Quantum Connections between M-theory and Superstrings

Michael B. Green

DAMTP, Cambridge University

IS N=8 SUPERGRAVITY FINITE?

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With **Pierre VANHOVE** and **Jorge RUSSO**
(**M. Gutperle**, S. Sethi, H. Kwon)

hep-th/0610299, hep-th/0611213, ……

Talk 1

Non-perturbative results at low orders in the derivative expansion for string theory in nine and ten dimensions. Interconnections with 11-dim. SUGRA.

Talk 2

Effects at higher loops. More generic.
Nonrenormalization conditions suggested by duality with M-theory. Relation to F-terms a la Berkovits.

What does this say about 4-dim N=8 SUGRA??

Significant CAVEAT !
Possible effects due to existence of massless wrapped M-branes in SUGRA limit ! Could modify perturbative expansion of N=8.

How far do Supersymmetry, Dualities and Unitarity constrain the effective M-theory action??

- Exact structure of perturbative and non-perturbative contributions to the derivative expansion
- Problem - no off-shell supersymmetry
- Explicit example of ten-dimensional type II supersymmetry, SL(2,Z) duality.

How much can be learned about QUANTUM **Maximal Supergravity??**

Concentrate on four-graviton scattering

derivatives of curvature (zero fluxes, fixed dilaton)

$R^4; D^4R^4; \ldots; D^{2k}R^4; \ldots$ ← linearized curvature

Type II

- A **tiny** subsector of the full action
- Investigate scalar field dependence of higher derivative terms
- Solutions of Poisson equations on moduli space
- Succession of perturbative "non-renormlization" conditions.
- *A fascinating pattern of zeta functions and multiple zeta functions (Euler-Zagier sums) - interconnections with multi-loop Feynman diagrams.*

PERTURBATIVE "DATA"

String perturbation theory:

four-graviton TREE-LEVEL and ONE-LOOP

4-Graviton Scattering in type II

Tree-level (Virasoro-Shapiro) amplitude:

$$A_4^{tree} = e^{-2\phi} \hat{R}^4 \, T(s,t,u)$$

($\hat{R}$ =linearized Weyl curvature)

characteristic of tree level

$$T = \frac{4}{\alpha'^3 stu} \frac{\Gamma(1-\alpha' s)\,\Gamma(1-\alpha' t)\,\Gamma(1-\alpha' u)}{\Gamma(1+\alpha' s)\,\Gamma(1+\alpha' t)\,\Gamma(1+\alpha' u)}$$

Derivative expansion:

Easy to expand in an infinite series of powers of s, t, u,
giving infinite series of $D^{2k} R^4$ terms with specific coefficients

- 'Transcendentality ?' (also for multi-loop SUGRA tomorrow)

4-Graviton Scattering in type II

Tree-level (Virasoro-Shapiro) amplitude:

$$A_4^{tree} = e^{-2\phi} \hat{R}^4 \, T(s,t,u)$$

($\hat{R}$ =linearized Weyl curvature)

Einstein-Hilbert; R^4; D^4R^4; D^6R^4

$$T = \frac{4}{\alpha'^3 stu} + 2\zeta(3) + \zeta(5)\alpha'^2(s^2+t^2+u^2) + \frac{2\zeta(3)^2}{3}\alpha'^3(s^3+t^3+u^3)$$

D^8R^4; $D^{10}R^4$

$$+ \frac{\zeta(7)}{2}\alpha'^4(s^2+t^2+u^2)^2 + \frac{4\zeta(3)\zeta(5)}{5}\alpha'^5(s^5+t^5+u^5)$$

$$+ \zeta(9)\alpha'^6\left(\frac{2}{27}(s^3+t^3+u^3)^2 + \frac{1}{4}(s^2+t^2+u^2)^3\right)$$

$$+ \frac{4\zeta(3)^3}{27}\alpha'^6(s^2+t^2+u^2)^3 + \cdots$$

$D^{12}R^4$

first degeneracy

Genus-one amplitude:

$$I = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} F(\tau, \bar{\tau}; s, t, u)$$

$$A_4^{genus-one} = \hat{R}^4 \, I(s,t,u)$$

Integral of modular function

Surprisingly little know about derivative expansion

- Difficult to perform integrals over toroidal world-sheet
  - evaluate multiple discrete momentum sums (Harmonic sums and Multiple Zeta Variables)

- Separate non-analytic massless thresholds from analytic terms.
  - compactify to NINE dimensions to avoid log thresholds

Intriguing pattern of coefficients

Genus-one amplitude:

$$I = \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2^2} F(\tau, \bar\tau; s, t, u)$$

