Physics of Higgs Factories (contribution to Snowmass)

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Abstract. We outline the unique role of a muon collider as a Higgs factory for Higgs boson resonance production in s-channel. Physics examples include the precision measurements of the Higgs mass and total width, the discrimination power for a Higgs boson in different models such as in SM or MSSM, spin and coupling determination via $h \to \tau^- \tau^+$ mode, differentiation of two nearly degenerate heavy Higgs bosons by an energy scan, and an extended Higgs sector and possible CP-violation couplings. Complementary to Higgs physics at the LHC and LC, the muon collider Higgs factory could play a crucial role in many ways.

I INTRODUCTION

A muon collider with c.m. energy centered at the Higgs boson mass offers a unique opportunity to produce Higgs bosons in the s-channel and thereby measure the Higgs masses, total width and several partial widths to very high precision. In the event that only a SM-like Higgs boson is discovered and its properties measured at the Tevatron, the LHC, and a LC, it may prove essential to build a muon collider to fully explore the Higgs sector. In particular, the very narrow width of a Standard Model (SM) Higgs bosons cannot be measured *directly* at the Large Hadron Collider (LHC) or at a future Linear Collider (LC). Furthermore, there are regions of parameter space for which it will be impossible for either the LHC or a LC to discover the heavier Higgs bosons of supersymmetry or, in the case of a general two-Higgs-doublet or more extended model, Higgs bosons of any mass with small or zero VV coupling.

The prospects for doing physics at a future Higgs factory should be discussed in light of recent experimental data. While by no means definitive, recent experimental results point in promising directions for Higgs factories. First, there is the $\sim 2.9\sigma$ statistical evidence from LEP [1–5] for a Higgs boson near $m_H \simeq 115$ GeV. Such a mass is in the optimal range for study at a Higgs factory and it is for such a low mass that the muon collider factory option would add the most information to data from the LHC and a LC. First, 115 GeV is sufficiently above the Z-pole that the background from Z production and decay to $b\bar{b}$ is not so large, and the mass is sufficiently below the WW^* threshold that the decay width remains small and the ability of the muon collider to achieve a very narrow beam energy spread can be exploited. Second, it is for masses below 120 GeV that the LC will have difficulty getting a precision measurement of the Higgs to WW^* branching ratio and, thus, have a large error for the indirect determination of the total width. Of course, a Higgs boson in this mass range, and having substantial VV coupling, is also the most natural interpretation of current precision electroweak data. On the theoretical side, a Higgs mass of ~ 115 GeV is very suggestive of supersymmetry. In the Minimal Supersymmetric Model (MSSM) such a mass is near the theoretical upper limit of $m_H < 130$ GeV, and would indicate a value of the supersymmetry parameter $\tan\beta$ substantially above 1 (assuming stop masses $\lesssim 1$ TeV).

The consistency of a Higgs with mass ~ 115 GeV with a large $\tan\beta$ supersymmetry scenario would mesh nicely with recent evidence for an anomalous magnetic moment of the muon [6] that deviates from the Standard Model prediction. The 2.6σ discrepancy is naturally accounted for provided $\tan\beta$ is constrained to be relatively large (and superparticle masses are not too heavy). More specifically, a supersymmetric interpretation of this discrepancy with the SM prediction implies the following relationship between the mass scale $\tilde{m}$ of supersymmetric particles contributing to the one-loop anomalous magnetic moment diagram and $\tan\beta$ [7],

$$\tan\beta\left(\frac{100\ \mathrm{GeV}}{\tilde{m}}\right)^2 = 3.3 \pm 1.3$$

Furthermore, if the anomalous magnetic moment is explained by supersymmetry the value of the Higgs mass parameter μ of supersymmetric models has a sign which is consistent with the constraints from the radiative decays, $b \to s\gamma$. Thus, a consistent picture begins to emerge suggesting low-energy supersymmetry with a Higgs boson in the right range.

While these recent experimental data are not definitive, they do point to an interesting scenario whereby a muon collider might prove essential to the understanding of the Higgs sector of a supersymmetric model. The muon collider could play an crucial role in precision tests of the Higgs sector in at least two ways: (1) accurately measuring the properties of a light SM-like Higgs boson and distinguishing it from a supersymmetric Higgs bosons, and (2) discovering heavy Higgs bosons of supersymmetry and accurately measuring their properties.

II MUON COLLIDERS

Muon colliders have a number of unique features that make them attractive candidates for future accelerators [8]. The most important and fundamental of these derive from the large mass of the muon in comparison to that of the electron. This leads to: a) the possibility of extremely narrow beam energy spreads, especially at beam energies below 100 GeV; b) the possibility of accelerators with very high energy; c) the possiblity of employing storage rings at high energy; d) the possibility of using decays of accelerated muons to provide a high luminosity source of neutrinos (under active consideration as reviewed elsewhere); e) increased potential for probing physics in which couplings increase with mass (as does the SM $h_{SM} f\overline{f}$ coupling).

Here our focus is on the Higgs sector. The relatively large mass of the muon compared to the mass of the electron means that the coupling of Higgs bosons to $\mu^+\mu^-$ is very much larger than to e^+e^-, implying much larger s-channel Higgs production rates at a muon collider as compared to an electron collider [see Fig. 1]. For Higgs bosons with a very small MeV scale width, such as a light SM Higgs boson, production rates in the s-channel are further enhanced by the muon collider's ability to achieve beam energy spreads comparable to the tiny Higgs width. In addition, there is little bremsstrahlung, and the beam energy can be tuned to one part in a million through continuous spin-rotation measurements [9]. Due to these important qualitative difference between the two types of machines, only muon colliders can be advocated as potential s-channel Higgs factories capable of determining the mass and decay width of a Higgs boson to very high precision [10,11]. High rates of Higgs production at e^+e^- colliders rely on substantial VVHiggs coupling for the Z+Higgs (Higgstrahlung) or WW $\to$Higgs (WW fusion) reactions, In contrast, a $\mu^+\mu^-$ collider can provide a factory for producing a Higgs boson with little or no VV coupling so long as it has SM-like (or enhanced) $\mu^+\mu^-$ couplings.

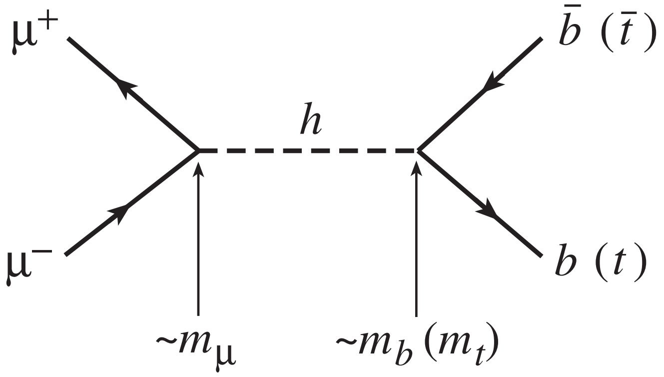

FIGURE 1. Feynman diagram for s-channel production of a Higgs boson.

Of course, there is a tradeoff between small beam energy spread, $\delta E/E = R$, and luminosity. Current estimates for yearly integrated luminosities (using $\mathcal{L} = 1 \times 10^{32}\mathrm{cm}^{-2}\mathrm{s}^{-1}$ as implying $L = 1\ \mathrm{fb}^{-1}/\mathrm{yr}$) are: $L_{\mathrm{year}} \gtrsim 0.1, 0.22, 1\,\mathrm{fb}^{-1}$ at $\sqrt{s} \sim 100$ GeV for beam energy resolutions of $R = 0.003\%, 0.01\%, 0.1\%$, respectively; $L_{\mathrm{year}} \sim 2, 6, 10\ \mathrm{fb}^{-1}$ at $\sqrt{s} \sim 200, 350, 400$ GeV, respectively, for $R \sim 0.1\%$. Despite this, studies show that for small Higgs width the s-channel production rate (and statistical significance over background) is maximized by choosing R to be such that $\sigma_{\sqrt{s}} \lesssim \Gamma_h^{\rm tot}$. In particular, in the SM context for $m_{h_{SM}} \sim 110$ GeV this corresponds to $R \sim 0.003\%$.

If the $m_h \sim 115$ GeV LEP signal is real or if the interpretation of the precision electroweak data as an indication of a light Higgs boson (with substantial VV coupling) is valid, [1] then both e^+e^- and $\mu^+\mu^-$ colliders will be valuable. In this scenario the Higgs boson would have been discovered at a previous higher energy collider (possibly a muon collider running at high energy), and then the Higgs factory would be built with a center-of-mass energy precisely tuned to the Higgs boson mass.[2] The most likely scenario is that the Higgs boson is discovered at the LHC via gluon fusion ($gg \to H$) or perhaps earlier at the Tevatron via associated production ($q\bar{q} \to WH, t\bar{t}H$), and its mass is determined to an accuracy of about 100 MeV. If a linear collider has also observed the Higgs via the Higgs-strahlung process ($e^+e^- \to ZH$), one might know the Higgs boson mass to better than 50 MeV with an integrated luminosity of 500 fb^{-1}. The muon collider would be optimized to run at $\sqrt{s} \approx m_H$, and this center-of-mass energy would be varied over a narrow range so as to scan over the Higgs resonance (see Fig. 2 below).

