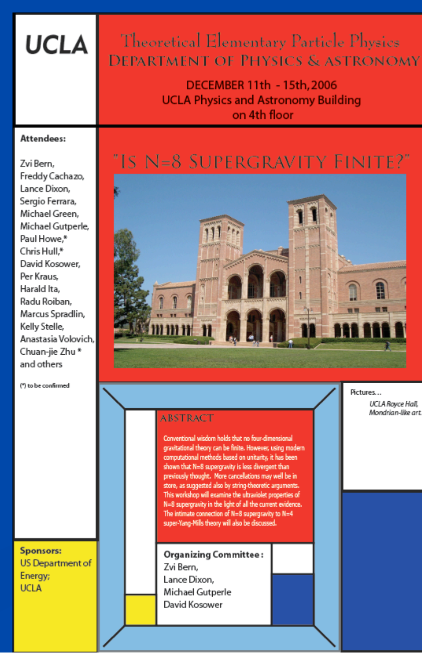
Relation Between Perturbative Gravity and Gauge Theories

Zvi Bern, UCLA

Review of work with Lance Dixon, Dave Dunbar, Maxim Perelstein and Joel Rozowsky



Outline

- Nonrenormalizability of gravity theories.
- The Kawai-Lewellen-Tye (KLT) relations between open and closed string tree amplitudes.
- KLT relations in field theory.
- Lagrangian interpretation.
- Loop amplitudes in gravity theories via unitarity.
- $N = 8$ supergravity from $N = 4$ sYM.
- Application to UV properties of gravity theories.

Non-Renormalizability of Quantum Gravity

Discussed in Kelly Stelle's talks

- Power counting suggests that theories of gravity are not renormalizable – dimensionful coupling.
- Non-renormalizablity of pure Einstein gravity confirmed by explicit two loop calculations. Goroff and Sagnotti; van de Ven
- Supergravity is better behaved. Many authors concluded that supergravity theories would diverge at three loops
  Deser, Kay and Stelles; Kallosh; Howe and Stelle; Green, Schwarz and Brink, etc.
- $N = 8$ supergravity appears better behaved.
  ZB, Dixon, Dunbar, Perelstein ,Rozowsky; Howe and Stelle; talk from Stelle.
- Hints $N = 8$ theory is finite. Talks from Dixon, Ita, Green, Roiban.

Key Issue: Coefficient of divergences can vanish dues to a hidden symmetry or dynamical reason.

We can investigate this with modern computation tools.

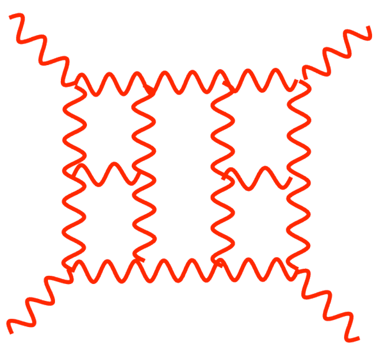
How *not* to Investigate Gravity Divergences

With Feynman diagrams the graviton vertex is a mess:

$$G_{3\mu\alpha,\nu\beta,\sigma\gamma}(k_1,k_2,k_3) =$$

$$\begin{aligned}
&\mathrm{sym}[-\tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma}) - \tfrac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma}) + \tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma})\\
&+P_6(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma}) + 2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma}) - P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma})\\
&+P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta}) + 2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma})\\
&+2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha}) - 2P_3(k_1\cdot k_2\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})]
\end{aligned}$$

Suppose we wanted to check if $N = 8$ supergravity is finite using Feynman diagrams:



First potential divergence is expected at 5 loops

This single diagram has $\sim 10^{30}$ terms prior to evaluating any integrals.
Impossible to evaluate via diagrams!

String Theory Intuition

Basic string theory fact:

$$\text{closed string} \sim (\text{left-mover open string}) \times (\text{right-mover open string})$$

In field theory limit this should imply:

$$\text{gravity} \sim (\text{gauge theory}) \times (\text{gauge theory})$$

- **How do we make this precise?**
- **How can we apply this to the quantum theory?**
- **What can this teach us about (super) gravity?**
- **Can we see these relations from the Lagrangian?**

Connection of Gravity and Gauge Theory

At tree level Kawai, Lewellen and Tye presented a relationship between closed and open string amplitudes.
In field theory limit, relationship is between gravity and gauge theory

$$M_4^{\rm tree}(1,2,3,4) = s_{12} A_4^{\rm tree}(1,2,3,4)\, A_4^{\rm tree}(1,2,4,3)\,,$$

$$M_5^{\rm tree}(1,2,3,4,5) = s_{12}s_{34} A_5^{\rm tree}(1,2,3,4,5) A_5^{\rm tree}(2,1,4,3,5) + s_{13}s_{24} A_5^{\rm tree}(1,3,2,4,5)\, A_5^{\rm tree}(3,1,4,2,5)$$

Gravity amplitude

Color stripped gauge theory amplitude

where we have stripped all coupling constants

$$\mathcal{A}_4^{\rm tree} = g^2 \sum\nolimits_{\rm non-cyclic} {\rm Tr}(T^{a_1}T^{a_2}T^{a_3}T^{a_4}) A_4^{\rm tree}(1,2,3,4)$$