Integral of modular function

$$A_4^{genus-one} = \hat{R}^4 I(s,t,u)$$

D^6R^4 $D^{10}R^4$

Analytic part – subtract threshold cuts

$$I_{an} = 1 + \frac{\alpha'^3}{3}\zeta(3)(s^3+t^3+u^3) + \frac{232\alpha'^5}{225}\zeta(2)\zeta(5)(s^5+t^5+u^5)$$

$$+\alpha'^6\zeta(3)^2\zeta(2)\left(\frac{29}{135}(s^3+t^3+u^3)^2 + \frac{23}{180}(s^2+t^2+u^2)^3\right) + \ldots$$

$D^{12}R^4$

- NO s^2 or s^4 terms.
- NO D^4R^4 or D^8R^4 terms !

Schematically :

MBG,Vanhove hep-th/9910056;
MBG, Russo,Vanhove hep-th/06?????

Toroidal world-sheet complex structure τ

$$I(s;t;u) = \int_F \frac{d^2\tau}{\tau_2^2}\sqrt{\tau_2}\,\Gamma(1;1)\int\prod_{i=1}\frac{d^2\nu_i}{\tau_2}\exp\Big(\sum_{ij}k_i D(\nu_i-\nu_j)k_j\Big)$$

Lattice of KK charges and winding nos.
- compactification on circle radius r

Scalar propagator $D(\nu_i-\nu_j)$

Expand in powers of $s;t;u$

$$I(s;t;u) \sim \sum_n \int (k_i\cdot k_j D_{ij})^n$$

Integrate over positions ν_i and over τ

- "Unfolding trick" gives integration over rectangle.
- Pick out term that behaves as 1/r ~ r and survives in ten dimensions.

Integrals are expressed in terms of harmonic sums

(thanks to Don Zagier)

$$S(m;n) = \sum_{k_1,\dots,k_m \neq 0} \frac{(-1)^{\left(\sum_{1\le i\le m} k_i\right)}}{|k_1 \cdots k_m|\,(|k_1| + \cdots + |k_m|)^n}$$

$$= m! \sum_{\substack{a_1,\dots,a_r \in \{1,2\} \\ a_1+\cdots+a_r = m-2}} 2^{2(r+1)-m-n}\, \zeta(n+2; a_1; \cdots; a_r)$$

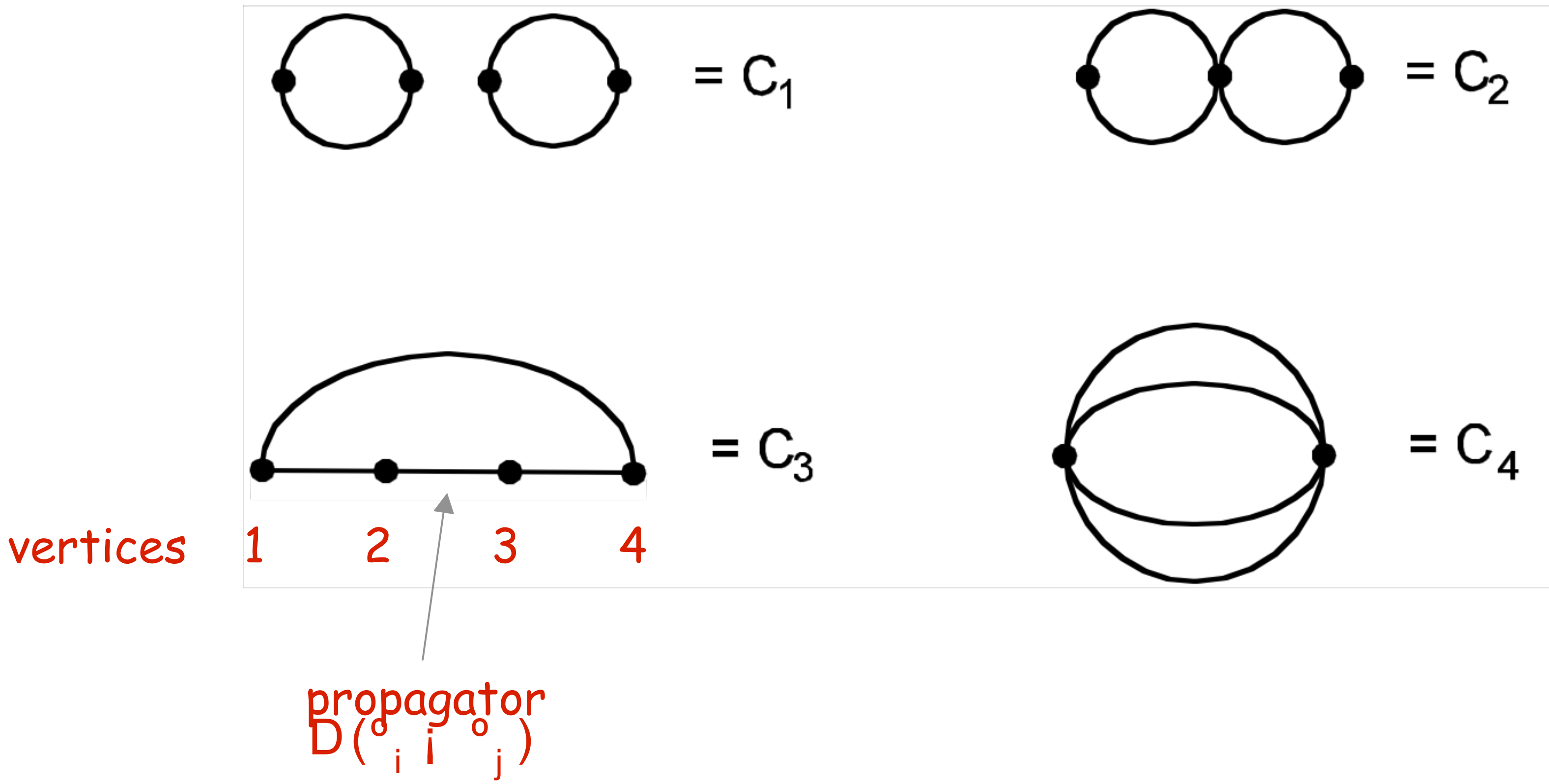
where ζ are Multiple Zeta Values :

