UV behaviour of $\mathcal{N} = 8$ supergravity: Hints from Perturbative String Theory

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The plan

- Low energy limit, divergences, cutoff, etc
- String perturbation theory: NSR vs. GS vs. Berkovits
  - NSR/GS amplitudes
  - summary of known results
- Brief outline of the Berkovits' formalism
- Nonrenormalization theorems
- Implications for 4-dimensional physics
  - dimensional reduction, subtleties, etc.

Some references:

On Loop Corrections To String Theory Effective Actions.
R.R. Metsaev, A.A. Tseytlin, Nucl.Phys.B298:109,1988.

The Geometry Of String Perturbation Theory
Eric D'Hoker, D.H. Phong, Rev.Mod.Phys.60:917,1988.

Two-loop superstrings VI: Non-renormalization theorems and the 4-point function
Eric D'Hoker, D.H. Phong, Nucl.Phys.B715:3-90,2005

N=4 Yang-Mills And N=8 Supergravity As Limits Of String Theories
Michael B. Green, John H. Schwarz, Lars Brink, Nucl.Phys.B198:474-492,1982

The one loop five graviton scattering amplitude and its low-energy limit
J.Lee Montag, Nucl.Phys.B393:337-360,1993; hep-th/9205097

Multiloop amplitudes and vanishing theorems using the pure spinor formalism for the superstring Nathan Berkovits, JHEP 0409:047,2004

Pure spinor formalism as an N=2 topological string
Nathan Berkovits, JHEP 0510:089,2005

New higher-derivative R^4 theorems Nathan Berkovits; hep-th/0609006

Quantum Properties Of Higher Dimensional And Dimensionally Reduced Supersymmetric Theories, E.S. Fradkin, A.A. Tseytlin, Nucl.Phys.B227:252,1983.

- quantum field theory from quantum string theory is nontrivial
- 10d: field theory effective action is divergent − need regulator
- Compare w/ string theory: need to identify/introduce same cutoff

1-loop:

Open: introduce cutoff in proper-time integral

Closed: built-in fundamental domain $\tau_1 \in [-\frac{1}{2}, \frac{1}{2}]$, $\tau_2 \in [\pi\alpha' \sqrt{1-\tau_1^2}, \infty)$

- Limits: low energy | string UV

	low energy	string UV
Closed:	$\alpha' \to 0$, $\kappa_{10} = g_s \alpha'^4$, Λ-fixed	α', g_s-fixed, $\Lambda \to \infty$

Not immediatey clear that the two limits commute: decoupling?

◇ Tree-level expectation: α' decouples string massive states

◇ Quantum level: $\lim_{\alpha' \to 0}$ string loops = loops of $\alpha' \to 0$ string theory

- Obviously true if low energy limit is a finite theory
- checked at 1 and 2 loops by explicit calculations; no all-loop argument

⋆ mass of string states same as cutoff $M^2 \sim \Lambda^2 \sim \frac{1}{\alpha'}$

Consequence:

1 loop : UV divergences of effective action reproduce UV divergences of low energy field theory indeed (Metsaev, Tseytlin)

$$\Gamma_{10} \propto \int d^{10}x\sqrt{g}\left[\frac{1}{\kappa_{10}^2}\left(R + \alpha'^3 R^4\right) + \Lambda^2 R^4 + \ldots\right]$$

— but finite parts may in principle differ

$L \geq 2$: differences may appear in divergent terms (don't at 2-loops)

Obvious questions:

Is it possible to argue convincingly to all loop orders that the leading UV divergence is unaffected by the potential noncommutativity of the $\alpha' \to 0$ and $\Lambda \to \infty$ limits?

◇ relation between 10d and 4d divergences? (Fradkin, Tseytlin)

◇ is it possible to find the $\alpha' \to 0$ limit of the integrand?

String theory; various formulations

NSR	GS	Berkovits (min/non-min)
world sheet susy	space-time susy	space-time susy
fields:		
X^μ, ψ^μ, b, c, β, γ +2d supergravity	X^μ, $\theta^{I\alpha}(z,\bar{z})$, ghosts +2d gravity	X^μ, $\theta^\alpha(z)$, p_α, $\tilde{\theta}^\alpha(\bar{z})$, $\tilde{p}_\alpha$ $\lambda_\alpha(z)$, w^α, $\tilde{\lambda}_\alpha(\bar{z})$, $\tilde{w}^\alpha$ $\overline{\lambda}_\alpha(z)$, $\overline{w}^{\alpha *}$, $\tilde{\overline{\lambda}}_\alpha(\bar{z})$, $\widetilde{\overline{w}}^{\alpha}$** $r_\alpha(z)$, s^{α} *, $\tilde{r}_\alpha(\bar{z})$, $\tilde{s}^\alpha$ **
bc systems, some with fractional weight	interacting CFT	bc syst., $(0,1)$ weights
geometric path integral	geometric path integral	"conformal gauge" prescription/consistency

