# Non-Renormalization Theorems in Super Yang-Mills and Supergravity Theories

UCLA Workshop on Ultraviolet properties of N=8 Supergravity

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Some litterature:

Grisaru & Siegel, Nucl. Phys. B201 (1982) 292.

Howe, Stelle & Townsend
"Miraculous Ultraviolet Cancellations in Supersymmetry made Manifest" Nucl. Phys. B236 (1984) 125

Howe + Stelle
"Ultraviolet Divergences in Higher Dimensional Supersymmetric Yang-Mills Theories" Phys. Lett. 137B (1984) 175

Howe + Stelle
"The Ultraviolet Properties of Supersymmetric Field Theories" (review) Int. J. Mod. Phys. A4 (1989) 1871

## Various scenarios for ultraviolet finiteness

At a given order in a perturbation expansion, one could find

A) No supersymmetric extension of a given non-supersymmetric potential counterterm exists.
(Eg. completion of $\int d^4x \sqrt{-g}\, R_{\mu\rho\nu\sigma} R^{\rho\lambda\sigma\tau} R^{\mu}{}_{\rho}{}^{\nu}{}_{\tau}$ at $L=2$ loops in $D=4$)

B) The supersymmetric completion of a given non-supersymmetric counterterm exists, but has vanishing value when subjected to the theory's coupled classical field equations.
(Eg. $L=1$, $D=4$ $\int d^4x \sqrt{-g}\,(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - T_{\mu\nu})^2$)

i) in this case, one can remove such a divergence by field redefinition renormalizations without adding new invariants to the action.

C) "Ordinary SUSY miraculous cancellations":
a valid non-vanishing-on-shell counterterm exists, but its coefficient is zero as a result of a non-renormalization theorem.
(Eg. F term invariants like $\int d^4x\, d^2\theta\, \phi^n$, $\bar{D}_{\dot\alpha} \phi = 0$)

D) Truly miraculous cancellations

¿ $N=8$ SG in $D=4$ ?

# The UV Finiteness of N=4, D=4 SYM

The 16-supercharge N=4 SYM theory has the component action

Gliozzi, Scherk & Olive
Brink, Schwarz & Scherk

$$I = \frac{1}{g^2}\int d^4x \, \mathrm{Tr}\Big\{ -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}D^\mu\bar{\phi}^{ij}D_\mu\phi_{ij} - \tfrac{i}{2}\bar{\lambda}^i\gamma^\mu D_\mu\lambda_i$$
$$+\tfrac{1}{2}\bar{\lambda}^i[\lambda^{cj},\phi_{ij}] - \tfrac{1}{2}\bar{\lambda}^c_i[\lambda_j,\bar{\phi}^{ij}]$$
$$+\tfrac{1}{8}[\phi_{ij},\phi_{kl}][\bar{\phi}^{ij},\bar{\phi}^{kl}]\Big\}$$

$i,j = 1,\dots,4$

containing fields $A_\mu$, $\lambda_i = \gamma_5\lambda_i$, $\phi_{ij} = \frac{1}{2}\epsilon_{ijkl}\bar{\phi}^{kl}$

This theory has, at the classical level,
- manifest $SU(4)$, of which 9 generators are chiral
- 16 nonlinear Poincaré supersymmetries with an algebra closing on the standard N=4 algebra only on-shell.
- N=4 conformal supersymmetry

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There are 2 basic types of argument for the UV finiteness of the D=4 quantum theory based on the above classical N=4 action:

1) Based on anomalies and symmetry preservation

Ferrara & Zumino
Sohnius & West

2) Based on non-renormalization theorems
   - for 8 supercharge Lorentz covariant Feynman rules — Howe, Stelle & Townsend
   - for 8 supercharge light-cone Feynman rules

Mandelstam
Brink, Lindgren & Nilsson

## 1) Anomaly/symmetry arguments

- original form, based on $SU(4)$ preservation:

  – The $N=4$ theory has $SU(4)$ but not $U(4)$ susy automorphism symmetry, owing to the self-conjugacy constraint of the 6 scalars,

$$\phi_{ij} = \tfrac{1}{2}\epsilon_{ijkl}\bar{\Phi}^{kl}$$

  – Under $N=1, D=4$ supersymmetry, UV divergences giving rise to a non-vanishing beta function would cause the appearance of a whole $N=1$ supermultiplet of anomalies:

| $T^{\mu}{}_{\mu}$ | $\gamma^{\mu} S_{\mu}$ | $\partial^{\mu} j^{(5)}_{\mu}$ |
|---|---|---|
| stress tensor trace anomaly | $\gamma$-trace anomaly of SUSY current | some axial current anomaly |

  – So a non-vanishing trace anomaly would be accompanied by an anomaly in one of the 4 $SU(4)$ chiral currents.