$$\zeta(s_1; \cdots; s_r) = \sum_{n_1 > n_2 > \cdots > n_r \ge 1} \frac{1}{n_1^{s_1} \cdots n_r^{s_r}}$$

Reduce to polynomials in zeta functions at least up to depth 14 ??

e.g. $$\zeta(3;1;1;1) = \frac{3}{4}\zeta(6) - \frac{1}{2}\zeta(3)^2$$

For example, at order $s^4 R^4$



Rapidly escalating series of diagrams –

up to s^6R^4 (but all orders calculation within sight?).

What is the non-perturbative completion ??

SL(2,Z) - invariant effective IIB action

Einstein-Hilbert

Higher-derivative interactions

$$®0^{4} S = S^{(0)} + ®0^{3} S^{(3)} + ®0^{5} S^{(5)} + ®0^{6} S^{(6)} + ®0^{7} S^{(7)} + :::$$

SL(2,Z) - invariant effective IIB action

(string frame)

Einstein-Hilbert   **Higher-derivative interactions**

$$\alpha'^4 S = \int d^{10}x \sqrt{g}\left(e^{-2\phi} R + \alpha'^3 e^{-\phi/2} Z_{\frac{3}{2}} R^4 + \alpha'^5 e^{\phi/2} Z_{\frac{5}{2}} D^4 R^4 + \dots \right)$$

note $\tau_2 \equiv e^{-\phi}$

$$Z_s = \sum_{(m,n)\neq(0,0)} \frac{\tau_2^{s}}{|m+n\tau|^{2s}} \qquad \text{SL(2,Z)-invariant}$$

$$\sim 2\zeta(2s)\tau_2^{s} + (\dots)\zeta(2s-1)\tau_2^{1-s} + \sum_{k\neq 0}\mu(k;s)\left(e^{2\pi i k\tau} + c.c.\right)\left(1 + O(\tau_2^{-1})\right)$$

TREE-level terms   **Genus-**$(s-\frac{1}{2})$ term   **D-INSTANTON** terms with pert. corrections

Non-renormalization at higher genus

Laplace equation on moduli space

$$\nabla^2_{-} Z_s = s(s-1) Z_s$$

Consequence of supersymmetry

invariance of action

$$(\delta^{(0)} + \alpha'^3 \delta^{(3)} + \dots)(S^{(0)} + \alpha'^3 S^{(3)} + \dots) = 0$$

on-shell algebra

$$[\delta;\delta]\epsilon = [\delta^{(0)} + \alpha'^3 \delta^{(3)} + \dots ; \delta^{(0)} + \alpha'^3 \delta^{(3)} + \dots]\epsilon$$

$$= -\,\cdot P\,\epsilon + \epsilon \text{ eqn. of motion}$$

But this is difficult to implement generally in absence of off-shell superspace formalism.

