Muon Bunch Compressor
Based on a Low RF Ring Cooler

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I INTRODUCTION

One of the most serious problems concerning the construction of a Higgs factory (or any other muon collider) is the creation of very short high intensity muon bunches [1]. A precooling part including pion producing target, decay channel, and low RF phase rotation system, can provide a muon bunch of 6-10 m length in the best case [2], whereas a 200 MHz cooling channel requires at least 10-15 times shorter in bunch length. A strong emittance exchange combined with the beam cooling appears to be the most reasonable method to this end. A ring cooler proposed in Ref. [3]– [4] is considered in this paper. From a technical standpoint, a low frequency and high gradient accelerating system is the critical part in this scheme. Assumed is an 8 MHz and 3 MeV/m RF system that provides capture in the bucket of 10 m length, acceleration, and—with appropriate absorbers— a reasonable cooling rate / emittance exchange at modest beam loss caused by muon decays. Longitudinal cooling factor 6-40 is achievable in this system, depending on used approximations. Transverse nonlinearity of bending magnets, as well as dependence of the revolution frequency on transverse momentum, are the most serious causes of the degradation. These and other effects are investigated including some measures to improveme the cooler performance.

II DESCRIPTION OF THE COOLER

The ring cooler described in the paper [4] is taken as a basis for the designed bunch compressor. Schematic and parameters of the compressor are given in Fig. 1 and Table 1. It consists of $8\times 45^\circ$ dipoles, 4 short straight sections (SS), and 4 long SS. The long SS containing RF cavities and liquid hydrogen absorbers are intended for transverse cooling of muons. Wedge absorbers for an emittance exchange are placed in the short SS where there is large dispersion. Thus, there are 4 periods, each including the bending part and the straight section.

Layout of the bending part is displayed in Fig. 2. It consists of 2 bending magnets and 2 solenoids with opposite direction fields. Besides forming a circular orbit, this part provides transverse focussing and dispersion required for the emittance

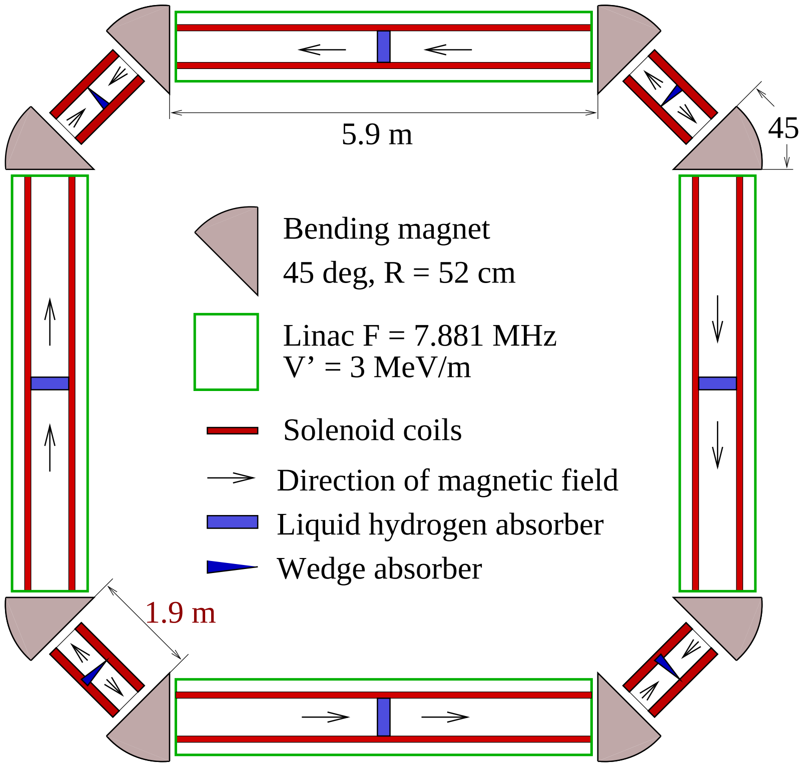

FIGURE 1. Schematic of the ring cooler

TABLE 1. Parameters of the ring cooler

1.	Circumference	34.475
2.	Nominal energy (total)	250 MeV
3.	Number of bending magnets	8
4.	Bending angle	45^0
5.	Bending radius	52 cm
6.	Bending field	1.453 T
7.	Normalized field gradient	0.5
8.	Length of short SS	1.902 m
9.	Length of long SS	5.900 m
10.	Axial field of long solenoid	2.055 T
11.	β-function at nominal energy	0.735 m
12.	Revolution/RF frequency	7.881 MHz
13.	RF harmonic number	1
15.	Accelerating gradient	3 MeV/m
16.	Synchronous phase	30°
17.	Main absorber	LH,30 cm
18.	Wedge absorber	$dE/dy = 0.2$ MeV/cm