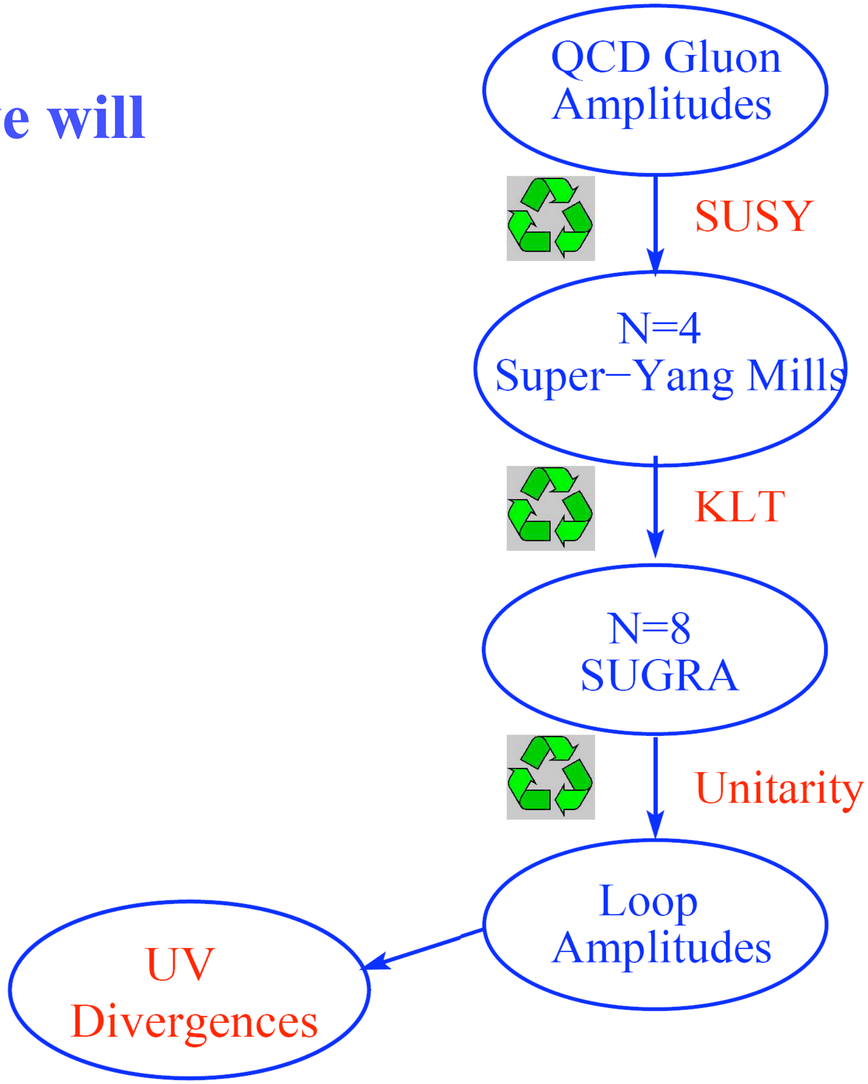
Full gauge theory amplitude

Holds for any external states.
See review: gr-qc/0206071

Progress in gauge theory can be imported into gravity theories

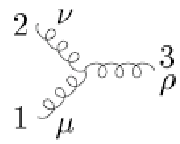
Alternative Strategy

Instead of off-shell methods we will use on-shell methods.

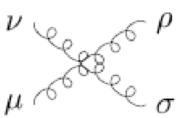


Connection of Gravity and Gauge Theory

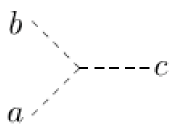
Color ordered Feynman rules



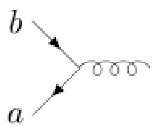
$$= \frac{i}{\sqrt{2}}(k_1 - k_2)^\rho \eta^{\mu\nu} + \text{cyclic}$$



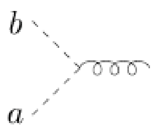
$$= i\eta^{\mu\rho}\eta^{\nu\sigma} - \frac{i}{2}\left(\eta^{\mu\nu}\eta^{\rho\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho}\right)$$



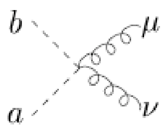
$$= f^{abc}$$



$$= i\delta^{ab}\gamma^\mu$$



$$= i\delta^{ab}(k_1 - k_2)^\mu$$



$$= \frac{i}{2}\delta^{ab}\eta^{\mu\nu}$$

Gravity tree amplitudes can be obtained from gauge theory Feynman rules.

Spinors

Xu, Zhang and Chang
Berends, Kleis and Causmaeker
Gastmans and Wu
Gunion and Kunszt
& many others

Spinor helicity for gluon polarization vectors:

$$\varepsilon_\mu^+(k;q) = \frac{\langle q^- |\, \gamma_\mu \,| k^- \rangle}{\sqrt{2}\, \langle q\, k \rangle}, \qquad \varepsilon_\mu^-(k,q) = \frac{\langle q^+ |\, \gamma_\mu \,| k^+ \rangle}{\sqrt{2}\, [k\, q]}$$

This is for convenience in *D* = 4, but this talk won't rely on this

All required properties of circular polarization satisfied:

$$\epsilon_i^2 = 0, \qquad k \cdot \epsilon_i = 0, \qquad \epsilon_i^+ \epsilon_i^- = -1$$

Changes in reference momentum q equivalent to on-shell gauge transformations:

$$\epsilon^{ab} \lambda_{ja} \lambda_{lb} \longleftrightarrow \langle j\, l \rangle = \langle k_{j\,-} | k_{l\,+} \rangle = \sqrt{2 k_j \cdot k_l}\; e^{i\phi}$$

$$\epsilon_{\dot a \dot b} \tilde\lambda_j^{\dot a} \tilde\lambda_l^{\dot b} \longleftrightarrow [j\, l] = \langle k_{j\,+} | k_{l\,-} \rangle = -\sqrt{2 k_j \cdot k_l}\; e^{-i\phi}$$

Graviton polarization tensors are squares of these:

$$\epsilon_{\mu\nu}^+ = \epsilon_\mu^+ \epsilon_\nu^+ \qquad\qquad 2 = 1 + 1$$

Gravity and Gauge Theory Amplitudes

Berends, Giele, Kuijf; Bern, De Freitas, Wong

$$M_4^{\rm tree}(1_h^-, 2_h^-, 3_h^+, 4_h^+) = \left(\frac{\kappa}{2}\right)^2 s_{12} A_4^{\rm tree}(1_g^-, 2_g^-, 3_g^+, 4_g^+) \times A_4^{\rm tree}(1_g^-, 2_g^-, 4_g^+, 3_g^+)$$

$$= \left(\frac{\kappa}{2}\right)^2 s_{12} \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle} \times \frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,4\rangle\,\langle 4\,3\rangle\,\langle 3\,1\rangle}$$

$$M_4^{\rm tree}(1_g^-, 2_{\bar q}^-, 3_q^+, 4_h^+) = g\frac{\kappa}{2} s_{12} A_4^{\rm tree}(1_g^-, 2_{\bar q}^-, 3_q^+, 4_g^+) \times A_4^{\rm tree}(1_s^-, 2_s^-, 4_g^+, 3_s^+)$$

$$= g\frac{\kappa}{2} \frac{\langle 1\,2\rangle^3\,\langle 1\,3\rangle}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle} \times T^{a_1} \frac{[4\,3]\,\langle 3\,2\rangle}{\langle 2\,4\rangle}$$

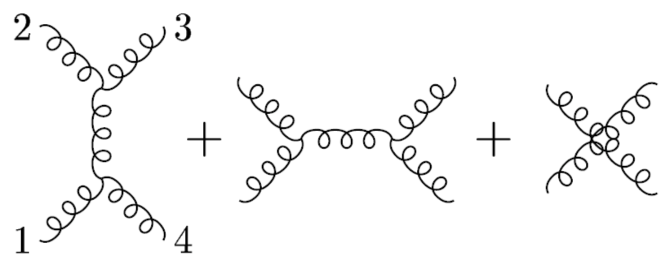
- **Very general property: tree amplitudes in gravity theories can be obtained from gauge theory ones.**

