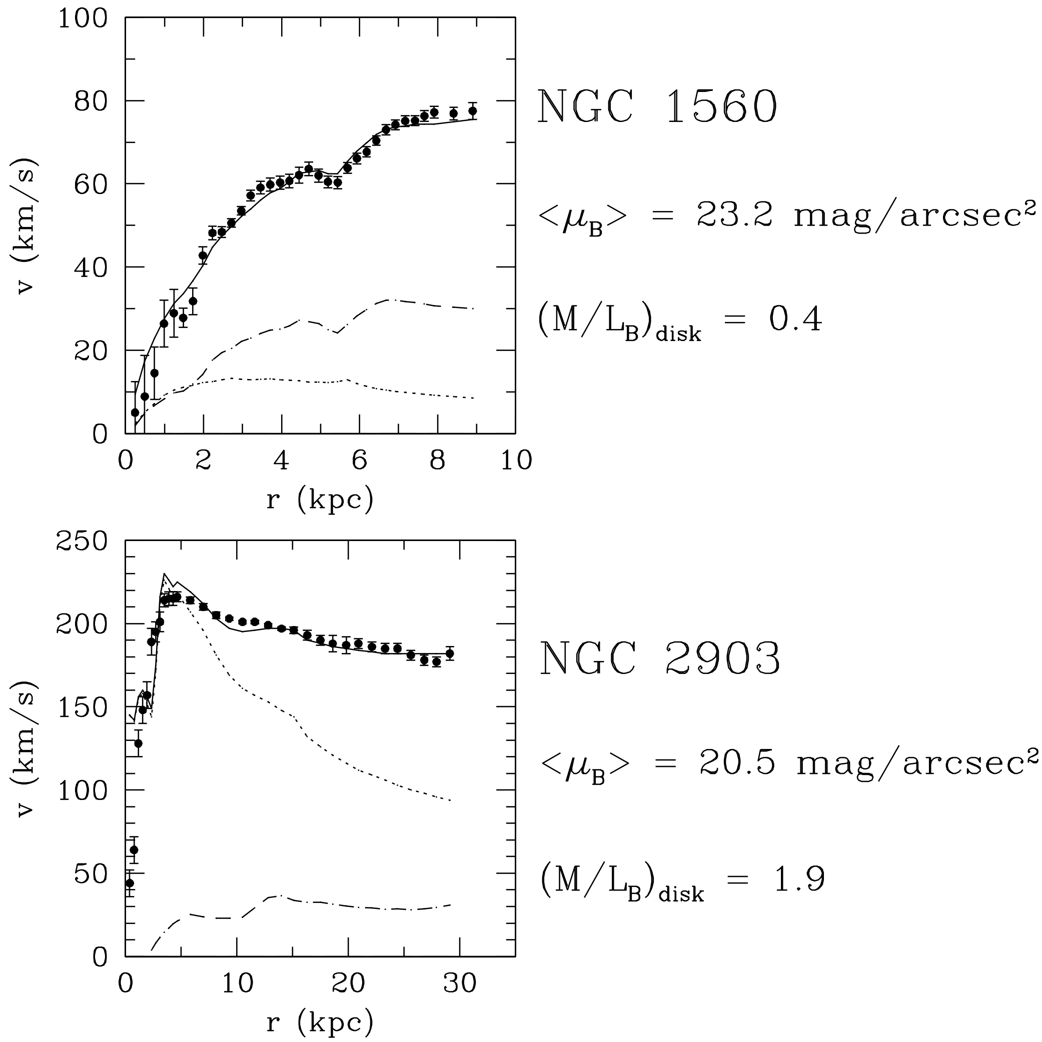
Equating this to the centripetal acceleration $a = v^2/r$ one finds

$$v^4 = GMa_0 = constant \; !$$

In other words, for sufficiently low values of the acceleration, this theory predicts *flat rotation curves*.

Interestingly the value of the MOND acceleration $a_0 = 1.2 \times 10^{-8}$ cm/s^2 is of the same order as cH_0!

MOND is able to successfully reproduce the rotation curves of galaxies without invoking dark matter.