exchange. Therefore, gradient magnets with a normalized field gradient of 0.5 are used here, providing beta-function of $R\sqrt{2} \simeq 73.5$ cm in both directions. To get the same beta-function at the dipole, the field at the solenoid is then constrained to $|B_{\text{solenoid}}| = B_{\text{dipole}}\sqrt{2} \simeq 2.05$ T. Of course, for the alternate solenoid used in this section $|B_{\text{solenoid}}|$ cannot be constant. However, real magnetic field plotted in Fig. 3 has the same matrix as an "ideal" solenoid with $|B| = const$ and instantaneous field flip, i.e. it provides a perfect matching at least for equilibrium particles. Note that the bending magnets are considered as magnetic mirrors at the calculation of

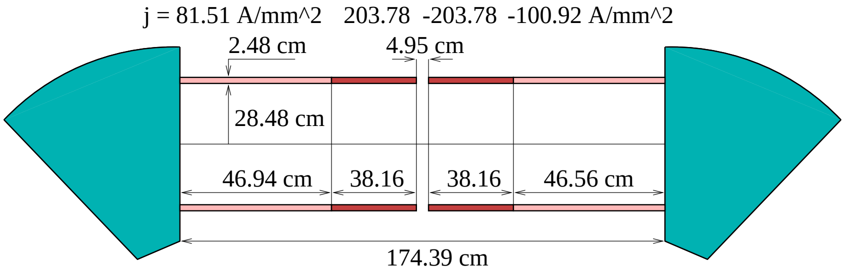

FIGURE 2. Layout of the short straight section

the solenoid field. Transverse field of the bending magnets is introduced analytically including nonlinear part to satisfy Maxwell equations. Calculation of 3D magnetic field for both solenoids and bending magnets has to be performed as a next step.

Two-component dispersion function $D_{x,y}$ of this section is also plotted in Fig. 3. With an appropriate choice of the solenoid length, the field is localized only in bending part, providing a dispersion-free long SS. The function D_x is the same in all short SS, while D_y changes in sign in any subsequent section. Vertical wedge absorbers are placed in the center of the bending sections for emittance exchange. Direction of the wedges also changes in any subsequent section. Material of the wedge absorbers is LiH, and the gradient of ionization energy loss is 0.2 MeV/cm corresponding to the wedge angle of 6.8°.

The long SS are designed very schematically because an external view and construction of low frequency accelerating system are unclear at present. It is assumed that the acceleration is performed with a 7.881 MHz traveling wave having a gradient of 3 MeV/m. There are two linacs of 280 cm length in each long SS, and a liquid hydrogen absorber of 30 cm length is placed at the center of SS providing an energy loss of muons 8.95 MeV for transverse cooling. A homogeneous solenoidal field of 2.055 T strength is used for transverse focusing. It produces the same $\beta = 73.5$ cm in all sections including absorber areas. This is acceptable because transverse cooling is not a design goal but longitudinal cooling is achieved through emittance exchange, and the transverse emittance remains almost constant at a rather large value.

III SIMULATION

Cooling of a single bunch with Gaussian distribution was studied for first for estimation of the cooler performance. Initial parameters of the beam w.r.t. the center of the long SS are given in the Table 2. The simulation was performed with varying values of the parameters to estimate the role of all the factors and to lay down a way to improve the system. 10000 muons was used for all simulations.