  – However, $SU(4)$ is expected to be anomaly free
  $\Rightarrow T^{\mu}{}_{\mu} = 0$

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- Another version of this argument uses a manifestly $N=1, D=4$ supersymmetric formalism Grisaru, Roček & Siegel; Stelle

$$I = \int d^4x\, d^2\theta\, \mathrm{Tr}\left\{\frac{1}{64 g^2} W^{\alpha} W_{\alpha}\right\} + \int d^4x\, d^4\theta\, \mathrm{Tr}\left\{e^{-gV}\bar{\Phi}^i e^{gV}\phi_i\right\}$$

$$+ \frac{ig}{3!}\int d^4x\, d^2\theta\, \mathrm{Tr}\, \underbrace{\left\{\epsilon^{ijk}\phi_i[\phi_j,\phi_k]\right\}}_{\text{F term, cannot be renormalized}} + \text{c.c.}$$

$$W_{\alpha} = \bar{D}^2\left(e^{-gV} D_{\alpha} e^{gV}\right)$$

  – Assuming the 16 supersymmetries are unbroken allows only one overall wavefunction renormalization and a coupling constant ren. $g_0 = Z_g g \quad V_0 = Z^{1/2} V \quad \phi_{0i} = Z^{1/2}\phi_i$ (can also justify this by assuming $SO(4)$ unbroken)

But in the background field method, $Z_g Z^{1/2} = 1$, while from F-term non-ren $Z_g Z^{3/2} =$
$\Rightarrow Z_g = Z = 1$

The anomaly/symmetry arguments all depend on the <u>assumption</u> that certain symmetries are preserved in the quantum theory.

Clearly better is an argument that relies only on linearly realized symmetries that can be maintained manifestly in the quantization procedure.

<u>Non-renormalization theorems</u>

The basic non-renormalization theorem in supersymmetry is that for F-term invariants built from chiral superfields.

- Use the <u>background field method</u>, splitting $\phi$ into background and quantum parts:

$$\underset{\text{total}}{\phi^i} = \underset{\text{background}}{\varphi^i} + \underset{\text{quantum}}{Q^i}$$

- solve the chirality constraint on $Q$ by introducing a quantum <u>prepotential</u>

$$Q^i = \bar{D}^2 X^i \qquad \bar{D}^2 = \bar{D}^{\dot\alpha}\bar{D}_{\dot\alpha}$$

where $X^i$ has a gauge symmetry $\delta X^i = \bar{D}^{\dot\alpha}\bar{\Lambda}^i_{\dot\alpha}$ (since $\bar{D}\bar{D}\bar{D}=0$)

- Expanding the Wess-Zumino action

$$I = \int d^4x\, d^4\theta\, \bar{\Phi}_i \phi^i + \lambda_{ijk} \underbrace{\int d^4x\, d^2\theta\, \phi^i\phi^j\phi^k}_{\text{F term}}$$

into background and quantum parts, find that all terms except the pure background terms can be written as <u>full superspace integrals</u> $\int d^4x\, d^4\theta\, f(\varphi, X)$

Eg. $\int d^4x\, d^2\theta\, \varphi \bar{D}^2 X \bar{D}^2 X = \int d^4x\, d^4\theta\, \varphi X \bar{D}^2 X$

- BFM renormalizations are read from infinities involving background $\varphi^i$ only $\Rightarrow \int d^4x\, d^4\theta\, f(\varphi, \bar{\varphi})$. So F-term is not renormalized.

# SYM Non-renormalization theorems

Wess, Zumino,
Iliopoulos, Ferrara, Piguet
Grisaru, Roček & Siegel

Suppose we are given a theory with N-extended Supersymmetry (i.e. 4N supercharges) which can be quantized using M-extended superfields (i.e. 4M linearly realized supersymmetries). We do not assume M=N.

