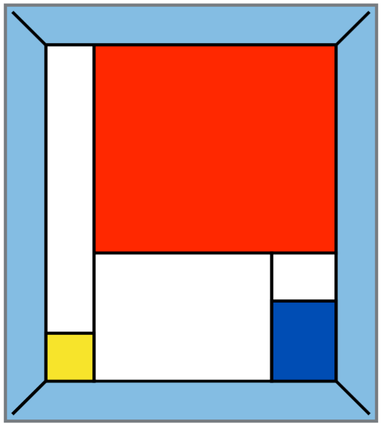
N=8 Supergravity at three loops and beyond



Z. Bern, L.D., R. Roiban

hep-th/0611086

UCLA workshop, “Is N=8 Supergravity Finite?”

December 12, 2006

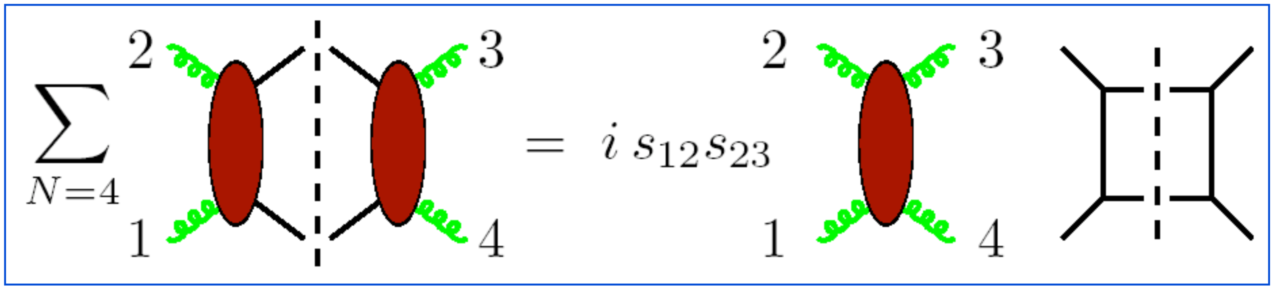
Outline

- Review power-counting inferred from iterated 2-particle cuts
- "No triangle" structure hypothesized at one loop, combined with higher-particle cuts, suggests that this power counting is too conservative, missing cancellations at 3 loops and beyond
- What can we say about the full 3 loop amplitude?
    - Nonplanar topologies allowed by no-triangle hypothesis
    - Information from nonplanar, "non-rung-rule" contributions to N=4 super-Yang-Mills theory
    - Some analysis of 3-particle cuts

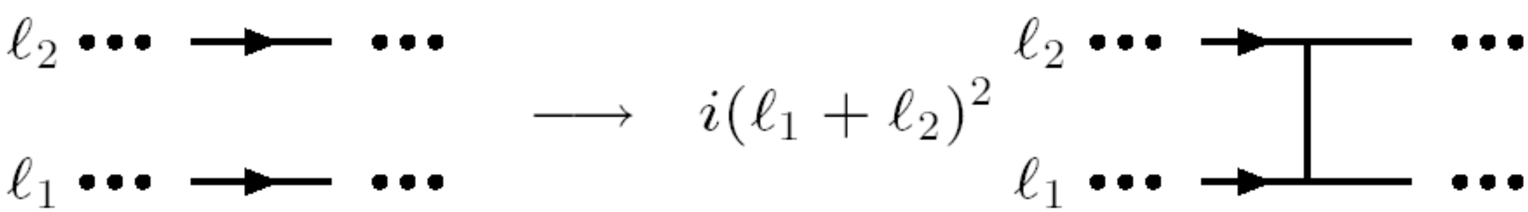
Unitarity and N=4 SYM

Many higher-loop contributions to *gg* → *gg* scattering deduced from a simple property of the 2-particle cuts at one loop

Bern, Rozowsky, Yan (1997)



Leads to "rung rule" for easily computing all contributions which can be built by iterating 2-particle cuts



Unitarity and N=8 Supergravity

Using KLT relations, Kawai, Lewellen, Tye (1986)

N=8 supergravity 4-point amplitudes are "squares" of N=4 SYM amplitudes

Bern, LD, Dunbar, Perelstein Rozowsky (1998)

→ N=8 2-particle cutting equation:

$$\sum_{N=8 \text{ states}} M_4^{\text{tree}}(-\ell_1, 1, 2, \ell_2) \times M_4^{\text{tree}}(-\ell_2, 3, 4, \ell_1)$$