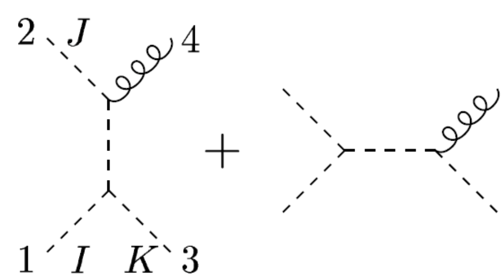
- **Even holds for higher derivative terms in effective actions.**

Bjerrum-Bohr

Connection of Gravity and Gauge Theory

$$M_4^{\rm tree}(1_g^-, 2_g^-, 3_g^+, 4_h^+) = g\frac{\kappa}{2} s_{12} A_4^{\rm tree}(1_g^-, 2_g^-, 3_g^+, 4_g^+)$$
$$\times\, A_4^{\rm tree}(1_s^I, 2_s^J, 4_g^+, 3_s^K)$$
$$= g\frac{\kappa}{2}\frac{\langle 1\,2\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,\langle 4\,1\rangle} \times f^{IJK}\frac{[4\,3]\,\langle 3\,2\rangle}{\langle 2\,4\rangle}$$





Lagrangians

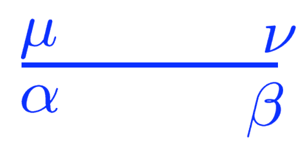
Consider the Einstein-Hilbert and Yang-Mills Lagrangians

$$\mathcal{L}_{\text{gravity}} = \sqrt{g}\, R\,, \qquad \mathcal{L}_{\text{YM}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu}$$

Lorentz indices from left and right gauge theories do not mix

Not obvious how gravity is the square of Yang-Mills.

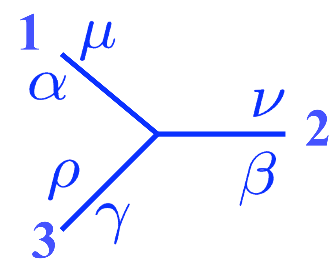
de Donder gauge propagator



$$P_{\mu\alpha;\nu\beta}(k) = \frac{1}{k^2}\left(\frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta} + \frac{1}{2}\eta_{\mu\beta}\eta_{\alpha\nu} - \frac{1}{D-2}\eta_{\mu\alpha}\eta_{\nu\beta}\right)$$

term is evil **Contracts left and right indices**

de Donder three-vertex



$$G^{\text{de Donder}}_{3,\mu\alpha,\nu\beta,\rho\gamma} \sim k_1 \cdot k_2 \eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\rho\gamma} + \text{mess}$$

- **Can we find a field variables and gauge choices such that left-right factorization of Lorentz indices obvious? More or less.**
- **Can we find Lagragian so gravity ~ (gauge theory)^2 manifest? Not known.**

W. Siegel
Z.B. and A. Grant

Rewriting the Lagrangian

$$L_{\mathrm{Einstein}} = \sqrt{-g}\,R \qquad\qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Two h term: $$L_2 = -\frac{1}{2}h_{\mu\nu}\partial^2 h^{\mu\nu} + \frac{1}{4}h^{\mu}{}_{\mu}\partial^2 h^{\nu}{}_{\nu}$$ **de Donder gauge**

Propagator:

evil term

$$P_{\mu\alpha;\nu\beta}(k) = \frac{1}{k^2}\left(\frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta} + \frac{1}{2}\eta_{\mu\beta}\eta_{\alpha\nu} - \frac{1}{D-2}\eta_{\mu\alpha}\eta_{\nu\beta}\right)$$

Trick: introduce dilaton

Left and right indices mix

$$L_2 = -\frac{1}{2}h_{\mu\nu}\partial^2 h^{\mu\nu} + \frac{1}{4}h^{\mu}{}_{\mu}\partial^2 h^{\nu}_{\nu} - \frac{1}{2}\phi\partial^2\phi$$

Apply field redefinition:

$$h_{\mu\nu} \to h_{\mu\nu} + \frac{\eta_{\mu\nu}\phi}{\sqrt{2}}, \qquad\qquad \phi \to h^{\mu}_{\mu} + \sqrt{\frac{D-2}{2}}\,\phi$$

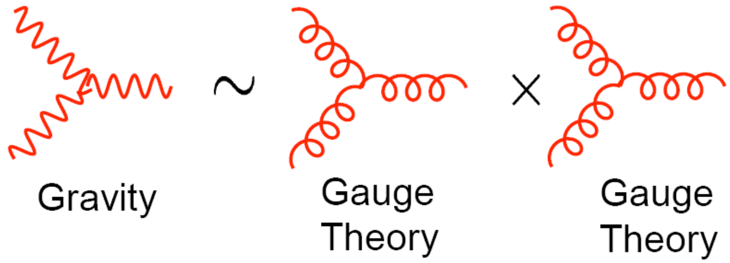
Kinetic term is now clean:

Left and right indices don’t mix

$$L_2 = -\frac{1}{2}h_{\mu\nu}\partial^2 h^{\mu\nu} + \frac{1}{2}\phi\partial^2\phi$$

Rewriting the Vertices

We want a graviton three vertex that factorizes:



Left and right Lorentz indices don't mix

Choose special field variables and gauges:

$$g_{\mu\nu} = e^{\sqrt{\frac{2}{D-2}}\kappa\phi} e^{\kappa h_{\mu\nu}} \equiv e^{\sqrt{\frac{2}{D-2}}\kappa\phi}\left(\eta_{\mu\nu}+\kappa h_{\mu\nu}+\frac{\kappa^2}{2}h_{\mu\rho}h_{\rho\nu}+\cdots\right)$$

checked through six points

$$\phi \to -\sqrt{\frac{2}{D-2}}\left[\left(\phi+\frac{1}{2}\mathrm{Tr}\,h\right)+\kappa\left(\frac{1}{4}\phi^2-\frac{1}{8}\mathrm{Tr}\,(h^2)\right)+\kappa^2\left(\frac{1}{12}\phi^3-\frac{1}{8}\phi\,\mathrm{Tr}\,(h^2)\right)+\cdots\right]$$

