Non-renormalization in M-theory and Maximal Supergravity

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IS N=8 SUPERGRAVITY FINITE?

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With **Pierre VANHOVE** and **Jorge RUSSO**
hep-th/0610299, hep-th/0611213

Higher-derivative interactions ??

Yesterday's discussion:
The complete coupling dependence of type II string theory $D^{2k}R^4$ interactions up to k=3 can be obtained by compactifying one and two-loop 11-dim. Supergravity on two-torus.

Higher-derivative interactions can arise
either from higher-derivative expansion of two-loop eleven-dimensional supergravity,
or from higher-loop supergravity contributions.

We do not expect to get exact coefficients beyond low orders (what order??). Nevertheless, as far as we can determine them, we find impressive systematics in both IIA and IIB. Also generalization of IIB Poisson equations And suggestive nonrenormalization conditions in IIA.

Higher-order expansion of two-loop amplitude

Certain higher – genus terms come by expanding TWO-LOOP supergravity to higher orders in S, T, U

RECALL 11-dimensions $S = R_{11}\, s$ String Theory

$$\left(G_{10} = R_{11}^{-1}\, g_{10} \right)$$

Higher order in IIA (compactify on circle)

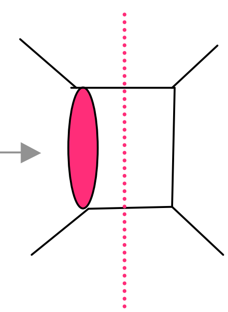
e.g., Expanding the finite (non-zero winding) contribution $D^8 R^4$
to one further power of S, T, U i.e.

Dimensionally $R_{11}^{¡4} D^8 R^4$ ´ $s^4 D^8 R^4$ » $[¤]^{20}$

Result $\frac{8½2}{45}$ ³(3) ³(1) $s^4 R^4$ genus - 1 in IIA

Represents log s - expected from unitarity
(use dimensional regularization).

Unitarity cut
contribution from tree-level ³(3) R^4 term.



No analytic $s^4 R^4$ term (cf one string loop)

Expanding to yet one further power of S :

$$\frac{1}{1620} R_{11}^{3} \zeta(3)\zeta(4) s^{5} R^{4}$$

Genus – 2 in IIA

(recall $R_{11}^{3} = e^{2\phi_A}$)

And so on .....

All orders expansion of SUGRA two-loop amplitude is now straightforward and contains rich string structure.

Higher-order in IIB: (torus volume $V \to 0$)

Suggestive generalizations of Poisson equations for higher derivative interactions. Motivated by structure of SUSY and by expansion of two-loop amplitude to higher orders.

$$S^{(k+3)} = \alpha'^{k-1} \int \sqrt{g}\, E_{(k)}\, D^{2k} R^4$$

Note: $E_{(0)} = Z_{\frac{3}{2}}$ $\quad E_{(2)} = Z_{\frac{5}{2}}$ $\quad E_{(3)} = E_{(\frac{3}{2};\frac{3}{2})}$

where $E_{(k)}$ satisfies coupled Poisson equations (schematic)

$$E_k = \sum_{i=1}^{n} E_k^i$$

$$(\nabla^2 - \lambda_i)\, E^{r_1}_{k_1} = \sum c^{r_1;r_2;r_3}_{k_1;k_2;k_3}\, E^{r_2}_{k_2}\, E^{r_3}_{k_3}$$

constants

Example:

Expansion of two loops on torus to order $s^4 R^4 \sim D^8 R^4$

Gives $1/r_B$ contribution in nine dimensions

$$\int d^9x \sqrt{-g^{(9)}}\, r_B^{-1}\, E_4(\Omega;\bar\Omega)\, D^8 R^4$$

with

$$E_4(\Omega;\bar\Omega) = \sum_{i=1}^{4} E_4^i(\Omega;\bar\Omega)$$

$$\begin{aligned}
\Delta_\Omega\, E_4^1(\Omega;\bar\Omega) &= 0; \\
(\Delta_\Omega - 6)\, E_4^2(\Omega;\bar\Omega) &= 2 Z_{\frac{3}{2}} Z_{\frac{1}{2}}; \\
(\Delta_\Omega - 20);\, E_4^3(\Omega;\bar\Omega) &= 9 Z_{\frac{3}{2}} Z_{\frac{1}{2}}; \\
(\Delta_\Omega - 42)\, E_4^4(\Omega;\bar\Omega) &= 20 Z_{\frac{3}{2}} Z_{\frac{1}{2}}
\end{aligned}$$

The exact expression presumably receives contributions from higher loops but the general structure of these equations is plausibly in accord with supersymmetry and other constraints.