*&** L&R (constrained) non-minimal systems; fix zero-mode issues

- become the same in light-cone gauge

- Scattering in the NSR formalism

$$\mathbf{A}_{i_1\ldots i_n} = \sum_{h=0}^{\infty} \int \frac{D(E\Omega)\ \delta(T)}{\text{Vol (Symm)}} \int DX^\mu\ V_{i_1}\ldots V_{i_n}\ e^{-I_m}$$

$$\text{Symm} = \text{sDiff}(\Sigma) \times \text{sWeyl}(\Sigma) \times \text{sU}(1)(\Sigma)$$

$$I_m = \frac{1}{4\pi} \int_\Sigma d^{2|2}\mathbf{z}\, E\, \mathcal{D}_+ X^\mu \mathcal{D}_- X^\mu \qquad E \equiv \text{sdet} {E_M}^A$$

$$X^\mu \equiv x^\mu + \theta\psi_+^\mu + \bar\theta\psi_-^\mu + i\theta\bar\theta F^\mu \quad ; \quad {E_m}^a \equiv {e_m}^a + \theta\gamma^a\chi_m - \frac{i}{2}\theta\bar\theta A {e_m}^a$$

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- Reliable superspace gauge fixing

(Verlinde, Verlinde)
(d'Hoker, Phong)

$$\mathbf{A}_{i_1\ldots i_n} = \int_{s\mathcal{M}} |dm^A|^2 \int D(XBC) \left| \prod_A \delta(\langle H_A|B\rangle)\right|^2 V_{i_1}\ldots V_{i_n}\ e^{-I}$$

$$I \equiv \frac{1}{2\pi}\int_\Sigma d^{2|2}\mathbf{z}E \left(\frac{1}{2}\mathcal{D}_+X^\mu\mathcal{D}_-X_\mu + B\mathcal{D}_-C + \bar{B}\mathcal{D}_+\bar{C}\right)$$

$$s\mathcal{M}_h \equiv \{E_M{}^A, \Omega_M + \delta(T)\}/\text{sDiff} \times \text{sWeyl} \times sU(1)\ ;\ \text{dim} = \begin{cases} (0|0) & h=0 \\ (1|0)_e \text{ or } (1|1)_o & h=1 \\ (3h-3|2h-2) & h\geq 2 \end{cases}$$

$$(H_A)_-{}^z \equiv (-)^{A(M+1)}E_-{}^M \frac{\partial E_M{}^z}{\partial m^A} = \bar{\theta}(\mu_A - \theta\chi_A)\Big|_{\text{WZ}}$$

Main issue: what is the measure on the super-moduli space?

1 loop: long history; analytic continuation & convergence

- consequences presumably incorporated in string-based method for gravity

(Bern, Dunbar, Shimada; Dunbar, Norridge,...)

- 4-graviton amplitude & $\mathcal{N}=8$ limit (Green, Schwarz)

$$\mathcal{A}_4^{1\,\text{loop}} = \Gamma_4^{(1\text{PI})} \prod_{i=1}^{4} \epsilon^{\mu_i\nu_i} \mathcal{K}_{\mu_1\mu_2\mu_3\mu_4} \overline{\mathcal{K}}_{\nu_1\nu_2\nu_3\nu_4} \qquad \mathcal{K} \propto p^4$$

$$\Gamma_4^{(1\text{PI})}\Big|_{\mathcal{F}_2} \propto \int_{\pi\alpha'}^{\infty} \frac{d\tau_2}{(\tau_2)^2} \int_0^1 \left[\prod_{i=1}^4 d\beta_i\right] \delta\Big(1 - \sum_{j=1}^4 \beta_j\Big) \times \left[\ e^{\tau_2 \Phi(p_{12},p_{23})} + e^{\tau_2 \Phi(p_{12},p_{13})} + e^{\tau_2 \Phi(p_{23},p_{13})}\right] + \mathcal{O}(\alpha')\,.$$

$$\Phi(p_{12},p_{23}) = p_{12}^2 \beta_1\beta_3 + p_{23}^2 \beta_2\beta_4 \quad ; \quad s=-p_{12}^2 \;;\; t=-p_{23}^2 \;;\; u=-p_{13}^2 \;;\; p_{ij}=p_i+p_j$$