Using a messy gauge fixing we get vertices with desired property:

$$L_3 = \kappa\left[\frac{1}{2}h_{\mu\nu}h_{\rho\sigma,\mu\nu}h_{\rho\sigma}+h_{\nu\mu}h_{\rho\mu,\sigma}h_{\rho\sigma,\nu}\right] - \frac{\kappa}{2}h_{\mu\nu,\kappa}h_{\mu\nu,\kappa}\phi$$

Left and right indices do not mix

comma means derivative

- **This construction has been pushed to five-point vertices.**
- **Does *not* explain KLT, just factorization of Lorentz indices**

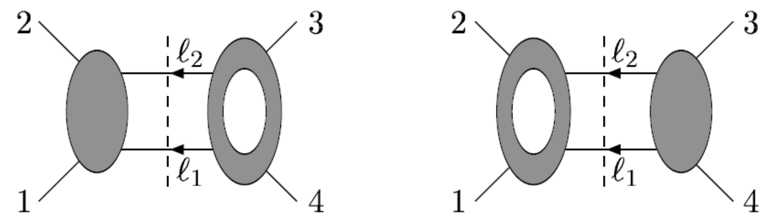
Derivation of Kawai-Lewellen-Tye amplitude relations starting from Lagragian is an open problem.

But this should not stop us from using KLT relations!

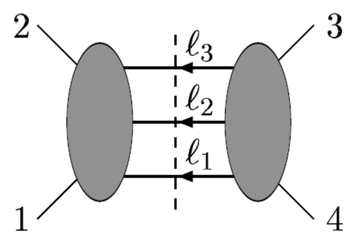
Bern, Dixon, Dunbar and Kosower

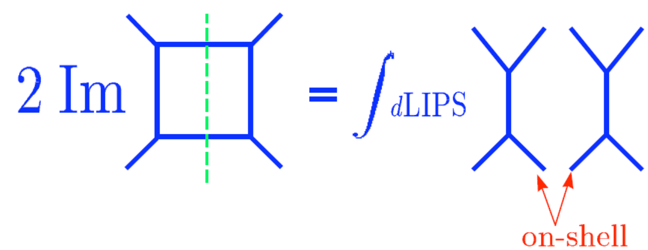
Onwards to Loops: Unitarity Method

Two-particle cut:



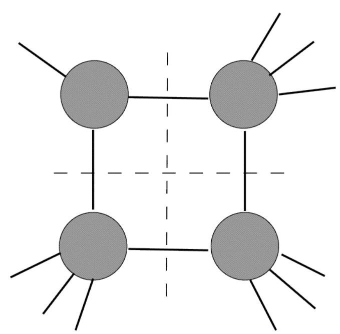
Three- particle cut:





Generalized unitarity:

Bern, Dixon and Kosower



Multiple cuts are an extremely efficient way to determine coefficients of integrals

One-loop box integrals always easy.

Britto, Cachazo and Feng; Cachazo and Buchbinder; see Johansson's talk

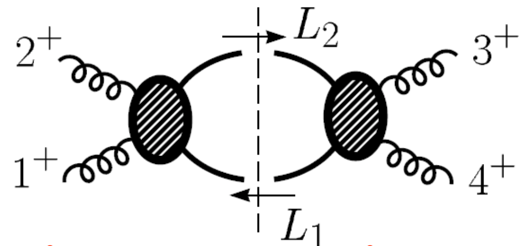
Generalized cut interpreted as cut propagators not canceling.

A number of recent improvements to method

Britto, Buchbinder, Cachazo and Feng; Berger, Bern, Dixon, Forde and Kosower; Britto, Feng and Mastrolia

Onwards to Loops

So far everything we have discussed is tree level.
To answer questions of divergences in quantum gravity we need loops.



Unitarity method provides a machinery for turning tree amplitudes into loop amplitudes.

Talks from Dixon, Ita, Johansson, Kosower,, Spradlin, Volovich,

Apply KLT to unitarity cuts:

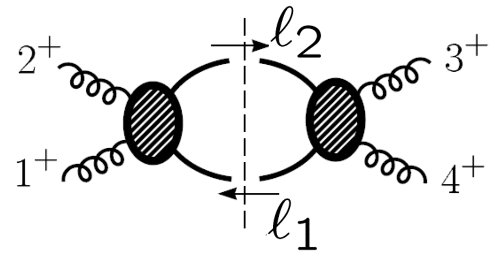
$$\sum_{\text{states}} M^{\text{tree}}_{\text{gravity}} \times M^{\text{tree}}_{\text{gravity}} = \left(\sum_{\text{states}} A^{\text{tree}}_{\text{gauge}} \times A^{\text{tree}}_{\text{gauge}} \right) \times \left(\sum_{\text{states}} A^{\text{tree}}_{\text{gauge}} \times A^{\text{tree}}_{\text{gauge}} \right)$$

Unitarity cuts in gravity theories can be reexpressed as sums of products of unitarity cuts in gauge theory.

Allows advances in gauge theory to be carried over to gravity.

$N = 8$ Supergravity from $N = 4$ Super-Yang-Mills

Using unitarity and KLT we express cuts of $N = 8$ supergravity amplitudes in terms of $N = 4$ amplitudes.



$$\sum_{N=8 \text{ states}} M_4^{\text{tree}}(-\ell_1, 1, 2, \ell_2) \times M_4^{\text{tree}}(-\ell_2, 3, 4, \ell_1)$$
$$= s^2 \sum_{N=4 \text{ states}} \left(A_4^{\text{tree}}(-\ell_1, 1, 2, \ell_2) \times A_4^{\text{tree}}(\ell_2, 3, 4, -\ell_1) \right) \times \sum_{N=4 \text{ states}} \left(A_4^{\text{tree}}(\ell_2, 1, 2, -\ell_1) \times A_4^{\text{tree}}(\ell_1, 3, 4, -\ell_2) \right)$$

Key formula for $N = 4$ Yang-Mills two-particle cuts:

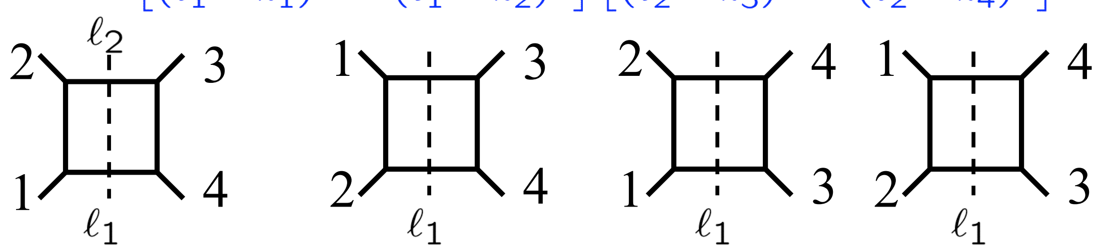
$$\sum_{N=4 \text{ states}} A_4^{\text{tree}}(-\ell_1, 1, 2, \ell_2) \times A_4^{\text{tree}}(-\ell_2, 3, 4, \ell_1) = -\frac{st A_4^{\text{tree}}(1,2,3,4)}{(\ell_1 - k_1)^2 (\ell_2 - k_3)^2}$$

Key formula for $N = 8$ supergravity two-particle cuts:

Note recursive structure!