III HIGGS PRODUCTION

The production of a Higgs boson (generically denoted h) in the s-channel with interesting rates is a unique feature of a muon collider [10,11]. The resonance cross section is

$$\sigma_h(\sqrt{s}) = \frac{4\pi\Gamma(h\to\mu\bar{\mu})\,\Gamma(h\to X)}{\left(s-m_h^2\right)^2 + m_h^2\left(\Gamma_{\rm tot}^h\right)^2}\,. \tag{1}$$

In practice, however, there is a Gaussian spread ($\sigma_{\sqrt{s}}$) to the center-of-mass energy and one must compute the effective s-channel Higgs cross section after convolution assuming some given central value of $\sqrt{s}$:

$$\overline{\sigma}_h(\sqrt{s}) = \frac{1}{\sqrt{2\pi}\,\sigma_{\sqrt{s}}}\int \sigma_h(\sqrt{\hat{s}})\,\exp\left[\frac{-\left(\sqrt{\hat{s}}-\sqrt{s}\right)^2}{2\sigma_{\sqrt{s}}^2}\right] d\sqrt{\hat{s}} \;\;\overset{\sqrt{s}=m_h}{\simeq}\;\; \frac{4\pi}{m_h^2}\,\frac{{\rm BF}(h\to\mu\bar{\mu})\,{\rm BF}(h\to X)}{\left[1+\frac{8}{\pi}\left(\frac{\sigma_{\sqrt{s}}}{\Gamma_h^{\rm tot}}\right)^2\right]^{1/2}}\,. \tag{2}$$

It is convenient to express $\sigma_{\sqrt{s}}$ in terms of the root-mean-square (rms) Gaussian spread of the energy of an individual beam, R:

$$\sigma_{\sqrt{s}} = (2~{\rm MeV})\left(\frac{R}{0.003\%}\right)\left(\frac{\sqrt{s}}{100~{\rm GeV}}\right)\,. \tag{3}$$

From Eq. (1), it is apparent that a resolution $\sigma_{\sqrt{s}} \lesssim \Gamma_h^{\rm tot}$ is needed to be sensitive to the Higgs width. Further, Eq. (2) implies that $\overline{\sigma}_h \propto 1/\sigma_{\sqrt{s}}$ for $\sigma_{\sqrt{s}} > \Gamma_h^{\rm tot}$ *and* that large event rates are only possible if $\Gamma_h^{\rm tot}$ is not so large that ${\rm BF}(h\to\mu\overline{\mu})$ is extremely suppressed. The width of a light SM-like Higgs is very small (e.g. a few MeV for $m_{h_{SM}} \sim 110$ GeV), implying the need for R values as small as $\sim 0.003\%$ for studying a light SM-like h. Fig. 2 illustrates the result for the SM Higgs boson of an initial centering scan over $\sqrt{s}$ values in the vicinity of $m_{h_{SM}} = 110$ GeV. This figure dramatizes: a) that the beam energy spread must be very small because of the very small $\Gamma_{h_{SM}}^{\rm tot}$ (when $m_{h_{SM}}$ is small enough that the $WW^\star$ decay mode is highly suppressed); b) that we require the very accurate *in situ* determination of the beam energy to one part in a million through the spin precession of the muon noted earlier in order to perform the scan and then center on $\sqrt{s} = m_{h_{SM}}$ with a high degree of stability.

If the h has SM-like couplings to WW, its width will grow rapidly for $m_h > 2m_W$ and its s-channel production cross section will be severely suppressed by the resulting decrease of ${\rm BF}(h\to\mu\mu)$. More generally, any h with

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[1] Even in a two-doublet extension of the minimal one-doublet SM Higgs sector, parameters can be chosen so that the only light Higgs boson has no VV coupling and yet good agreement with precision electroweak data maintained [19].

[2] If the higher energy muon collider has already been constructed, this would simply require construction of a small storage ring tuned to the appropriate energy.

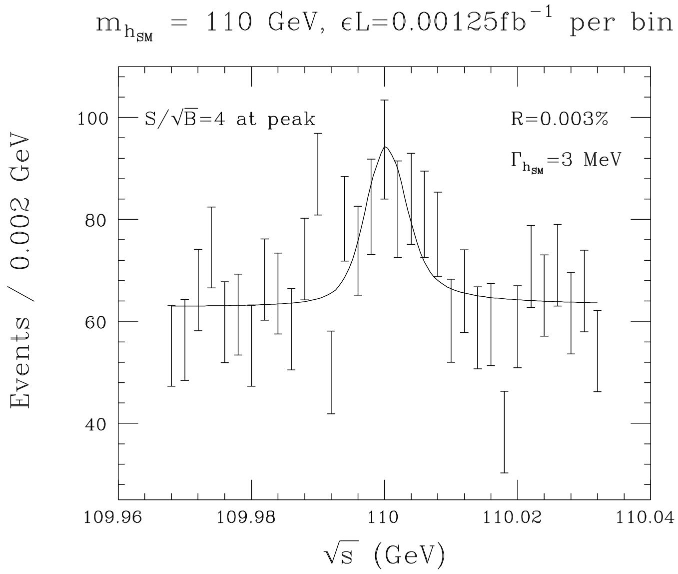

FIGURE 2. Number of events and statistical errors in the $b\bar{b}$ final state as a function of $\sqrt{s}$ in the vicinity of $m_{h_{SM}} = 110$ GeV, assuming $R = 0.003\%$, and $\epsilon L = 0.00125$ fb^{-1} at each data point.

SM-like or larger $h\mu\mu$ coupling will retain a large s-channel production rate when $m_h > 2m_W$ only if the hWW coupling becomes strongly suppressed relative to the $h_{SM}WW$ coupling.

The general theoretical prediction within supersymmetric models is that the lightest supersymmetric Higgs boson h^0 will be very similar to the h_{SM} when the other Higgs bosons are heavy. This 'decoupling limit' is very likely to arise if the masses of the supersymmetric particles are large (since the Higgs masses and the superparticle masses are typically similar in size for most boundary condition choices). Thus, h^0 rates will be very similar to h_{SM} rates. In contrast, the heavier Higgs bosons in a typical supersymmetric model decouple from VV at large mass and remain reasonably narrow. As a result, their s-channel production rates remain large.

For a SM-like h, at $\sqrt{s} = m_h \approx 115$ GeV and $R = 0.003\%$, the $b\bar{b}$ rates are

$$\text{signal} \approx 10^4 \text{ events} \times \mathrm{L}(\mathrm{fb}^{-1})\,, \tag{4}$$

$$\text{background} \approx 10^4 \text{ events} \times \mathrm{L}(\mathrm{fb}^{-1})\,, \tag{5}$$

The SM Higgs cross sections and backgrounds are shown in Fig. 3 for $R = 0.003\%$ and $m_{h_{SM}}$ values such that the dominant decay mode is $b\bar{b}$.

IV THE MUON COLLIDER ROLE

An assessment of the need for a Higgs factory requires that one detail the unique capabilities of a muon collider versus the other possible future accelerators as well as comparing the abilities of all the machines to measure the same Higgs properties. Muon colliders and a Higgs factory in particular would only become operational after the LHC physics program is well-developed and quite possibly after a linear collider program is mature as well. So one important question is the following: if a SM-like Higgs boson and, possibly, important physics beyond the Standard Model have been discovered at the LHC and perhaps studied at a linear collider, what new information could a Higgs factory provide?

The s-channel production process allows one to determine the mass, total width, and the cross sections $\overline{\sigma}_h(\mu^+\mu^- \to h \to X)$ for several final states X to very high precision. The Higgs mass, total width and the cross sections can be used to constrain the parameters of the Higgs sector. For example, in the MSSM their precise values will constrain the Higgs sector parameters m_{A^0} and $\tan\beta$ (where $\tan\beta$ is the ratio of the two vacuum expectation values (vevs) of the two Higgs doublets of the MSSM). The main question is whether these constraints will be a valuable addition to LHC and LC constraints.

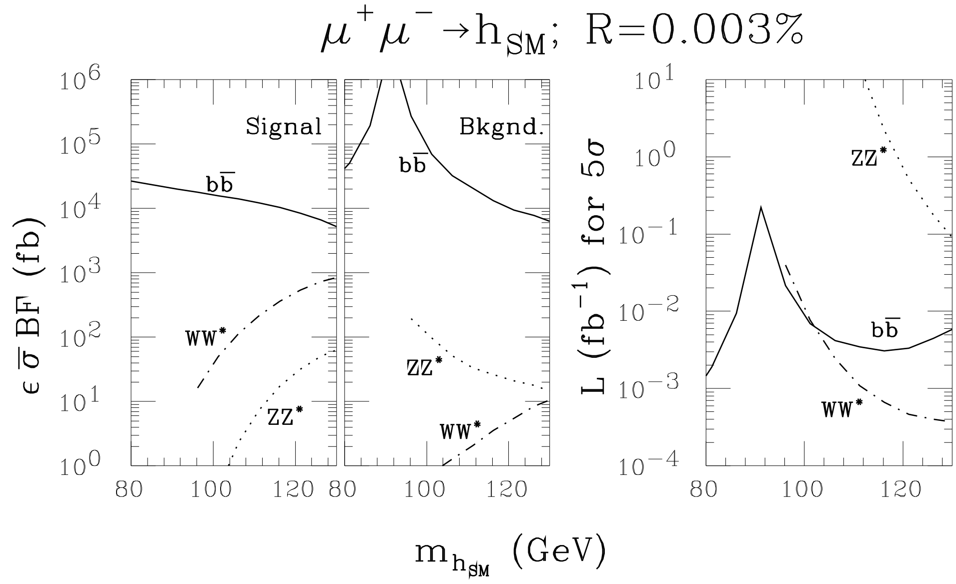

FIGURE 3. The SM Higgs cross sections and backgrounds in $b\bar{b}$, WW^* and ZZ^*. Also shown is the luminosity needed for a 5 standard deviation detection in $b\bar{b}$. From Ref. [10].

The expectations for the luminosity available at linear colliders has risen steadily. The most recent studies assume an integrated luminosity of some 500 fb^{-1} corresponding to 1-2 years of running at a few$\times$100 fb^{-1} per year. This luminosity results in the production of greater than 10^4 Higgs bosons per year through the Bjorken Higgs-strahlung process, $e^+e^- \to Zh$, provided the Higgs boson is kinematically accessible. This is comparable or even better than can be achieved with the current machine parameters for a muon collider operating at the Higgs resonance; in fact, recent studies have described high-luminosity linear colliders as "Higgs factories," though for the purposes of this report, we will reserve this term for muon colliders operating at the s-channel Higgs resonance.

A linear collider with such high luminosity can certainly perform quite accurate measurements of certain Higgs parameters such as the Higgs mass, couplings to gauge bosons, couplings to heavy quarks, etc. [17]. Precise measurements of the couplings of the Higgs boson to the Standard Model particles is an important test of the mass generation mechanism. In the Standard Model with one Higgs doublet, this coupling is proportional to the particle mass. In the more general case there can be mixing angles present in the couplings. Precision measurements of the couplings can distinguish the Standard Model Higgs boson from one from a more general model and can constrain the parameters of a more general Higgs sector.

TABLE 1. Achievable relative uncertainties for a SM-like $m_h = 110$ GeV for measuring the Higgs boson mass and total width for the LHC, LC (500 fb^{-1}), and the muon collider (0.2 fb^{-1}).

	LHC	LC	$\mu^+\mu^-$
m_h	9×10^{-4}	3×10^{-4}	$1-3\times10^{-6}$
$\Gamma_h^{\rm tot}$	>0.3	0.17	0.2

The accuracies possible at different colliders for measuring m_h and $\Gamma_h^{\rm tot}$ of a SM-like h with $m_h \sim 110$ GeV are given in Table 1. Once the mass is determined to about 1 MeV at the LHC and/or LC, the muon collider would employ a three-point fine scan [10] near the resonance peak. Since all the couplings of the Standard Model are known, $\Gamma_{h_{SM}}^{\rm tot}$ is known. Therefore a precise determination of $\Gamma_h^{\rm tot}$ is an important test of the Standard Model, and any deviation would be evidence for a nonstandard Higgs sector. For a SM Higgs boson with a mass sufficiently below the $WW^\star$ threshold, the Higgs total width is very small (of order several MeV), and the only process where it can be measured *directly* is in the s-channel at a muon collider. Indirect determinations at the LC can have higher accuracy once m_h is large enough that the $WW^\star$ mode rates can be accurately measured, requiring $m_h > 120$ GeV. This is because at the LC the total width must be determined indirectly by measuring a partial width and a branching fraction, and then computing the total width,

$$\Gamma_{tot} = \frac{\Gamma(h \to X)}{BR(h \to X)} , \tag{6}$$

for some final state X. For a Higgs boson so light that the $WW^\star$ decay mode is not useful, then the total width measurement would probably require use of the $\gamma\gamma$ decays [18]. This would require information from a photon collider as well as the LC and a small error is not possible. The muon collider can measure the total width of the Higgs boson much better, a very valuable input for precision tests of the Higgs sector.