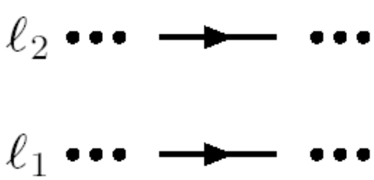
$$= i\, stu\, M_4^{\text{tree}}(1,2,3,4) \left[\frac{1}{(\ell_1 - k_1)^2} + \frac{1}{(\ell_1 - k_2)^2} \right] \left[\frac{1}{(\ell_2 - k_3)^2} + \frac{1}{(\ell_2 - k_4)^2} \right]$$

square

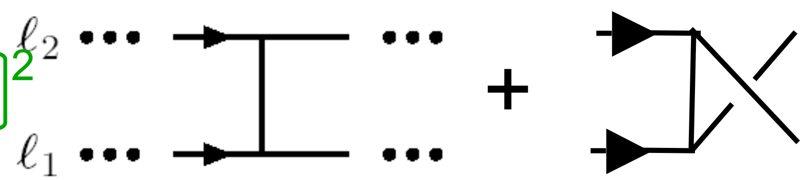
cancel in iteration

$$\frac{1}{t} + \frac{1}{u} = \frac{t+u}{tu} = \frac{-s}{tu}$$

Leads to N=8 rung rule:



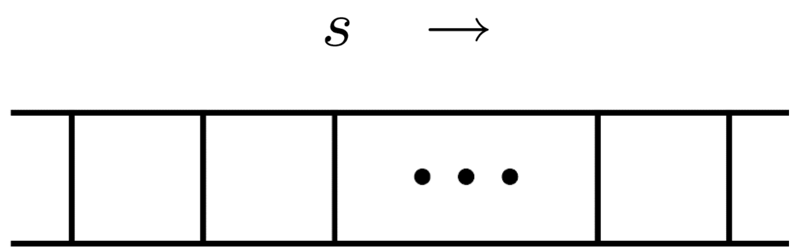
$$\longrightarrow \quad i[(\ell_1 + \ell_2)^2]^2$$



Ladder diagrams (Regge-like)

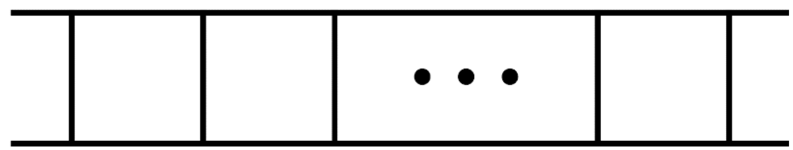
In N=4 SYM

$$st\, A_4^{\text{tree}} \times s^{L-1}$$



In N=8 supergravity

$$stu\, M_4^{\text{tree}} \times s^{2(L-1)}$$

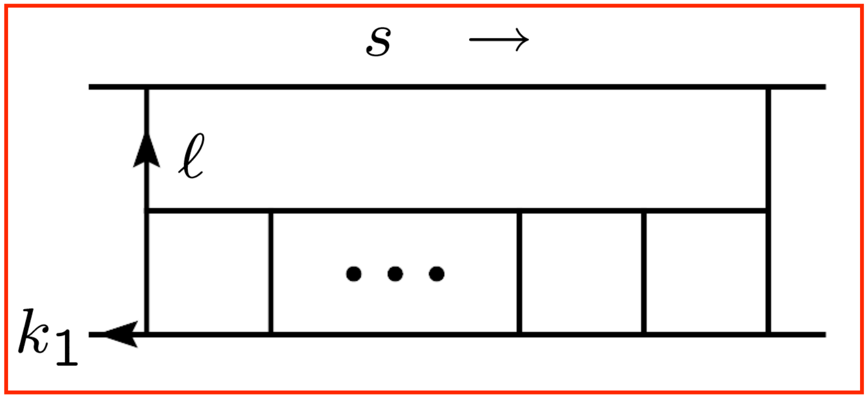


Extra s^L in gravity from "charge" = energy

More UV divergent diagrams

N=4 SYM

$$st\, A_4^{\text{tree}} \times t \times [(\ell + k_1)^2]^{L-2}$$



N=8 supergravity

$$stu\, M_4^{\text{tree}} \times t^2 \times [(\ell + k_1)^2]^{2(L-2)}$$

Integral in *D* dimensions scales as

$$\mathcal{I} \sim \int d^{DL}\ell \, \frac{(\ell^2)^{2(L-2)}}{(\ell^2)^{3L+1}}$$

→ Critical dimension D_c for log divergence obeys

$$\frac{D_c L}{2} + 2(L-2) = 3L + 1 \quad \Longrightarrow$$

$$D_c = 2 + \frac{10}{L}$$ N=8

$$D_c = 4 + \frac{6}{L}$$ N=4 SYM

BDDPR (1998)

Is this power counting correct?