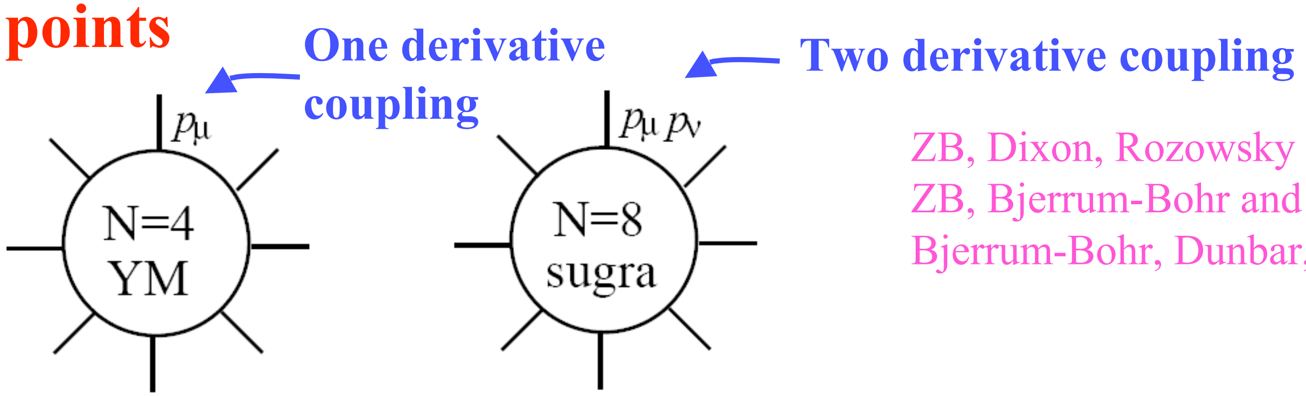
$$\sum_{N=8 \text{ states}} M_4^{\text{tree}}(-\ell_1, 1, 2, \ell_2) \times M_4^{\text{tree}}(-\ell_2, \ell_3, -\ell_4, \ell_1)$$
$$= istu\, M_4^{\text{tree}}(1,2,3,4) \left[\frac{1}{(\ell_1 - k_1)^2} + \frac{1}{(\ell_1 - k_2)^2} \right] \left[\frac{1}{(\ell_2 - k_3)^2} + \frac{1}{(\ell_2 - k_4)^2} \right]$$

Generates all contributions with s-channel cuts.



Applications at One Loop

Surprising cancellations not explained by susy are found beyond four points



ZB, Dixon, Rozowsky and Perelstein
ZB, Bjerrum-Bohr and Dunbar
Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager

Two derivative coupling means $N = 8$ should have a worse power counting relative to $N = 4$ super-Yang-Mills theory.

However, explicit five and six-point computations exhibit cancellations so that effectively the power counting in $N = 8$ supergravity is the same as for $N = 4$ super-Yang-Mills amplitudes.

“no-triangle hypothesis”

See Harald Ita’s talk: **strong evidence of additional one-loop cancellation are general.**

See Lance Dixon’s talk: **implications for higher loops.**

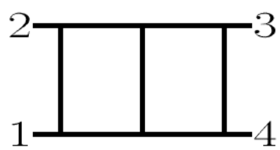
Applications at Two Loops

ZB, Dixon, Dunbar, Perelstein, Rozowsky

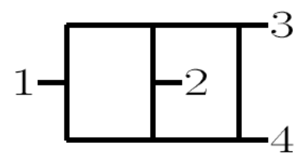
At two loops we computed two- and three-particle cuts giving the complete expression.

$$s = (k_1 + k_2)^2$$
$$t = (k_1 + k_4)^2$$
$$u = (k_2 + k_4)^2$$

Scalar double box integrals



planar



nonplanar

+ perms

planar

$$\mathcal{M}_4^{\text{2-loop}} = \left(\frac{\kappa}{2}\right)^6 stu\, M_4^{\text{tree}}(1,2,3,4)\big(s^2\mathcal{I}_4^{\text{2-loop,P}}(s,t) + s^2\mathcal{I}_4^{\text{2-loop,P}}(s,u)$$
$$+ s^2\mathcal{I}_4^{\text{2-loop,NP}}(s,t) + s^2\mathcal{I}_4^{\text{2-loop,NP}}(s,u) + \text{ cyclic}\big)$$

nonplanar

Comments:

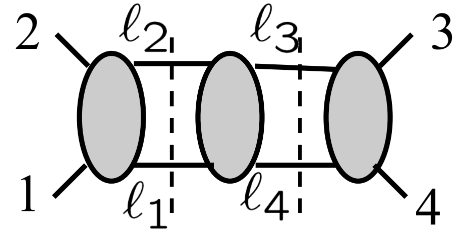
- **Values of integrals in terms of polylogs near $D = 4$ known.**

  Smirnov; Tausk

- **Originally, two-particle cuts in D dimensions, but three-particle cuts in 4 dimensions (with some arguments why it didn't matter).**
- **Later confirmation of these (and $N = 4$) results using D dimensional three-particle cuts. Complete calculation.**

Two-Particle Cuts

We iterate the two particle cuts



$$s = (k_1 + k_2)^2$$
$$t = (k_1 + k_4)^2$$
$$u = (k_2 + k_4)^2$$

One-loop evaluation

$$\sum_{N=8 \text{ states}} M_4^{\text{tree}}(-\ell_1, 1, 2, \ell_2) \times M_4^{\text{tree}}(-\ell_2, \ell_3, -\ell_4, \ell_1)$$