What is the complete list of order $1/\alpha'$ interactions ??

Partial list implied by linearized SUSY

Constrained scalar superfield : ©(x; µ)

(Howe, West)

- + µ¤ + µ¡ ¹ º ½µ G¹ º ½ + µ¡ ¹ º ½ ¡ ½ D¹ ª º + µ¡ ¹ º ½µ µ¡ ¿¾ µ R¹ º ¿¾ + ¢¢¢

	Dilatino	Third-rank field	Gravitino	Weyl tensor	
2	3=2	1	1=2	0	$q_{U(1)}$

R- charge

Interactions : Z d16µ d10x F [©(x; µ)]

e.g.

$$\int d^{10}x \; \frac{@^{16}}{@\bar{-}^{16}} F \Big|_{-=-_0} \; ¤^{16} \; ; \qquad \int d^{10}x \; \frac{@^{4}}{@\bar{-}^{4}} F \Big|_{-=-_0} \; R^4$$

$$F_{¤^{16}} = \frac{@^{12}}{@\bar{-}^{12}} F_{R^4} \Big|_{-=-_0}$$

$$- \; ! \; \frac{a+b-}{c+d-}$$

Nonlinear - SL(2,Z) covariant :

$$\int d^{10}x \, f^{(0;0)}(-,\bar{-}) \, R^4 \qquad \int d^{10}x \, f^{(12;¡12)}(-,\bar{-}) \, ¤^{16}$$

$$f^{(0;0)} = Z_{3=2} \qquad q_{U(1)} = 24$$

$f^{(w;w^0)}$ transforms with holomorphic and antiholomorphic weights $(w; w^0)$

$$f^{(w;w^0)} \; ! \; f^{(w;w^0)} (c+d-)^{w} (c+d\bar{-})^{w^0}$$

Covariant derivatives:

$$D_w f^{(w,-w)} = f^{(w+1,-w-1)} \qquad \bar{D}_w f^{(w,-w)} = f^{(w-1,-w+1)}$$

$$\overset{\lambda^{16}}{\downarrow} \qquad\qquad \overset{R^4}{\downarrow}$$

$$f^{(12,-12)} = (D)^{12} f^{(0,0)} = \sum_{(m,n)\neq(0,0)} \frac{\tau_2^{3/2}}{|m+n\tau|^3}\left(\frac{m+n\bar{\tau}}{m+n\tau}\right)^{12}$$

Again contains tree-level, genus-one and D-instanton terms

Similarly for many other interactions at order $1/\alpha'$.

BUT
Complete list of interactions requires full `nonlinear' SUSY analysis - Challenging!

Higher derivative interactions ??

Impose M-theory/String Theory duality on
Eleven-Dimensional Supergravity

Duality with 11-dimensonal M-theory

CLASSICALLY: M-theory on T^2 is dual to type II on a circle - radius r_A or r_B

(Schwarz, Aspinwall, Witten)

$$r_B = \frac{1}{R_{10}\sqrt{R_{11}}} = r_A^{-1}$$

Torus volume:

$$V = R_{10} R_{11} = \exp\left(\frac{1}{3}\phi_B\right) r_B^{-\frac{4}{3}}$$

Complex structure: $\tau = \tau_1 + i\tau_2 =$ Complex IIB coupling

$$\tau_1 = C^{(0)} = C_9^{(1)}; \qquad \tau_2 = \frac{R_{10}}{R_{11}} = \exp\left(-\phi_B\right) = r_A \exp\left(-\phi_A\right).$$

Type IIB in d=10: $V \to 0$ ⟶ $r_B \to \infty$

Type IIA in d=10: $R_{10} \to \infty$ ⟶ $r_A \to \infty$

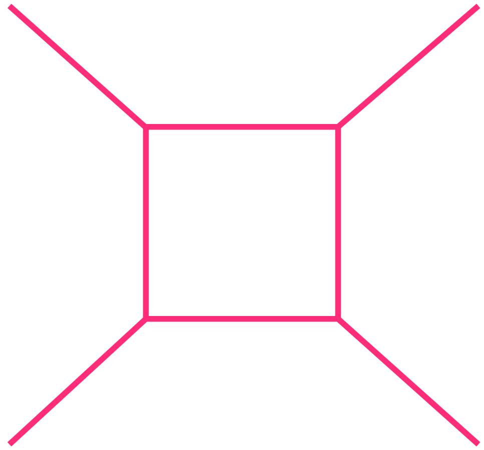
What about Quantum Effects ??