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- 4-graviton amplitude & $\mathcal{N}=8$ limit (Green, Schwarz)

- 5-graviton amplitude (Frampton, Kikuchi, Ng; Lam, Li)

  - low energy limit ($D=10$; $\mathcal{A}_5=\Gamma_5\,\mathcal{K}_5\cdot\epsilon\cdot\overline{\mathcal{K}}_5$ $\mathcal{K}_5\propto p^5$) (Montag)

$$\begin{aligned}\Gamma_5^{(1\mathrm{PI})}\Big|_{\mathcal{F}_2} = {} & 2\pi\kappa^5\int_{\pi\alpha'}^{\infty}\frac{d\tau_2}{(\tau_2)^2}\int_0^1\Big[\textstyle\prod_{i=1}^5 d\beta_i\Big]\delta\Big(1-\textstyle\sum_{j=1}^5\beta_j\Big)\\ & \times\Big[\;e^{\tau_2\Phi(p_{12},p_{23},p_{34},p_{45},p_{51})}+e^{\tau_2\Phi(p_{12},p_{23},p_{35},p_{54},p_{41})}\\ & +e^{\tau_2\Phi(p_{12},p_{24},p_{43},p_{35},p_{51})}+e^{\tau_2\Phi(p_{12},p_{24},p_{45},p_{53},p_{31})}\\ & +e^{\tau_2\Phi(p_{12},p_{25},p_{53},p_{34},p_{41})}+e^{\tau_2\Phi(p_{12},p_{25},p_{54},p_{43},p_{31})}\\ & +e^{\tau_2\Phi(p_{13},p_{32},p_{24},p_{45},p_{51})}+e^{\tau_2\Phi(p_{13},p_{32},p_{25},p_{54},p_{41})}\\ & +e^{\tau_2\Phi(p_{13},p_{34},p_{42},p_{25},p_{51})}+e^{\tau_2\Phi(p_{13},p_{35},p_{52},p_{24},p_{41})}\\ & +e^{\tau_2\Phi(p_{14},p_{42},p_{23},p_{35},p_{51})}+e^{\tau_2\Phi(p_{14},p_{43},p_{32},p_{25},p_{51})}\Big]+\mathcal{O}(\alpha')\end{aligned}$$

$$\Phi(p_{12},p_{23},p_{34},p_{45},p_{51})=p_{12}\beta_1\beta_3+p_{23}\beta_2\beta_4+p_{34}\beta_3\beta_5+p_{45}\beta_4\beta_1+p_{51}\beta_5\beta_2$$

1 loop: long history; analytic continuation & convergence

- consequences presumably incorporated in string-based method for gravity

(Bern, Dunbar, Shimada; Dunbar, Norridge,...)

- 4-graviton amplitude & $\mathcal{N} = 8$ limit (Green, Schwarz)
- 5-graviton amplitude (Frampton, Kikuchi, Ng; Lam, Li)
  - low energy limit ($D = 10$; $\mathcal{A}_5 = \Gamma_5 \, \mathcal{K}_5 \cdot \epsilon \cdot \overline{\mathcal{K}}_5$ $\mathcal{K}_5 \propto p^5$) (Montag)

- Same UV behaviour as that of 4-graviton amplitude
  - 1PI and 1PR components (required by worldsheet duality)
  - pentagons with q^2 numerator factors but p^{10} outside
  - result assigned to cancellations due to 10d gauge invariance
  - early hint of no-triangle hypothesis at 5-pt 1-loop?
  - similar with unitarity-reconstruction of 10d integrand

(Bern, Dixon, Perelstein, Rozowsky)

- N-graviton amplitude (even spin structure) (d'Hoker, Phong;...)
  - comments on low energy limit $(D=10)$ even spin structure (no ϵ) (Montag)

- Same UV behaviour as that of 4-graviton amplitude (any helicity)
  - unclear how many momenta come out (min 8); gravity $\simeq \partial^2\phi^3$
  - Argument based on counting powers of τ_2: $\dfrac{\tau_2^N}{\tau_2^{6+(N-4)}} = \dfrac{1}{\tau_2^2}$
    - ◇ $5+(N-4)$ from rescaling of q; N from integration over z_i
  - N-gons with q^{2N-8} numerator factors $\mapsto$ boxes?
  - Perhaps an early hint of no-triangle hypothesis?
  - 1PI and 1PR components (required by worldsheet duality)
- Not clear what happens with odd spin structures (parity–odd), but field theory intuition suggests that they are less divergent

2 loops: technical complications: (..., d'Hoker, Phong,...)