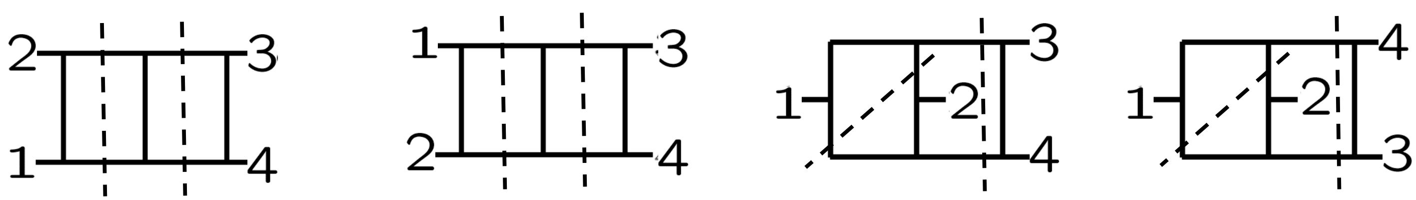
$$= istu\, M_4^{\text{tree}}(-\ell_4, 1, 2, \ell_3) \left[\frac{1}{(\ell_1 - k_1)^2} + \frac{1}{(\ell_1 - k_2)^2}\right] \left[\frac{1}{(\ell_2 - \ell_3)^2} + \frac{1}{(\ell_2 + \ell_2)^2}\right]$$

Two-loop evaluation

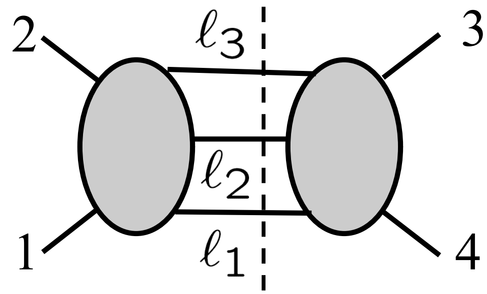
$$M_4^{\text{2-loop}}(1,2,3,4)\Big|_{s\text{-cut}} = M_4^{\text{1-loop}}(-\ell_4, 1, 2, \ell_3) \times M_4^{\text{1-tree}}(-\ell_3, 3, 4, \ell_4)$$

Restore loop integration and include non-planar cuts

$$M_4^{\text{2-loop}}(1,2,3,4)\Big|_{s\text{-cut}} = stu M_4^{\text{tree}} s^2 \Big(\mathcal{I}_4^{\text{2-loop,P}}(s,t) + \mathcal{I}_4^{\text{2-loop,P}}(s,u) + \mathcal{I}_4^{\text{2-loop,NP}}(s,t) + \mathcal{I}_4^{\text{2-loop,NP}}(s,u)\Big)\Big|_{s\text{-cut}}$$



Three-Particle Cuts



Use KLT tree relations:

$$M_5^{\rm tree}(\ell_1,1,2,\ell_3,\ell_2) = i\,(\ell_1+k_1)^2(\ell_3+k_2)^2 A_5^{\rm tree}(\ell_1,1,2,\ell_3,\ell_2)\,A_5^{\rm tree}(1,\ell_1,\ell_3,2,\ell_2) + \{1\leftrightarrow 2\}$$

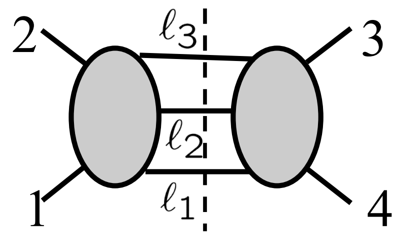
$$M_5^{\rm tree}(-\ell_3,3,4,-\ell_1,-\ell_2) = i\,(\ell_3-k_3)^2(\ell_1-k_4)^2 A_5^{\rm tree}(-\ell_3,3,4,-\ell_1,-\ell_2)\,A_5^{\rm tree}(3,-\ell_3,-\ell_1,4,-\ell_2) + \{3\leftrightarrow 4\}\,.$$

$$\begin{aligned}\sum_{N=8\ \text{states}} & M_5^{\rm tree}(1,2,\ell_3,\ell_2,\ell_1) M_5^{\rm tree}(3,4,-\ell_1,-\ell_2,-\ell_3) \\ &= -(\ell_1+k_1)^2(\ell_3+k_2)^2(\ell_3-k_3)^2(\ell_1-k_4)^2 \\ &\quad\times\Big[\sum_{N=4\ \text{states}} A_5^{\rm tree}(\ell_1,1,2,\ell_3,\ell_2) A_5^{\rm tree}(-\ell_3,3,4,-\ell_1,-\ell_2)\Big] \\ &\quad\times\Big[\sum_{N=4\ \text{states}} A_5^{\rm tree}(1,\ell_1,\ell_3,2,\ell_2) A_5^{\rm tree}(3,-\ell_3,-\ell_1,4,-\ell_2)\Big] \\ &\quad + \{1\leftrightarrow 2\} + \{3\leftrightarrow 4\} + \{1\leftrightarrow 2,\,3\leftrightarrow 4\}\end{aligned}$$

supergravity

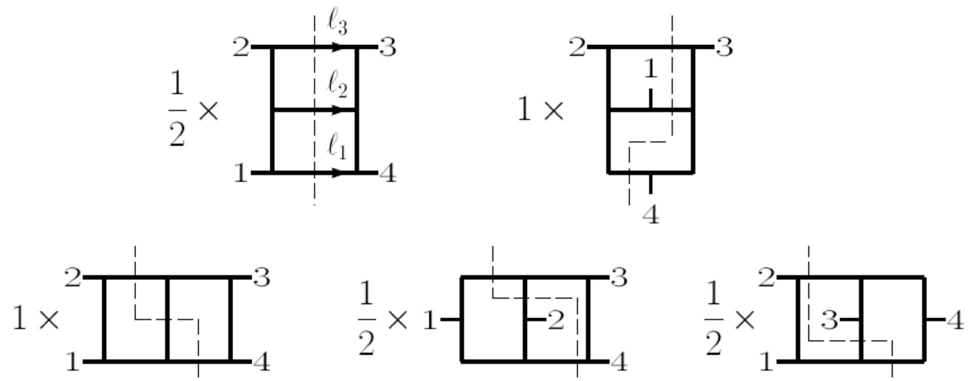
super-Yang-Mills already evaluated so import results