In fact, a muon collider of limited luminosity can remain more than competitive with LHC + LC for discriminating between the SM h_{SM} and some SM-like h even for m_h values such that the LC obtains a good measurement of $WW^\star$ rates. As it happens, for $X = b\bar{b}$ there is a fortuitous compensation that results in $\overline{\sigma}_h(\mu^+\mu^- \to h \to b\bar{b})$ being almost completely independent of the somewhat uncertain b quark mass. Very roughly, larger m_b means larger $\mathrm{BF}(h \to b\bar{b})$ but also larger $\Gamma_h^{\rm tot}$. The latter implies a smaller convoluted cross section $\overline{\sigma}_h(\mu^+\mu^- \to h)$ (i.e. before including the branching ratio). Further, larger $\Gamma_h^{\rm tot}$ mean less damping because of beam energy spread. The result is that $\overline{\sigma}_h(\mu^+\mu^- \to h \to b\bar{b})$ is essentially independent of the input m_b value (within reasonable limits) [13].

As a result, the precise measurement of $\overline{\sigma}_h(\mu^+\mu^- \to h \to b\bar{b})$ at a muon collider might provide the best single discriminator between the SM Higgs and a SM-like Higgs. This is nicely illustrated in the context of the MSSM. For a Higgs mass of 110 GeV, and assuming a typical soft-supersymmetry-breaking scenario, Fig. 4 shows the resulting excluded regions of m_{A^0} for the (a) LHC+LC, (b) with a muon collider with 0.2 fb^{-1} integrated luminosity, and (c) with a muon collider with 10 fb^{-1} integrated luminosity. Some comments on these results are appropriate. First, one should note that the measurement of $\Gamma_h^{\rm tot}$ (± 0.5 MeV, i.e $\pm 20\%$) at the muon collider is not nearly so powerful a discriminator as the $\pm 3.5\%$ ($\pm 0.5\%$) measurement of $\overline{\sigma}_h(\mu^+\mu^- \to h \to b\bar{b})$ at $L = 0.2$ fb^{-1} (10 fb^{-1}). Second, as m_h increases and the $WW^\star$ decay mode becomes more prominent, much more accurate determinations of partial width ratios and the total width become possible at LHC+LC and the LHC+LC exclusion regions move rapidly to higher m_{A^0}, but at best becoming comparable to the 0.2 fb^{-1} muon collider exclusion regions. Third, the conclusion that with higher luminosities than the 0.1 fb^{-1} per year currently envisioned for the Higgs factory this discriminator would have incredible sensitivity to m_{A^0} assumes that systematic errors for the absolute cross section will be smaller than the statistical errors. Fourth, we should note that there are high $\tan\beta$ scenarios in which decoupling sets in very early in m_{A^0} and no machine would be able to set a lower bound on m_{A^0} above masses for which it would be readily produced in H^0A^0 pair production at a $\sqrt{s} = 500$ GeV LC. Finally, if there was a very light neutralino such that $h^0 \to \chi^0\chi^0$ decays were possible, this would be known ahead of time and the $\mu^+\mu^- \to h^0 \to b\bar{b}$ rate prediction within the SUSY context would have to be corrected to very high precision to account for these additional decays. SUSY loop corrections to the $b\bar{b}$ coupling might also have to be accounted for to high precision if the SUSY spectrum turns out to be light. But these last two caveats also apply to the LC measurements.

Given the above sensitivity, the next question is the extent to which parameters of the superymmetric Higgs sector can be determined with some reasonable level of precision. To study this, an input MSSM model was assumed with $m_{h^0} = 110$ GeV, $m_{A^0} = 400$ GeV, $\tan\beta = 10$ and $A_t = \mu = M_{\rm SUSY} = 1$ TeV. Various observables were computed as a function of m_{A^0} and $\tan\beta$. The extremely precise measurement of m_{h^0} at a muon collider basically determines m_{A^0} as a function of $\tan\beta$ (or vice versa) given the fixed SUSY breaking scenario. The $b\bar{b}$ event rate and, to a lesser extent, $\Gamma_{h^0}^{\rm tot}$ determine the location along the line allowed by the fixed value of m_{h^0}. The results are illustrated in Fig. 5 for the above sample model [14]. If one allows for variation in the (probably constrained by LHC data) SUSY breaking parameters, the muon collider lines turn into ellipses. Equivalently, strong constraints from the muon collider measurements are imposed on the multi-dimensional MSSM parameter space that includes m_A and $\tan\beta$ and the SUSY-breaking parameters. Clearly, a very powerful reduction in the allowed values would result from the measurements of m_{h^0}, $\overline{\sigma}(\mu^+\mu^- \to h^0 \to b\bar{b})$ and $\Gamma_{h^0}^{\rm tot}$. As before, for higher m_{h^0} (such that the $WW^\star$ mode becomes well measured), LHC+LC constraints improve significantly, but would never equal those of even the 0.1 fb^{-1} muon collider constraints.

Another important probe of the physics of a light h that is only possible at a muon collider is the possibility of measuring $\Gamma(h \to \mu^+\mu^-)$. Typically, the muon collider data must be combined with LC and/or LHC data to extract this very fundamental coupling. If the h is SM-like then the following determinations are possible.

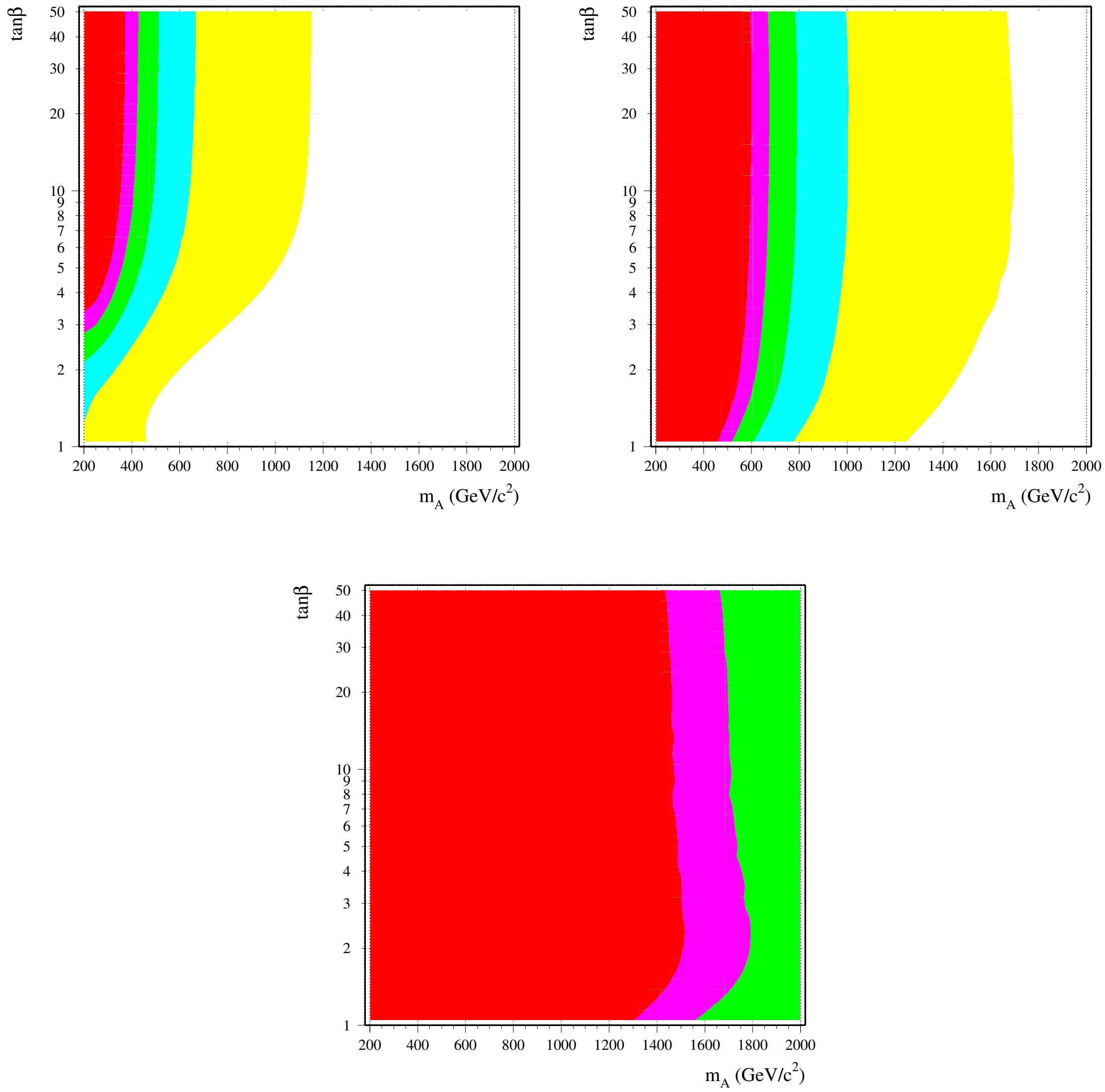

FIGURE 4. The $m_{A^0} - \tan\beta$ discrimination from the measurements at (a) LHC(300 fb^{-1})+LC(500 fb^{-1}), (b) 0.2 fb^{-1} at a muon collider, and (c) 10 fb^{-1} at a muon collider. The exclusion regions (starting from the left) are $> 5\sigma$, $4-5\sigma$, $3-4\sigma$, $2-3\sigma$, and $1-2\sigma$. From Ref. [13].