ONE LOOP in 11 dimensions on T^2

Sum all Feynman diagrams:

$\hat{R}$ » k k h

$\hat{R}^4$



Box diagram of
SCALAR field theory

Sum over windings of loop around $\hat{m}, \hat{n}$ cycles of T^2
Winding numbers

¤ 3 V

$\hat{m} = \hat{n} = 0$

divergence in zero winding number sector
suppressed in limit V ! 0

$V^{¡\frac{1}{2}}$

from non-zero windings

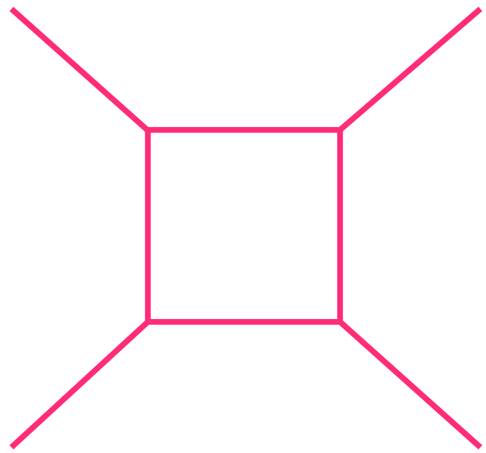
What about Quantum Effects ??

ONE LOOP in 11 dimensions on T^2

Sum all Feynman diagrams:

$\hat{R} \sim k\,k\,h$

$\hat{R}^4$



Box diagram of **SCALAR** field theory

(i) LOW ENERGY $S, T, U \to 0$

$$A = \sum_{(\hat{m},\hat{n}) \neq (0,0)} \frac{\tau_2^{\frac{3}{2}}}{|\hat{m} + \hat{n}\tau|^3} V^{-\frac{1}{2}} \hat{R}^4 = Z_{\frac{3}{2}} V^{-\frac{1}{2}} \hat{R}^4$$

TEN-DIMENSIONAL IIB limit:

$$\xrightarrow[V\to 0]{} \frac{1}{\alpha'} \int d^{10}x \sqrt{-g_B}\, R^4\, e^{-\frac{1}{2}\phi_B}\, Z_{\frac{3}{2}}(\tau,\bar{\tau})$$

$Z_{\frac{3}{2}}(\tau,\bar{\tau})$ contains TREE - LEVEL and GENUS-ONE perturbative terms together with non-perturbative D-instantons

D-instanton contributions correspond beautifully, via AdS/CFT, to the instanton contributions in N=4 SU(N) Yang-Mills theory.

(ii) HIGHER ORDERS in S, T, U :

Infinite series of terms in IIA limit: $(r_A \to \infty)$

$$c_h \, e^{(h-1)\phi_A} D^{2h} R^4$$

string genus, h

Finite coefficients c_h determined **exactly** by one-loop supergravity (see later)

(no contribution from L > 2 loops)

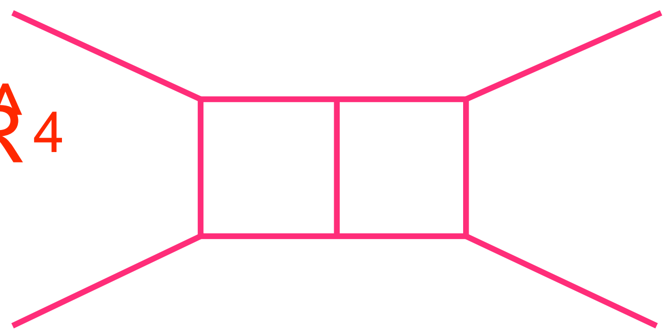
HIGHER LOOP 11-DIM. SUGRA

(Bern, Dixon, Dunbar, Perelstein, Rozowsky)