In ten dimensions the exact function E_4 must satisfy many constraints.

E_4 must contain genus-one and genus-two terms with correct $\log -\,^{2}$ threshold corrections (unitarity).

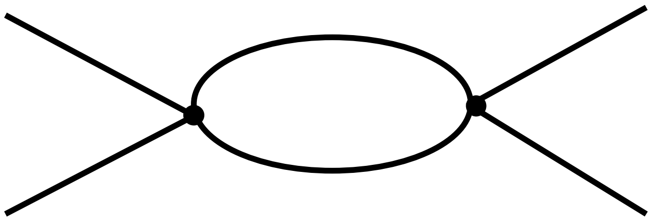
Also a genus-four term that should match IIA.

Three loops in 11 dimensions

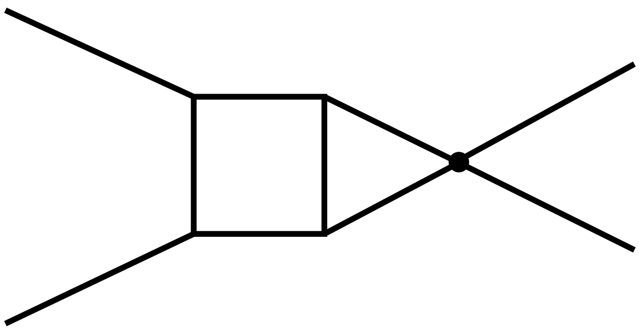
Λ^{29} naïve divergence

$\frac{1}{R_{11}^3}$ s » [¤]^3

Diagrams that contribute to tree-level IIA

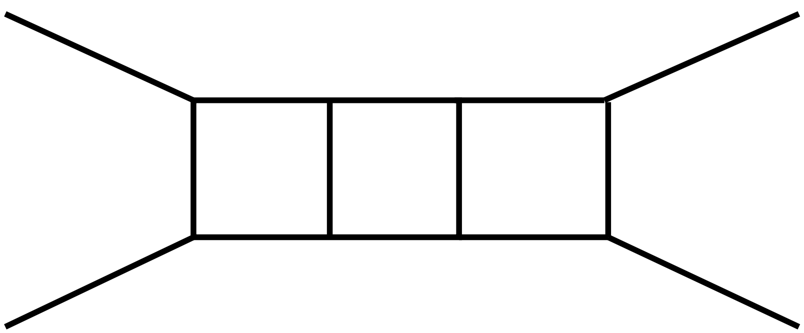


$\frac{1}{R_{11}^3}\, s^4 R^4 ¤^6$ ³(7)



$\frac{1}{R_{11}^3}\, s^5 R^4 ¤^3$ ³(3)³(5)

Explicit calculations (MBG, Russo, Vanhove).
viz. pattern of zeta functions



$\frac{1}{R_{11}^3}\, s^6 R^4$ ³(3)^3 or ³(9) ??