Use of the background field method in M-extended superspace leads to the following:

1) Quantum corrections to the effective action $\Gamma$ may always be expressed as a <u>full superspace integral</u> $\int d^4x\, d^{4M}\theta\ \Delta\mathcal{L}$

2) There are complications at the one-loop level arising from "ghost for ghost" problems from prepotential gauge fixing. But for $L \geq 2$ loops, things settle down.

3) For $L \geq 2$ loops, the integrand $\Delta\mathcal{L}$ must be expressed in terms of background field <u>superspace connections</u> $A_\alpha(x,\theta)$, $A_\mu(x,\theta)$.

- Inside loops, one encounters prepotentials for the quantum fields $\underbrace{a_\alpha = e^{-gV} D_\alpha e^{gV}}_{\text{e.g. for } M=1}$. But external lines are always background, and for them prepotentials are not introduced.
- This is the SYM analogue of having $\varphi$ background chiral fields appear in $\int d^4x\, d^4\theta\, f(\varphi,\bar\varphi)$ in the W-Z model.

4) Counterterms for UV divergences must be <u>local</u> in x-space. So it is illegal to try to write $\int d^2\theta\, f(\varphi)$ as $\int d^4\theta \frac{D^2}{\Box} f(\varphi)$.

<u>Upshot</u>: $L \geq 2$ counterterms that can only be written as integrations $f(A_\alpha)$ over <u>submanifolds</u> of M-superspace are <u>excluded</u>.

# N=4 SYM in M=2 superfields

Howe, Stelle, Townsend

When decomposed into M=2 multiplets, the N=4 SYM theory splits into M=2 SYM ($A_\mu, \lambda_1, \lambda_2, \phi_{12}$ (complex)) and an M=2 hypermultiplet ($\phi_{1i=3,4}$ (complex), $\lambda_3, \lambda_4$)

- For M=2 SYM, one has superspace connections

$$\mathcal{A}_A = (\underbrace{\mathcal{A}_\alpha{}^i, \bar{\mathcal{A}}_{\dot\alpha i}}_{\alpha,\dot\alpha = 1,2}, \underbrace{\mathcal{A}_\mu}_{\mu = 0,1,2,3})$$

From the superspace covariant derivative $\mathcal{D}_A = D_A + ig\mathcal{A}_A$ one derives the M=2 SYM field strength

$$\mathcal{F}_{AB} = D_A\mathcal{A}_B - (-1)^{AB} D_B \mathcal{A}_A + i[\mathcal{A}_A, \mathcal{A}_B\} + t_{AB}{}^C \mathcal{A}_C$$

flat s.space torsion $\quad t_{\alpha}{}^i{}_{\dot\beta j}{}^\mu = -i\,\sigma^\mu_{\alpha\dot\beta}\,\delta^i_j$ , 0 otherwise

- Constraints : to obtain the correct component field content, the M=2 field strength is subject to

$$\mathcal{F}^{ij}_{\alpha\beta} = \epsilon_{\alpha\beta}\epsilon^{ij}\bar{W} \qquad \mathcal{F}_{\alpha}{}^i{}_{\dot\beta j} = 0$$

– the remaining field strength superfield $W$ is covariantly chiral, $\bar{\mathcal{D}}_{\dot\alpha i} W = 0$

The M=2 SYM action is naturally written as a chiral M=2 superspace integral

$$I = \int d^4x\, d^4\theta\, \mathrm{Tr}(W^2) + \text{c.c.}$$

- Note that since $W \sim \bar{D}_{\dot\alpha i}\bar{\mathcal{A}}_{\dot\beta j}$ , there is no way this action can be rewritten as a full $\int d^4x\, d^8\theta$ M=2 superspace action involving just the connections $\mathcal{A}_A$

Quantum part of $\mathcal{N}=2$ SYM connection: prepotential

linearized level: $a_\alpha{}^i = D^3_\alpha{}^i \bar{D}^{2kl} V_{kl} + \cdots$

for which the prepotential has the gauge transformation

$$\delta V_{ij} = D_\alpha{}^k \Lambda^\alpha{}_{ijk} + \cdots$$

- Nonlinear extensions of these, plus gauge-fixing procedures, attention to infrared divergence issues, ghosts, etc, all were dealt with. H.S.T.

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<u>Hypermultiplet sector</u>

A key point in formulating the N=4 SYM theory in $\mathcal{N}=2$ superfields was finding an $\mathcal{N}=2$ off-shell formulation of the hypermultiplet.