$D_c = 2 + \frac{10}{L}$ N=8 $\Leftrightarrow$ $stu\, M_4^{\text{tree}} \times t^2 \times [(\ell + k_1)^2]^{2(L-2)}$

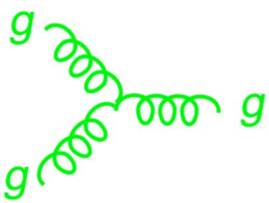
$\Leftrightarrow$ $D^4 R^4$ potential counterterm at every loop order $L \geq 2$

Reasons to reexamine whether it might be too conservative:

- Superspace-based speculation that *D=4* case diverges only at *L=6*, not *L=5* Howe, Stelle, hep-th/0211279; K. Stelle, at this workshop
- Multi-loop string calculations seem not to allow D^4R^4 past *L*=2. Berkovits, hep-th/0609006
- String/M duality arguments with similar conclusions, suggesting possibility of finiteness Green, Russo, van Hove, hep-th/0610299; M. Green, at this workshop
- No triangle hypothesis for 1-loop amplitudes Bjerrum-Bohr et al,, hep-th/0610043 H. Ita, at this workshop

No-triangle power counting at one loop

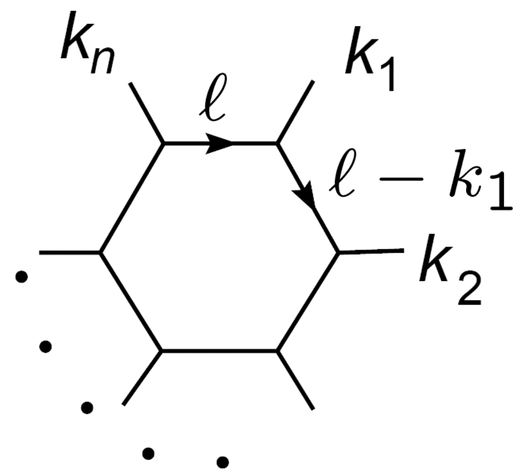
generic gauge theory (spin 1)



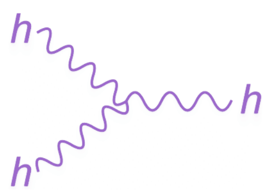
$$\supset \ \ell^\mu \, \eta^{\nu\rho} + \cdots$$

$$\Rightarrow \quad (\ell^\mu)^n$$

N=4 SYM $\Rightarrow \quad (\ell^\mu)^{n-4}$



generic gravity (spin 2)



$$\supset \ \ell^{\mu_1} \, \ell^{\mu_2} \, \eta^{\nu_1\rho_1} \, \eta^{\nu_2\rho_2} + \cdots$$

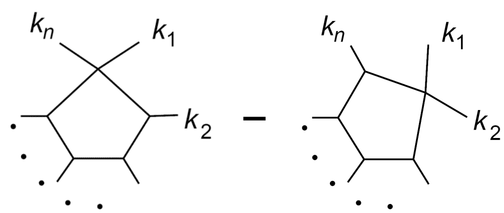
$$\Rightarrow \quad (\ell^\mu)^{2n}$$

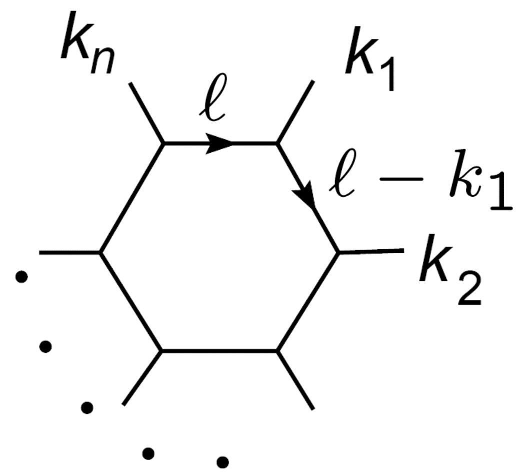
N=8 supergravity $\overset{??}{\Rightarrow} \quad (\ell^\mu)^{2(n-4)}$

evidence that it is better

No-triangle power counting (cont.)