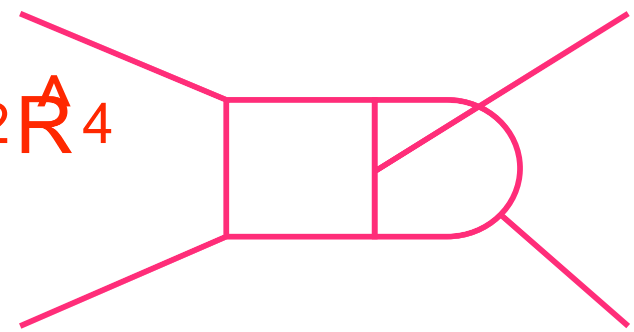
- TWO LOOPS – factor out overall D^4R^4

  resulting in **scalar** field theory diagrams

$S^2\hat{R}^4$



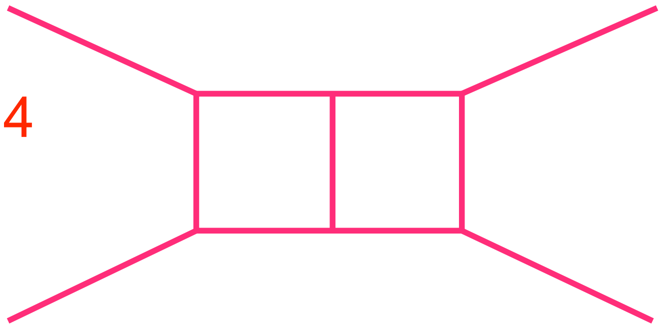
$S^2\hat{R}^4$



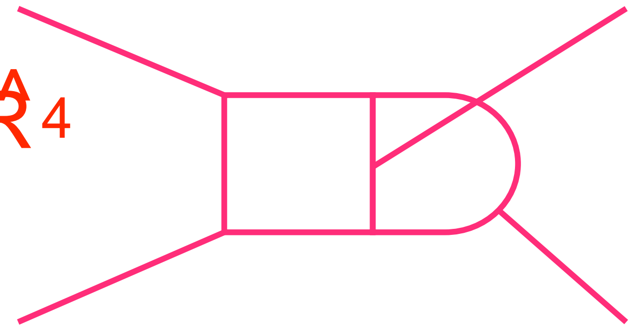
\+ T and U diagrams

TWO LOOPS on T^2

$S^2\hat{R}^4$



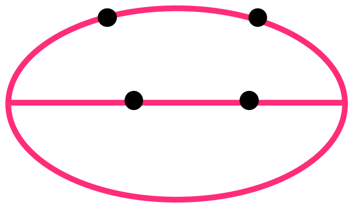
$S^2\hat{R}^4$



+ T and U diagrams

Each loop winds around either cycle:

Winding numbers $\hat{m}_1, \hat{m}_2, \hat{n}_1, \hat{n}_2$



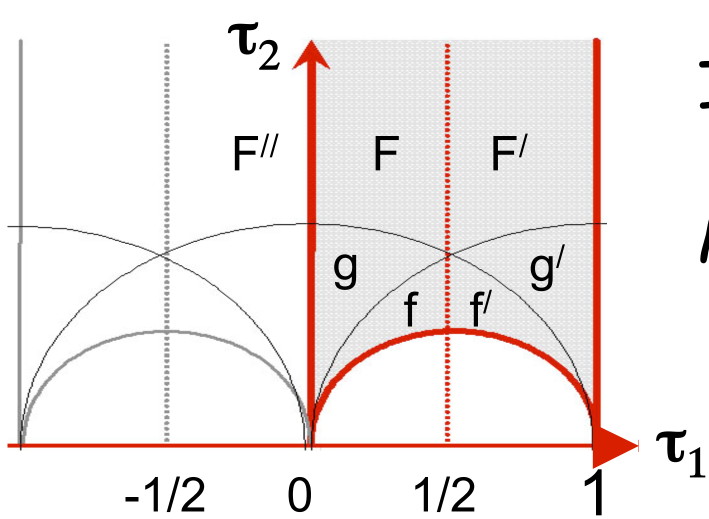
Integration over three Schwinger parameters

½ , ¸ , ¾ ! V , ¿$_1$, ¿$_2$

where ¿$_1$ = ½ / (½+¸) ; ¿$_2$ = p ¢ / (½+¸) ; V = p ¢

¢ = ½¸ + ½¾ + ¸¾

c.f. Complex structure and volume of torus !



Integrate over 3 fundamental domains

Map to F + F$^{//}$ = F

Low energy limit: S; T; U ! 0

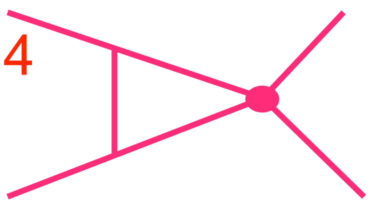
$G_{IJ} m_I m_J = V \frac{|m_1 + m_2 - |^2}{-_2}$