- issue: local gauge slice dependence of prescriptions
- For even spin structures:
  - gauge-fixed measure $d\mu[\delta]$, independent of gauge slice
  - unique relative phases making $\sum_\delta \eta_\delta d\mu[\delta]$ a modular form
  - $\sum_\delta \eta_\delta d\mu[\delta]$ vanishes point-by-point
- explicit vanishing of $N \leq 3$-pt amplitudes for graviton multiplet
- 4-point amplitude
  - ◇ type II: nren. R^4; Heterotic: nren. F^4, $(F^2)^2$, F^2R^2 and R^4
  - ◇ type II: $\mathcal{A}_4 \propto p^{12}$ (improves older arguments that $A_4^{2-\text{loop}} \propto p^8$)
  - ◇ agrees with $SL(2,\mathbb{Z})$ duality predictions (d'Hoker, Gutperle)
- squares with explicit unitarity reconstruction of 10d integrand

$L > 2$ loops: ◇ Info unavailable; need assumptions

Review of the Berkovits' formalism

Goal: Covariantly quantize space-time susy strings in 10d

◇ main difficulty w/ GS: 2nd class constraints $\leftrightarrow$ κ symmetry

- κ symmetry $\Longrightarrow$ half of each fermion is unphysical

$\Longrightarrow$ covariance requires infinitely many ghosts $\sum_{n\geq 0}(-1)^n = \frac{1}{2}$

- Different approach:

◇ declare $Q^2 = 0$ as fundamental and search for solutions with the same "matter" fields as the GS string (perhaps in first order formalism): 10 nonchiral X-s, 16 $\theta^\alpha(z)$, $p_\alpha(z)$ and 16 $\tilde{\theta}^\alpha(\bar{z})$, $\tilde{p}_\alpha(\bar{z})$

◇ add "ghost" fields such that $(c_L^{\text{total}}, c_R^{\text{total}}) = (0, 0)$

- $c_L^{\text{matter}} = 10 - 2\times 16 = -22 \rightarrow$ e.g. 11 weight $(0, 1)$ bc systems

◇ no "regular" representation of $SO(9, 1)$ – add *nonlinear* constraints

◇ no direct relation to κ-symmetry ghosts

Projection: commuting subalgebra of 10d susy algebra & SYM eom

$$\{D_\alpha, D_\beta\} \propto (\gamma^\mu)_{\alpha\beta} P_\mu \quad \rightarrow \quad \lambda^\alpha \lambda^\beta \{D_\alpha, D_\beta\} = 0$$

"Projector": $\lambda^\alpha \lambda^\beta (\gamma^\mu)_{\alpha\beta} = 0$ – covariant constraint

- With background: $D^V_\alpha = D_\alpha + V_\alpha$

$$\lambda^\alpha \lambda^\beta \{D^V_\alpha, D^V_\beta\} \propto_{\text{lin}} \lambda^\alpha \lambda^\beta D_{(\alpha} V_{\beta)} \propto \lambda^\alpha \lambda^\beta \gamma^{\mu_1\mu_2\mu_3\mu_4\mu_5}_{\alpha\beta} \mathcal{F}_{\mu_1\mu_2\mu_3\mu_4\mu_5}$$

$\longrightarrow$ SYM equations of motion (Howe)

- Interestingly: $\lambda^\alpha = \lambda^+(1, \epsilon_{abcde} u^{bc} u^{de}, u^{ab}) = (\mathbf{1}, \bar{\mathbf{5}}, \mathbf{10})$ of $U(5) \subset SO(10)$

  $\rightarrow$ 11 independent; covariant constraint (but no covariant solution)

- use components of λ^α as the missing "ghosts"

  introduce conjugate momenta; assemble them in $w_\alpha = (\mathbf{1}, \bar{\mathbf{5}}, \mathbf{10})$

- $Q = \int \lambda^\alpha d_\alpha$ (type I/Heterotic); type II: 2 copies Q_L and Q_R

  - $d_\alpha = p_\alpha + \gamma^\mu_{\alpha\beta} \partial x_\mu \theta^\beta + \frac{1}{2} \gamma^\mu_{\alpha\beta} \gamma_{\mu\gamma\delta} \theta^\beta \theta^\gamma \partial \theta^\delta$ – ws realization of D_α

Main advantage: 32 manifest $Q^I_\alpha \mapsto$ full power of superspace

In spite of non-covariant decomposition $\lambda = (\mathbf{1}, \bar{\mathbf{5}}, \mathbf{10})$.....