$$\mathcal{I}_n[2\ell \cdot k_1] \equiv \int \frac{d^D\ell \;\; 2\ell \cdot k_1}{\ell^2(\ell - k_1)^2 \cdots}$$
$$= \int \frac{d^D\ell \;\; [\ell^2 - (\ell - k_1)^2]}{\ell^2(\ell - k_1)^2 \cdots}$$
$$= \mathcal{I}^{(1)}_{n-1}[1] - \mathcal{I}^{(2)}_{n-1}[1]$$





N=4 SYM, $(\ell^\mu)^{n-4}$

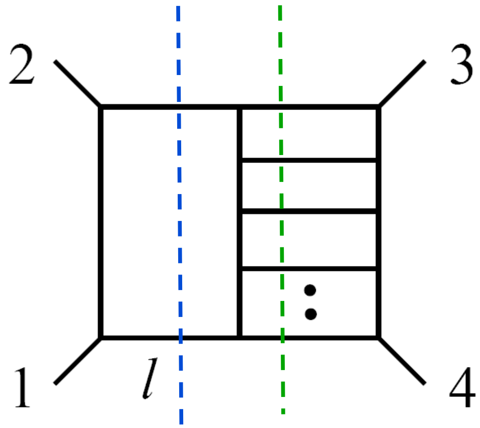
pentagon linear in ℓ^μ → scalar box with no triangle

generic pentagon quadratic in ℓ^μ → linear box → scalar triangle

But all N=8 amplitudes inspected so far, with 5,6,~7,… legs,
contain no triangles → more like $(\ell^\mu)^{n-4}$ than $(\ell^\mu)^{2(n-4)}$

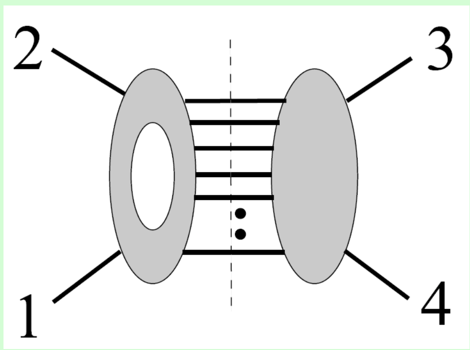
see talk by Ita

A key *L*-loop topology



2-particle cut exposes Regge-like ladder topology, containing numerator factor of $[(l+k_4)^2]^{2(L-2)}$

L-particle cut exposes one-loop (*L*+2)-point amplitude – but $[(l+k_4)^2]^{2(L-2)}$ would (heavily) violate the no-triangle hypothesis

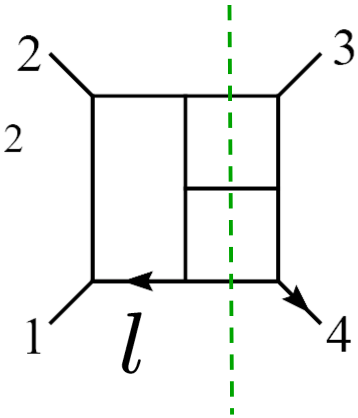


Three-loop case

3 loops interesting because it is first order for which:

- N=4 SYM & N=8 SUGRA might have a different critical dimension
- the full amplitude isn't known yet

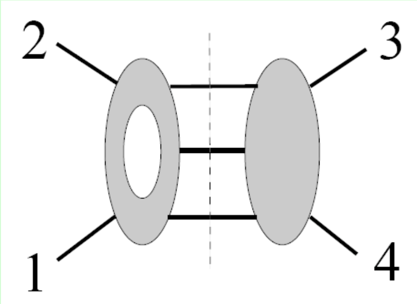
$s^2[(l+k_4)^2]^2$



3-particle cut exposes one-loop 5-point amplitude with $[(l+k_4)^2]^2$

– violates no-triangle hypothesis

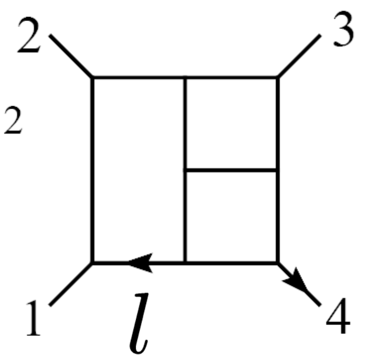
– which for 5-point case is a fact



Bern, LD, Perelstein, Rozowsky, hep-th/9811140

Three-loop case (cont.)