Three-Particle Cuts



very similar to $N = 4$ sYM

Resulting cut integral structures

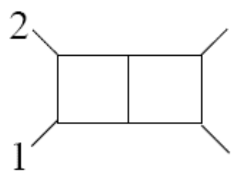


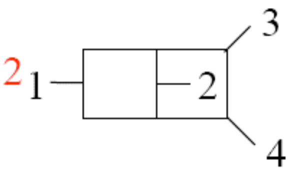
$$
\begin{aligned}
& stuM_4^{\rm tree} \int \frac{d^D\ell_1}{(2\pi)^D}\frac{d^D\ell_2}{(2\pi)^D}\,\frac{1}{\ell_1^2\,\ell_2^2\,\ell_3^2} \\
& \times \bigg\{ \bigg[\bigg(\frac{1}{2}\,\frac{t^2}{(\ell_1+k_1)^2(\ell_3+k_2)^2(\ell_3-k_3)^2(\ell_1-k_4)^2} + \frac{t^2}{(\ell_2+k_1)^2(\ell_3+k_2)^2(\ell_3-k_3)^2(\ell_1-k_4)^2} \\
& \quad + \frac{s^2}{(\ell_1+\ell_2)^2(\ell_2+\ell_3)^2(\ell_3+k_2)^2(\ell_1-k_4)^2} + \frac{1}{2}\,\frac{s^2}{(\ell_2+\ell_3)^2(\ell_3+k_1)^2(\ell_2+k_2)^2(\ell_1-k_4)^2} \\
& \quad + \frac{1}{2}\,\frac{s^2}{(\ell_1+\ell_2)^2(\ell_3+k_2)^2(\ell_2-k_3)^2(\ell_1-k_4)^2} \bigg) + \text{perms}(\ell_1,\ell_2,\ell_3) \bigg] \\
& \qquad\qquad + \{1\leftrightarrow 2\} + \{3\leftrightarrow 4\} + \{1\leftrightarrow 2,\, 3\leftrightarrow 4\} \bigg\} \bigg|_{\ell_i^2=0},
\end{aligned}
$$

Two-Loop $N = 8$ Amplitude

Z.B., Dixon, Dunbar, Perelstein and Rozowsky

From two- and three-particle cuts we get the $N = 8$

$(s\,K)^2$ 

$+ \; (s\,K)^2$ 

$+$ perms

where $K = st A_4^{\rm tree}$ ← **Yang-Mills tree**

$$\mathcal{M}_4^{\text{2-loop},\ D=7-2\epsilon}\Big|_{\rm pole} = \frac{1}{2\epsilon\ (4\pi)^7}\frac{\pi}{3}(s^2+t^2+u^2)\ stu M_4^{\rm tree},$$

$$\mathcal{M}_4^{\text{2-loop},\ D=9-2\epsilon}\Big|_{\rm pole} = \frac{1}{4\epsilon\ (4\pi)^9}\frac{-13\pi}{9072}(s^2+t^2+u^2)^2\ stu M_4^{\rm tree},$$

$$\mathcal{M}_4^{\text{2-loop},\ D=11-2\epsilon}\Big|_{\rm pole} = \frac{1}{48\epsilon\ (4\pi)^{11}}\frac{\pi}{5791500}\Big(438(s^6+t^6+u^6) - 53s^2t^2u^2\Big)\ stu M_4^{\rm tree}.$$

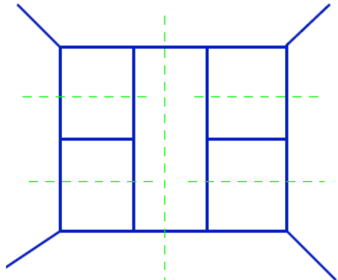
Note: theory diverges at one loop in $D = 8$

Counterterms are derivatives acting on Bel-Robinson tensor

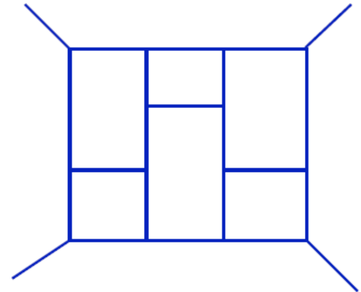
$$t_8 t_8 R^4 \equiv t_8^{\mu_1\mu_2\cdots\mu_8}\, t_8^{\nu_1\nu_2\cdots\nu_8}\, R_{\mu_1\mu_2\nu_1\nu_2}\, R_{\mu_3\mu_4\nu_3\nu_4}\, R_{\mu_5\mu_6\nu_5\nu_6}\, R_{\mu_7\mu_8\nu_7\nu_8}$$

For D=5, 6 the amplitude is finite contrary to traditional superspace power counting.

Iterated Two-Particle Cuts to All Loop Orders



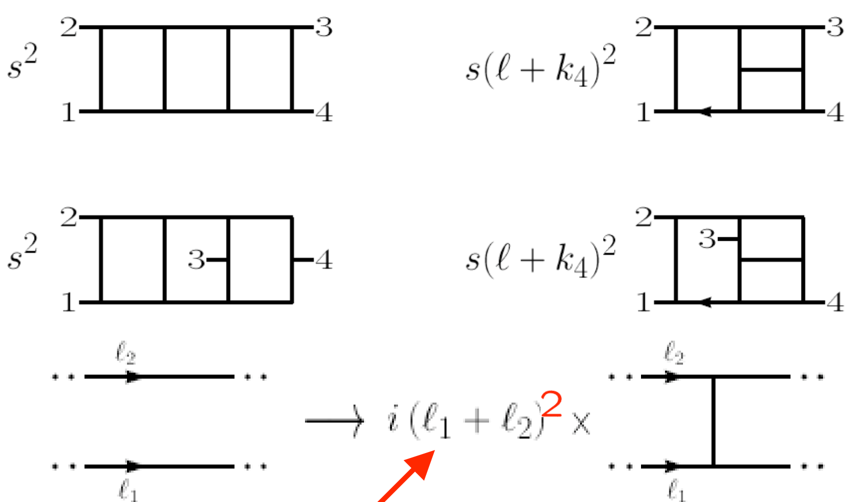
constructible from
iterated 2 particle cuts



not constructible from
iterated 2 particle cuts

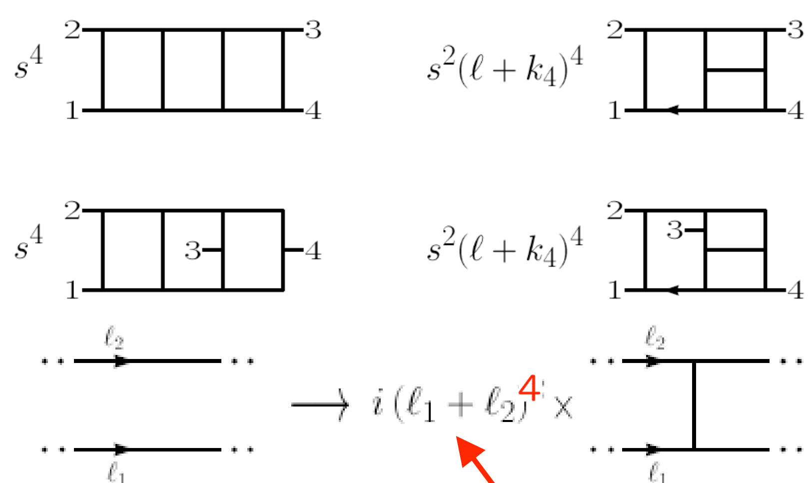
Results from iterated two-particle cuts

$\mathcal{N} = 4$ super-Yang-Mills



strong evidence this
is correct factor

$\mathcal{N} = 8$ supergravity



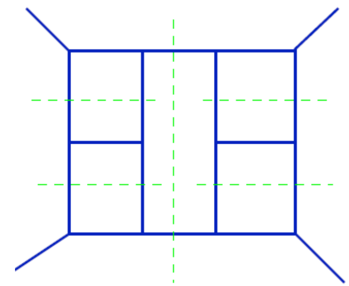
less certain

$(\ell + k_4)^2 = 2\ell \cdot k_4$??