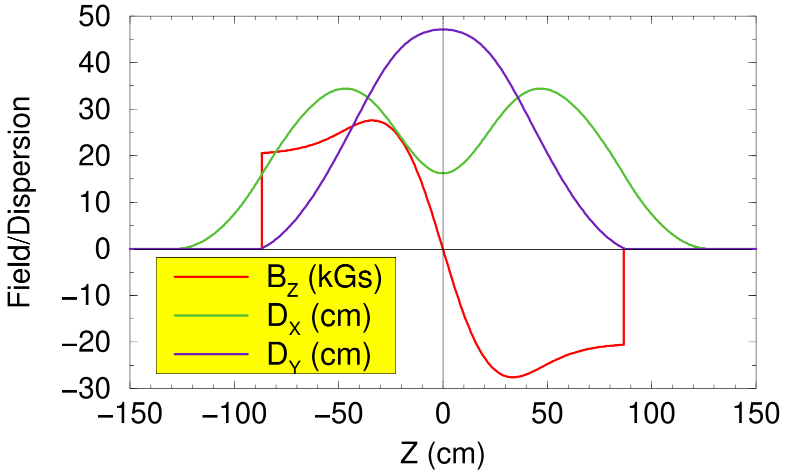

FIGURE 3. Magnetic field and dispersion at the short straight section

TABLE 2. R.m.s. sizes of the injected beam

Horizontal size (cm)	6.40
Vertical size (cm)	6.40
Longitudinal size (cm)	200.
Horizontal momentum (MeV/c)	20.0
Vertical momentum (MeV/c)	20.0
Energy spread (MeV)	35.0
Horizontal emittance (cm)	1.21
Vertical emittance (cm)	1.21
Longitudinal emittance (cm)	66.3
3D-emittance (cm^3)	97.1

A Linear approximation without chromaticity

Chromatic and nonlinear effects are ignored in this simulation except for the nonlinearity of the accelerating field. (It is a conventional approximations for "normal" accelerators.) The results are presented in Fig. 4 and Fig. 5. Fig. 4 shows the evolution of the bunch emittance and transmission. By 25 turns, the longitudinal emittance decreases from 66.3 cm to 1.6 cm (cooling factor 41), and 6D emittance – from 97.1 cm^3 to 1.2 cm^3 (cooling factor 81). Transmission is 54%, and the only cause of the particle loss is muon decay.

Fig. 5 represents the longitudinal phase space at the injection (red points) and after 25 turns (blue spot at the center). After an appropriate matching the bunch is quite acceptable for a high RF cooler (f.e. 200 MHz).

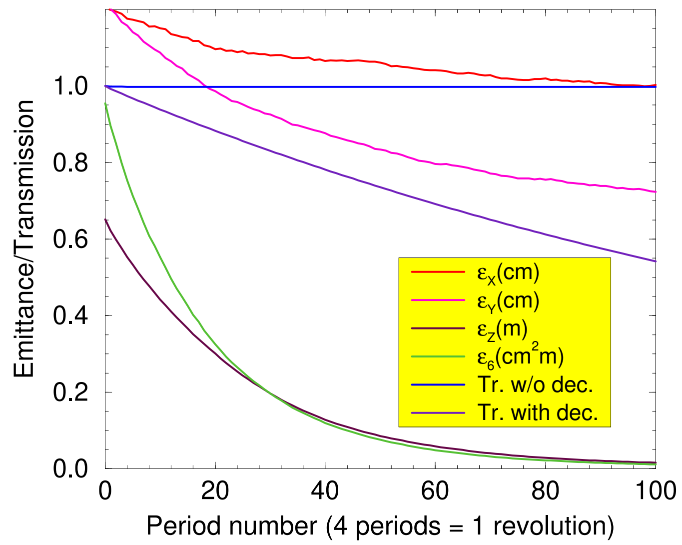

FIGURE 4. Evolution of emittance and transmission at the cooling (linear approximation without chromaticity).