- all "useful" operators are constructed out of λ
- OPE algebra is covariant
  - ◇ $N^{\mu\nu} = \lambda\gamma^{\mu\nu}w$ & $\widetilde{N}^{\mu\nu} = \tilde{\lambda}\gamma^{\mu\nu}\tilde{w}$ obey Lorentz algebra
  - ◇ $J^{\mu\nu} = \theta\gamma^{\mu\nu}p + N^{\mu\nu}$ has the same level as in NSR
- bosonization: conf gauge NSR $\leftrightarrow$ part of Berkovits' fields

• Rest: add further fields and gauge "completing" them to $SO(10)$ representations: unchanged cohomology: add quartets

$$\left.\begin{array}{ll} \delta_{\text{BRST}}\tilde{\chi} = \tilde{\gamma} & \delta_{\text{BRST}}\tilde{\gamma} = 0 \\ \delta_{\text{BRST}}\tilde{\beta} = \tilde{\mu} & \delta_{\text{BRST}}\tilde{\mu} = 0 \end{array}\right\} \quad Q \to Q + \tilde{\gamma}\tilde{\mu} \quad \text{\& reorganize} \quad Q = \lambda^\alpha d_\alpha$$

• reverse: gauge-fix to lightcone (Berkovits; Berkovits, Marchioro)

States and vertex operators: $Z^M = (X^\mu, \theta^\alpha, \tilde{\theta}^\alpha)$

$$QV = 0 \qquad \{Q, U\} = \partial V \qquad \text{impose on–shell conditions}$$

SYM : $V_{\mathsf{YM}}{=}\lambda^\alpha A_\alpha(x,\theta)$; $U_{\mathsf{YM}} = \partial\theta^\alpha A_\alpha + \Pi^\mu B_\mu + d_\alpha W^\alpha + N_{\mu\nu} F^{\mu\nu}$

sugra : $V_g{=}V_{\mathsf{YM}}\widetilde{V}_{\mathsf{YM}}$; $U_{\mathsf{g}} = U_{\mathsf{YM}}\widetilde{U}_{\mathsf{YM}} \propto e^{ikx}$

Objections:

- not quite geometric; constructed directly with gauge-fixed diffeo invariance (conformal gauge)
- missing diffeo ghosts – in particular b; $\nexists$ negative ghost nr. op.
  $\longrightarrow$ need some prescription to go to higher loops
- The minimal version has $11_L + 11_R$ chiral bosons; similar to NSR need picture changing operators to kill zero-mode integrals

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◇ genus g: $\#_0(\lambda) = 11$ and $\#_0(w) = 11g$; organize in λ^α, $N^{\mu\nu}$, J_λ

◇ e.g. $Z \propto \delta(J_\lambda)$ $J_\lambda = \lambda^\alpha w_\alpha$ & fix coefficient from $QZ = 0$ & $\partial Z = Q\Lambda$

◇ no neg. ghost # op; absorb Z in antighost: $\{Q, \tilde{b}\} = T\, Z_B$

Advantage: fields = worldsheet scalars $\mapsto$ no supermoduli space

$$\mathcal{A}_N = \int d^{3g-3}\tau \langle \Big| \prod_{j=1}^{3g-3} (\int dw_j \mu_j(w_j) \tilde{b}_{B_j}(w_j)) \times$$

$$\times \prod_{P=3g-2}^{10g} Z_{B_P}(w_P) \prod_{R=1}^{g} Z_J(v_R) \prod_{I=1}^{11} Y_{C_I}(y_I) \Big|^2 \prod_{r=1}^{N} \int dz_r U(z_r) \rangle$$

$$Z_B = B_{\mu\nu}(\lambda\gamma^{\mu\nu} d)\delta(B_{\rho\sigma}N^{\rho\sigma}) \qquad Z_J = (\lambda^\alpha d_\alpha)\delta(J_\lambda) \qquad Y_{C_I} = C_{I\alpha}\theta^\alpha \delta(C_{I\beta}\lambda^\beta)$$

schematically :
$$\tilde{b}_B = B(d^2\Pi + dN\partial\theta + N^2 + N\Pi^2)\delta(BN)$$
$$+ B^2(d^4 + d^2N\Pi + N^2\Pi^2 + N^2 d\partial\theta)\delta'(BN)$$
$$+ B^3(d^4N + d^2N^2\Pi)\delta''(BN) + B^4(d^4N^2)\delta'''(BN)$$