1) $$\Gamma(h \to \mu^+\mu^-) = \frac{[\Gamma(h\to\mu^+\mu^-)\mathrm{BF}(h\to b\bar{b})]_{\mu\mathrm{C}}}{\mathrm{BF}(h\to b\bar{b})_{\mathrm{NLC}}};$$

2) $$\Gamma(h \to \mu^+\mu^-) = \frac{[\Gamma(h\to\mu^+\mu^-)\mathrm{BF}(h\to WW^\star)]_{\mu\mathrm{C}}}{\mathrm{BF}(h\to WW^\star)_{\mathrm{NLC}}};$$

3) $$\Gamma(h \to \mu^+\mu^-) = \frac{[\Gamma(h\to\mu^+\mu^-)\mathrm{BF}(h\to ZZ^\star)]_{\mu\mathrm{C}}\Gamma_h^{\mathrm{tot}}}{\Gamma(h\to ZZ^\star)_{\mathrm{NLC}}};$$

4) $$\Gamma(h \to \mu^+\mu^-) = \frac{[\Gamma(h\to\mu^+\mu^-)\mathrm{BF}(h\to WW^\star)\Gamma_h^{\mathrm{tot}}]_{\mu\mathrm{C}}}{\Gamma(h\to WW^\star)_{\mathrm{NLC}}}.$$

Using the above, a determination of $\Gamma(h \to \mu^+\mu^-)$ with accuracy $\pm 4\%$ would be possible for an $L = 0.4$ fb^{-1} scan and LC(200 fb^{-1}) data. In the MSSM context, such precision means that one would have 3σ or greater difference between the expectation for the h_{SM} vs. the result for the h^0 if $m_{A^0} \leq 600$ GeV, assuming

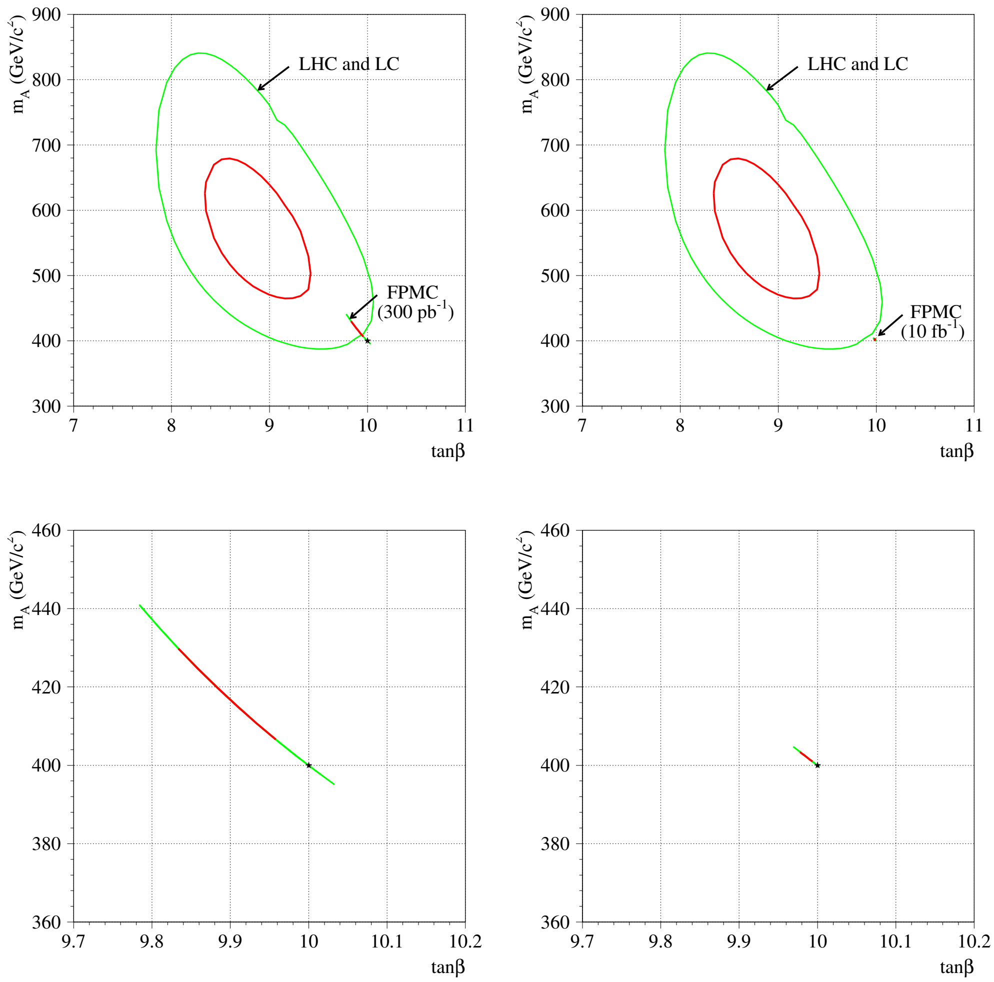

FIGURE 5. The implications of the h^0 scan for the MSSM $[m_{A^0}, \tan\beta]$ parameter space, assuming all other SUSY parameters are known. In (a) and (b) the LHC+LC expectations (1 and 2 σ ellipses) are compared to the (far superior) muon collider results for $L = 0.1$ fb^{-1} and $L = 10$ fb^{-1}, respectively. The LHC+LC ellipses collapse effectively to lines, see (c) and (d) for $L = 0.1$ fb^{-1} and $L = 10$ fb^{-1} respectively, because the Higgs mass is determined with extreme precision at a muon collider. The remaining uncertainty is associated primarily with the errors for the $b\overline{b}$ final state rate and, to a lesser extent, $\Gamma_h^{\rm tot}$. From Ref. [13].

$m_{h^0} \lesssim 135$ GeV (the MSSM upper limit). Further, this is an absolutely direct and model independent determination of $\Gamma(h^0 \to \mu^+\mu^-)$ that for certain has no systematic theoretical uncertainties. Finally, we reiterate that the muon collider provides the only probe of this 2nd generation lepton coupling and would thus be one of the few checks of the the SM or MSSM explanation of lepton masses.

To summarize, if a Higgs is discovered at the LHC or possibly earlier at the Fermilab Tevatron, attention will turn to determining whether this Higgs has the properties expected of the Standard Model Higgs. If the Higgs is discovered at the LHC, it is quite possible that supersymmetric states will be discovered concurrently. The next goal for a linear collider or a muon collider will be to better measure the Higgs boson properties to determine if everything is consistent within a supersymmetric framework or consistent with the Standard Model. A Higgs factory of even modest luminosity can provide uniquely powerful constraints on the parameter space of the supersymmetric model via its very precise measurement of the light Higgs mass, the highly accurate determination of the total rate for $\mu^+\mu^- \to h^0 \to b\overline{b}$ (which has almost zero theoretical systematic uncertainty due to its insensitivity to the unknown m_b value) and the moderately accurate determination of the h^0's total width. Adding in the model-independent determination of the $h^0\mu^+\mu^-$ coupling constant (not incorporated in

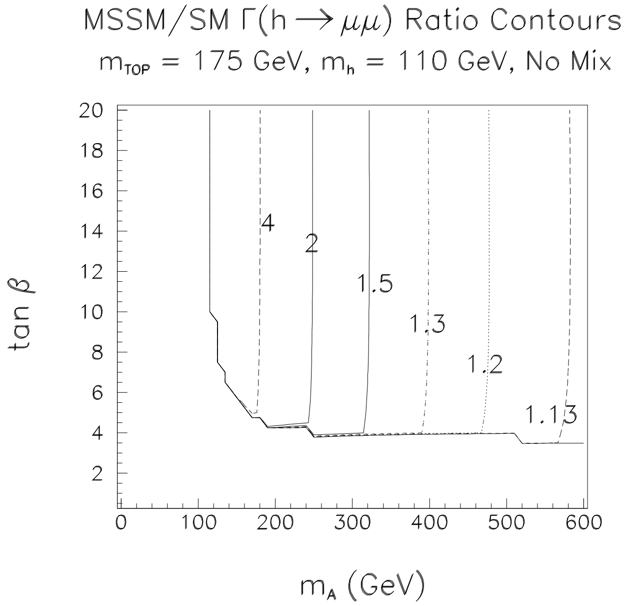

FIGURE 6. Contours in $(m_{A^0}, \tan\beta)$ parameter space for $\Gamma(h^0 \to \mu^+\mu^-)/\Gamma(h_{SM} \to \mu^+\mu^-)$. We have assumed a no-mixing SUSY scenario and employed $m_{h^0} = m_{h_{SM}} = 110$ GeV. For maximal mixing, there is little change in the contours — only the size of the allowed range is altered. From [21].

the figures and earlier estimates) would increase the precision of the constraints.

V $H \to \tau^+\tau^-$

A particularly important channel is the $\tau^-\tau^+$ final state [15]

$$\mu^-\mu^+ \to \tau^-\tau^+. \tag{7}$$

In the SM at tree level, this s-channel process proceeds in two ways, via γ/Z exchange and Higgs boson exchange. The former involves the SM gauge couplings and presents a characteristic FB (forward-backward in the scattering angle) asymmetry and a LR (left-right in beam polarization) asymmetry; the latter is governed by the Higgs boson couplings to $\mu^-\mu^+, \tau^-\tau^+$ proportional to the fermion masses and is isotropic in phase space due to spin-0 exchange. The unambiguous establishment of the $\tau^-\tau^+$ signal would allow a determination of the relative coupling strength of the Higgs boson to b and τ and thus test the usual assumption of $\tau - b$ unification. The angular distribution would probe the spin property of the Higgs resonance.

The differential cross section for $\mu^-\mu^+ \to \tau^-\tau^+$ via s-channel Higgs exchange can be expressed as

$$\frac{d\sigma_h(\mu^-\mu^+ \to h \to \tau^-\tau^+)}{d\cos\theta} = \frac{1}{2}\bar{\sigma}_h\ (1 + P_-P_+) \tag{8}$$

where θ is the scattering angle between μ^- and τ^-, $P_\mp$ the percentage longitudinal polarizations of the initial $\mu^\mp$ beams, with $P = -1$ purely left-handed, $P = +1$ purely right-handed and $P = 0$ unpolarized.