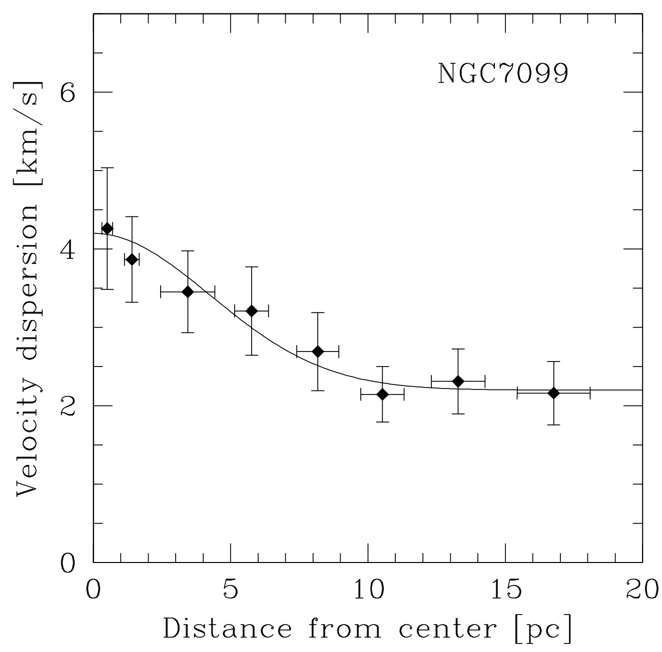


The points show the observed 21 cm line rotation curves of a low surface brightness galaxy, NGC 1560 and a high surface brightness galaxy, NGC 2903. The dotted and dashed lines are the Newtonian rotation curves of the visible and gaseous components of the disk and the solid line is the MOND rotation curve. The only free parameter is the mass-to-light ratio of the visible component. [From Sanders, astro-ph/0601431]

By placing an artificial planetary system in space (APSIS) we could test MOND since, for planets orbiting a ‘sun’ of mass 10 gm:

1. The inner planets ($r < 8$ cm) are expected to show Keplerian behaviour $v \propto r^{-1/2}$.

2. The outer planets ($r > 8$ cm) would have lower accelerations than the MOND value and so their circular velocity would be independent of their radial distance, if MOND were correct.



Thus the rotation curve of APSIS could be used to rule out (or confirm) MOND !

The orbital period of a planet in MOND $T \propto r$ is quite different from the Keplerian prediction $T \propto r^{3/2}$!

This can be used to test the theory using an Artificial Planetary System in Space (APSIS).

Testing Gravity Theories with APSIS

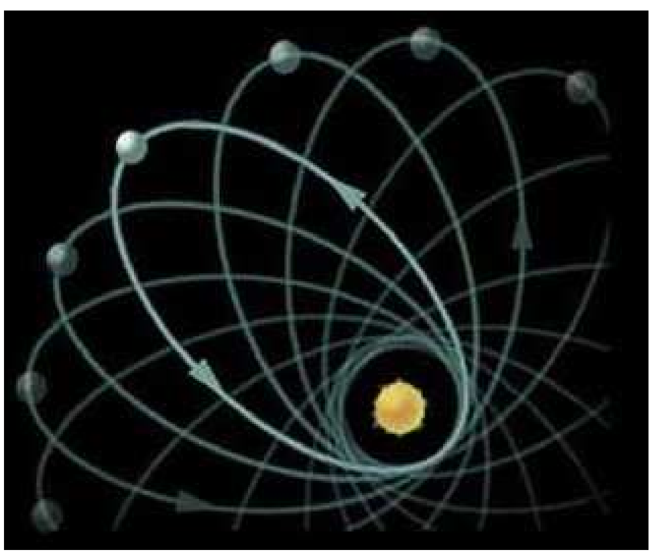
- Extra-dimensions and MOND predict departures from Kepler's $T \propto r^{3/2}$ law:

MOND predicts $T \propto r$, while $T \propto f(r)$ in theories with extra dimensions.

This can be tested by 'planets' in circular orbits at varying distances from the center.

- MOND and extra-dimensional theories also predict a periapsis shift for a planet moving in an elliptical orbit.

For MOND this effect is huge: ~ 1 radian per revolution !!



The APSIS experiment can therefore be used to probe gravity on small scales ($\sim 1\mu m$) as well as low accelerations ($< 1.2 \times 10^{-8}$ cm/s^2).

Spacecraft Design — some preliminary ideas

The APSIS experiment will consist of an inner satellite which will include a planetary system ('proof mass') in free fall and shielded from air drag, solar pressure, etc.

The inner satellite will also include an observing apparatus monitoring the motion of the 'planets' and communication equipment.