- The Fayet-Sohnius formulation in terms of an $SU(2)$ doublet superfield $\phi^i$ satisfying $D_\alpha^{(i}\phi^{j)} = \bar{D}_{\dot\alpha}^{(i}\phi^{j)} = 0$ is on-shell, i.e. the constraints imply the field equations. This formulation does not allow an off-shell extension with a finite set of auxiliary fields.
- However, the <u>relaxed hypermultiplet</u> H.S.T. does the trick: $D_{\alpha(i} L_{jk)} = D_\alpha{}^k L_{ijkl}$ $\quad D_{\alpha(i} L_{jklm)} = 0$

$$D_\alpha{}^i D_{\beta}{}^j L = [D_\alpha{}^i, \bar{D}_{\dot\alpha i}] L = 0$$

– these constraints can be solved by introducing prepotentials for the quantum fields

$$L^{ij} = \tfrac{3}{4} \bar{D}^{ij} D^{3k}_\alpha \rho^\alpha{}_k + \tfrac{1}{2} \bar{D}_k{}^{(i} D^{j)k} D_\alpha{}^l \rho^\alpha{}_l + \text{c.c.}$$

$$L^{ijkl} = \tfrac{2}{5} D^{(ij} \bar{D}^{kl)} [D_\alpha{}^m \rho^\alpha{}_m + \bar{D}^{\dot\alpha}{}_m \bar{\rho}_{\dot\alpha}{}^m]$$

$$L = D^{ij} \bar{D}^{kl} X_{ijkl} \qquad X_{ijkl} = X_{(ijkl)}$$

The free hypermultiplet action for the quantum fields is then written as a full $N=2$ superspace integral

$$I_{HM} = \int d^4x\, d^8\theta \left[ \left( L_{ij} D_\alpha{}^i \rho^{\alpha j} + c.c. \right) + L^{ijkl} X_{ijkl} \right]$$

leading to the equations of motion

$$L_{ijkl} = 0 \qquad D_{\alpha j} L^{ij} = D_\alpha{}^i L$$

which contain the usual equations of motion for the hypermultiplet component fields.

- Note: the scalars appear here as a triplet $L_{ij}|_{\theta=0}$ and a singlet $L|_{\theta=0}$ instead of a complex doublet as in the F.S. hypermultiplet.

- As for the $N=1$ Wess-Zumino multiplet, solving the off-shell constraints in terms of prepotentials introduces <u>gauge invariances</u> for the hypermultiplet prepotentials.

- These h.m. pregauge invariances lead to a non-renormalization theorem for the hypermultiplet itself:

  $I_{HM}$ cannot be written as a full $\int d^4x\, d^8\theta$ $N=2$ superspace integral involving the constrained quantities $L^{ij}$, $L^{ijkl}$, $L$ in terms of which the <u>background</u> fields must appear. ("No background prepotentials" rule)

  $\Rightarrow$ $I_{HM}$ is also ruled out as an UV counterterm.

Hence: $N=4$, $D=4$ SYM is UV <u>finite</u>.

A basic question: what $N$-extension can one quantize with?

Rivelles & Taylor
Roček & Siegel

Consider again $N=4$ SYM at the linearized level $\leftrightarrow$ quadratic action. Suppose there were a full $N=4$ off-shell formulation with a finite set of auxiliary fields.

Write the action schematically as $I = \int d^4x\, \Phi\, \mathcal{D}\, \Phi$ where $\mathcal{D}$ is some differential operator.

- If $I$ is invariant under $N=4$ linearly realized supersymmetry closing off-shell, then so is

$$I' = \frac{1}{m^2} \int d^4x\, \Phi\, \mathcal{D}\, \Box\, \Phi$$

- Decompose then the propagating states (good + ghost) of $I + I'$ into the known on-shell supermultiplets.
- Note that for fermions $\int \bar{\lambda} \not{\partial} \lambda + \frac{1}{m^2} \int \bar{\lambda} \not{\partial} \Box \lambda$ yields 2 massless "physical" good states + 4 massive ghost states.
- For auxiliary fermion pairs, one gets four massive good and four massive ghost states.
- auxiliary bosons produce massive states, but we don't need this
- Now count massive fermionic states in total, making the assumption that auxiliary fermions always occur in pairs.
- Each massive $N=4$ multiplet contains an integral multiple of 128 fermionic states.