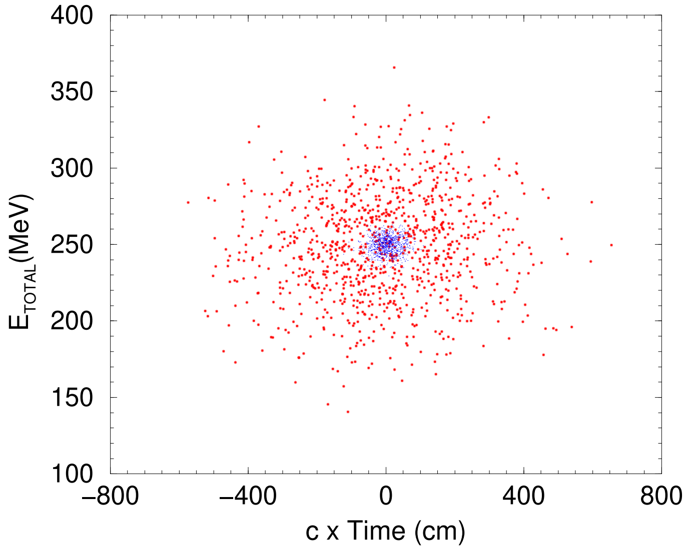

FIGURE 5. Longitudinal phase space before (red) and after the cooling (blue).

B Linear approximation with chromaticity

All chromatic effects are taken into account in this simulation and transverse motion is considered in paraxial approximation as before. The results presented in

Fig. 6 and Fig. 7 are somewhat worse than the previous ones. Now, the longitudinal r.m.s. emittance decreases after 25 turns from 66.3 cm to 2.3 cm (cooling factor 29), and 6D emittance – from 97.1 cm^3 to 2.4 cm^3 (cooling factor 40). Transmission is 44% with decay and 81% without decay. Thus, 19% of particles are lost in this approximation because of the machine imperfection.

Longitudinal phase space after 25 turns is shown in the right in Fig. 7. For comparison, the same is plotted for previous case – without chromaticity (left). It is seen that the chromaticity causes an additional beam halo which can explain the increased emittance. Possibly, the distinction will be not so marked if an acceptance cut is applied. This problem has to be studied further specifically.

Probably, rather high particle loss in this case is an effect of linear betatron resonances at non-equilibrium energies. In support of this the dependence of β-function on total energy is plotted in Fig. 8. Note that in the used linear approximation $\beta_x = \beta_y$ because the bending magnets produce the same focusing in both directions. Several resonances are seen in Fig. 8 with the most serious one at E = 236 MeV. Additionally, resonance excitation of dispersion has to be taken into account. Actually, the long straight sections are dispersion-free only in linear approximation in $\Delta p/p$ what would mean an achromatic system. The dispersion function with higher order corrections is non-zero in long SS which is shown in Fig. 9. Dependence of dispersion on energy at the center of the long SS is plotted in Fig. 8 demonstrating a resonance behavior also (another components of the dispersion are zero in this point because of symmetry of the system.)

One can suppose on this basis that a suppression both the resonances and the nonlinear dispersion is a way to improve the bunch compressor.

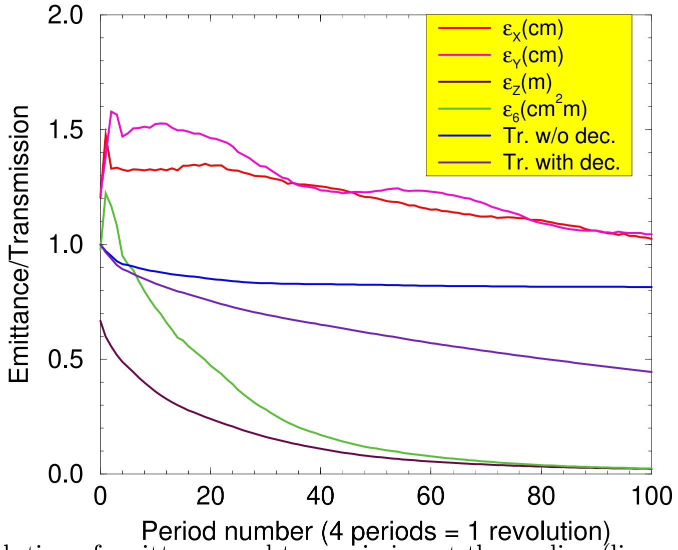

FIGURE 6. Evolution of emittance and transmission at the cooling (linear approximation with chromaticity).