$$s^2\left[(l+k_4)^2\right]^2$$

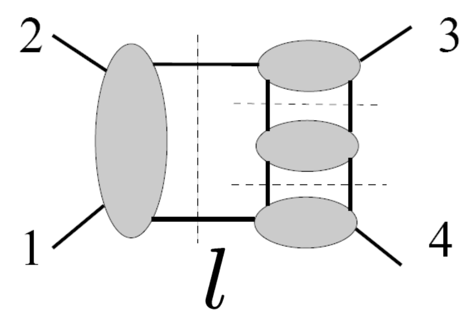


numerator factor might "really" be

$$s^2\,[2l\cdot k_4]^2$$

because $(l+k_4)^2 = l^2 + 2l\cdot k_4$

and the iterated 2-particle cut, by which this integral was detected, assumes that $l^2 = 0$



However, even the second form violates the no-triangle restriction

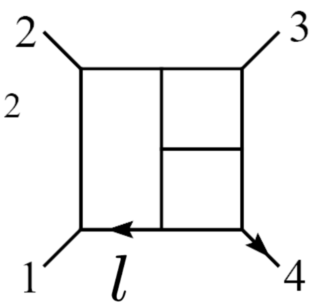
(but not $(\ell^\mu)^{2(n-4)}$)

Montag (1992)

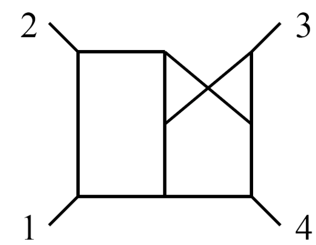
Three-loop case (cont.)

Something else must cancel the bad "left-loop" behavior of this contribution. But what?

$$s^2\left[(l+k_4)^2\right]^2$$



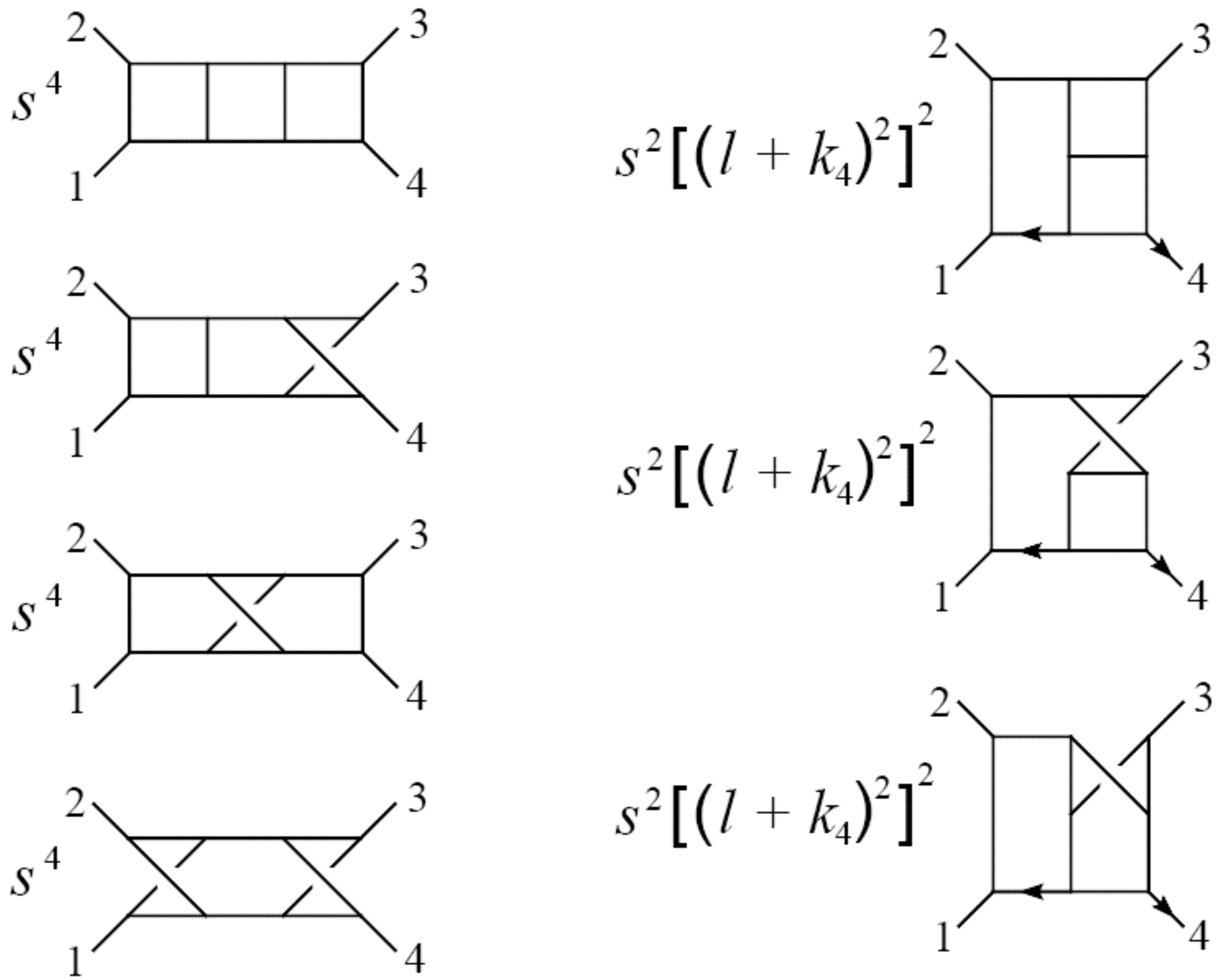
Maybe other "rung-rule" contributions detectable via 2-particle cuts, such as