Power Counting To All Loop Orders

From '98 paper:

Z.B., Dixon, Dunbar, Perelstein and Rozowsky



- **Assumed iterated 2-particle cuts give the generic UV behavior**
- **Assumed no cancellations with other uncalculated terms.**
- **Assumed numerator factors are similar to YM^2 ones.**

- **No evidence was found that more than 12 powers of loop momenta come out of the integrals.**
- **This is precisely the number of loop momenta extracted from the integral at two loops.**

Elementary power counting for 12 loop momenta coming out of the integral gives finiteness condition:

$$\boxed{D < \frac{10}{L} + 2} \qquad (L > 1)$$

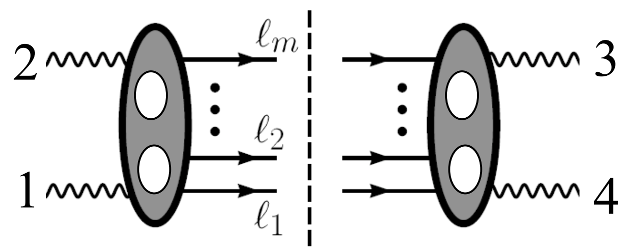
In $D = 4$ finite for $L < 5$.
L is number of loops.

MHV Cuts to All Loop Orders

MHV amplitudes satisfy very simple on-shell susy Ward identities:

- Simple relative factors.
- Same relative factors at any loop order.

Suppose we limit ourselves to MHV crossing the cuts.



$$s = (k_1 + k_2)^2$$

The sum over all MHV contributions crossing the cuts gives a numerator that can be collected to:

$$[(\ell_1 + \ell_2 + \ldots + \ell_m)^2]^8 = s^8 \longleftarrow$$

Momenta come out of the loop integral

Finiteness condition $D < \frac{10}{L} + 2$

Accounting for all factors factors gives agreement with iterated two-particle cuts.

Because only susy used a limited number of numerator momenta come out of the loop integrals.

How reliable is the finiteness bound?

$$\boxed{D < \frac{10}{L} + 2} \qquad (L > 1)$$

Because of assumptions this power count is *not* a proof (except at 2 loops).

Can the theory be more divergent? Seems unlikely:

- Two-loop calculations prove extra cancellations compared to known-superspace power counting.
- It seems very unlikely that fewer powers of loop momenta come out of the integral at 3 or more loops than at 2 loops.

Can the theory be less divergent or even finite?

- For this to be true need additional cancellations beyond those visible in iterated 2 particle cuts or in MHV cuts.

Interpretation of $N = 8$ Results

ZB, Dixon, Dunbar, Perelstein, Rozowsky
hep-th/9802162

’98 analysis conclusion:

- **No evidence from iterated 2 particle cuts or from MHV cuts of any UV cancellation which get stronger as number of loops increases.**

However, for $N = 8$ supergravity to be UV finite there must be cancellations which get stronger as the number of loops increases.

Recent insight:

Today, however, we have evidence of such cancellations:
To be discussed in talks by Dixon and Ita.

“no-triangle hypothesis”

Summary of 1998 $N = 8$ Situation

ZB, Dixon, Dunbar, Perelstein, Rozowsky

Finiteness Condition: $$\boxed{D < \frac{10}{L} + 2} \qquad (L > 1)$$

Evidence:

- ***Complete*** **calculation at 2 loops confirms this. Improved UV behavior compared to earlier superspace power counting (no assumptions).**
- **Interated two-particle cuts give above finiteness bound.**
- ***All*** **cuts, but with MHV only crossing the cut gives same bound.**

But some assumptions:

- **Iterated 2-particle cuts or MHV cuts control the UV behavior.**
- **Rung rule numerator factors are squares of $N = 4$ factors.**

$$(l - k_1)^2 \to [(l - k_1)^2]^2 \text{ not } [2l \cdot k_1]^2$$

- **No additional cancellations with other terms.**

Summary

- Modern computational methods provide a powerful way to study UV divergences of gravity theories:
  - -- unitarity method
  - -- Kawai Lewellen Tye relations between tree amplitudes.
- Explicit two-loop examples where maximally supersymmetric gravity shown to be less divergent than predicted by known-superspace power counting.
- Recent advances in computing $N = 4$ super-Yang-Mills theory amplitudes can be imported into $N = 8$ calculations.

Is $N = 8$ supergravity ultraviolet finite?

In talks from Ita and Dixon we will hear about additional unexpected cancellations. String hints will be discussed in talks of Green and Roiban.

UCLA

Theoretical Elementary Particle Physics
DEPARTMENT OF PHYSICS & ASTRONOMY

DECEMBER 11th - 15th, 2006
UCLA Physics and Astronomy Building
on 4th floor

Attendees:

Zvi Bern,
Freddy Cachazo,
Lance Dixon,
Sergio Ferrara,
Michael Green,
Michael Gutperle,
Paul Howe,*
Chris Hull,*
David Kosower,
Per Kraus,
Harald Ita,
Radu Roiban,
Marcus Spradlin,
Kelly Stelle,
Anastasia Volovich,
Chuan-jie Zhu *
and others

(*) to be confirmed

"IS N=8 SUPERGRAVITY FINITE?"

Pictures...
UCLA Royce Hall, Mondrian-like art.

ABSTRACT

Conventional wisdom holds that no four-dimensional gravitational theory can be finite. However, using modern computational methods based on unitarity, it has been shown that N=8 supergravity is less divergent than previously thought. More cancellations may well be in store, as suggested also by string-theoretic arguments. This workshop will examine the ultraviolet properties of N=8 supergravity in the light of all the current evidence. The intimate connection of N=8 supergravity to N=4 super-Yang-Mills theory will also be discussed.

Organizing Committee:
Zvi Bern,
Lance Dixon,
Michael Gutperle
David Kosower

Sponsors:
US Department of Energy;
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