The differential cross section for the SM background is given by the γ/Z contributions

$$\frac{d\sigma_{SM}}{d\cos\theta} = \frac{3}{8}\sigma_{QED}A[1 - P_+P_- + (P_+ - P_-)A_{LR}](1 + \cos^2\theta + \frac{8}{3}\cos\theta A_{FB}^{eff}). \tag{9}$$

Here the effective FB asymmetry factor is

$$A_{FB}^{eff} = \frac{A_{FB} + P_{eff}A_{LR}^{FB}}{1 + P_{eff}A_{LR}}, \tag{10}$$

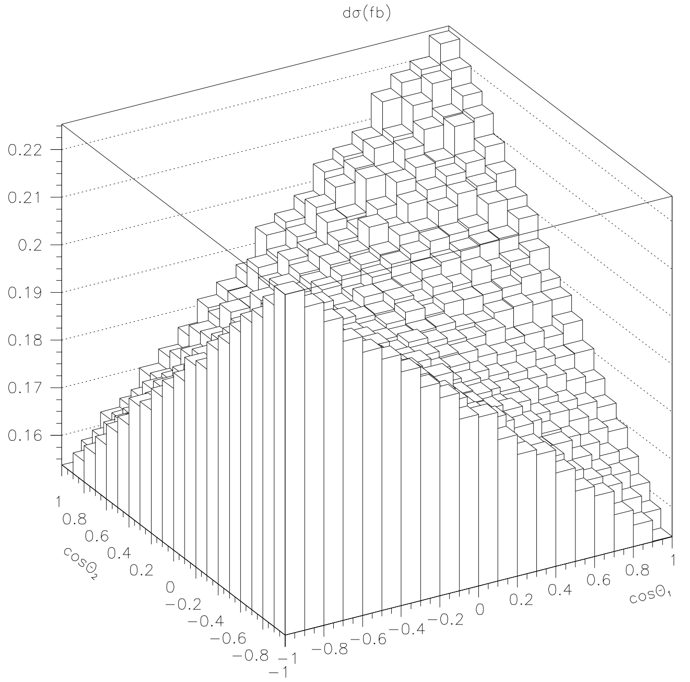

FIGURE 7. Double differential distribution for $\mu^-\mu^+ \to h \to \tau^-\tau^+ \to \rho^-\nu_\tau\rho^+\bar{\nu}_\tau$. $\sqrt{s} = m_h = 120$ GeV is assumed. Initial $\mu^\mp$ beam polarizations are taken to be $P_- = P_+ = 0.25$. The Higgs production cross section is convoluted with Gaussian energy distribution for a resolution $R = 0.05\%$.

with the effective polarization

$$P_{eff} = \frac{P_+ - P_-}{1 - P_+ P_-}, \tag{11}$$

and

$$A_{LR}^{FB} = \frac{\sigma_{LR+RL\to LR} - \sigma_{LR+RL\to RL}}{\sigma_{LR+RL\to LR} + \sigma_{LR+RL\to RL}}. \tag{12}$$

A_{FB}, A_{LR} are the standard asymmetries. For the case of interest where initial and final state particles are leptons, $A_{LR} = A_{LR}^{FB}$.

From the cross section formulas of Eqs. (8) and (9), the enhancement factor of the signal-to-background ratio (S/B) due to the beam polarization effects is

$$\frac{S}{B} \sim \frac{1 + P_- P_+}{1 - P_- P_+ + (P_+ - P_-)A_{LR}}. \tag{13}$$

The final state polarization configurations of $\tau^-\tau^+$ from the Higgs signal and the SM background are very different. There is always a charged track to define a kinematical distribution for the decay. In the τ-rest frame, the normalized differential decay rate can be written as

$$\frac{1}{\Gamma}\frac{d\Gamma_i}{d\cos\theta} = \frac{B_i}{2}(a_i + b_i P_\tau \cos\theta) \tag{14}$$

where θ is the angle between the momentum direction of the charged decay product in the τ-rest frame [16] and the τ-momentum direction, B_i is the branching fraction for a given channel i, and $P_\tau = \pm 1$ is the τ helicity. For the two-body decay modes, a_i and b_i are constant and given by

$$a_\pi = b_\pi = 1, \tag{15}$$

$$a_i = 1 \quad \text{and} \quad b_i = -\frac{m_\tau^2 - 2m_i^2}{m_\tau^2 + 2m_i^2} \quad \text{for} \quad i = \rho,\ a_1. \tag{16}$$

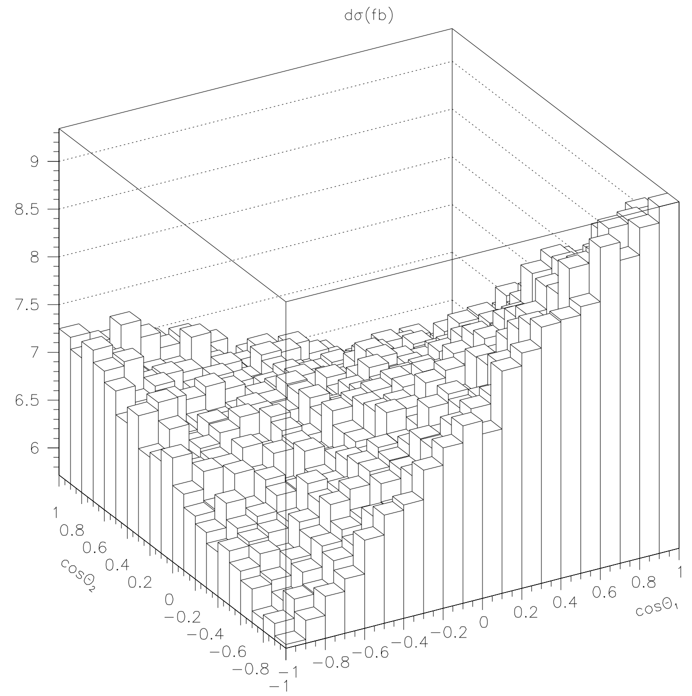

FIGURE 8. Double differential distribution for $\mu^-\mu^+ \to \gamma^*/Z^* \to \tau^-\tau^+ \to \rho^-\nu_\tau\rho^+\bar{\nu}_\tau$. $\sqrt{s} = 120$ GeV is assumed. Initial $\mu^\mp$ beam polarizations are taken to be $P_- = P_+ = 0.25$. The SM production cross section is convoluted with Gaussian energy distribution for a resolution $R = 0.05\%$.

For the three-body leptonic decays, the $a_{e,\mu}$ and $b_{e,\mu}$ are not constant for a given three-body kinematical configuration and are obtained by the integration over the energy fraction carried by the invisible neutrinos. One can quantify the event distribution shape by defining a "sensitivity" ratio parameter

$$r_i = \frac{b_i}{a_i}. \tag{17}$$

For the two-body decay modes, the sensitivities are $r_\pi = 1$, $r_\rho = 0.45$ and $r_{a_1} = 0.007$. The $\tau \to a_1\nu_\tau$ mode is consequently less useful in connection with the τ polarization study. As to the three-body leptonic modes, although experimentally readily identifiable, the energy smearing from the decay makes it hard to reconstruct the $\tau^-\tau^+$ final state spin correlation.

The differential distribution for the two charged particles (i, j) in the final state from $\tau^-\tau^+$ decays respectively can be expressed as

$$\frac{d\sigma}{d\cos\theta_i\, d\cos\theta_j} \sim \sum_{P_\tau=\pm 1} \frac{B_i B_j}{4}\ (a_i + b_i P_{\tau^-}\cos\theta_i)(a_j + b_j P_{\tau^+}\cos\theta_j), \tag{18}$$

where $\cos\theta_i$ $(\cos\theta_j)$ is defined in τ^- (τ^+) rest frame as in Eq. (14). For the Higgs signal channel, $\tau^-\tau^+$ helicities are correlated as LL $(P_{\tau^-} = P_{\tau^+} = -1)$ and RR $(P_{\tau^-} = P_{\tau^+} = +1)$. This yields the spin-correlated differential cross section

$$\frac{d\sigma_h}{d\cos\theta_i\, d\cos\theta_j} = (1 + P_-P_+)\sigma_h\ \frac{B_i B_j}{4}\ [a_i a_j + b_i b_j \cos\theta_i\cos\theta_j], \tag{19}$$

We expect that the distribution reaches maximum near $\cos\theta_i = \cos\theta_j = \pm 1$ and minimum near $\cos\theta_i = -\cos\theta_j = \pm 1$. How significant the peaks are depends on the sensitivity parameter in Eq. (17). Here we simulate the double differential distribution of Eq. (19) for $\mu^-\mu^+ \to h \to \tau^-\tau^+ \to \rho^-\nu_\tau\rho^+\bar{\nu}_\tau$ and the result is shown in Fig. 7. Here we take $\sqrt{s} = m_h = 120$ GeV for illustration. The Higgs production cross section is convoluted with Gaussian energy distribution for a resolution $R = 0.05\%$. We see distinctive peaks in the

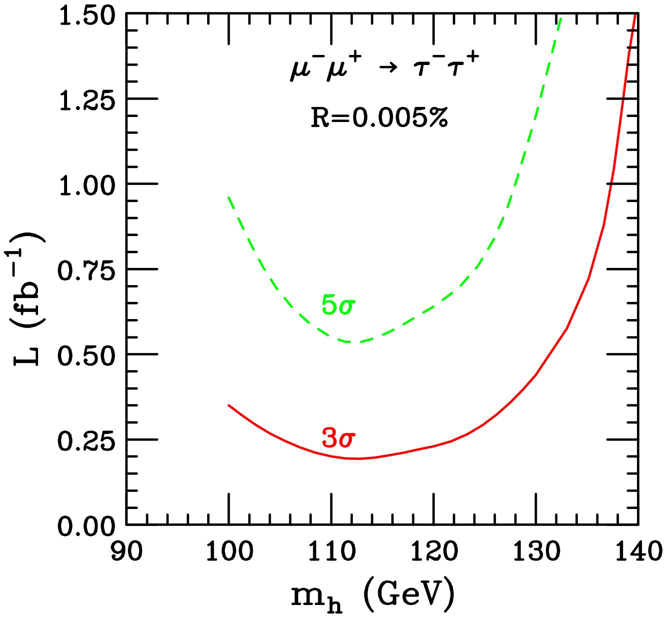

FIGURE 9. Integrated luminosity (in fb^{-1}) needed for observing the two-body decay channels $\tau \to \rho\nu_\tau$ and $\tau \to \pi\nu_\tau$ at 3σ (solid) and 5σ (dashed) significance. Beam energy resolution $R = 0.005\%$ and a 25% polarization are assumed.

distribution near $\cos\theta_{\rho^-} = \cos\theta_{\rho^+} = \pm 1$, as anticipated. In this demonstration, we have taken $\mu^\mp$ beam polarizations to be $P_- = P_+ = 25\%$, which is considered to be natural with little cost to beam luminosity.

In contrast, the SM background via γ^*/Z^* produces $\tau^-\tau^+$ with helicity correlation of LR ($P_{\tau^-} = -P_{\tau^+} = -1$) and RL ($P_{\tau^-} = -P_{\tau^+} = +1$). Furthermore, the numbers of the left-handed and right-handed τ^- at a given scattering angle are different because of the left-right asymmetry, so the initial muon beam polarization affects the $\tau^-\tau^+$ spin correlation non-trivially. Summing over the two polarization combinations in $\tau^-\tau^+$ decay to particles i and j, we have

$$\frac{d\sigma_{SM}}{d\cos\theta_i\, d\cos\theta_j} = (1 - P_- P_+)\sigma_{SM}\ (1 + P_{eff} A_{LR}) \times$$
$$\frac{B_i B_j}{4}[(a_i a_j - b_i b_j \cos\theta_i \cos\theta_j) + A_{LR}^{eff}(a_i b_j \cos\theta_j - a_j b_i \cos\theta_i)]. \tag{20}$$

The final state spin correlation for $\mu^-\mu^+ \to \gamma^*/Z^* \to \tau^-\tau^+$ decaying into $\rho^-\rho^+$ pairs is shown in Fig. 8. The maximum regions near $\cos\theta_{\rho^-} = -\cos\theta_{\rho^+} = \pm 1$ are clearly visible. Most importantly, the peak regions in Figs. 7 and 8 occur exactly in the opposite positions from the Higgs signal. We also note that the spin correlation from the Higgs signal is symmetric, while that from the background is not. The reason is that the effective LR-asymmetry in the background channel changes the relative weight of the two maxima, which becomes transparent from the last term in Eq. (20).