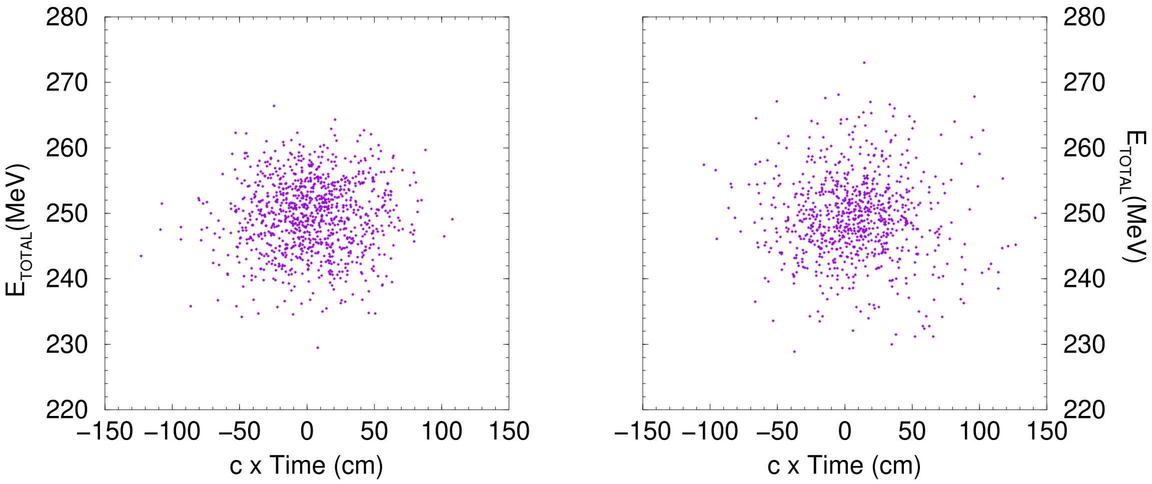

FIGURE 7. Longitudinal phase space after 25 turns. Comparison of the cooling without chromaticity (left) and with chromaticity (right).

C Transverse nonlinearity and chromaticity

This simulation is performed with all chromatic and nonlinear effects included *except for dependence of the revolution frequency on transverse momentum of a particle*. Beam parameters in dependence on number of turns are plotted in Fig. 10. The longitudinal cooling is almost the same as in the previous simulation: r.m.s. emittance decreases after 25 turns from 66.3 cm to 2.0 cm (cooling factor 33). However, an increase in transverse emittance by a factor of about 1.3 is observed now; as a result, 6D emittance changes from 97.1 cm^3 to 4.9 cm^3 (cooling factor 20). Transmission is also less in this case having only 30% with decay and 55% without decay.

Because the only new factor is the transverse nonlinearity, probably nonlinear betatron resonances are the cause for this degradation. An additional investigation

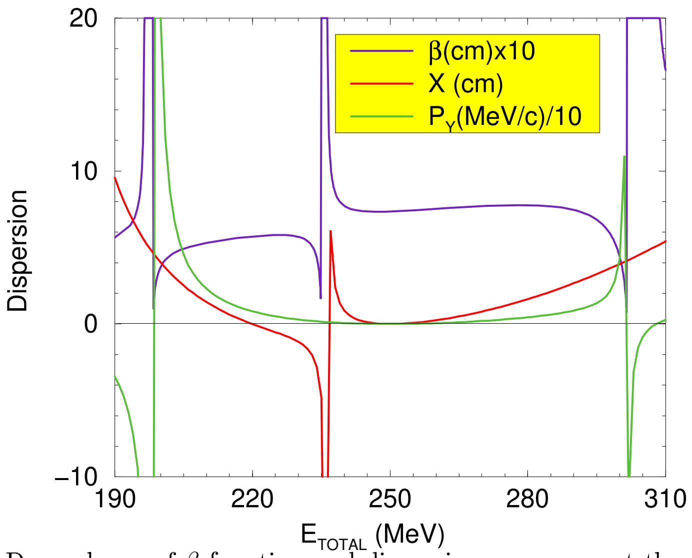

FIGURE 8. Dependence of β-function and dispersion on energy at the center of long SS.

