Extending Luminosity in μ^+-μ^- Collider Scenarios

In Table 1 we present proposed parameters for μ^+-μ^- colliders as developed for the 1999 status report[1]. These parameters are used as starting points for the discussion of the potential of future μ^+-μ^- colliders. These parameters and their luminosity numbers were developed under the constraints that:

- The parameters were within reasonable reach of a collider facility, using feasible parameters of a proton source, target and $\pi \rightarrow \mu$ collection system, ionization cooling, recirculating-linac acceleration and a collider ring.
- The parameters would provide at least the minimal luminosity needed for the exploration of new physics in the proposed colliders.
- The physics reach of the proposed facility would be comparable (or better, in Higgs physics) than that of other concurrently proposed facilities, such as e^+-e^- colliders.

Since then the posted potential luminosities of proposed e^+-e^- colliders have increased through incremental improvements, without any fundamental changes in the collider approaches. We would now like to explore the potential of extending the parameters of the 1999 baseline scenarios to higher luminosity; firstly, under the consideration that the same general design approaches are used but that the operating parameters are incrementally extended to increase luminosity, and, secondly, under the possibilities that substantially different approaches are used for some facility components to obtain dramatically increased luminosities. In this section we only address the possibility of incremental improvements.

The formula for luminosity is:

$$L = \frac{f_0 n_S n_b \gamma_\mu N_\mu{}^2}{4\pi \varepsilon_N \beta^*}, \qquad [1]$$

where n_S is the luminosity lifetime (in turns) in the collider, n_b is the number of colliding bunches in each beam, N_μ is the number of muons per bunch, $\gamma_\mu = (E_\mu/m_\mu)$, ε_N is the normalized emittance, and β^* is the collider focusing parameter, with the beam size at collisions given by $\sigma^2 = \varepsilon_N \beta^*/\gamma_\mu$.

An important parameter that limits luminosity in colliders is the beam-beam tune shift (per collision), which is:

$$\Delta\nu = \frac{3 r_\mu N_\mu}{2\varepsilon_N \gamma_\mu^2} \qquad [2]$$

where r_μ = is the muon electromagnetic radius. In the status report, it was assumed that we were limited to $\Delta\nu < 0.05$, which is the limit in medium-energy circular e^+-e^- colliders. However, in a μ^+-μ^- collider, beam storage is required for only ~1000 turns and the beams are continuously decaying. Simulations show that $\Delta\nu$ as large as 0.015 or more can be used without difficulty.[2]

Assuming that $\Delta\nu$ is a controlling limitation, luminosity can be written as:

$$L = \frac{f_0 n_S n_B \gamma_\mu^3 N_\mu \Delta\nu}{6\pi\beta^*}. \qquad [3]$$

In this section we will now consider the implications of increasing L from the values in Table 1, considering eqs [1-3].

Increasing $\Delta\nu$ implies increasing N_μ or decreasing ε_N. The total number of muons is $N_{tot} = n_B N_\mu$. If $n_B > 1$, N_μ can be increased by combining bunches (without increasing N_{tot}). The high-energy scenarios of table 1 (0.4 and 3 TeV) have two bunches (each) of μ^+ and μ^-, so these bunches could be combined, doubling the luminosity. Reducing ε_N requires more transverse cooling (or emittance exchange). The beams in the 0.1 TeV scenarios can probably receive further transverse cooling; cooling to $\varepsilon_N = 10^{-4}$ m-rad should be possible without increasing the longitudinal emittance.

The total number of muons $N_{tot} = n_B N_\mu$ can be increased by increasing the power of the initial proton beam on the production target, and/or by increasing the efficiency of the μ collection and cooling system. The 4 MW proton source is already at fairly high intensity, but successful design and/or operation at that intensity will suggest incremental improvements and an increase to ~10 MW will eventually be possible. In this increase it is uncertain how much one would rely on more protons per pulse and how much would be obtained by increasing the pulsing frequency.

The number of storage turns n_s is set by the collider circumference and beam energy; it could be slightly increased by using higher-field bending magnets to shorten the circumference.

The other remaining parameter is β^*, the focussing strength at the interaction point. This parameter has several constraints related to beam parameters, and substantial reduction in β^* may require added beam cooling. In general it is limited by the "hourglass effect" to approximately the bunch length and in the luminosity calculations we have assumed it to be equal to the bunch length. This limit could be reduced by reducing the bunch length, which implies a lower longitudinal emittance and/or increasing collision energy spreads, which are limited by the lattice chromaticity acceptances. β^* can actually be reduced below σ_z, with continuing luminosity increase.