Related to our discussion – 0-mode counting=pointwise cancellation

- 0-, 1-, 2-, 3-point amplitudes vanish to all genera

- first contribution can be $\partial^4 R^4$ – similarly with NSR

Can one do better? $\mapsto$ Non-minimal pure spinor superstring

(Berkovits; Berkovits, Nekrasov)

Idea: Eliminate the need for picture-changing operators

i.e. eliminate zero modes

◇ λ_0^α & $\bar\lambda_0^\alpha \mapsto a_\lambda^\alpha = \lambda_0^\alpha + i\bar\lambda_0^\alpha$ & $(a_0^\dagger)^\alpha = \lambda_0^\alpha - i\bar\lambda_0^\alpha$

(Berkovits, Hatsuda, Siegel – NSR)

◇ extra fields but same spectrum $\mapsto$ quartets

- New field content:

$$\lambda^\alpha(z),\ w_\alpha\ \ \&\ \ \underbrace{\overbrace{\overline{\lambda}_\alpha(z),\ \overline{w}^\alpha}^{\text{bosons}},\ r_\alpha(z),\ s^\alpha}_{11\text{quartets}}\ \&\ \text{constraints: } \overline{\lambda}\gamma^\mu\overline{\lambda} = 0\ \ \overline{\lambda}\gamma^\mu r = 0$$

$$Q_{\text{non-min}} = \int dz\,(\lambda^\alpha d_\alpha + \overline{w}^\alpha r_\alpha)$$

Several BRST invariants:

$$\overline{N}_{\mu\nu} = \frac{1}{2}(\overline{w}\gamma_{\mu\nu}\overline{\lambda} - s\gamma_{\mu\nu}r) \qquad J_{\overline{\lambda}} = \overline{w}\overline{\lambda} - sr \qquad T_{\overline{\lambda}} = \overline{w}\partial\overline{\lambda} - s\partial r$$

$$S = s\overline{\lambda} \qquad J_r = rs \qquad \Phi = \overline{w}r \qquad S_{\mu\nu} = \frac{1}{2}s\gamma_{\mu\nu}\overline{\lambda}$$

$\longrightarrow$ no correction to level of Lorentz alg. & central charge

◇ From the existing fields construct an $\mathcal{N} = 2$ algebra w/ the same properties as that of "critical topological strings

- $U(1)$ generator may be taken $J_{\text{ghost}} = w_\alpha\lambda^\alpha - \overline{w}^\alpha\overline{\lambda}_\alpha$ up to $Q\Lambda$

- b-ghost$\equiv$ spin-2 generator of this algebra!

◇ Forget that the non-minimal fields have been added to deal with the zero-modes and use the technology developped for amplitude calculations for topological strings (Bershadsky, Cecotti, Ooguri, Vafa) (Antoniadis, Gava, Narain, Taylor)

◇ same vertex operators as before; $V_{\text{closed}} = V_{\text{open}}\overline{V}_{\text{open}}$

Non-minimal amplitude prescription: (Berkovits)

- similar to bosonic string – no supermoduli space

$$\mathcal{A}_N = \int [d^2\tau]_{3g-3} \langle \Big| \prod_{j=1}^{3g-3} \int (dy_j \mu_j(y_j) b(y_j)) \Big|^2 \prod_{r=1}^{N} \int d^2 z_r \, U(z_r) \, |\mathcal{N}|^2 \rangle$$

$$\langle\rangle \equiv \int [d\lambda][d\overline{\lambda}][dr][dw^I][d\overline{w}^I][ds^I] \sim \lambda^{3g+8}\overline{\lambda}^{11}$$

- $b(y)$–composite b-ghost, $\{Q, b\} = T$; – needs regularization

$$b = s^\alpha \partial \overline{\lambda}_\alpha + \frac{\overline{\lambda}(2\Pi d - N\partial\theta - J_\lambda \partial\theta - \partial^2\theta)}{4(\overline{\lambda}\lambda)} + \frac{(\overline{\lambda}\gamma^{mnp} r)(d\gamma_{mnp} d + 24 N_{mn}\Pi_p)}{192(\overline{\lambda}\lambda)^2}$$
$$- \frac{(r\gamma_{mnp} r)(\overline{\lambda}\gamma^m d) N^{np}}{16(\overline{\lambda}\lambda)^3} + \frac{(r\gamma_{mnp} r)(\overline{\lambda}\gamma^{pqr} r) N^{mn} N_{qr}}{128(\overline{\lambda}\lambda)^4}$$