The outer satellite will be equipped with sensors which precisely measure its position with respect to the planetary system and a set of micro-propulsion thrusters that automatically control its position, based on feedback from sensors.

This will ensure that the satellite remains centered about the free-falling planetary system.

The APSIS experiment will be controlled by means of a dedicated computer which will monitor the drag-free motion and log data. The payload electronics will include a radiation sensor and an external magnetic field sensor.

Much of the technology required for APSIS has already been developed in connection with the LISA Pathfinder (LP) mission and the Gravity Probe B gyroscope experiment (GP-B).

"LISA Pathfinder will put two test masses in a near-perfect gravitational free fall and control and measure their motion with unpredecented accuracy."

(http://www.rssd.esa.int/index.php?project=LISAPATHFINDER)

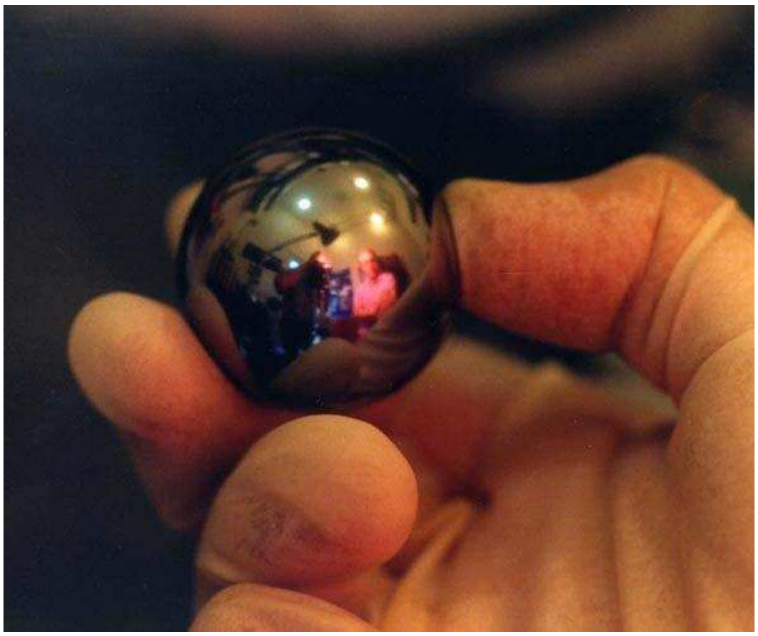
For this purpose LP has developed state-of-the-art technology consisting of inertial sensors, laser metrology system, drag-free control system and an ultra-precise micro-propulsion system.

In APSIS and in LP all non-gravitational forces that push masses away from geodesics need to be minimized.

Superconducting shielding can eliminate external magnetic and electric disturbances, while low temperature and pressure can reduce most thermal disturbances. For GP-B the background magnetic field near the experiment was reduced to $< 10^{-7}$ gauss while the probe pressure was $< 10^{-14}$ atm.

It is important that the APSIS 'planets' be spherical and homogeneous.

The designers of the GP-B experiment have achieved sphericity of the gyroscope spheres less than 40 atomic layers from perfect. The GP-B spheres are homogeneous to 3 parts in 10^7 !



A gentle release mechanism for the test masses is essential for APSIS just as it is for the LISA Pathfinder.

In LP one hopes to release a ~ 5 cm test mass with an accuracy of 60 μm and a release speed of less than 5 μm/s (18 mm per hour) !

As in LP and GP-B, the APSIS planets will be shielded from non-gravitational forces and discharged at regular intervals using fibre-coupled UV lamps.

• In GP-B a stream of pure Helium gas is used to speed up the gyroscopes. The same could be used to maneuver APSIS planets.

• One could also use low intensity lasers to correct the planetary orbit and techniques borrowed from Astrometry to determine the planetary orbit.

• Tidal effects from masses associated with the spacecraft are an important source of systematics:

By adding counterweights at appropriate points of the spacecraft we could try and make the APSIS experiment gravitationally invisible !

Spinning the spacecraft about the orbital plane of the planets will average tidal forces which are spacecraft based (Nobili *et al.* , 1987–1990).

Literature

1. Varun Sahni, Yuri Shtanov, *APSIS: An Artificial Planetary System in Space to probe extra-dimensional gravity and MOND* , Int.J.Mod.Phys.D17:453-466, 2008. [gr-qc/0606063]

2. A. M. Nobili, *et al.* , *A proposed manmade planetary system in space to measure the gravitational constant,* ESA Journal **14**, 389 (1990).