In Table S-1, we display the baseline luminosity parameters of the 1999 status report. In table S-2 we generate some "upgraded" luminosity numbers. The upgraded numbers are obtained by increasing the number of muons by 2.5 times, which implies more proton power (~2 $\times$) and greater collection efficiency (~ 1.25 $\times$), and by incremental improvements (~0.7 $\times$ emittance and β^* reduction) in cooling and focusing. We have not enforced the assumption that 6-D emittances must be the same; this is probably realistic since the final emittance exchanges for minimizing ε_t are expected to increase 6-D emittances. In the tables we assume $\beta^* = \sigma_z$ so that lowering β^* requires reduced longitudinal emittance; this is a bit pessimistic since β^* could be reduced to ~half of σ_z and luminosity would increase by ~50%, offsetting additional cooling requirements. We have also not decreased collider circumferences, which would proportionately increase luminosity. However, the power, focusing, and cooling upgrades would increase luminosity by an order of magnitude.

As a caveat, the "upgraded" luminosities would require improvements on μ^+-μ^- collider technologies which are not yet fully established, and are certainly less established than the status report numbers. However the present discussion indicates that the status report luminosities are not absolute limits, and would be subject to incremental improvement in an operating collider.

An intriguing question is whether there are “advanced” technologies which could increase μ^+-μ^- collider performance beyond the status report baselines. Among these are “low-energy” cooling for stopped (or nearly stopped) muons, optical stochastic cooling, beam-beam force neutralization at the interaction point, plasma lens focusing (in the cooling or in the collider), etc. Possible gains by such methods are discussed by B. King et al.[]

Table S-1 Initial $\mu^+-\mu^-$ Collider parameters (“Higgs factory” and 3TeV Colliders) [ref. 1]

	Beam properties at capture	100GeV high-resolution Higgs Factory	100GeV high-luminosity Higgs Factory	3TeV High-Energy Collider	
Collision Energy		100	100	3000	GeV
E_μ/beam	0.1	50	50	1500	GeV
σ_x	10 cm	295 μm	86 μm	3.2 μm	
$\sigma_{x'}$	70	2.1	2.1	1.1	mr
σ_z	680	14.1	4.1	0.3	cm
σ_p / p	0.65	3×10^{-5}	0.0012	0.0016	
ε_{xN}	~20	0.29	0.085	0.05	mm
ε_{zN}	~1000	2.02	24	70	mm
ε_{6N}	$\sim 4 \times 10^5$	0.17	0.17	0.17	mm^3
β^*		14.1	4.1	0.3	cm
$\Delta\nu$		0.015	0.051	0.044	
n_{tuens}		450	450	785	
F		15	15	15	Hz
N_b		1	1	2	Bunch/beam/spill
N		4×10^{12}	4×10^{12}	2×10^{12}	Particles/bunch
L		10^{31}	1.2×10^{32}	7×10^{34}	cm^{-2}-s^{-1}

Table S-2 “High-Luminosity” $\mu^+-\mu^-$ Collider parameters (“Higgs factory” and 3TeV Colliders)

	Beam properties at capture	100GeV high-resolution Higgs Factory	100GeV high-luminosity Higgs Factory	3TeV High-Energy Collider	
Collision Energy		100	100	3000	GeV
E_μ/beam	0.1	50	50	1500	GeV
σ_x	10 cm	160 μm	53 μm	2.2 μm	
$\sigma_{x'}$	70	1.6	2.6	1.1	mr
σ_z	680	10	2	0.2	cm
σ_p / p	0.65	3×10^{-5}	0.0011	0.0016	
ε_{xN}	~20	0.12	0.065	0.035	mm
ε_{zN}	~1000	1.4	10	45	mm
ε_{6N}	$\sim 4 \times 10^5$	0.02	0.045	0.055	mm^3
$\Delta\nu$		0.044	0.083	0.157	
F		15	15	15	Hz
N_b		2	2	2	Bunch/beam/spill
N		5×10^{12}	5×10^{12}	5×10^{12}	Particles/bunch
L		1.1×10^{32}	1.0×10^{33}	1×10^{36}	cm^{-2}-s^{-1}

References

[1] C. M. Ankenbrandt et al., Phys. Rev. STAB 2, 081001 (1999).
[2] M. Furman, BF-19/CBP-Note-169/LBL-38563, 1996
[3] B. King, ed. "High-energy $\mu^{+}-\mu^{-}$Colliders," in progress, (2001).