We next estimate the luminosity needed for signal observation of a given statistical significance. The results are shown in Fig. 9. The integrated luminosity (L in fb^{-1}) needed for observing the characteristic two-body decay channels $\tau \to \rho\nu_\tau$ and $\tau \to \pi\nu_\tau$ at 3σ (solid) and 5σ (dashed) significance is calculated for both signal and SM background with $\sqrt{s} = m_h$. Beam energy resolution $R = 0.005\%$ and a 25% $\mu^\pm$ beam polarization are assumed.

We estimate the statistical error on the cross section measurement. If we take the statistical error to be given by

$$\epsilon = \frac{\sqrt{S+B}}{S} = \frac{1}{\sqrt{L}}\,\frac{\sqrt{\sigma_S + \sigma_B}}{\sigma_S}, \tag{21}$$

summing over both $\rho\nu_\tau$ and $\pi\nu_\tau$ channels for $R = 0.005\%$, a 25% beam polarization with 1 fb^{-1} luminosity, we obtain

$$\begin{array}{lcccc} \sqrt{s}=m_h \text{ (GeV)} & 100 & 110 & 120 & 130 \\ \epsilon\ (\%) & 27 & 21 & 23 & 32 \end{array} \tag{22}$$

The uncertainties on the cross section measurements determine the extent to which the $h\tau^-\tau^+$ coupling can be measured.

In summary, we have demonstrated the feasibility of observing the resonant channel $h \to \tau^-\tau^+$ at a muon collider. For a narrow resonance like the SM Higgs boson, a good beam energy resolution is crucial for a clear signal. On the other hand, a moderate beam polarization would not help much for the signal identification. The integrated luminosity needed for a signal observation is presented in Fig. 9. Estimated statistical errors for the $\mu^-\mu^+ \to h \to \tau^-\tau^+$ cross section measurement are given in Eq. (22). We emphasized the importance of final state spin correlation to purify the signal of a scalar resonance and to confirm the nature of its spin. It is also important to carefully study the $\tau^-\tau^+$ channel of a supersymmetric Higgs boson which would allow a determination of the relative coupling strength of the Higgs to b and τ.

VI HEAVY HIGGS BOSONS

As discussed in the previous section, precision measurements of the light Higgs boson properties might make it possible to not only distinguish a supersymmetric boson from a Standard Model one, but also pinpoint a range of allowed masses for the heavier Higgs bosons. This becomes more difficult in the decoupling limit where the differences between a supersymmetric and Standard Model Higgs are smaller. Nevertheless with sufficiently precise measurements of the Higgs branching fractions, it is possible that the heavy Higgs boson masses can be infered. A muon collider light-Higgs factory might be essential in this process.

In the context of the MSSM, m_{A^0} can probably be restricted to within 50 GeV or better if $m_{A^0} < 500$ GeV. This includes the $250 - 500$ GeV range of heavy Higgs boson masses for which discovery is not possible via H^0A^0 pair production at a $\sqrt{s} = 500$ GeV LC. Further, the A^0 and H^0 cannot be detected in this mass range at either the LHC or LC in $b\bar{b}H^0, b\bar{b}A^0$ production for a wedge of moderate $\tan\beta$ values. (For large enough values of $\tan\beta$ the heavy Higgs bosons are expected to be observable in $b\bar{b}A^0, b\bar{b}H^0$ production at the LHC via their $\tau^+\tau^-$ decays and also at the LC.)

A muon collider can fill some, perhaps all of this moderate $\tan\beta$ wedge. If $\tan\beta$ is large the $\mu^+\mu^-H^0$ and $\mu^+\mu^-A^0$ couplings (proportional to $\tan\beta$ times a SM-like value) are enhanced thereby leading to enhanced production rates in $\mu^+\mu^-$ collisions. The most efficient procedure is the operate the muon collider at maximum energy and produce the H^0 and A^0 (often as overlapping resonances) via the radiative return mechanism. By looking for a peak in the $b\bar{b}$ final state, the H^0 and A^0 can be discovered and, once discovered, the machine $\sqrt{s}$ can be set to m_{A^0} or m_{H^0} and factory-like precision studies pursued. Note that the A^0 and H^0 are typically broad enough that $R = 0.1\%$ would be adequate to maximize their s-channel production rates. In particular, $\Gamma \sim 30$ MeV if the $t\bar{t}$ decay channel is not open, and $\Gamma \sim 3$ GeV if it is. Since $R = 0.1\%$ is sufficient, much higher luminosity ($L \sim 2-10$ fb^{-1}/yr) would be possible as compared to that for $R = 0.01\% - 0.003\%$ required for studying the h^0.

In short, for these moderate $\tan\beta$–$m_{A^0} \gtrsim 250$ GeV scenarios that are particularly difficult for both the LHC and the LC, the muon collider would be the only place that these extra Higgs bosons can be discovered and their properties measured very precisely.

In the MSSM, the heavy Higgs bosons are largely degenerate, especially in the decoupling limit where they are heavy. Large values of $\tan\beta$ heighten this degeneracy as shown in Fig. 10. A muon collider with sufficient energy resolution might be the only possible means for separating out these states. Examples showing the H and A resonances for $\tan\beta = 5$ and 10 are shown in Fig. 11. For the larger value of $\tan\beta$ the resonances are clearly overlapping. For the better energy resolution of $R = 0.01\%$, the two distinct resonance peaks are still visible, but become smeared out for $R = 0.06\%$.

A precise measurement on the heavy Higgs boson masses could provide a powerful window on radiative corrections in the supersymmetric Higgs sector [22]. Supersymmetry with gauge invariance in the MSSM implies the mass-squared sum rule

$$m_h^2 + m_H^2 = m_A^2 + m_Z^2 + \Delta\ , \tag{23}$$

where Δ is a calculable radiative correction (the tree-level sum rule results from setting $\Delta = 0$). Solving for the mass difference

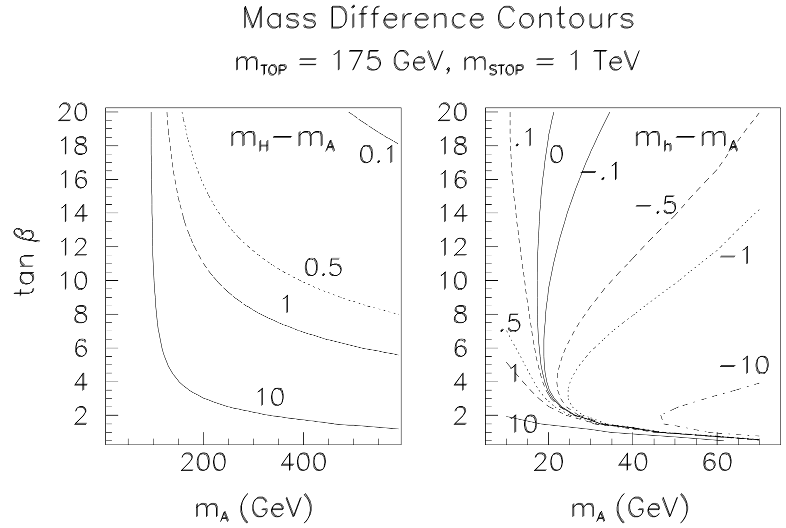

FIGURE 10. Contours of $m_H - m_A$ (in GeV) in the $(m_H, \tan\beta)$ parameter space. Two-loop/RGE-improved radiative corrections are included taking $m_t = 175$ GeV, $m_{\tilde{t}} = 1$ TeV, and neglecting squark mixing. *include only the left panel?*

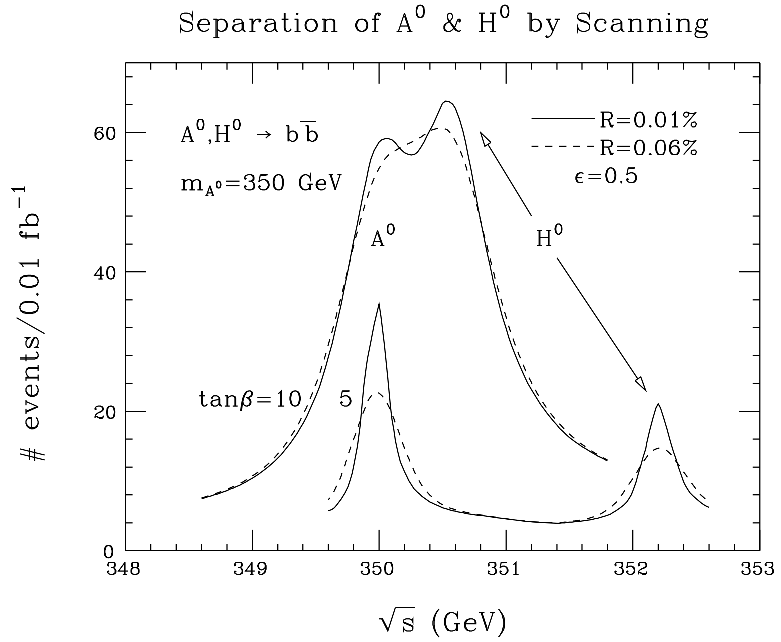

FIGURE 11. Separation of A and H signals for $\tan\beta = 5$ and 10. From Ref. [10].

$$m_A - m_H = \frac{m_h^2 - m_Z^2 - \Delta}{m_A + m_H}\,, \tag{24}$$

one obtains a formula involving observables that can be precisely measured. For example the error on the m_Z is just 2.2 MeV from the LEP measurements [20], and the light Higgs mass can be measured to less than an MeV in the s-channel. The masses of and the mass difference between the heavy Higgs states H and A can also be measured precisely by s-channel production. The ultimate precision that can be obtained on the masses of the H and A depends strongly on the masses themselves and $\tan\beta$. But a reasonable expectation is that a scan through the resonances should be able to determine the masses and the mass-difference to some tens of MeV [22]. Altogether these mass measurements yield a prediction for the radiative correction Δ which is calculable in terms of the self-energy diagrams of the Higgs bosons [23].

Finally it will be especially interesting to measure the branching ratios of these heavy Higgs bosons and compare to the theoretical predictions. For $\tan\beta \gtrsim 5$ the H^0, A^0 decay more often into $b\bar{b}$ than into $t\bar{t}$. There is a substantial range of parameter space where significant numbers of events involving both types of decays will be seen and new type of determination of $\tan\beta$ will be possible. If supersymmetric particle masses are below $\sim m_{A^0}/2$, then the branching ratios for A^0, H^0 decays to the many distinguishable channels provide extremely powerful constraints on the soft-supersymmetry-breaking parameters of the model [24–26].