3. A. D. Alexeev *et al.* , Grav. Cosmol. **5**, 67 (1999) [gr-qc/0002088]. etc.

Thank You !!!

Tidal Effects

The ratio of the tidal force F_t due to an external mass M located at a distance $\mathcal{R}$ from our 'planetary system' and the gravitational force $F_g \simeq Gm^2/r^2$ between 'planets' is

$$\frac{F_t}{F_g} \simeq 2\frac{M}{m}\left(\frac{r}{\mathcal{R}}\right)^3 \ .$$

Clearly $F_t/F_g \ll 1$ if $r/R \ll (m/M)^{1/3}$.

So miniaturization will greatly help to minimize tidal forces between the experiment and its environment.

In a geostationary orbit $F_t/F_g < 0.2$ for a planet of mass 1kg in a 10 cm orbit.

At L2 the tidal influence of the Earth further decreases to $F_t/F_g < 4 \times 10^{-6}$.

Therefore the planetary system is stable !

- For mass $M \simeq 100$ kg associated with spacecraft and planet mass $m = 1$kg, $F_t/F_g < 1 \Rightarrow r < \mathcal{R}/5$.

The planetary orbit must be much smaller than the size of the spacecraft.

Tidal effects from masses associated with the spacecraft are an important source of systematics:

By adding counterweights at appropriate points of the spacecraft we could try and make the APSIS experiment gravitationally invisible !

Spinning the spacecraft may be necessary because it would average forces which are body-based in the spin plane which is also the orbital plane of the planets (Nobili *et al.* , 1987–1990).

Corrections to Newton's law for extended spherical bodies (planets)

The Newtonian potential *for point sources* with corrections due to the modified gravity is

$$
\begin{aligned}
U(R) &= -\frac{Gm_1 m_2}{R} + U_{\text{mod}}(R)\,, \\
U_{\text{mod}}(R) &= -\frac{Gm_1 m_2}{R}\left(\frac{r_0}{R}\right)^{n+1}\,. \qquad (4)
\end{aligned}
$$

For extended spherical bodies of radii r_1 and r_2 separated by a distance R, the correction term $U_{\text{mod}}(R)$ becomes

$$
\tilde{U}_{\text{mod}}(R) = U_{\text{mod}}(R)\left[1 + \frac{(n+1)(n+2)}{10}\left(\frac{r_1^2 + r_2^2}{R^2}\right) + \ldots\right]\,.
$$

The leading order correction is of order $(r_i/R)^2 \ll 1$ and is quite small.

The eccentricity for a 2-body system of identical masses

$$e = \sqrt{1 + \frac{2E\mathcal{M}}{\mu\alpha^2}}$$

where $E < 0$ is the total energy, $\mathcal{M}$ is the angular momentum, $\alpha = Gm^2$ and μ is the reduced mass.

So to place 2 bodies in an orbit with prescribed eccentricity we need to know the total energy and the angular momentum.

Dark Matter or MOND ?

While dark matter successfully explains several different sets of observations, its present avatar — cold dark matter — is currently facing an increasing number of observational challenges including galaxy cores which appear to be shallower than the cuspy cores predicted by CDM, and an over-abundance of dwarf galaxies predicted by this scenario.

It may be that traditional remedies to these problems (baryonic feedback, making the dark matter 'warm' instead of 'cold', etc.) may alleviate some of the tension between theory and observations.

But it could also be that the current situation warrants a more fundamental revision of our understanding of the basic laws governing gravity.

Many different varieties of Dark Matter: cold dark matter (axion or WIMP ?), warm dark matter, fuzzy dark matter, frustrated dark matter, self-interacting dark matter, etc.

The experimental search for dark matter depends crucially upon its nature. Different experiments may be required to either detect dark matter or to rule it out.

In contrast, a single experiment (APSIS) is sufficient to test MOND !

Clusters of galaxies appear to be posing problems for MOND as well as dark matter !

Problem for MOND: Clusters may still require some amount of non-baryonic matter (Eg. Bullet cluster) [arXiv:08064319].

Problem for Dark Matter: Multiwavelength observations of the cluster Abell 520 (cosmic train wreck) show that its dark matter distribution coincides with its X-ray emitting gas but not with the galaxy distribution !