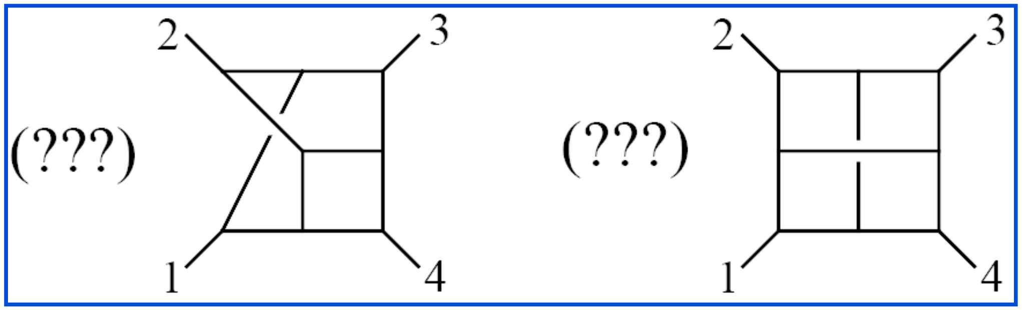
Maybe contributions that only appear when the 3-particle cuts (or maybe 4-particle cuts) are evaluated.

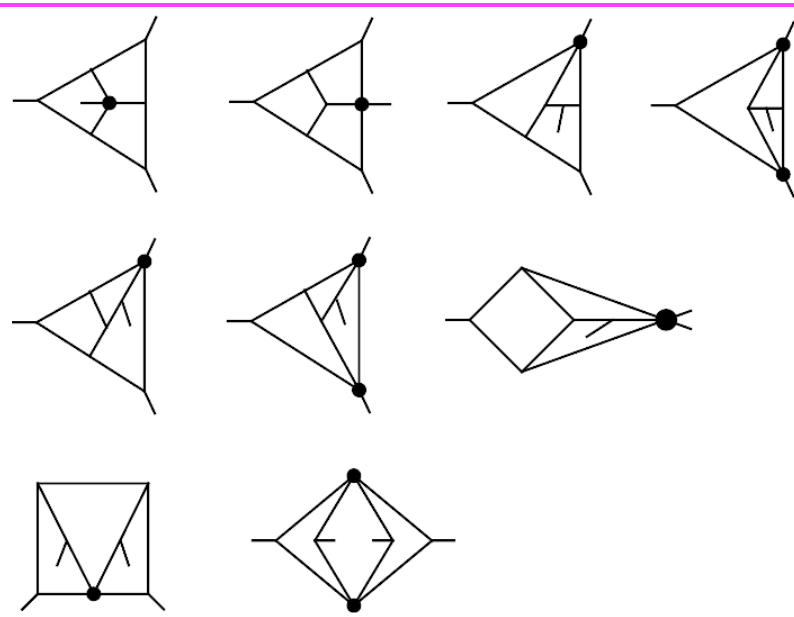
What topologies are possible, assuming no triangle subgraphs?