Represents all 3-loop diagrams

Some higher-loop counterterm diagrams

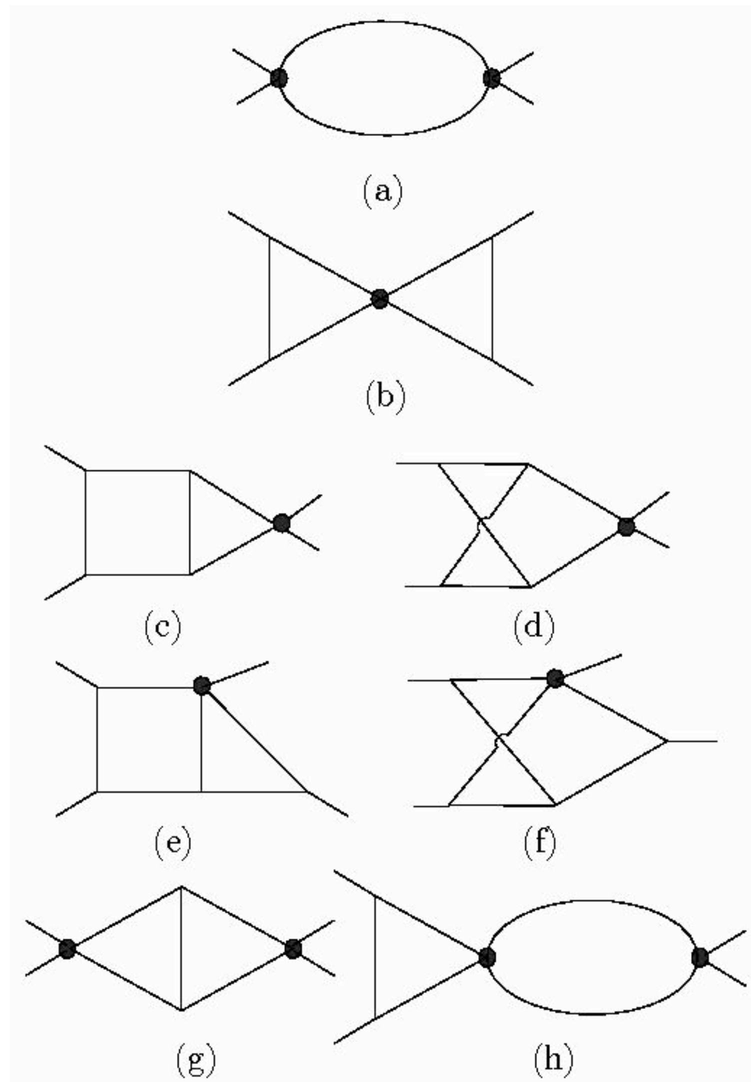
We have evaluated all these diagrams to low orders in S,T,U. Leads to a pattern of modular functions for IIB theory.

(a) Contributes to $Z_{\frac{7}{2}} D^8 R^4$

(e) and (f) contribute to $D^6 R^4$
BUT in NINE dimensions.
Does not contaminate two-loop results.

(b)-(d), (g), (h) are either $D^{10} R^4$ or $D^{12} R^4$

NO contributions to $D^4 R^4$



- How far can we determine the complete four-graviton effective action ??

  Complete nonlinear Poisson equation ??

- Generalizations – e.g R^N, …… ??

  Supersymmetric completion ??

What about higher-loop 11d supergravity??

(MBG, J.Russo, P.Vanhove hep-th/0610299) (general loops)

Little known of details beyond two loops – but duality with string theory provides important constraints.

GENUS-L 11- dim. SUGRA on circle

Non-renormalization conditions for $D^{2k}R^4$:

i) Four-graviton amplitude in 11-dim. SUGRA compactified on circle of radius R_{11}, L loops, momentum cutoff cancelled by counterterms

$$S_L^{(3+\beta_L)} = \ell^{9L-6-2\beta_L} \int d^{11}x \sqrt{-G^{10}}\, R_{11}\, D^{2\beta_L} \hat{R}^4$$

$\beta_1 = 0; \ \beta_2 = 2;$

$\beta_L \geq 2 \ (L > 2)$

ii) Expanding to higher powers of S,T,U : (powers of $R_{11}^2 D^2$)

$$S_L^{(3+\beta_L+v)} = \ell^{9L-6-2\beta_L-w} \int d^{10}x \sqrt{-G^{(10)}}\, R_{11}^{1-w} (R_{11}^2 D^2)^v D^{2\beta_L} \hat{R}^4$$

$w \geq 0$ (sub)divergence

In terms of Type IIA string variables

using $R_{11}^3 \to e^{2\phi_A} \quad G_{\mu\nu} \to R_{11}^{-1} g_{A\,\mu\nu}$

$$S_L^{(k+3)A} \sim \alpha^{9L-6-2q-2k} \int d^{10}x \sqrt{-g_A}\, e^{2(h-1)\phi_A} D^{2k} R^4$$

$q = \frac{w}{2} - v \qquad k = \frac{-}{L} + v$

Where string genus $h = k + 1 - \frac{2-}{3L} - \frac{w}{3}$

Assume $h \in Z$ and $0 \le h$

$(w \ge 0)$

leading to strong bound:

$$h < k - \frac{1}{3}$$

(when $L > 1$)

$$h = k$$

(when $L = 1$)

Lift to eleven dimensions:

Only terms independent of R_{11} survive $R_{11} \; ! \;\; 1$ limit

Selects only terms with $k = 3h \; ¡ \;\; 3$

e.g. $R^4 \; ; \qquad D^6 R^4 ; \qquad D^{12} R^4 \; ; \qquad : : :$

IIA genus $h = 1 \qquad h = 2 \qquad h = 3$

(also $D^{6n} R^{4+3m}$ and other interactions of same dim.)