Hence, for $n$ auxiliary fermion pairs, one would have

$$4n + 4.4 = 128p \text{ ghost massive states}$$

$$4n = 128q \text{ good massive states}$$

where $p, q \in \mathbb{Z}$.

Subtract: $p - q = \frac{1}{8}$. Not possible for finite $p, q \in \mathbb{Z}$

For <u>finite</u> numbers of auxiliary fields, the maximum M-extension of $D=4$ supersymmetry allowed by the previous argument for $N=4$ SYM is $M=2 \leftrightarrow 8$ supercharges

- This has been achieved in two ways:
  - $M=2$ Lorentz covariant quantization HST
  - $SU(4)$ covariant light-cone quantization Mandelstam; Brink, Lindgren + Nilsson

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As we have seen, the $M=2$ superspace Feynman rules are sufficient to show cancellation of the $N=4$ SYM (logarithmic $\sim \frac{1}{d-4}$) divergences at $L \geq 2$ loops.

- At $L=1$ loop, one needs to do an explicit calculation, owing to ghost-for-ghost difficulties in the $M=2$ SYM. (At one loop <u>only</u>, it is convenient to use prepotentials for the background fields as well to stop the ghost-for-ghost problems, but this weakens the non-renormalization theorem for $L=1$.)

- At one loop, one finds, for $m_i$ hypermultiplets in representations $R_i$ of the $M=2$ SYM gauge group $G$, the $\beta$ function

$$\beta_{L=1}(g) = \frac{2g^2}{16\pi^2}\left(\sum_i m_i T(R_i) - C_2(G)\right)$$

$T(R_i)$: trace over squares of the $R_i$ rep. matrices

$C_2(G)$: quadratic Casimir for gauge group $G$

- $\beta_{L=1}$ vanishes for $N=4$ SYM ($m=1$, $R=$ adjoint) as well as for other models with $M=2$ supersymmetry only. Howe, Stelle, West
- For example, in the fundamental representation of $SU(N)$, $T(R)=\frac{1}{2}$, $C_2(SU(N))=N$ so for $m=2N$, one also has a finite theory.

# Divergences in Supergravity

The structure of supergravity divergences follows the pattern of the SYM discussion above:

- At $L \geq 2$ loops, counterterms must be
  1) written in terms of the M-superspace superveilbeins $E_M{}^A$ $M = (\mu, \alpha^i, \dot{\alpha}_j)$ world $\underline{A}$ tangent space for the background fields. These are subject to various torsion constraints in M-extended superspace, but in the background field method these constraints are solved by introducing prepotentials for the quantum parts of the fields only (at $L \geq 2$ loops).
  2) full superspace integrals $\int d^4x \, d^{4M}\theta$
  3) fully gauge invariant under background gauge transformations.

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The main question for pure N-extended supersymmetry is: <u>what is M?</u> (Max. linearly realizable supersymmetry.)

Counting arguments for the massive fermion modes in $\int \bar{\Phi} \theta \Phi + \frac{1}{m^2} \int \bar{\Phi} \theta \Box \Phi$ models yield the following possibilities, assuming <u>finite</u> sets of auxiliary fields:

$$N = 8, 6, 5, 4, 3, 2, 1$$
$$M = 4, 3, 3, 2, 2, 2, 1$$

## The role of the non-manifest full N-extended supersymmetry

Above, we have concentrated on the non-renormalization theorem for the maximally linearly realizable M-extended supersymmetry.

However, supergravity theories with N-extended supersymmetry on-shell do have some requirements from the full N-supersymmetry.

- Slavnov–Taylor identities for the local N-supersymmetry are highly nonlinear and can involve mixing between different orders in perturbation theory except ⇒ at the first order with non-vanishing UV divergences.
- At this first divergent order, one gets an understandable constraint on counterterms if one imposes the classical field equations so the classical action does not mix with the counterterm.

— then, counterterms must be invariant under the full N-extended supersymmetry

- There is no requirement on writing the on-shell N-supersymmetric counterterm as a full $\int d^4x\, d^{4N}\theta$ superspace integral, however.

— there are a range of "superaction" formulas for N-extended supersymmetry involving integration of constrained integrands over submanifolds of N-superspace.