Torus metric

$A = D^4 \hat{R}^4 \sum_{\{m_I ; n_I\}} \int_0^1 dV V^3 \int_F \frac{d^2 ¿}{¿_2^2} e^{i \, ¼ V \frac{G_{IJ}}{¿_2} [(m + ¿ n)^I (m + \bar{¿} n)^J]}$

viz: mapping of world-sheet torus into target space

S2 $\hat{R}^4$



¤3

Naïve divergence

subdivergence proportional to $\int_{\frac{5}{2}}$

$[¤]^{20} » ¤^3 V^{i \frac{5}{2}} D^4 R^4$

$[¤]^5$ $[¤]^{12}$

Counterterm determined by 1-loop result

M-theory on T^2

$$S_{D^4R^4} = \int d^9x \sqrt{-G}\, V\, D^4R^4\, V^{-\frac{5}{2}}\, Z_{\frac{5}{2}}(\Omega, \bar{\Omega}) + O(V)$$

Ten-dimensional IIB

$$\xrightarrow[V \to 0]{} \alpha'^{0} \int d^{10}x \sqrt{-g_B}\, D^4R^4\, e^{\frac{1}{2}\phi_B}\, Z_{\frac{5}{2}}(\Omega, \bar{\Omega})$$

$Z_{\frac{5}{2}}(\Omega, \bar{\Omega})$ contains TREE - LEVEL and GENUS-TWO
coinciding with recent two-loop string results
terms together with non-perturbative D-instantons.

* * Absolute normalization fixed.

(MBG, P.Vanhove, hep-th/0510027)

D^6R^4 interaction

Expand two-loop supergravity amplitude to next order in S, T, U.

Contribution to effective action (in $V \to 0$ limit)

$$S_{D^6R^4} = \alpha'^2 \int d^{10}x \sqrt{-g}\, e^{\phi}\, E_{(\frac{3}{2},\frac{3}{2})}(\Omega,\bar\Omega)\, D^6R^4$$

Recall tree-level $\zeta(3)^2$ - no longer Eisenstein series.

Expansion of two-loop 11-dim. supergravity to order D^6R^4

$$A = D^6 \hat{R}^4 \sum_{(m_I;n_I)} \int_0^\infty dV\, V^2 \int_F \frac{d^2\tau}{\tau_2^2} H(\tau,\bar\tau)\, e^{-\pi V G_{IJ}[(m+\tau n)^I (m+\bar\tau n)^J]/\tau_2}$$

$e^{-\pi E}$

$$= D^6 \hat{R}^4 V^{-3} E_{(\frac{3}{2},\frac{3}{2})}(\Omega,\bar\Omega)$$

prefactor satisfies

$$\tau_2^2(\partial^2_{\tau_1} + \partial^2_{\tau_2}) H = 12 H - 12\, \tau_2\, \delta(\tau_1)$$

Apply Laplace operator

$$\Delta_\Omega = \Omega_2^2\, \partial_\Omega \partial_{\bar\Omega}$$

- Use $\Delta_\Omega e^{-\pi E} = \Delta_\tau e^{-\pi E}$

- Integrate by parts to show that $E_{(\frac{3}{2},\frac{3}{2})}$ satisfies a Laplace equation with source.

Laplace equation with source

$$\nabla^2_{\Omega} E_{(\frac{3}{2},\frac{3}{2})} = 12 E_{(\frac{3}{2},\frac{3}{2})} - 6 Z_{\frac{3}{2}} Z_{\frac{3}{2}}$$

$Z_{\frac{3}{2}}$ - coefficient of R^4

Analyze Fourier modes:

$$E_{(\frac{3}{2},\frac{3}{2})}(\Omega;\Omega^1) = E^{(0)}_{(\frac{3}{2},\frac{3}{2})}(\Omega_2) + \sum_{k\neq 0} E^{(k)}_{(\frac{3}{2},\frac{3}{2})}(\Omega_2)\, e^{2ik\pi\Omega_1}$$

D-instantons

Equate powers of Ω_2 on LHS and RHS of Laplace equation

Laplace equation with source

$$\nabla^2_{\Omega} E_{(\frac{3}{2},\frac{3}{2})} = 12 E_{(\frac{3}{2},\frac{3}{2})} - 6 Z_{\frac{3}{2}} Z_{\frac{3}{2}}$$

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D-instantons

tree, genus-one, genus-two

$$E^{(0)}_{(\frac{3}{2},\frac{3}{2})} = 4\zeta(3)^2\Omega_2^{3} + 8\zeta(2)\zeta(3)\Omega_2 + \frac{48}{5}\zeta(2)^2\Omega_2^{-1} + \frac{32}{63}\zeta(2)\Omega_2^{-3} + (\dots)e^{-4\pi\Omega_2}$$

genus-three - subtle!

D-instanton PAIRS, charges k, -k

Tree-level and genus-one coefficients agree with those of Type IIB string perturbation theory.

Genus-two coefficient has not yet been extracted from string theory.