VII HIGGS THRESHOLD MEASUREMENT

The mass, width and spin of a SM-like Higgs boson can also be determined by operating either a muon collider or a linear collider at the Zh production threshold. The rapid rise in the production near the threshold is sensitive to the Higgs mass [27]. Furthermore the spin of the Higgs boson can be determined by examining the rise in the cross section near threshold. However, these measurements require tens of inverse femtobarns to provide a useful measurement of the mass (<100) MeV. These threshold measurements can be performed at a LC; with 100 fb^{-1} of integrated luminosity, an error of less than 100 MeV can be achieved [27] for $m_h < 150$ GeV. This is comparable to the other methods at energies above threshold. The only means to reduce the experimental error on the Higgs mass further to below 1 MeV is to produce the Higgs in the s-channel at a muon collider.

The shape of the $\ell^+\ell^- \to Zh$ threshold cross section can also be used to determine the spin and to check the CP=+ property of the Higgs [28]. These threshold measurement become of interest for a muon collider in the case where at least a hundred inverse femtobarns of luminosity is available.

VIII NON-EXOTIC, NON-SUPERSYMMETRIC SM HIGGS SECTOR EXTENSIONS

Although the standard interpretation of precision electroweak data is that there should be a light Higgs boson with SM-like VV couplings, alternative Higgs sector models can be constructed in which a good fit to the precision data is obtained even though the Higgs boson with large VV coupling is quite heavy (~ 1 TeV). The simplest such model [19] is based upon the CP-conserving general two-Higgs-doublet model. The large $\Delta S > 0$ and $\Delta T < 0$ coming from the heavy Higgs with large VV coupling is compensated by an even larger $\Delta T > 0$ coming from a small ($\Delta M \sim 1$ GeV is sufficient) mass splitting between the $H^\pm$ and the other heavy neutral Higgs boson. The result is a shift in the $\Delta S > 0$, $\Delta T > 0$ direction (relative to the usual $m_{h_{SM}} \sim 100$ GeV scenario in the SM) that remains well within the current 90% CL ellipse in the S,T plane. The first signal for this type of scenario would be discovery of a heavy SM-like Higgs boson at the LHC. Consistency with precision electroweak data would require this type of scenario or some other exotic new physics scenario.

Models of this type cannot arise in the supersymmetric context because of constraints on the Higgs self couplings coming from the SUSY structure. They require a special 'non-decoupling' form for the potential that could arise in models where the two-doublet Higgs sector is an effective low energy description up to some scale Λ of order 10 TeV or so. For these special potential forms, there is typically also a Higgs boson $\widehat{h}$ ($\widehat{h}$ =decoupled-h^0 or $\widehat{h} = A^0$) with $m_{\widehat{h}} < 500$ GeV and no tree-level VV coupling. It's primary decay modes would be to $b\overline{b}$ or $t\overline{t}$ (depending upon its mass) and its $\mu^+\mu^-$ coupling would be proportional to $\tan\beta$. For a substantial range of $\tan\beta$, this $\widehat{h}$ could not be detected at either the LHC or the LC [19]. In particular, at the LC even the $e^+e^- \to Z^\star \to Z\widehat{h}\widehat{h}$ process (the quartic coupling being of guaranteed strength) would only allow $\widehat{h}$ discovery up to 150 GeV (250 GeV) for $\sqrt{s} = 500$ GeV (800 GeV) [29].

The muon collider could be the key to discovering such a $\widehat{h}$. By running at high energy, the radiative return tail for $E_{\mu^+\mu^-}$ might result in production of a detectable number of events. In particular, if $\tan\beta > 5$, operation at maximal $\sqrt{s}$ with $R = 0.1\%$ would guarantee that the $\widehat{h}$ would be detected as a 4σ or higher bump in the bremsstrahlung tail of the $m_{b\overline{b}}$ distribution after 3 to 4 years of running. Alternatively, a scan could be performed to look for the $\widehat{h}$. The scan procedure depends upon how $\Gamma^{\rm tot}_{\widehat{h}}$ depends on $m_{\widehat{h}}$ in that one must always have R such that $\sigma_{\sqrt{s}} \lesssim \Gamma^{\rm tot}_{\widehat{h}}$; the luminosity expected for the required R must then be employed. Further, one must use steps of size $\Gamma^{\rm tot}_{\widehat{h}} \sim \sigma_{\sqrt{s}}$. For $2m_t > m_{\widehat{h}} > 150$ GeV, $\widehat{h} \to b\overline{b}$ and $\Gamma^{\rm tot}_{\widehat{h}} \sim 0.05 - 0.1$ GeV unless $\tan\beta < 1$. For $m_{\widehat{h}} > 2m_t$, $\Gamma^{\rm tot}_{\widehat{h}}$ rises to at least 1 GeV or so. As result, it would be possible to employ

$R = 0.05 - 0.1\%$ or so for $m_{\widehat{h}} < 2m_t$ rising to $R = 0.5 - 1\%$ for $m_{\widehat{h}} > 2m_t$. In a $3-4$ year program, using earlier quoted nominal yearly L's for such R's as function of $\sqrt{s}$, we could imagine devoting:

- $L = 0.003$ fb^{-1} to 2000 points separated by 0.1 GeV in $\sqrt{s} = 150-350$ GeV range — the total luminosity required would be $L = 4$ fb^{-1} or about 3 years of operation. One would find (4σ level) the $\widehat{h}$ in the $b\overline{b}$ state if $\tan\beta \gtrsim 4-5$.
- $L = 0.03$ fb^{-1} to each of 100 points separated by 0.5 GeV in the $\sqrt{s} = 350-400$ GeV range — the corresponding total luminosity used is $L = 3$ fb^{-1} or about 1/2 year of operation. For $\tan\beta > 6$ (< 6), one would find the $\widehat{h}$ in $b\overline{b}$ ($t\overline{t}$) final state.
- $L = 0.01$ fb^{-1} to each of 100 points separated by 1 GeV in the $\sqrt{s} = 400-500$ GeV range — the total luminosity employed would be $L = 1$ fb^{-1}, or about 1/10 year. For $\tan\beta > 7$ (< 8), one would detect the $\widehat{h}$ in the $b\overline{b}$ ($t\overline{t}$) final state.

In this way, the muon collier would detect the $\widehat{h}$ if $m_{\widehat{h}} < 2m_t$ and $\tan\beta \gtrsim 5$ or if $m_{\widehat{h}} > 2m_t$ for any $\tan\beta$. Once discovered, $\sqrt{s} = m_{\widehat{h}}$ could be chosen for the muon collider and it would be possible to study the $\widehat{h}$ properties in detail.

IX CP VIOLATION

A muon collider can probe the CP properties of a Higgs boson produced in the s-channel. One can measure correlations in the $\tau^+\tau^-$ final state or, if the Higgs boson is sufficiently heavy, in the $t\overline{t}$ final state [31,32]. In the MSSM at tree-level the Higgs states h^0, H^0, and A^0 are CP eigenstates, but it has been noted recently that sizable CP violation is possible in the MSSM Higgs sector through loop corrections involving the third generation squarks [33,34]. As noted earlier, in the MSSM the two heavy uncharged Higgs bosons are almost degenerate with a mass splitting comparable or less than their widths with the H^0 being CP-even and the A^0 being CP-odd. The CP-violating neutral Higgs boson mixing can cause a mass splitting larger than their widths, and sufficient resolution on the masses of the H^0 and A^0 bosons together with the mass sum rule is a probe of this physics. Various CP asymmetries in the $t\overline{t}$ final state can be observed as well, and a muon collider is an ideal place to look for these effects.

The most ideal means for determing the CP nature of a Higgs boson at the muon collider is to employ transversely polarized muons. For h production at a muon collider with muon coupling given by the form $\overline{\mu}(a + ib\gamma_5)\mu h$, the cross section takes the form

$$\begin{aligned}\overline{\sigma}_h(\zeta) &= \overline{\sigma}_h^0 \left(1 + P_L^+ P_L^- + P_T^+ P_T^- \left[\frac{a^2-b^2}{a^2+b^2}\cos\zeta - \frac{2ab}{a^2+b^2}\sin\zeta\right]\right) \\ &= \overline{\sigma}_h^0 \left[1 + P_L^+ P_L^- + P_T^+ P_T^- \cos(2\delta + \zeta)\right] ,\end{aligned} \tag{25}$$

where $\overline{\sigma}_h^0$ is the polarization average convoluted cross section, $\delta \equiv \tan^{-1}\frac{b}{a}$, P_T (P_L) is the degree of transverse (longitudinal) polarization, and ζ is the angle of the μ^+ transverse polarization relative to that of the μ^- as measured using the the direction of the μ^-'s momentum as the $\hat{z}$ axis. Of course, if there is no P_T there would be sensitivity to $\overline{\sigma}_h^0 \propto a^2 + b^2$ only. Only the $\sin\zeta$ term is truly CP-violating, but the $\cos\zeta$ term also provides significant sensitivity to a/b. Ideally, one would isolate $\frac{a^2-b^2}{a^2+b^2}$ and $\frac{-2ab}{a^2+b^2}$ by running at fixed $\zeta = 0, \pi/2, \pi, 3\pi/2$ and measuring the asymmetries (take $P_T^+ = P_T^- \equiv P_T$ and $P_L^\pm = 0$)

$$\begin{aligned}\mathcal{A}_I &\equiv \frac{\overline{\sigma}_h(\zeta=0) - \overline{\sigma}_h(\zeta=\pi)}{\overline{\sigma}_h(\zeta=0) + \overline{\sigma}_h(\zeta=\pi)} = P_T^2\frac{a^2-b^2}{a^2+b^2} = P_T^2\cos 2\delta \, , \\ \mathcal{A}_{II} &\equiv \frac{\overline{\sigma}_h(\zeta=\pi/2) - \overline{\sigma}_h(\zeta=-\pi/2)}{\overline{\sigma}_h(\zeta=\pi/2) + \overline{\sigma}_h(\zeta=-\pi/2)} = -P_T^2\frac{2ab}{a^2+b^2} = -P_T^2\sin 2\delta \, .\end{aligned}$$

If $a^2 + b^2$ is already well determined, and the background is known, then the error in these assymetries cam be approximated as $\frac{\mathcal{A}^2}{[\delta\mathcal{A}]^2} \propto P_T^4 L$, which points to the need for the highest possible transverse polarization, even if some sacrifice in L is required.