- $\mathcal{N} = e^{\{Q,\chi\}}$–regularization factor for zero-mode integrals
  - formal proof that amplitude is independent of choice of χ
  - pick: $\mathcal{N} = e^{-\overline{\lambda}_\alpha \lambda^\alpha - r_\alpha \theta^\alpha - \frac{1}{2} N^I_{\mu\nu} \overline{N}^{\mu\nu}_I - J^I \overline{J}_I - (\overline{\lambda} s^I)(\lambda d^I) - \frac{1}{4}(s^I \gamma^{\mu\nu} \overline{\lambda})(d^I \gamma_{\mu\nu} \lambda)}$
  - may contribute θ zero-modes

Nonrenormalization theorem for 4-graviton amplitude:

$U_{\text{closed}} = U_{\text{open}}\overline{U}_{\text{open}} \qquad U_{\text{open}} = A_M \partial Z^M + W^\alpha d_\alpha + F_{\mu\nu} N^{\mu\nu}$

Idea: ⋆ naive counting of zero-modes (no regularization)

⋆ reliable as long as there is no need for regularization

chiral zero-mode census:

d_α	θ^α	r	s	λ	w	$\overline{\lambda}$	$\overline{w}$
$16g$	16	11	$11g$	11	$11g$	11	$11g$

their origin:

	d_α	θ^α	r	s
U	4	$2g+4$	–	–
$\mathcal{N}$	$11g$	$12-2g$	$12-2g$	$11g$
b	$5g-4$	N/A	$2g-1$	–

◇ the logic:

min. $\#\partial \mapsto$ min. $\#\theta$ from $U \mapsto$ max. $\#\theta$ from $\mathcal{N} \mapsto$

$\mapsto$ max. $\#r$ from $\mathcal{N} \mapsto$ min. $\#r$ from b

Consequences for the structure of amplitudes/effective action:

- Gravity: $(\partial_\theta \partial_{\tilde{\theta}})^{2g+4} W^4_{\text{grav}} \sim \partial^{2g} R^4$
- SYM: $\partial_\theta^{2g+4} W^4_{\text{YM}} \sim \partial^g F^4$

pole: $\dfrac{\lambda^{3g+8}\overline{\lambda}^{11}}{\overline{\lambda}^{2g-1}\lambda^{5g-4}} \mapsto g < 6$

Nonrenormalization theorem for 4-graviton amplitude:

Idea: $\star$ naive counting of zero-modes (no regularization)
$\star$ reliable as long as there is no need for regularization

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- Gravity: $(\partial_\theta \partial_{\tilde\theta})^{2g+4} W^4_{\text{grav}} \sim \partial^{2g} R^4$
- SYM: $\partial_\theta^{2g+4} W^4_{\text{YM}} \sim \partial^g F^4$

pole: $\frac{\lambda^{3g+8}\bar\lambda^{11}}{\bar\lambda^{2g-1}\lambda^{4g-4}} \mapsto g < 6$

Curiously – naive application of chiral counting leads to discrepancy

Explicit calculations of the 3-loop SYM integrand (Bern, Dixon, Smirnov)

$$\Gamma^{3-\text{loops}}_{d=10} \propto \partial^2 F^4$$

$\diamond$ Puzzling – zero-mode counting implies pointwise cancellation

component formalism: dimension-dependent cancellations?

Relations to 4d physics

- Compactification on T^6 with $R_i^2 \gg \alpha' \to 0$
  - distinguish integrand/amplitude from effective action

v1: compute/define string theory on $\mathbb{R}^{1,3} \times T^6$

v2: compute/define string theory on $\mathbb{R}^{1,9}$ & "compactify" the result

- Subtleties with v2:

2 steps:
- relate regulators – for proper-time cutoffs relate cutoffs
- relate higher-dim and 4-dim fields

Compare 1-loop effective actions (Fradkin, Tseytlin)

$$\Gamma_d = -\left[\frac{1}{d}\Lambda^d \mathcal{B}_0 + \cdots + \frac{1}{d-p}\Lambda^{d-p}\mathcal{B}_p + \frac{1}{2}\ln\frac{\Lambda^2}{\mu^2}\mathcal{B}_d + \text{finite}\right]$$

$\Lambda_4^4 = \Lambda_d^d \mathrm{Vol}(T^{d-4}) \mapsto$ 4d log divergences are related to Λ^{d-4} (SYM)

– presumably captures some differences d-dim and 4-dim q-integrals

$\mathcal{B}$ $\sim$heat kernel coeff's; differ due to changes in the kinetic operator