— the simplest example of this is the N=1 chiral superspace integral $\int d^4x\, d^2\theta\, F$ $\qquad \bar{D}_{\dot\alpha} F = 0$

# The $R^4$ counterterm

$N=1$ $R^4$ counterterm: Deser, Kay, Stelle

$D=4$ $N \geq 4$ supergravity has a field-strength superfield $W_{[ijkl]}$ satisfying the linearized constraints

$$D_\alpha{}^i W_{jklm} = -\frac{4}{N-3}\, \delta^i_{[j} D_\alpha{}^n W_{klm]n}$$

SU(N) indices

$$\bar{D}_{\dot\alpha i} W_{jklm} = \bar{D}_{\dot\alpha [i} W_{jklm]}$$

and, in addition, for $N=8$, $\bar{W}^{ijkl} = \frac{1}{4!}\epsilon^{ijklmnpq} W_{mnpq}$

At the linearized level, each component of $W_{ijkl}$ is a field $F$ with $2s$ symmetrized spinor indices of same chirality $F_{\alpha_1 \dots \alpha_{2s}}$ obeying $\partial_{\dot\alpha}{}^{\alpha_1} F_{\alpha_1 \dots \alpha_{2s}} = 0 = \Box F_{\alpha_1 \dots \alpha_{2s}}$.

These on-shell component field strengths describe massless spin $s$ for $s \leq 2$, including $N(N-1)(N-2)(N-3)/12$ $s=0$ scalars for $N \leq 7$, 70 for $N=8$.

- All these scalars arise from dimensional reduction of higher-dimensional gauge fields ($A_{\mu\nu\rho}$) or the metric.

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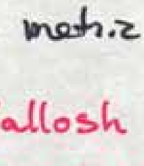
The $R^4$ counterterm is written for $N=8$ as a superaction integral over a subset of the $\theta_\alpha{}^i$ fermionic coordinates:

Kallosh
Howe, Stelle & Townsend

232848 rep. of SU(8):



$$\Delta I = \kappa^4 \int d^4x\, (d^{16}\theta)_{232848}\, (W^4)_{232848}$$

– one can also view the integrand as the highest-dim term in the square of $(W^2)_{1764}$ 1764:  SU(8) rep., which is the multiplet containing the Bel-Robinson tensor.

$$T_{\alpha\dot\alpha, \beta\dot\beta, \gamma\dot\gamma, \delta\dot\delta} = C_{\alpha\beta\gamma\delta}\, C_{\dot\alpha\dot\beta\dot\gamma\dot\delta}$$

Weyl tensor

- If one reduces the <u>manifest</u> supersymmetry to $N=4$ and uses the constraints on $W_{ijkl}$, one can write this as a full $\int d^{16}\theta$ $N=4$ superspace integral

<u>Resulting predictions for first divergent loop orders</u>
in $D=4$ N-extended supergravity

A general L-loop counterterm for a theory written in terms of M-extended superfields is of the form

$$I_M^{(L)} = \kappa^{2(L-1)} \int d^4x \, d^{4M}\theta \, \mathcal{L}_M^{(L)}$$

$$\dim \mathcal{L}_M^{(L)} = 2\,(L+1-M)$$

- On-shell, all superfields contributing to $\mathcal{L}_M^{(L)}$ have dimensions $\geq 0$, so one finds

$$L \geq M-1$$

- However for $N \leq 3$ the minimum dimension of $\mathcal{L}_M^{(L)}$ increases because there are no scalar fields (which have power-counting dimension 0):

$$\dim \mathcal{L}_M^{(L)} \geq 8 - 2N \qquad N \leq 3$$

Thus, we obtain the following limits on first divergence order purely from the superspace power counting:

| | |
|---|---|
| $L \geq 3$ | $N = M = 1$ |
| $L \geq 3$ | $N = M = 2$ |
| $L \geq 2$ | $N = 3 \quad M = 2$ |
| $L \geq 1$ | $N = 4 \quad M = 2$ |
| $L \geq 2$ | $N = 5, 6 \quad M = 3$ |
| $L \geq 3$ | $N = 8 \quad M = 4$ |

Requiring on-shell full N extended supersymmetry, however, gives the same bound in all cases:

$$L \geq 3$$

Thus, as far as one can see from superspace Feynman rules with <u>finite</u> sets of auxiliary fields, <u>all N-extended supergravities could diverge at 3 loops.</u>