Two main consequences

i) Non-renormalization of $D^{2k}R^4$ beyond

h loops in IIA string theory

(cf Up to k=5 by Berkovits `F-terms')

ii) $h = k$ term is given **exactly** by L=1

ONE - loop eleven-dimensional SUGRA.

Possible implications for supergravity:

- Genus-h string amplitude

$$A_4^h = \alpha'^{\beta_h - 1} e^{2(h-1)\phi} \sum_i S_i^{(\beta_h)} I_i^{(h)}(\alpha' s, \alpha' t, \alpha' u) R^4$$

$S^{\beta_h} \sim s^{\beta_h} + t^{\beta_h} + \ldots$

- If $\beta_h = h$ (ie $k \geq h$)

Low energy: $$A_4^{(k+3)} \sim \alpha'^{h-1} e^{2(h-1)\phi} (s^h + \ldots) R^4$$

- Low-energy field theory limit has factor of $D^{2h} R^4$ outside loop integral.

  - i.e., string duality implies subtle cancellations so loops are much more convergent

- Reduces degree of divergence of low energy supergravity

The supergravity limit

In supergravity we hold Newton's constant

$\kappa_{10}^2 = \alpha'^4 e^{2\phi}$ fixed and take limit $\alpha' \to 0$

Loop amplitude in ten dimensions

$$A_4^h \approx \kappa_{10}^{2(h-1)} \alpha'^{3-4h+\beta_h} S(\beta_h)\,(1 + O(\alpha' s))\, R^4$$

α' is interpreted as UV cut-off, $\alpha' = \Lambda^{-2}$

Now consider dimensional reduction to d dimensions.

Compactification of string theory on a torus

Consider a (10-d)-torus with all radii equal to R and fix the d-dimensional Newton constant

$$\kappa_d^2 = \alpha'^4 R^{d-10} e^{2\phi}$$

In order to decouple Kaluza-Klein modes and winding Modes (as well as massive string modes)

$$\alpha' \to 0; \quad R \to 0; \quad \alpha'/R \to 0$$

so we may set $R = r\sqrt{\alpha'}$

Genus-h loop amplitude behaves as

$$A_4^d \sim \kappa_d^{2(h-1)} \ell^{(d-2)h-2\beta_h-6} s^{\beta_h} R^4$$

Ultra-violet finite if $$(d - 2)h - 2\beta_h - 6 < 0$$

$(h > 1)$

i.e., IF

(i) $\beta_h = 2$ all h (conservative) (Bern, Dixon, Dunbar, Perelstein, Rozowsky)

Finite when $d < 2 + \frac{10}{h}$ (up to five loops)

(ii) $\beta_h = h$ for $h = 1; 2; 3; 4; 5$ (Berkovits F-term analysis)

$\beta_h = 6$ $h > 5$

Finite when $d < 2 + \frac{18}{h}$ (up to EIGHT loops)

F-term analysis generalizes to M-graviton scattering $M < 16 - 2h$

(ii) $\bar{h} = h$ All loops

Finite when $d < 4 + \frac{6}{h}$ $(h > 1)$

cf N=4 Yang-Mills

(Bern, Dixon, Dunbar, Perelstein, Rozowsky)

Finite in four dimensions all loops

Also generalizes to M-graviton scattering

BUT !!

There are two possible problems, which might well be related.

1) The functions $I^{h}_{i}(\alpha' s, \alpha' t, \alpha' u; R)$ might have poles which were assumed to be absent.

They are almost certainly absent up to genus-5 (Berkovits).

If present, they would cancel powers of s,t,u, and lead to UV divergences.

Can we deduce the occurrence of such poles from more careful consideration of string theory ??

2) Rexamine the limit to N=8 supergravity

In addition to the perturbative supergravity states there are various types of `non-perturbative' states (Black Holes) in four-dimensional supergravity.

These can be seen by compactifying string theory on a 6-torus to d=4, leading to
Wrapped p-branes with p=1, ..., 6, Kaluza-Klein charges and Kaluza-Klein monopoles.

In the low energy limit, with fixed four-dimensional Newton constant towers of such states necessarily become massless.

This suggests that perturbative N=8 supergravity may be \`incomplete' since it does not take into accout the effect of such massless \`non-perturbative' states, which could have a profound effect on the supergravity perturbation expansion.

Smilar remarks were made by Hirosi Ooguri at this meeting