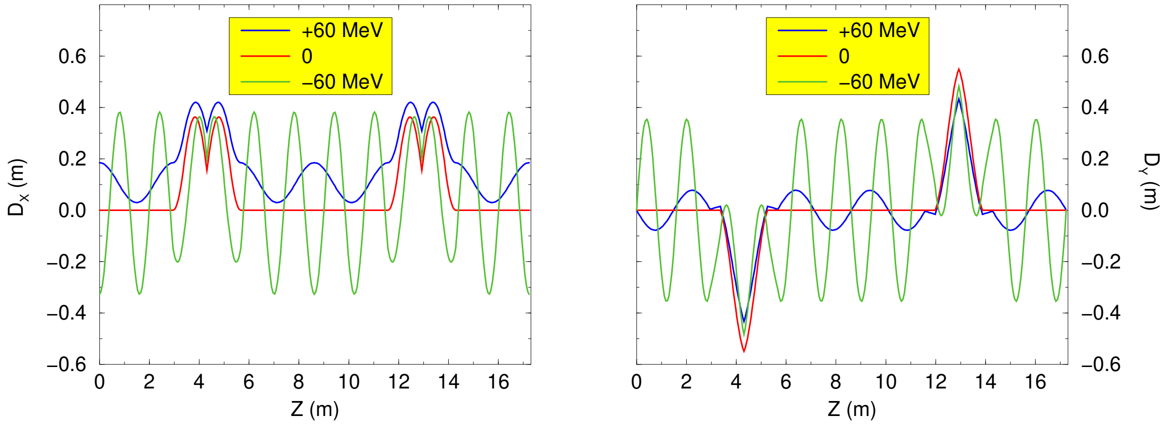

FIGURE 9. Dispersion function vs Z at various energy

has demonstrated that the bending magnets are mainly responsible for the resonances while the solenoid nonlinearity is almost negligible (in long SS, it is only kinematic effect because of dependence $p_z(p_t)$ at given energy). Therefore, some investigation, and if possible, correction of the dipole nonlinearities are required.

D Full simulation

The latest effect that has to be taken into account is dependence of revolution frequency on transverse momentum at a given energy. Corresponding simulation is performed at the same conditions as before; however, after the generation of Gaussian distribution the following correlation is introduced:

$$E = E_{ref}\sqrt{1 + \frac{1}{2}\left[\left(\frac{p_t}{mc}\right)^2 + \left(\frac{eBr}{2c}\right)^2\right]_{random}} + \Delta E_{random},$$

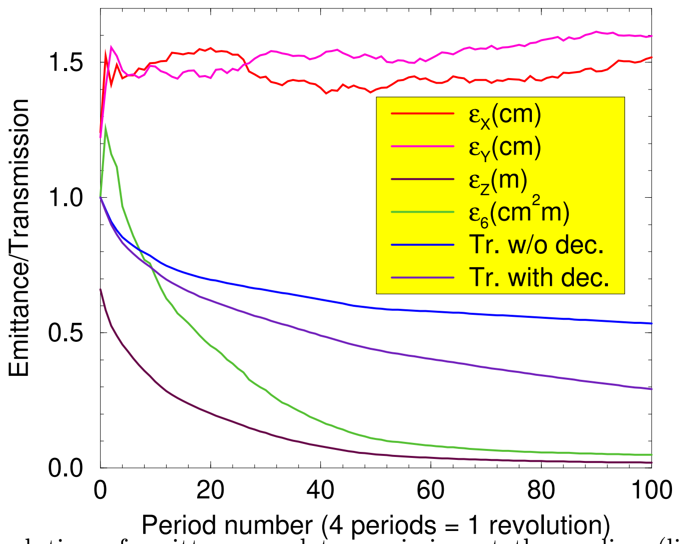

FIGURE 10. Evolution of emittance and transmission at the cooling (linear approximation with chromaticity).