If $A_{(d)} = (A_{(4)}, 0)$ and $g_d = \begin{pmatrix} g_{(4)} & 0 \\ 0 & \mathbb{1}_{d-4} \end{pmatrix}$ then $\mathcal{B}_p(\Delta_d) = \mathcal{B}_p(\Delta_4)$

◇ reduced theory may have divergences that break d-dim symmetries

- maximal susy glues all sectors

Higher loops:

$$\Gamma_d \sim \sum_{l=0}^{N} \hbar^l \sum_{k=1}^{l} \left(\sum_{p=0}^{d-1} \Lambda^{d-p} \mathcal{B}_p^{l,k} + \ln \frac{\Lambda^2}{\mu^2} \mathcal{B}_d^{(l,k)} \right)^k$$

$$\left\{ \Lambda^{d-4+k} \sum_{r<s} \alpha_r \left(\ln \frac{\Lambda^2}{\mu^2} \right)^r \right\}_{d-\mathrm{dim}} \mapsto \left\{ \Lambda^k \left(\ln \frac{\Lambda^2}{\mu^2} \right)^s \right\}_{4-\mathrm{dim}}$$

◇ seems unclear if there is an algorithmic way of relating counterterms of d and 4-dim theories if we start with full d simmetry

- potential dimension-dependent cancellations

Conclusion

△ "Reports of the death of supergravity are exagerations."

Stephen Hawking

△ P. string theory hints that $\mathcal{N} = 8$ sugra UV behaviour is better

△ Some hints are recovered from field theory considerations

△ ... but some are (much) better

- Subtleties remain in relating string and sugra EA, 10d & 4d, etc
- If $\mathcal{N} = 8$ supergravity is finite, what is its relation to string theory?

  - e.g. twistor/topological string interpretation? (Abu-Zeid, Hull, Mason)

  - what does one need to impose to reconstruct string theory?

- N-graviton amplitude (even spin structure) (d'Hoker, Phong;...)

$$\mathcal{A}_N^{1\,\text{loop}} = \frac{\kappa^N}{2^{10}(\alpha')^{12-N}} \int_{\mathcal{F}} \frac{d^2\tau}{\tau_2} \int \left[\prod_{i=1}^N \epsilon_i^{\mu_i\nu_i} d\overline{\eta}_{i\mu_i} d\eta_{i\nu_i} d^2\theta_i d^2 z_i\right]$$
$$\times \exp\left[-\frac{i\pi\alpha'}{2}\sum_{ij}{}' p_i\cdot p_j G_{11}^{ij}\right]\int d^{10}q \exp\left[-\frac{\pi\tau_2}{4\alpha'}q^2\right]$$
$$\times \left|\sum_{a\times b=0}(-)^{a+b}\left\{\frac{\Theta_{ab}(0|\tau)}{[\eta(\tau)]^3}\right\}^4 \exp\left[\widetilde{K}_{ab}(\overline{\eta},\overline{\theta};q)\right]\right|^2$$

$$\widetilde{K}_{ab}(\overline{\eta},\overline{\theta}) = -i\pi\alpha'\sum_{ij}{}'\Big\{p_i\cdot p_j\overline{\theta}_i\overline{\theta}_j S_{ab}^{ij} + \overline{\eta}_i\cdot\overline{\eta}_j S_{ab}^{ij} - 2p_i\cdot\overline{\eta}_j\overline{\theta}_i S_{ab}^{ij}$$
$$-2p_i\cdot\overline{\eta}_j\overline{\theta}_j\left[(\partial_j G_{11}^{ij}) - \frac{1}{2i\tau_2}(z_{ij}-\overline{z}_{ij})\right] - 2\overline{\eta}_i\cdot\overline{\eta}_j\overline{\theta}_i\overline{\theta}_j\left[(\partial_i\partial_j G_{11}^{ij}) - \frac{1}{2i\tau_2}\right]\Big\}\,.$$
$$+i\pi\sum_j\left[q - \frac{i\alpha'}{\tau_2}\sum_i p_i(z_{ij}-\overline{z}_{ij})\right]\cdot\overline{\eta}_j\overline{\theta}_j$$

$$G_{11}^{ij} = -\frac{1}{2i\pi}\log\left|e^{-\pi y_{ij}^2/\tau_2}\frac{\Theta_{11}(z_{ij}|\tau)}{\eta(\tau)}\right|^2 \quad ; \quad S_{ab}^{ij}(z) = -\frac{1}{2i\pi}\frac{\Theta_{ab}(z_{ij}|\tau)\Theta_{11}'(0|\tau)}{\Theta_{ab}(0|\tau)\Theta_{11}(z_{ij}|\tau)}$$