N=8 3-loop rung-rule integrals



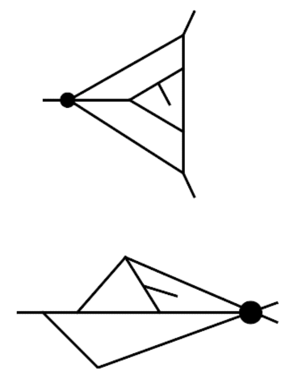
N=8 3-loop cubic non-rung-rule topologies





and their noncubic offspring

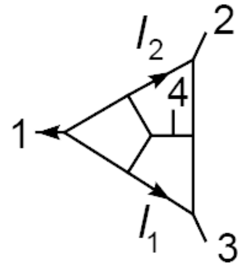
and the noncubic rung-rule offspring



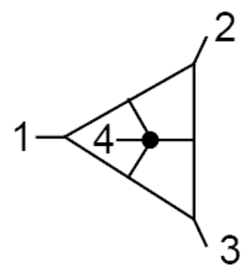
Can N=4 SYM provide more clues?

- For the non-rung-rule topologies, a simple “squaring” of numerator factors is probably too simple.
- Nevertheless, the structure of the nonplanar, subleading-in-N_c terms for N=4 SYM at 3 loops may give some hints:

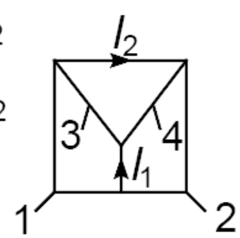
$$s_{12}(l_1+k_1)^2 - s_{13}(l_1+k_2)^2$$



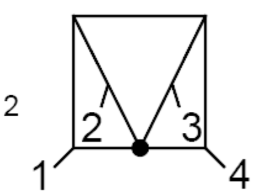
$$-\frac{1}{3}(s_{12}-s_{13})$$



$$s_{12}(l_2+k_1+k_3)^2 + s_{13}(l_1+k_1+k_2)^2 - s_{12}\,s_{13}$$



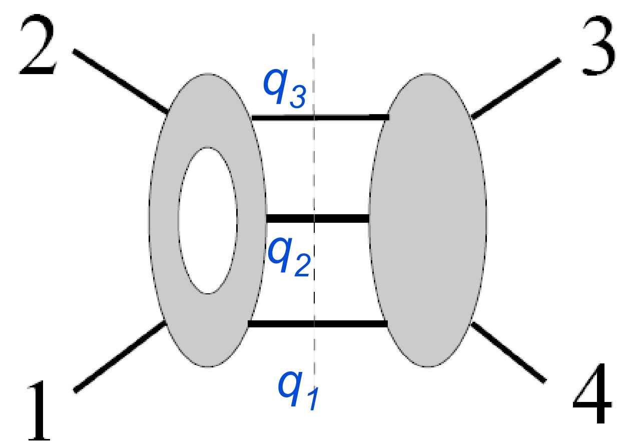
$$-s_{12}$$



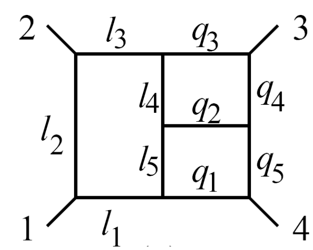
- Color here is not really assigned properly, but that doesn’t matter for the application to gravity.

Partial progress for N=8 at 3 loops

3-particle cut can be evaluated using KLT, and "KLT-like", representations of the tree, and 1-loop, 5-point amplitudes.
For example, the loop amplitude has the form