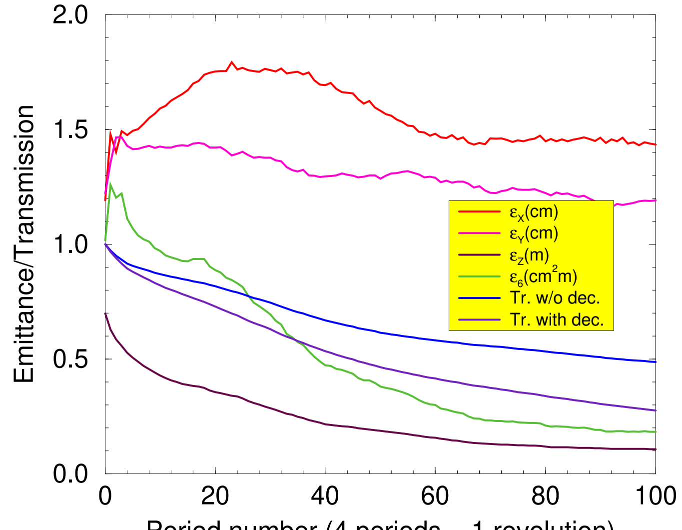

FIGURE 11. Evolution of emittance and transmission at the cooling (linear approximation with chromaticity).

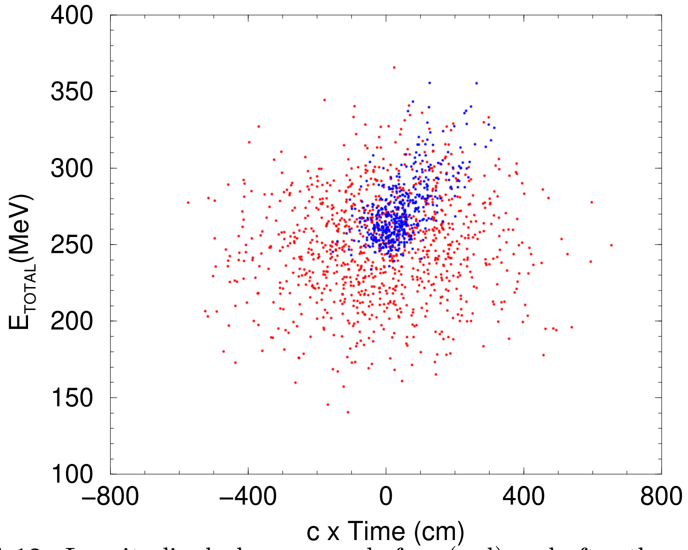

FIGURE 12. Longitudinal phase space before (red) and after the cooling (blue).

where E_{ref} =250 MeV. It is assumed that such a correlation should appear at a bunching at the ring which is not yet considered. The results presented in Fig. 11 and Fig. 12 are significantly worse than all the previous cases because of the deterioration of longitudinal characteristics. Now the longitudinal r.m.s. emittance decreases only to 10.8 cm and 6D emittance – to 18.4 cm^3. The cooling factor is about 6 for both cases because transverse emittance is finally almost the same as in the beginning. However, there is a considerable growth of the horizontal emittance in the first part of the cooling accompanied by more particle loss. Transmission after 25 turns is 28% with decay and 49% without decay.

Longitudinal phase space after 25 turns is shown in Fig. 12 by blue points. The main difference from Fig. 5 is a large energy spread caused by transverse momentum - energy correlation, which does not decrease at the cooling because the transverse momentum is almost constant. Apparently, transverse cooling should be added to this system to improve the situation. Preliminary investigation have demonstrated that a simple decrease of the wedge angle by factor 2 provides a small transverse cooling and increases the total cooling factor to $\sim$ 10. However, the considered machine is not designed for the transverse cooling because of large β-function at absorbers. Therefore, more radical changes are required including an adiabatic increase of magnetic field at song solenoids.

IV CONCLUSION

It is shown that a low RF ring cooler is capable to decrease a longitudinal emittance of a single bunch from 60-70 cm to 2-3 cm which satisfies the requirements of the Higgs factory. There are several factors causing a degradation of the cooler performances, and the most serious of them is the dependence of the revolution frequency on the transverse momentum. An additional energy spread arising at bunching is not suppressed in the considered version because r.m.s. transverse momentum of the beam is about constant at the cooling. In principle, this spread is reversible, and it should vanish after a transverse cooling. However, a modification of the cooler is required including an adiabatic decrease of beta-function.

Another serious factor is nonlinearity of transverse motion in the bending magnets, and efforts have to be undertaken to weaken this. Chromatic effects are less dangerous, though a suppression of linear betatron resonances at non-equilibrium energies, as well as nonlinear in $\Delta p/p$ dispersion, should be investigated more.

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