"Although a displacement between the X-ray gas and galaxy/dark matter distributions may be expected in a merger, a mass peak without galaxies cannot be easily explained within the current collisionless dark matter paradigm" – ApJ 668:806 (2007).

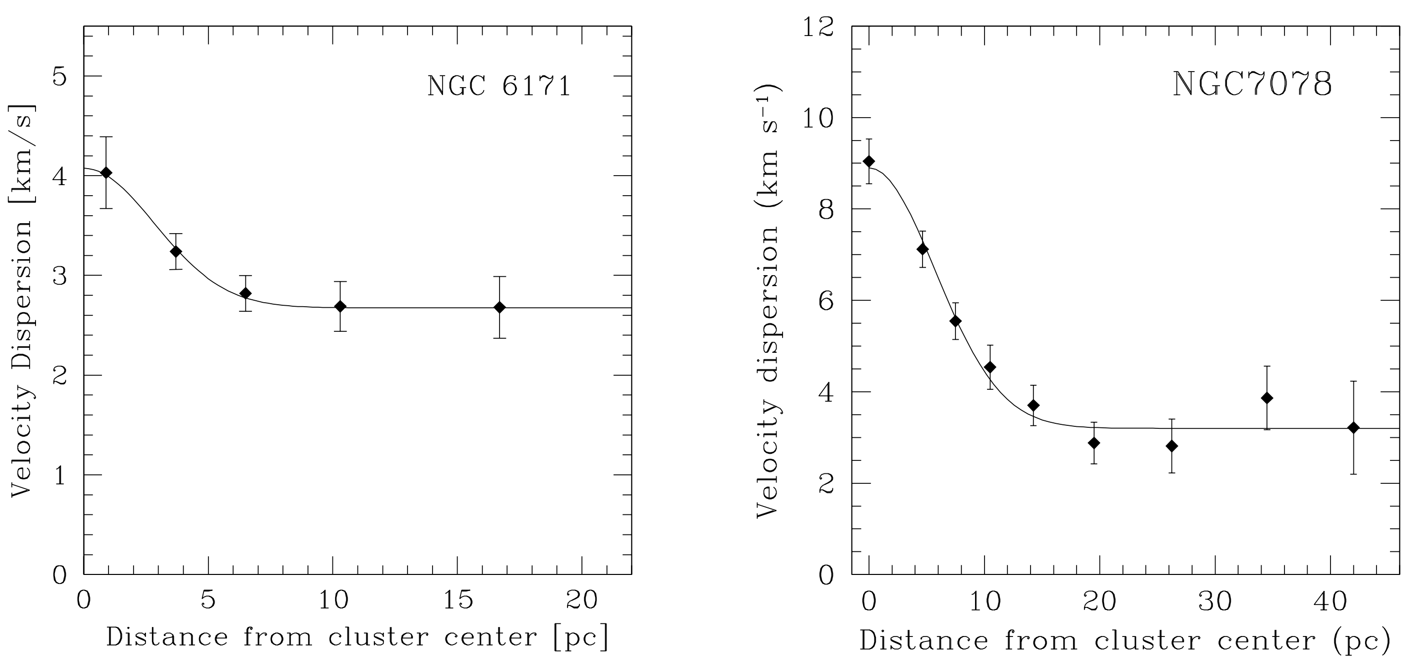
Open issues for Dark Matter

• The Fornax dwarf spheroidal appears to have a cored dark matter halo which poses a problem for cold dark matter which predicts a cuspy halo. [Goerdt et al. MNRAS 368, 1073 (2006)]

• Warm dark matter with a ~ 0.5 keV particle can explain this observation but runs into trouble when confronted with the flux anomalies of gravitationally lensed quasars. [Metcalf and Zhao ApJ 567, L5 (2002); Mirranda and Maccio, arXiv:0706.0896]

Curiously, flat rotation curves are found even for globular clusters – systems with which one does not ordinarily associate dark matter !

In these clusters the rotation curve flattens out at large radii where the local gravitational acceleration falls below the MOND value $a_o \sim 10^{-8}$ cm/s^2.



From Scarpa *et al.* [arXiv:0707.2459]

These observations also appear to be telling us that it is the local value of the acceleration which is important and not its global value. (The latter is larger than 10^{-8} cm/s^2 for these objects which lie within the Milky Way).

In this case one is tempted to test MOND by an experiment carried out within our solar system, such as APSIS.