Of course, in reality the precession of the muon spin in a storage ring makes running at fixed ζ impossible. A detailed study is required [35]. We attempt a brief outline. Taking $\vec{B} = -B\hat{y}$ we may write

$$s_{\mu^-} = P_H^- \left[\gamma(\beta,\hat{z})\cos\theta^- - (0,\hat{x})\sin\theta^-\right] + P_V^-(0,\hat{y}), \quad s_{\mu^+} = P_H^+ \left[\gamma(\beta,-\hat{z})\cos\theta^+ - (0,\hat{x})\sin\theta^+\right] + P_V^+(0,\hat{y}). \quad (26)$$

Here, $\hat{z}$ is the direction of the μ^- instantaneous momentum, P_H (P_V) is the horizontal (vertical, i.e. $\hat{y}$) degree of polarization, $P_H^\pm \cos\theta^\pm = P_L^\pm$, and $\sqrt{[P_H^\pm \sin\theta^\pm]^2 + [P_V^\pm]^2} = P_T^\pm$. For any setup for initial insertion into the storage ring, $\theta^\pm$ can be computed as functions of the turn number N_T counting starting with $N_T = 1$ the first time the bunch passes the IP. (For example, if the $\mu^\pm$ beams enter the storage ring with $\hat{P}_H^\pm = \hat{p}_{\mu^\pm}$, then $\theta^\pm(N_T) = \omega(N_T - 1/2)$. Here, $\omega = 2\pi\gamma\frac{g_\mu - 2}{2}$, with $\gamma = E/m_\mu$.) As a function of θ^- and θ^+, defining $c_- \equiv \cos\theta^-$ etc.,

$$\frac{\overline{\sigma}_h(\theta^+,\theta^-)}{\overline{\sigma}_h^0} = (1 + P_H^+ P_H^- c_+ c_-) + \cos 2\delta (P_V^+ P_V^- + P_H^+ P_H^- s_+ s_-) + \sin 2\delta (P_H^- P_V^+ s_- - P_H^+ P_V^- s_+). \quad (27)$$

This formula shows that by following the dependence of $\overline{\sigma}_h(\theta^+,\theta^-)$ on N_T, one can extract values for $\cos 2\delta$ and $\sin 2\delta$. In practice, it is best to run in several configurations. To approximate the $\zeta = 0$ configuration, one would choose $P_H^+ = P_H^- = P_H = 0.05$, $\theta^- = \theta^+$, $P_V^+ = P_V^- = \sqrt{P^2 - P_H^2}$. To approximate the $\zeta = \pi$ configuration, choose $P_H^+ = P_H^- = P_H = 0.05$, $\theta^- = \theta^+ + \pi$, $P_V^- = -P_V^+ = -\sqrt{P^2 - P_H^2}$. To emphasize the $\zeta = \pi/2$ and $\zeta = 3\pi/2$ configurations over many turns of the bunches, we choose $P_H^- = P$ ($P_V^- = 0$), $P_H^+ = P_H = 0.05$ and $P_V^+ = \sqrt{P^2 - P_H^2}$. To obtain an accurate measurement of δ, it is necessary to develop strategy for maximizing $\langle P_T^4 \rangle L$ by selecting only energetic muons to accelerate and combining bunches. Lack of space prevents a detailed description.

To gain a quantitative understanding of how successful such a strategy for determining the CP-nature of the h can be, let use define $(\hat{a},\hat{b}) = (a,b)/(gm_\mu/2m_W)$ and give contours at $\Delta\chi^2 = 1, 4, 6.635, 9$ in the $\delta = \tan^{-1}\frac{\hat{b}}{\hat{a}}$, $r = \sqrt{\hat{a}^2 + \hat{b}^2}$ parameter space. We define I as the proton source intensity enhancement relative to the standard value implicit for the earlier-given benchmark luminosities. We compare four cases: (i) the case of $P = 0.2$, $L = 0.15~\mathrm{fb}^{-1}$, which corresponds to $I = 1$ and the polarizaton level naturally achieved without any special selection against slow muons; (ii) we maintain the same proton intensity, $I = 1$, select faster muons to the extent that it becomes possible to merge neighboring muon bunches, leading to $P^m(I=1) \sim 0.39$ and $L = 0.075~\mathrm{fb}^{-1}$; (iii) we increase the proton source intensity by a factor of two, $I = 2$, while selecting faster muons and merging the bunches, corresponding to $P^m(I=2) \sim 0.48$ and $L = 0.075~\mathrm{fb}^{-1}$; finally (iv) we employ $I = 3$ and use so-called 'just-full bunches', corresponding to $P^f(I=3) \sim 0.45$ and $L = 0.15~\mathrm{fb}^{-1}$. Results in the case of a SM Higgs boson with $m_{h_{SM}} = 130$ GeV are presented in Fig. 12. One sees that a 30% (1σ) measurement of $\hat{b}/\hat{a}$ is possible without increased proton source intensity, using the simple technique of selecting fast muons and performing bunch merging. An $\lesssim 20\%$ measurement would require a moderately enhanced proton source intensity.

After a studying a number of cases, the overall conclusion of [35] is that this procedure will provide a good CP determination (superior to other techniques) provided one merges bunches and compensates for the loss of luminosity associated with selecting only energetic muons (so as to achieve high average polarization) by having a proton source that is at least two times as intense as that needed for the studies discussed in previous sections (that do not require large transverse polarization).

X CONCLUSIONS AND PLANNING FOR FUTURE FACILITIES

Around 2006 the LHC will begin taking data, hopefully revealing the path that particle physics will take in the next century. At the moment there are a few experimental hints suggesting that a Higgs boson might be just around the corner, and there are intriguing indications from the anomalous magnetic moment of the muon that supersymmetric particles may be easily detected at the LHC. This scenario would present a strong argument for the construction of a LC to study this interesting physics which would be at a scale light enough to be probed. A muon collider could play a crucial role in several ways. First, a s-channel light-Higgs factory would provide crucial precision measurements of the h^0 properties which in turn impose critical constraints on the masses of heavier Higgs bosons and other SUSY parameters. Among other things, the heavier Higgs bosons might be shown to definitely lie within reach of muon collider s-channel production. Further, it could

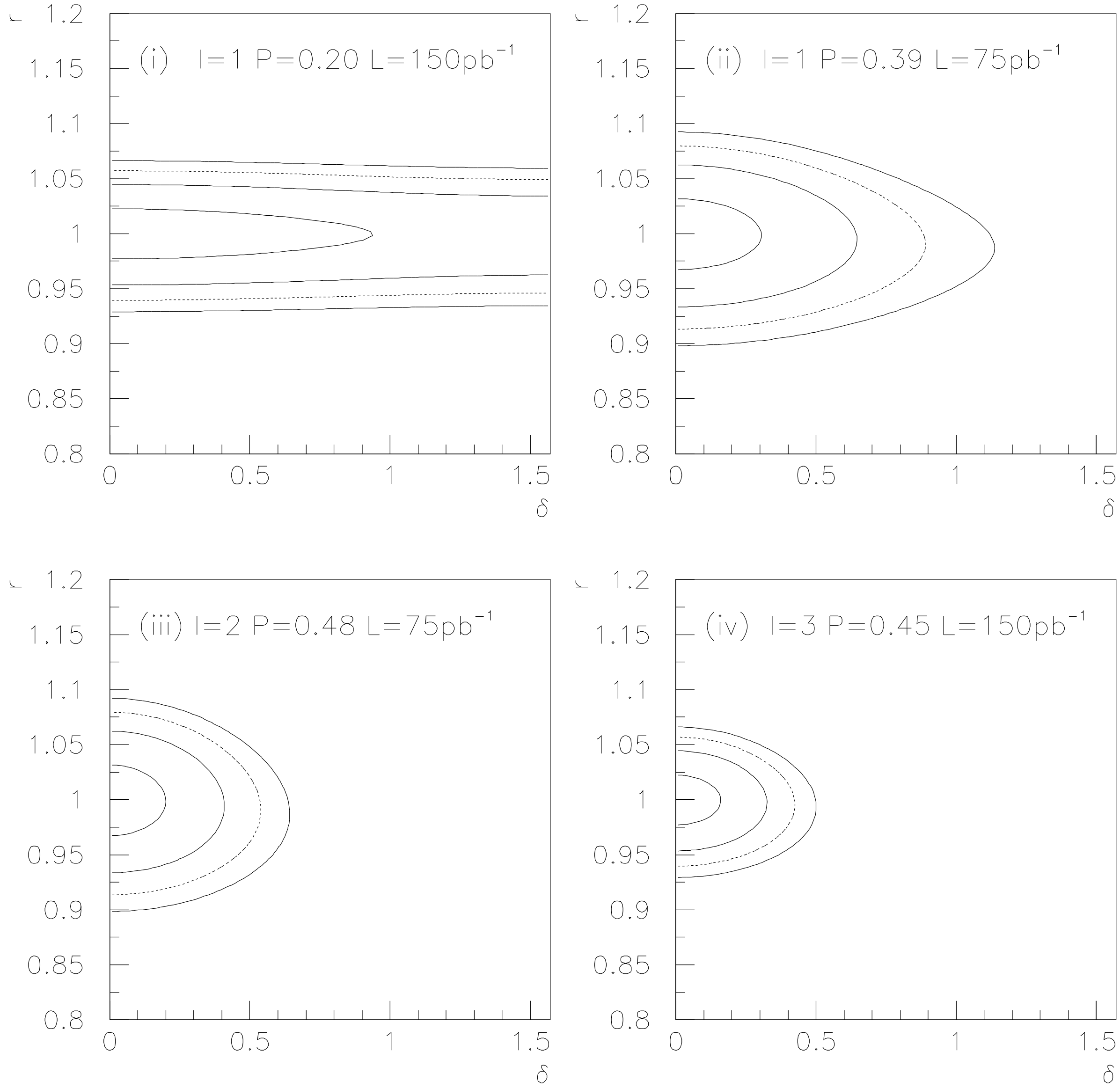

FIGURE 12. Contours at $\Delta\chi^2 = 1, 4, 6.635, 9$ for the $\hat{a}$ and $\hat{b}$ measurement for a SM Higgs ($\hat{a} = 1, \hat{b} = 0$) with $m_{h_{SM}} = 110$ GeV for the four luminosity/bunch-merging options outlined in the text. Here, $\delta = \tan^{-1}\frac{\hat{b}}{\hat{a}}$ and $r = \sqrt{\hat{a}^2 + \hat{b}^2}$. For small δ, $\frac{\hat{b}}{\hat{a}} \sim \delta$.

be that the heavier H^0 and A^0 cannot be detected at the LHC or LC (a scenario that arises, for example, for moderate $\tan\beta$ values and $m_{A^0} \sim m_{H^0} \gtrsim 250$ GeV). Since their detection in s-channel production at the muon collider would be relatively certain, the muon collider would be an essential component in elucidating the full physics of the Higgs sector. Further, there are even (non-supersymmetric) scenarios in which one only sees a SM-like Higgs as the LHC and LC probe scales below a TeV, but yet muon collider Higgs factory studies would reveal additional Higgs bosons. Finally, one should not forget that the muon collider might prove to be the best approach to achieving the highest energies possible in the least amount of time. Construction of a Higgs factory would be a vital link in the path to high energy.

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