Determines the 11-dimensional M-theory limit (independent of R_{11})

$$S = \mathrm{const.}\, l_{11}^3 \int d^{11}x \sqrt{-G}\, D^6 R^4$$

First contribution after R^4

Tree-level and genus-one coefficients agree with those of Type IIB string perturbation theory.

Genus-two coefficient has not yet been extracted from string theory.

Genus-three is precisely the same as the three-loop contribution to D^6R^4 in type IIA deduced by expanding eleven-dimensional **one-loop** amplitude.

From very different expression

D-instanton – anti D-instanton pairs with precisely determined coefficients.

WHAT ABOUT THREE STRING LOOPS?

Equating $\bar{\ }^{\,-\frac{3}{2}}$ terms in Laplace equation

$$(\Delta - 12)\,\bar{\ }^{\,-\frac{3}{2}} = 6 Z_{\frac{3}{2}}\, Z_{\frac{3}{2}}$$

$$0 \qquad = \qquad 0$$

homogeneous eqn.   no $^{-\frac{3}{2}}$ term

Coefficient β cannot be determined simply by matching LHS and RHS. Try different method:

Multiply Laplace eqn. by Z_4 and integrate

$Z_4 = 2\zeta(8)\tau_2^4 + \frac{5\pi}{8}\zeta(7)\tau_2^{-3} + \ldots$

$$\int_{F_L} \frac{d^2\tau}{\tau_2^2} Z_4 (\Delta E_{(\frac{3}{2},\frac{3}{2})} - 12 E_{(\frac{3}{2},\frac{3}{2})}) = -6 \int_{F_L} \frac{d^2\tau}{\tau_2^2} Z_4 Z_{\frac{3}{2}}^2 .$$

Integrating by parts and using $\Delta Z_4 = 12 Z_4$, together with the `unfolding' trick gives

$$= \frac{384\pi^2}{7} \int_0^\infty d\tau_2\, \tau_2^3 \sum_{k\neq 0} k^2 \sigma(|k|;\tfrac{3}{2})^2 K_1^2(2\pi|k|\tau_2)$$

$$= \frac{32}{7\pi^2} \sum_{k\geq 1} \frac{\sigma(k;\frac{3}{2})^2}{k^2} = \frac{16}{189}\pi^2\zeta(4)$$

Bessel fn

Ramanujan

$\sigma(k;s) = \sum_{m|k} m^{1-2s}$

This is precisely the same as the genus-three contribution to D^6R^4 in type IIA deduced by expanding eleven-dimensional ONE-LOOP amplitude !

- THREE LOOPS AND HIGHER in 11-DIM. SUGRA
  - can one factor out higher powers of D^2 ???

Many intriguing insights by
Bern, Dixon, Dunbar, Kosover, Smirnov, ...
Most complete for N=4 Yang-Mills.

Linear differential relations implied by supersymmetry

Review $O(\alpha'^3)$

(MBG, Sethi)

off-shell susy

$$\delta^{(0)} S^{(3)} + \delta^{(3)} S^{(0)} = 0$$

susy algebra

$$[\delta^{(0)}, \delta^{(3)}]\Phi = \frac{\partial S^{(3)}}{\partial \Phi}$$

eqn of motion

leads to

$$D Z^{(0,0)}_{\frac{3}{2}} = \frac{3}{2} Z^{(1,-1)}_{\frac{3}{2}}$$

$$\bar{D} Z^{(1,-1)}_{\frac{3}{2}} = \frac{1}{2} Z^{(0,0)}_{\frac{3}{2}}$$

so

$$\bar{D} D Z^{(0,0)}_{\frac{3}{2}} = \Delta_{\Omega} Z^{(0,0)}_{\frac{3}{2}} = \frac{3}{4} Z^{(0,0)}_{\frac{3}{2}}$$

Which is the Laplace equation previously motivated from eleven dimensions

At $O(®'^{6})$

susy

$$\pm^{(0)} S^{(6)} + \pm^{(3)} S^{(3)} + \pm^{(6)} S^{(0)} = 0$$

algebra

$$[\pm^{(0)};\pm^{(6)}]© + [\pm^{(3)};\pm^{(3)}]© = \frac{@S^{(6)}}{@©}$$

equation of motion

New effect:

Mixing of $S^{(3)}$ with $S^{(6)}$ leads to source term in Poisson eqn. for coefficient of D^6R^4 term.

What about higher-loop 11d supergravity??

(three or more loops)

Can higher-loop amplitudes contaminate these results?
The successes suggest they cannot.

Little known of systematics beyond two loops – but dimensional considerations are suggestive.