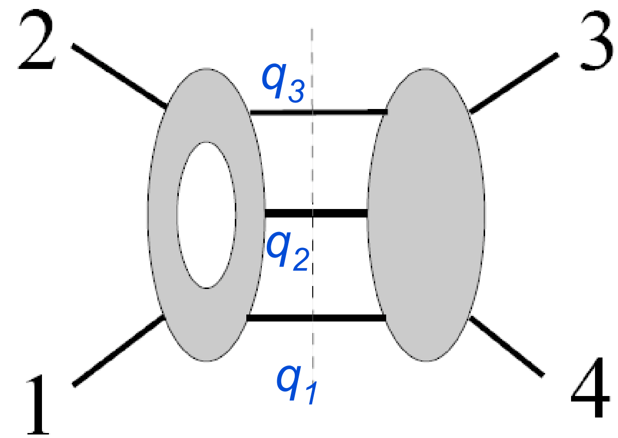
$$M_5^{\text{1-loop}}(1,2,q_1,q_2,q_3)$$
$$= -\frac{1}{2}\sum_{\text{perms}} s_{q_2 q_1} s_{12}^2 s_{2q_3}^2 A_5^{\text{tree}}(1,2,q_3,q_2,q_1)$$
$$\times A_5^{\text{tree}}(1,2,q_3,q_1,q_2)\int \frac{d^D l_1}{(2\pi)^D}\frac{1}{l_1^2 l_2^2 l_3^2 l_4^2} + \mathcal{O}(\epsilon)$$



Products of N=4 SYM trees from left and right side of cut give "traces" also encountered in the planar + nonplanar 3-loop N=4 SYM amplitude, simplifying sum over states.

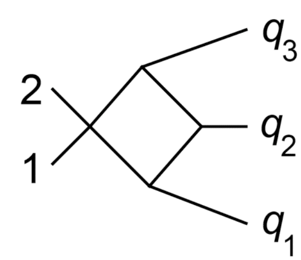
Partial progress at 3 loops (cont.)

- 3-particle cut for fixed q_i is sum over 1-loop pentagon-like integrals (many permutations).
- But pentagons are not independent of boxes, so we reduce them all to boxes.
- Then compare coefficients of boxes (and triangles) between the true 3-particle cut, and the 3-particle cut of an ansatz built off of the rung-rule diagrams.



Perelstein

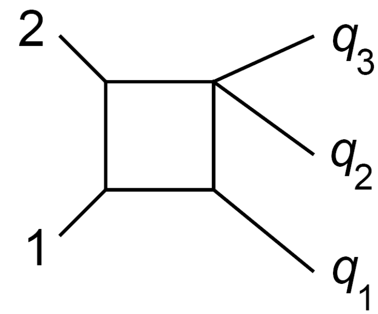
A box which works already (and so should not be corrected by additional terms in the ansatz)



But it seems somewhat trivial in that none of the non-rung-rule topologies are capable of generating it

Partial progress at 3 loops (cont.)

Another box which works already (and so should not be corrected)



Like the previous box, it seems that none of the non-rung-rule topologies can generate it

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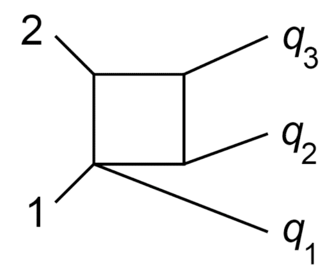
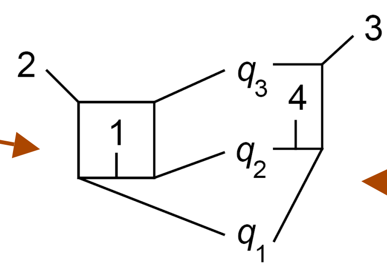
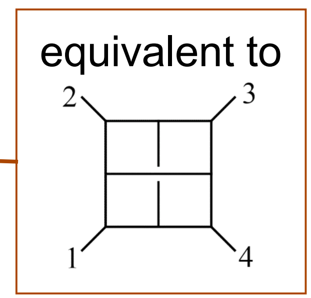
A box which doesn't work yet (and so should be corrected)



This one can come from non-rung-rule topologies





But what is the correct numerator factor?

Conclusions & Outlook

• Old power-counting formula from iterated 2-particle cuts predicted

$$D_c = 2 + \frac{10}{L}$$ N=8

• New evidence combining 3- and higher-particle "gedanken" cuts with no-triangle behavior of one-loop multi-leg N=8 amplitudes shows that there must be additional cancellations of some type.

• Will these cancellations reduce the overall degree of divergence at 3 loops? At higher loops? All the way to

$$D_c = 4 + \frac{6}{L}$$ N=4 SYM ??

• A complete representation of the 3-loop 4-graviton amplitude, consistent with all the multi-particle cuts, would go a long way toward addressing these important questions.